

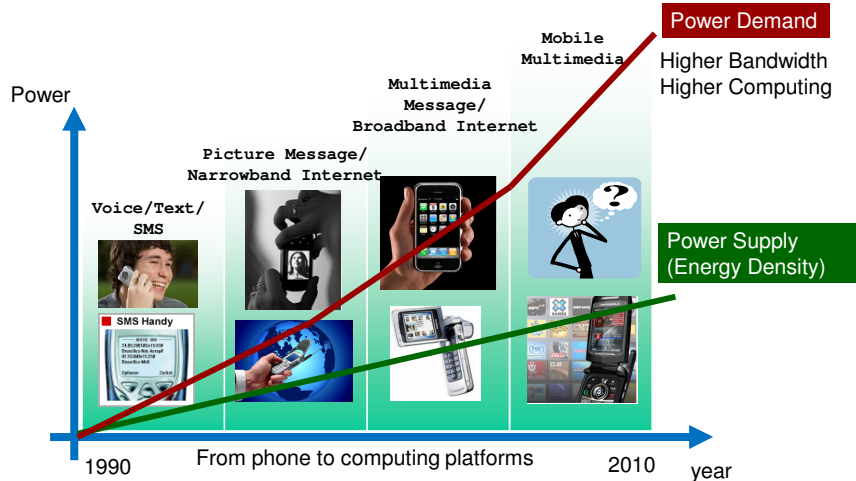
# Power-Aware Scheduling

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Lecture #10

## Outline

- Need for Power Management
  - DVS and DFS
- Constant Speed Energy Minimization
- *SysClock*
- *PM-Clock*
- *Dynamic PM-Clock*
- Effects of Hardware Limitations
- Evaluation

## Power Trend in Mobile Devices



The power supply & demand relationship is according to unpublished research by the Boston Consulting Group

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## Motivation

- The power consumption of CMOS circuits is given by

$$P \propto C_L * V_{dd}^2 * f$$

- $f \propto (V_{dd} - V_{th})^\alpha / V_{dd}$  ; For 0.25- $\mu$ m technology,  $\alpha \approx 1.3-1.5$

- Energy =  $P * \text{Delay} \approx k * f^{2/(\alpha-1)} \approx k * f^x$

➡ Running a CPU at lower speed consumes less energy

- Real-time tasks are required to complete at their deadline.

➡ Reducing the CPU speed to complete the task just before its deadline achieves the same performance but saves energy

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## Issue of Operating Voltage/Frequency

- The pre-dominant device technology is CMOS
- In CMOS, slower (faster) frequency means lower (higher) number of transitions
  - Hence, lower power is consumed
- **Power is proportional to  $V^2f$** 
  - $P \rightarrow$  Power
  - $V \rightarrow$  Voltage
  - $f \rightarrow$  Frequency
- Can reduce unit computation energy by reducing frequency and voltage
- Maximum gate delays inversely related to voltage

- Applies to processors.
- Can be applied to memory as well.
- Check specifications carefully.

## Scaling Techniques

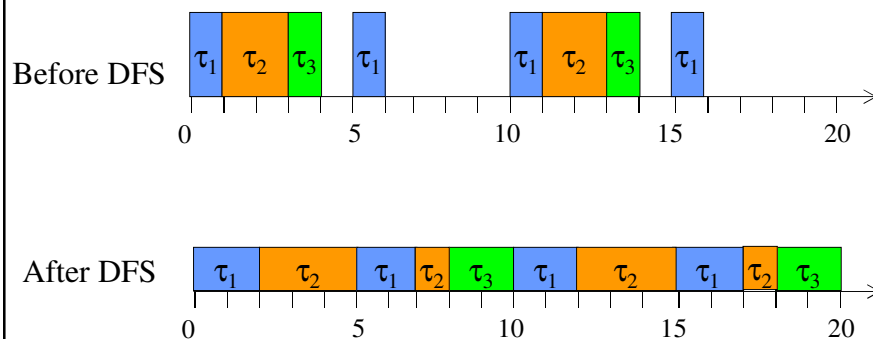
- **DFS: Dynamic Frequency Scaling**
  - Lower or increase clock frequency
  - With lower frequency, the same amount of work (# of instructions to be executed) takes longer
  - Hence, power may go down, but energy may remain the same
- **DVS: Dynamic Voltage Scaling**
  - Lower or increase clock voltage
  - As frequency increases, higher voltage is required
  - Conversely, as frequency is lowered, only a lower voltage is required
- **DVFS: Dynamic Voltage/Frequency Scaling**
  - Scale voltage up (down) when frequency is scaled up (down)
  - Needs processors supporting software adjustable PLL, voltage regulator
    - e.g., XScale, SpeedStep, PowerNow!, Crusoe

