ME/EE/CS 133a Homework 1

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10–07–24

1. Problem 1

- (a)

  There exists 5 degrees of freedom. A Rigid Body in 3D space has 6 possible Degrees of Freedom bieng (x, y, z) and (pitch, yaw, roll). A line segment has no thickness so its incapable of achieving the roll rotational DOF. A line segment is 2 dimensional.

- (b)

  There exists 5 degrees of freedom. A Rigid Body in 3D space has 6 possible Degrees of Freedom bieng (x, y, z) and (pitch, yaw, roll). A torus can move along translational axis (x, y, z) and is limited by 1 (rotating about its center) rotational axis (pitch, yaw, roll).

- (c)

  6 DOFs. 5 from part (a) and the 6th comes from the length of the segment. The length is an additional property of the line segment that can be varied independently, and thus contributes 1 extra degree of freedom to the total number needed to describe the segment in 3D space.

- (d)

  7 Dofs. 5 from part (b) similar to part (c) the major and minor axis add 2 additional DOFS.

2. Problem 2

- (a) Lets use Gublers formula for calculating the degrees of freedom. Formula states that

$$\text{DOF} = m(N) + \sum_{i=1}^{J} f_i$$

Using the formula:

$$\text{DOF} = 3(N) + \sum_{i=1}^{J} f_i = 3(2) - 2 = 4$$

Therefore, the system has 4 degree of freedom.

(b) Using the formula:

$$\text{DOF} = 3(N) + \sum_{i=1}^{J} f_i = 3(3) - 2(2) = 5$$

Thus, the system has 5 degree of freedom.

3. Problem 3

The human arm non iclusing wrist consists of 5 Fingers, 4 are identical and thumb is different. The DOFS per joint per finger group are as follows:

Thumb:

- The base of the thumb moves:

- *Up/down* (1 DOF)
- *Forward/backward* (1 DOF)

- The knuckle of the thumb moves:

- *Up/down* (1 DOF)
- *Left/right* (1 DOF)

- The thumb tip joint moves:

- *Up/down* (1 DOF)

Total for the thumb = 5 DOFs.

Fingers (Index, Middle, Ring, Pinky):

Each finger has the following joints:

- Knuckle:
  - *Up/down* (1 DOF)
  - *Left/right* (1 DOF)
- Middle joint of the finger:
  - *Up/down* (1 DOF)
- Fingertip joint:
  - *Up/down* (1 DOF)

So, each finger has 4 DOFs, and for 4 fingers:

$$4 \times 4 = 16\,\text{DOFs}.$$

Total Degrees of Freedom:

The total DOFs for the hand are:

$$16\,(\text{DOFs for 4 fingers}) + 5\,(\text{DOFs for the thumb}) = 21\,\text{DOFs}.$$

4. Problem 4

- (a)

  Given that the hips and pelvis are fixed, the legs and ankles can move freely. The degrees of freedom are:

  - **Ankles**: 1 DOF per leg (up/down movement).
  - **Knees**: 1 DOF per knee (up/down movement).
  - **Hip**: 1 rotational DOF for pedaling motion.

  Total DOFs:

  $$\text{Total DOFs} = 2\,(\text{ankles}) + 2\,(\text{knees}) + 1\,(\text{hip}) = 5\,\text{DOFs}.$$

- (b)

  When standing on the pedals, the hips and pelvis are not fixed. The degrees of freedom are:

  - **Ankles**: 1 DOF per leg (up/down movement).
  - **Knees**: 1 DOF per knee (up/down movement).
  - **Hips**: 3 DOFs per leg (Forward/backward, left/right, in/out rotation).

  Total DOFs:

  $$\text{Total DOFs} = 2\,(\text{ankles}) + 2\,(\text{knees}) + 6\,(\text{hips}) = 11\,\text{DOFs}.$$

5. Problem 5

Geometric Derivations

We start by using the Pythagorean theorem to solve for the distance r, angle β, and angle γ. Then, we calculate the angles $\theta_1, \theta_2, \theta_3, \theta_4$ as described below.