## Dynamic Frequency Scaling (DFS)

- [Weiser+94]
  - busy system  $\rightarrow$  increase frequency
  - idle system  $\rightarrow$  reduce frequency

## Using DFS

- “Static” DFS
- Taskset:  $\tau_1 = (1, 5)$      $\tau_2 = (2, 10)$      $\tau_3 = (1, 10)$   
 $U = 1/5 + 2/10 + 1/10 = 0.5$



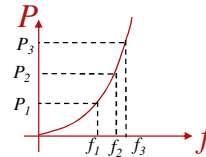
## The Problem

How to determine the CPU clock frequency to satisfy the schedulability of real-time task sets and save energy?

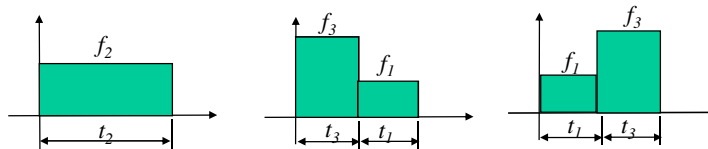
## Constant Speed Energy Minimization

- The power-frequency relationship is a convex function

$$- P = k f^x, 0 \leq f \leq f_{max}$$



- Energy is minimized if a workload with a given deadline is executed at *constant* speed



- $f_1 \leq f_2 \leq f_3; t_2 = t_1 + t_3; (f_2 * t_2) = (f_1 * t_1) + (f_3 * t_3)$
- Then  $p_2 * t_2 \leq p_1 * t_1 + p_3 * t_3$

## Brief Overview

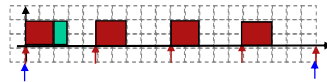
- Example of simple frequency-scaling algorithm for fixed-priority preemptive scheduling policies (RM, DM, FIFO, etc.)
- Static frequency scaling (SFS) algorithms
  - Optimal system-clock-freq assignment: one clock frequency for the entire task set
  - Task-clock-freq assignment: one clock frequency for each task
- Dynamic voltage scaling (DVS) algorithm
- Experimental results
- Effect of hardware limitation
  - Non-ideal power-frequency characteristic, finite operating frequencies

## System Model and Notation

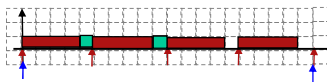
- Task requirement  $\{C, T, D\}$ 
  - $C$ : number of cycles
  - $T, D$ : period and deadline in time units
- Tasks are scheduled by the deadline-monotonic (DM) scheduling algorithm
- Assume there are  $n$  tasks,  $\tau_1$  to  $\tau_n$ , and  $D_1 \leq D_2 \leq \dots \leq D_n$
- $v_i$  denotes CPU clock frequency to execute task  $\tau_i$
- **Goal**: minimize energy usage of the task set within the hyper-period
  - Hyperperiod = LCM of all the periods