Pythagorean Theorem for r

The distance r from the origin to point B is calculated using the Pythagorean theorem:

$$r = \sqrt{X_B^2 + Y_B^2}$$

where:

$$X_B = d + c \cdot \cos(\phi)$$

and

$$Y_B = c \cdot \sin(\phi)$$

This represents the distance from the base pivot to point B in the four-bar linkage.

Law of Cosines for β

The angle β is derived using the law of cosines:

$$\cos(\beta) = \pm\frac{a^2 + r^2 - b^2}{2 \cdot a \cdot r}$$

Solving for β:

$$\beta = \pm\cos^{-1}\left(\frac{a^2 + r^2 - b^2}{2 \cdot a \cdot r}\right)$$

Angle γ

The angle γ can be found using the atan2 function, which accounts for the correct quadrant:

$$\gamma = \text{atan2}(Y_B, X_B)$$

This handles both the angle's direction and ensures the correct quadrant is considered.

Full Expression for θ

The angle θ is derived from the combination of γ and β, where:

$$\theta = \operatorname{atan2}(Y_B, X_B) \pm \cos^{-1}\left(\frac{a^2 + (X_B^2 + Y_B^2) - b^2}{2 \cdot a \cdot \sqrt{X_B^2 + Y_B^2}}\right)$$

with:

$$X_B = d + c \cdot \cos(\phi)$$
$$Y_B = c \cdot \sin(\phi)$$

Possibilities for θ

Finally, the angles θ_1 and θ_2 are calculated as (from original theta):

$$\theta_1 = \gamma + \beta$$
$$\theta_2 = \gamma - \beta$$

By adding 2π to each, we get θ_3 and θ_4:

$$\theta_3 = \theta_1 + 2\pi$$
$$\theta_4 = \theta_2 + 2\pi$$

Configuration Space Plots

The following figures show the configuration space of the four-bar linkage for different values of c:.

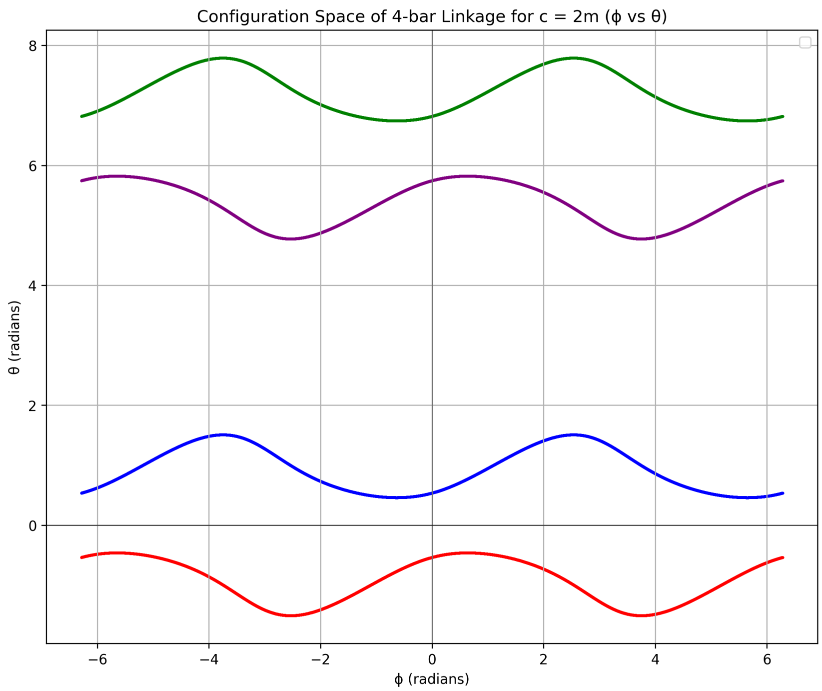

Figure 1: Configuration Space for $c = 2m$

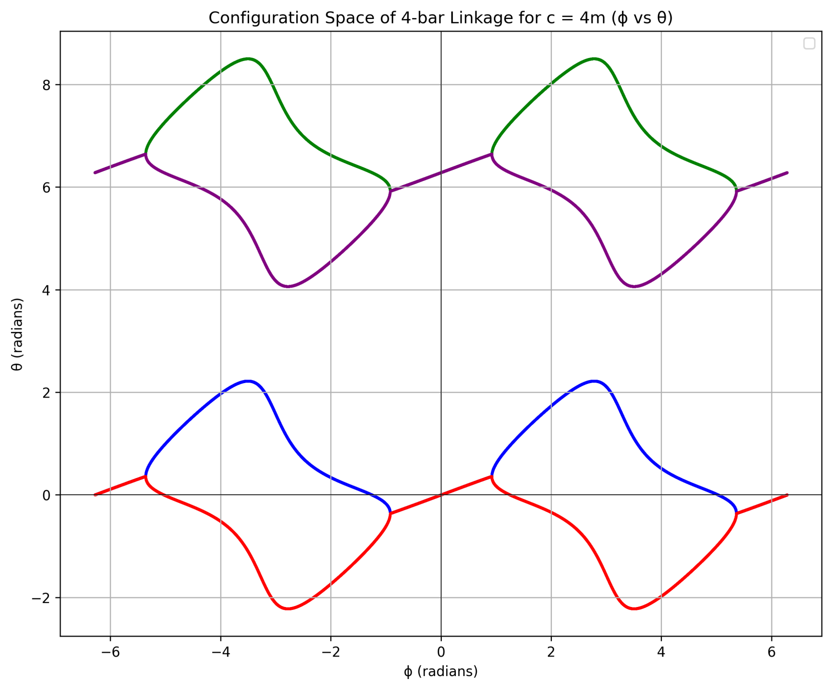

Figure 2: Configuration Space for $c = 4m$

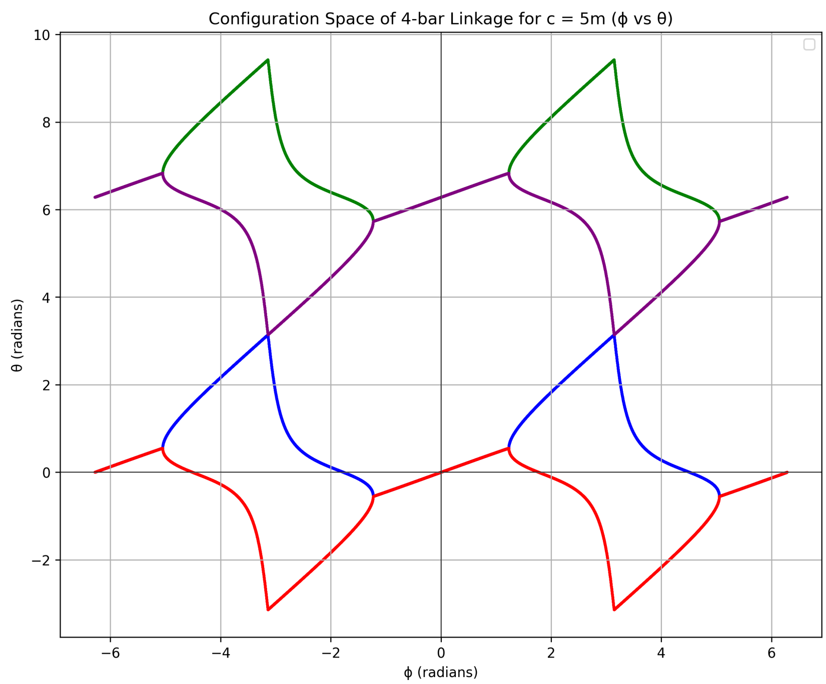

Figure 3: Configuration Space for $c = 5m$

Code (pyhton) Used:

```
import numpy as np
import matplotlib.pyplot as plt

a = 4
b = 5
d = 6

save_path = r"C:\\Users\\farha\\Desktop\\Caltech\\2024\\
    EE133a\\Hw1"

def calculate_theta(phi, c):
    XB = d + c * np.cos(phi)
    YB = c * np.sin(phi)
    r = np.sqrt(XB**2 + YB**2)
    cos_beta = (a**2 + r**2 - b**2) / (2 * a * r)
    beta = np.arccos(np.clip(cos_beta, -1, 1))
    gamma = np.arctan2(YB, XB)
    theta_1 = gamma + beta
    theta_2 = gamma - beta
    theta_3 = theta_1 + 2 * np.pi
    theta_4 = theta_2 + 2 * np.pi

    return theta_1, theta_2, theta_3, theta_4

phi_values = np.linspace(-2*np.pi, 2*np.pi, 10000)
c_values = [2, 4, 5]

for c in c_values:
    theta_1_values = []
    theta_2_values = []
    theta_3_values = []
    theta_4_values = []

    for phi in phi_values:
        theta_1, theta_2, theta_3, theta_4 =
            calculate_theta(phi, c)
        theta_1_values.append(theta_1)
        theta_2_values.append(theta_2)
        theta_3_values.append(theta_3)
        theta_4_values.append(theta_4)

    plt.figure(figsize=(10, 8))
```

```python
        plt.scatter(phi_values, theta_1_values, label=f'', color='blue', s=1, marker='o')
        plt.scatter(phi_values, theta_2_values, label=f'', color='red', s=1, marker='o')
        plt.scatter(phi_values, theta_3_values, label=f'', color='green', s=1, marker='o')
        plt.scatter(phi_values, theta_4_values, label=f'', color='purple', s=1, marker='o')