## Static Frequency Scaling Algorithm Example

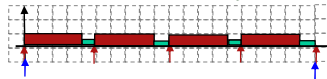
- $\tau_1\{2, 5, 4\}$  and  $\tau_2\{1, 20, 20\}$ ; Hyper-period ( $H$ ) = 20
- Assume  $f_{max} = 1$  cycle/time unit; Energy ( $E$ ) =  $k \cdot f^3 \cdot t$
- If the CPU runs at  $f_{max}$ ,  $E = k f_{max}^3 \cdot 9$



- System clock frequency assignment sets CPU speed to  $0.5 \cdot f_{max}$   
 $E = k (0.5 f_{max})^3 \cdot 18 = k f_{max}^3 \cdot 2.25$



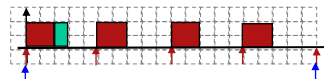
- Task clock frequency assignment sets  $v_1 = 0.5 f_{max}$ ,  $v_2 = 0.25 f_{max}$   
 $E = k (0.5 f_{max})^3 \cdot 16 + k (0.25 f_{max})^3 \cdot 4 = k f_{max}^3 \cdot 2.0625$



## Basic Intuition Behind Sys-Clock

- Consider the workload needed to complete a given task
  - The execution of higher priority tasks and itself
  - The energy to satisfy  $\tau_1$ 's deadline is obtained by running its higher priority tasks and itself with the lowest possible speed

$\tau_1\{2, 5, 4\}$  and  $\tau_2\{1, 20, 20\}$



– Workload for  $\tau_2$  ( $W_2$ ) =  $4 \cdot C_1 + C_2$  cycles

– Lowest possible freq.  $\Omega_2 = w_2/d_2 = (4 \cdot 2 + 1)/20 = 0.45$  cycles/time unit

- To maintain schedulability, all task constraints must be satisfied
  - With speed 0.45 cycles/time unit,  $\tau_1$  misses its deadline!!
  - Lowest possible freq.  $\Omega_1 = w_1/d_1 = 2/4 = 0.50$  cycles/time unit
- The CPU needs to run at, at least,  $\max(0.45, 0.50) = 0.50$  cycles/time unit to satisfy the task set

## Sys-Clock Algorithm [1]

- Consider a task's workload at critical instant
  - The task's execution and its preemption
- Workload is varied only when the task completes
  - Later the completion time, higher the preemption
  - Hence, **completing a task at its deadline is not always optimal !!**
- Apply constant speed energy minimization concept
  - Consider all possible completion times before deadline
  - Energy-minimizing freq of a task ( $\nu$ )= the lowest speed

## Sys-Clock Algorithm [2]

- For schedulability:

All task deadlines must be satisfied.

$$\text{Sys-Clock-Freq} = \text{MAX } [\nu] \text{ for all tasks}$$

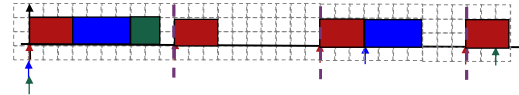
- Lower frequency → At least one task will miss deadline
- **Sys-Clock is optimal among single-clock-frequency schemes.**



## Workload Vs. Frequency

$\tau_1 \{3, 10, 10\},$   
 $\tau_2 \{4, 23, 23\},$   
 $\tau_3 \{2, 32, 32\},$

Run at  $f_{max} = 1 \text{ cycle/time unit}$

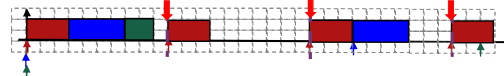


- The workload needed to satisfy each task's constraint varies with the CPU clock frequency
- $\Omega_3|_{t=10} = (C_1 + C_2 + C_3)/10 = (3+4+2)/10 = 0.9$
- $\Omega_3|_{t=20} = (2C_1 + C_2 + C_3)/20 = (6+4+2)/20 = 0.6$
- $\Omega_3|_{t=23} = (3C_1 + C_2 + C_3)/23 = (9+4+2)/23 = 0.652$
- $\Omega_3|_{t=30} = (3C_1 + 2C_2 + C_3)/30 = (9+8+2)/30 = 0.63$
- $\Omega_3|_{t=32} = (4C_1 + 2C_2 + C_3)/32 = (12+8+2)/32 = 0.6875$
- $\Omega_3 = \min(\Omega_3|_t) \text{ for all } t's = \min(0.9, 0.6, 0.65, 0.63, 0.69) = 0.6$
- $t$  is the ending time of an idle period before the deadline

## Sys-Clock Example

$\tau_1 \{3, 10, 10\}$   
 $\tau_2 \{4, 23, 23\}$   
 $\tau_3 \{2, 32, 32\}$

Run at  $f_{max} = 1 \text{ cycle/time unit}$



- $t$ 's are the endings of idle periods when the workload changes
- Compute freq. needed to complete a task at time  $t$
- For task  $\tau_1$ ,  $v_1$  is given by  $= (C_1 / T_1) = 3 / 10 = 0.3$
- For task  $\tau_2$ ,  $v_2$  is computed as
  - $\text{freq}(\tau_2) | t=10 = (C_1 + C_2)/10 = (3+4)/10 = 0.7$
  - $\text{freq}(\tau_2) | t=20 = (2C_1 + C_2)/20 = (6+4)/20 = 0.5$
  - $\text{freq}(\tau_2) | t=23 = (3C_1 + C_2)/30 = (9+4)/23 = 0.565$
- For task  $\tau_3$ ,  $v_3$  is computed as
  - $\text{freq}(\tau_3) | t=10 = (C_1 + C_2 + C_3)/10 = (3+4+2)/10 = 0.9$
  - $\text{freq}(\tau_3) | t=20 = (2C_1 + C_2 + C_3)/20 = (6+4+2)/20 = 0.6$
  - $\text{freq}(\tau_3) | t=30 = (3C_1 + 2C_2 + C_3)/30 = (9+8+2)/30 = 0.63$
  - ... (as on previous slide)
- Sys-clock freq  $= \max[v_1, v_2, v_3] = \max[0.3, 0.5, 0.6]$

$$v_2 = \min[ \text{freq}(\tau_2) \text{ for all } t ] \\ = \min[ 0.7, 0.5, 0.565 ] \\ = 0.5$$

$$v_3 = \min[ \text{freq}(\tau_3) \text{ for all } t ] \\ = \min[ 0.9, 0.6, 0.63 ] \\ = 0.6$$

# Sys-Clock Algorithm

**During admission control:**  
**for**  $i = 1$  **to**  $n$ :  
     $\alpha_i = \text{Compute\_alpha}(\tau_i)$   
    **if**  $(\alpha_i > 1)$  **return** fail  
**end for**  
 $\text{sys\_clock} = \text{MAX}_i(\alpha_i)$   
**set\\_system\\_clock**(sys\\_clock)

Sys-Clock is optimal among all (global) system clock frequency assignment algorithms

- If the CPU clock frequency is lower than Sys-Clock, at least one task will miss its deadline
- If the CPU clock frequency is higher than Sys-Clock, more energy will be consumed

**Compute\_alpha**( $\tau_i$ ):  
IN.BZP = true /\* computing busy period or idle period \*/  
 $\Gamma = C_i$  /\* time trace \*/  
 $\text{slack} = 0$  /\* the slack time \*/  
 $w = 0$  /\* the workload \*/  
 $t^{(w)} = 0$  /\* the beginning of the next busy period \*/  
 $\alpha = 2.0$  /\* the energy-minimizing clock frequency normalized to  $f_{max}$  \*/  
**while**  $(\Gamma \leq D_i)$   
    **if**(IN.BZP)  
         $\Delta = D_i - \Gamma$   
        **while**  $(\Gamma < D_i)$  and  $(\Delta > 0)$   
            /\*  $hp(\tau_i)$  is the set of tasks with priorities  $\geq \tau_i$ 's \*/  
             $\Gamma' = \text{slack} + \sum_{\tau_j \in hp(\tau_i)} (\lfloor \frac{\Gamma}{T_j} \rfloor + 1) * (C_j / f_{max})$   
             $\Delta = \Gamma' - \Gamma$   
             $\Gamma = \Gamma'$   
        **end while**  
        IN.BZP = false  
    **else**  
        idle.duration =  $\text{MIN}_{\tau_j \in hp(\tau_i)} [D_i - \Gamma, (\lfloor \frac{\Gamma}{T_j} \rfloor + 1) * T_j] - \Gamma$   
         $\Gamma += \text{idle.duration}$   
         $\text{slack} += \text{idle.duration}$   
         $t^{(w)} = \Gamma$   
         $w = t^{(w)} - \text{slack}$   
         $\alpha = \text{MIN}(\alpha, w / t^{(w)})$   
        IN.BZP = true  
    **end if**  
**end while**  
**return**  $\alpha$