        plt.xlabel('   (radians)')
        plt.ylabel('   (radians)')
        plt.title(f'Configuration Space of 4-bar Linkage for c = {c}m (   vs   )')
        plt.axhline(0, color='black', linewidth=0.5)
        plt.axvline(0, color='black', linewidth=0.5)
        plt.legend()
        plt.grid(True)

        plt.savefig(f"{save_path}config_space_c_{c}m.png", dpi=300, bbox_inches='tight')

    plt.show()
```

6. problem 6

Deriving Equations:

Given that:

$$l_1 = 5\,\text{m}, \quad l_2 = 1\,\text{m}, \quad l_3 = 2\,\text{m}$$

The angles for the joints are denoted by θ_1, θ_2, and θ_3. We aim to find the possible (x, y)-coordinates for the end-effector P by calculating the position for each case where the angles have different constraints.

Position of End-Effector P

We can find P by summing up the positions of the links as follows:

Coordinates for Link 1

$$X_1 = l_1 \cos(\theta_1), \quad Y_1 = l_1 \sin(\theta_1)$$

Coordinates for Link 2

For the second link:

$$X_2 = X_1 + l_2 \cos(\theta_1 + \theta_2), \quad Y_2 = Y_1 + l_2 \sin(\theta_1 + \theta_2)$$

Coordinates for Link 3 (End-Effector)

For the third link (end-effector point P):

$$X_3 = X_2 + l_3 \cos(\theta_1 + \theta_2 + \theta_3), \quad Y_3 = Y_2 + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

Substituting for X_2 and Y_2, we can express the position of the end-effector directly in terms of θ_1, θ_2, and θ_3:

$$X_3 = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$Y_3 = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

Workspace Calculation

Now its possible to calculate the workspace for the three cases (done numericaaly)

Plotting the Workspaces

Below are the workspaces for each of the three cases:

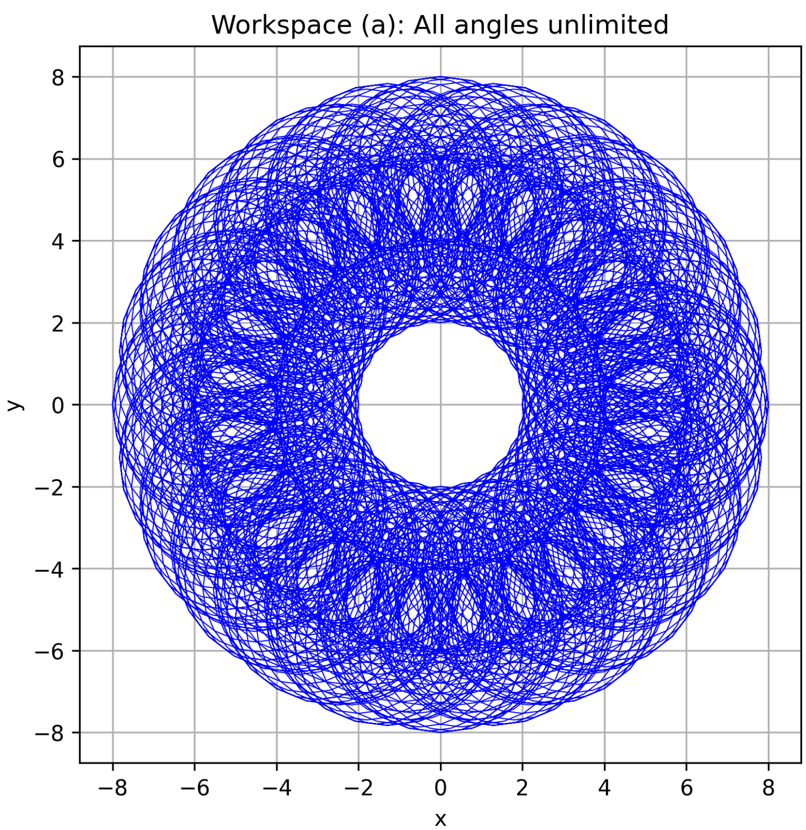

Figure 4: Workspace (a): All angles unlimited

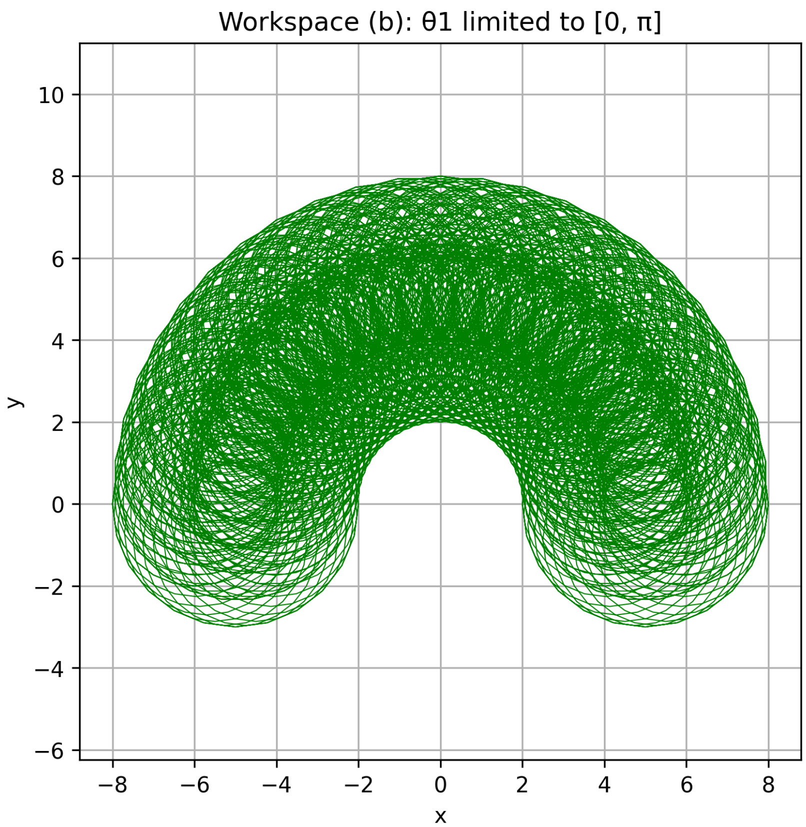

Figure 5: Workspace (b): θ_1 limited to $[0, \pi]$

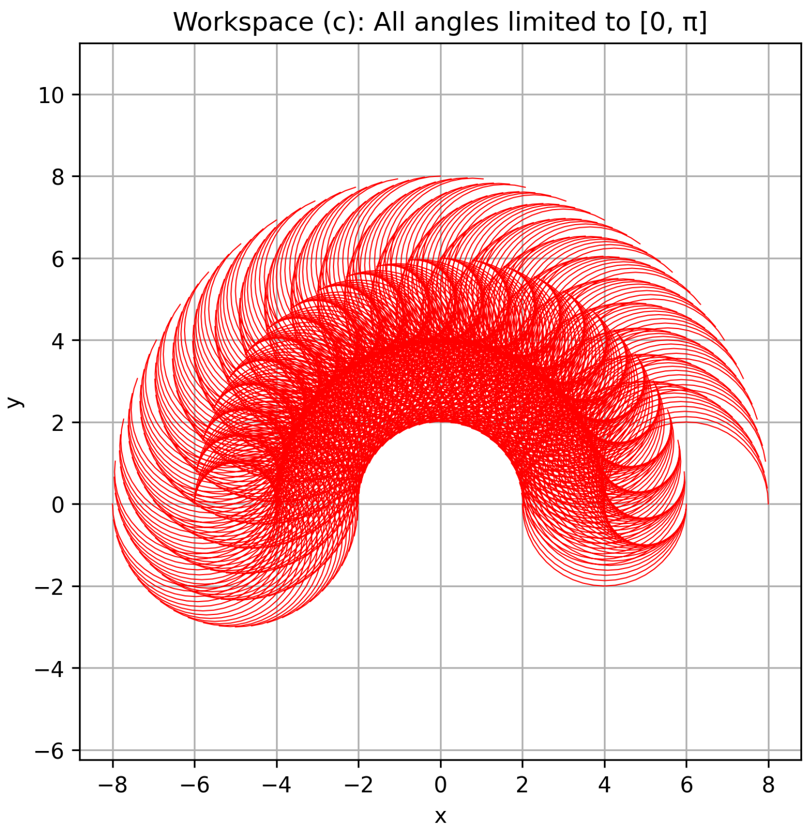

Figure 6: Workspace (c): All angles limited to $[0, \pi]$

Code (pyhton) Used:

```python
import numpy as np
import matplotlib.pyplot as plt

l1 = 5
l2 = 1
l3 = 2
def calculate_workspace(theta1_range, theta2_range, theta3_range, num_points=25):
    theta1_values = np.linspace(theta1_range[0], theta1_range[1], num_points)
    theta2_values = np.linspace(theta2_range[0], theta2_range[1], num_points)
    theta3_values = np.linspace(theta3_range[0], theta3_range[1], num_points)

    X = []
    Y = []
    for theta1 in theta1_values:
        for theta2 in theta2_values:
            for theta3 in theta3_values:
                x = l1 * np.cos(theta1) + l2 * np.cos(theta1 + theta2) + l3 * np.cos(theta1 + theta2 + theta3)
                y = l1 * np.sin(theta1) + l2 * np.sin(theta1 + theta2) + l3 * np.sin(theta1 + theta2 + theta3)
                if len(X) > 0 and np.sqrt((x - X[-1])**2 + (y - Y[-1])**2) > 1.0:
                    X.append(None)
                    Y.append(None)

                X.append(x)
                Y.append(y)

    return np.array(X), np.array(Y)