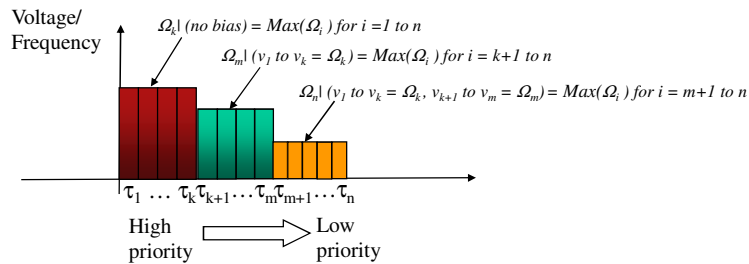
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# PM-Clock Algorithm

- Each task has its own clock frequency setting
- If a higher priority task needs higher frequency
  - a lower priority task's frequency can be reduced even more
    - Since the higher-priority task will complete faster, creating more slack
- PM-Clock is sub-optimal with much less complexity!
- $E_{\text{PM-Clock}} / E_{\text{optimal}} \approx 101\%$



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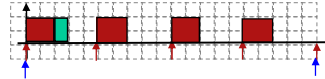
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## Task Clock Frequency Assignment

- Each task has its own clock frequency setting
- At each context switch, CPU scales its clock frequency based on the corresponding frequency of the next task
- PM-Clock frequency: Sys-Clock++
  - If a higher-priority task needs a higher frequency to satisfy its own deadline than  $\Omega_i$ , task  $\tau_i$  can reduce its frequency to reduce energy consumption.

$\tau_1 \{2, 5, 4\}$  and  $\tau_2 \{1, 20, 20\}$



–  $\Omega_1 = 2/4 = 0.5$

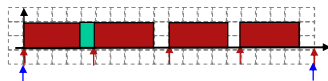
For  $\tau_2$ , its completion times to consider are given by 5, 10, 15, and 20 (i.e. whenever task  $\tau_1$  arrives and  $\tau_2$ 's own period/deadline). Other completion times must be considered in principle, but each one of them will yield worse energy savings than these discrete scheduling points.

Pick the best (low) frequency obtained among these options.

- $\Omega_2 = \min\{(2+1)/5, (2*2+1)/10, (3*2+1)/15, (4*2+1)/20\} = 9/20 = 0.45$
- $\rightarrow \tau_1$  must run at 0.5 while  $\tau_2$  needs to run the system only at 0.45
- $\rightarrow$  since  $\tau_1$  runs at 0.5,  $\tau_2$  can run at a frequency  $< 0.45$ !