def plot_and_save_workspace(X, Y, title, filename, color='blue'):
    plt.figure(figsize=(6, 6))
    plt.plot(X, Y, color=color, linewidth=0.5)
    plt.title(title)
    plt.xlabel('x')
    plt.ylabel('y')
```

```
        plt.axis('equal')
        plt.grid(True)
        plt.savefig(f'{filename}.png', dpi=300, bbox_inches='tight')
        plt.show()

    theta1_range_a = [0, 2*np.pi]
    theta2_range_a = [0, 2*np.pi]
    theta3_range_a = [0, 2*np.pi]
    X_a, Y_a = calculate_workspace(theta1_range_a, theta2_range_a, theta3_range_a)

    theta1_range_b = [0, np.pi]
    theta2_range_b = [0, 2*np.pi]
    theta3_range_b = [0, 2*np.pi]
    X_b, Y_b = calculate_workspace(theta1_range_b, theta2_range_b, theta3_range_b)

    theta1_range_c = [0, np.pi]
    theta2_range_c = [0, np.pi]
    theta3_range_c = [0, np.pi]
    X_c, Y_c = calculate_workspace(theta1_range_c, theta2_range_c, theta3_range_c)

    plot_and_save_workspace(X_a, Y_a, 'Workspace (a): All angles unlimited', 'workspace_case_a', color='blue')
    plot_and_save_workspace(X_b, Y_b, 'Workspace (b):  1 limited to [0,   ]', 'workspace_case_b', color='green')
    plot_and_save_workspace(X_c, Y_c, 'Workspace (c): All angles limited to [0,   ]', 'workspace_case_c', color='red')
```

7. Problem 7

Time Spent was around 5 hours.