## PM-Clock Example (cont'd.)

$\tau_1 \{2, 5, 4\}$  and  $\tau_2 \{1, 20, 20\}$



- Now, note that there are two frequencies involved, one for each task:

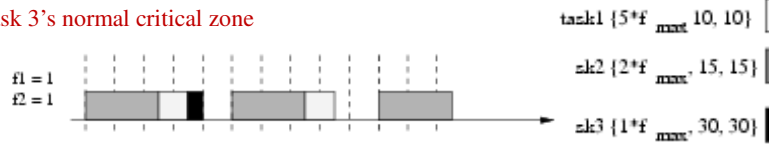
–  $v_1 = 0.5$

With  $\tau_1$  running at 0.5,  $\tau_2$  can complete at time 5, 10, 15 or 20

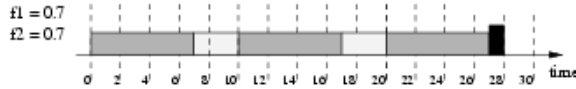
$$\begin{aligned}
 v_2 &= \min[1/(5-(2/0.5)), & /* \text{if } \tau_2 \text{ completes at } t=5 */ \\
 &\quad 1/(10-(2*2/0.5)), & /* \text{if } \tau_2 \text{ completes at } t=10 */ \\
 &\quad 1/(15-(3*2/0.5)), & /* \text{if } \tau_2 \text{ completes at } t=15 */ \\
 &\quad 1/(20-(4*2/0.5))] & /* \text{if } \tau_2 \text{ completes at } t=20 */ \\
 &= \min[1/(5-4), 1/(10-8), 1/(15-12), 1/(20-16)] \\
 &= \min[1, 0.5, 0.333, 0.25] \\
 &= 0.25
 \end{aligned}$$

## PM-Clock with 3 Tasks

Task 3's normal critical zone



Task 3's "inflated" critical zone



First, SysClock computations yield  $\epsilon_1 = 0.5$ ,  $\epsilon_2 = 0.7$  and  $\epsilon_3 = 0.67$ .

→  $v_1 = 0.7$ ,  $v_2 = 0.7$  and  $v_3$  can be improved to less than 0.67.

## General Algorithm for PM-Clock Calculation

```

// Assume a taskset has  $n$  tasks and  $D_1 \leq D_2 \leq \dots \leq D_n$ 
//  $v_i$  is the task clock frequency assigned by PM-Clock
During Admission Control:
For each task  $\tau_i$ :
     $v_i = 0$ ,  $\epsilon_i = \text{Energy-Min-Freq}(C_i, T_i, D_i)$ 
End for
For  $i = 1$  to  $n$ :
    //  $lp(\tau_i)$  are tasks with priorities  $\leq \tau_i$ 's priority
     $v_i = \text{lowest } f \text{ such that } (f/f_{max}) \geq \max_{\tau_j \in lp(\tau_i)} \epsilon_j$ 
    If ( $i \neq 1$ ) and ( $v_{i-1} > v_i$ ) then
         $v_i = 0$ 
        For  $j = i$  to  $n$ :  $\epsilon_j = \text{Inflated-f}(C_j, T_j, D_j)$  End for
    End if
     $v_i = \text{lowest } f \text{ such that } (f/f_{max}) \geq \max_{\tau_j \in lp(\tau_i)} \epsilon_j$ 
End for

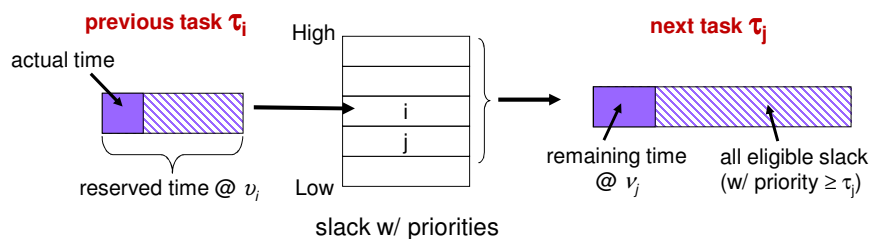
Inflated-f}(C_i, T_i, D_i):
    //  $i.\beta$  and  $\beta$  are inflated and scalable workload for time  $t$ 
    //  $\delta$  = scalable time, IN.BZP is the busy period flag
    //  $hp(\tau_i)$  are tasks with priorities  $\geq \tau_i$ 's priority
     $S = I = \beta = \Delta = \delta = 0$ ,  $\alpha = 1$ , IN.BZP = TRUE
     $\omega = C_i/f_{max}$ ,  $\omega' = 0$ 
    Do while ( $\omega < D_i$ )
        If (IN.BZP == TRUE) then
             $\Delta = D_i - \omega$ 
            Do while ( $\omega < D_i$ ) && ( $\Delta > 0$ )
                 $i.\beta = 0$ ,  $\beta = 0$ 
                For  $j = 1$  to  $n$ :
                    If ( $D_j \leq D_i$ ) && ( $v_j \neq 0$ ) then
                         $i.\beta = i.\beta + \frac{C_j}{v_j * f_{max}} * (\lfloor \frac{\omega}{T_j} \rfloor + 1)$ 
                    Else if ( $D_j \leq D_i$ ) && ( $v_j = 0$ ) then
                        // This task is still scalable
                         $\beta = \beta + \frac{C_j}{f_{max}} * (\lfloor \frac{\omega}{T_j} \rfloor + 1)$ 
                    End if
                End for
                 $\omega' = i.\beta + \beta + S$ ,  $\Delta = \omega' - \omega$ ,  $\omega = \omega'$ 
            End while
            IN.BZP = FALSE
        Eke
             $I = \min_{j \in hp(\tau_i)} \{ (T_j * \lceil \frac{\omega}{T_j} \rceil) - \omega, D_i - \omega \}$ 
             $S = S + I$ ,  $\omega = \omega + I$ ,  $t = \omega$ ,  $\delta = t - i.\beta$ 
            If ( $\frac{\beta}{\delta} < \alpha$ ) then  $\alpha = \frac{\beta}{\delta}$  End if
            IN.BZP = TRUE
        End if
    End while
    return  $\alpha$ 
    
```

## Dynamic Frequency Scaling

- Static Voltage Scaling (SFS) assumes that all tasks use worst-case cycles specified in their reserve requirements
- Dynamic Voltage Scaling (DFS)
  - Multimedia applications in the average case use resource less than 10% of their worst-case specifications
  - Detect the earlier completed task, transfer its slack to another low priority task
  - Kernel determines the dynamic frequency of a task based on its reserve specification and available slack

## Dynamic PM-Clock Algorithm

- A task may use less resource than reserved
  - Slack is transferred to first task w/ same or lower-priority
  - Reduce voltage/frequency on-the-fly



## Simulation Results

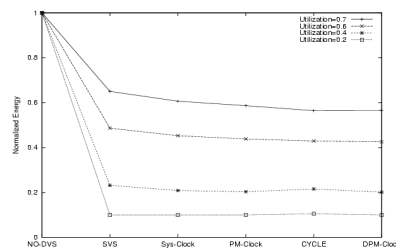
- Each task has a uniform probability of having a short (1-10 ms), medium (10-100 ms) or long (100-1000 ms) period.
- The task period is uniformly distributed in each range.
- Target Study
  - The effect of Variable Processor Utilization
  - The effect of BCEC/WCEC Ratio
- Comparing Sys-Clock, PM-Clock and Dynamic PM-Clock with SVS, CYCLE
  - SVS – System Clock Frequency that complete the task exactly at the deadline
  - CYCLE – More complex dynamic voltage scaling based on SVS with  $O(n^2)$  at every context switching
  - Sys-Clock, PM-Clock:  $O(Mn^2)$ , Dynamic PM-Clock: additional  $O(1)$  at every context switching

## Energy vs. System Utilization (1 of 2)

**BCEC/WCEC = 0.5**

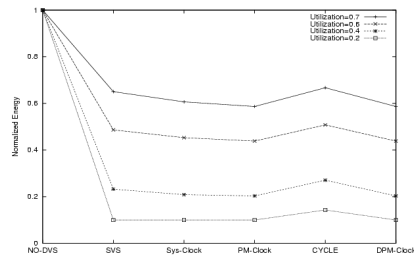
BCEC = best-case execution time

WCEC = worst-case execution time



## Energy vs. System Utilization (2 of 2)

**BCEC/WCEC = 1.0**

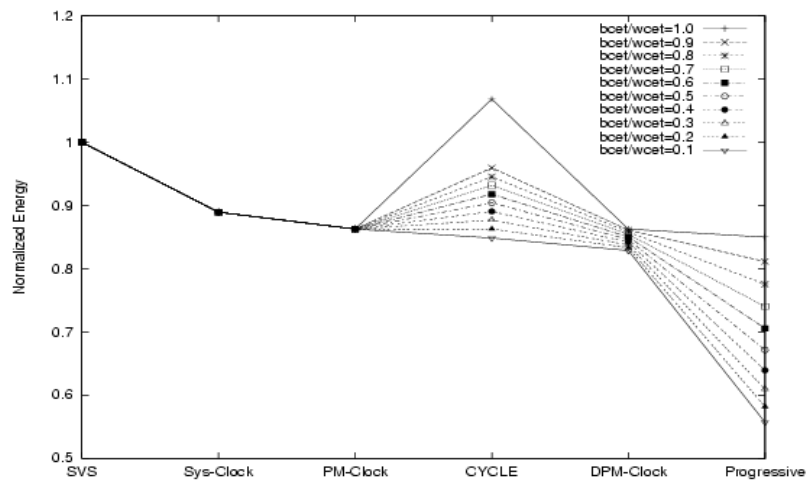


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## Energy vs. BCEC/WCEC



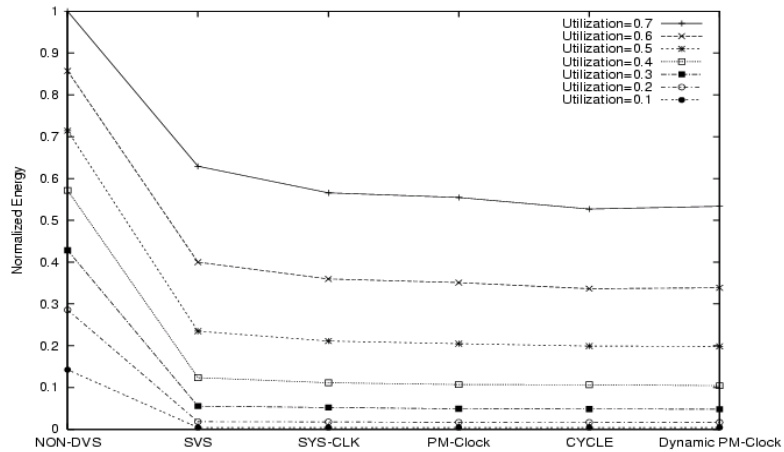
**System Utilization = 0.5**

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## The Effect of Processor Utilization at Highest Frequency

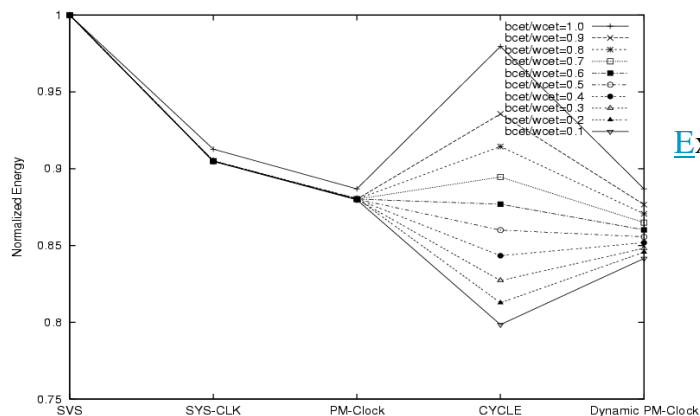


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## The Effect of BCET/WCET Ratio



Ratio of  
Best-Case vs  
Worst-Case  
Execution Times

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## Effect of Hardware Limitations

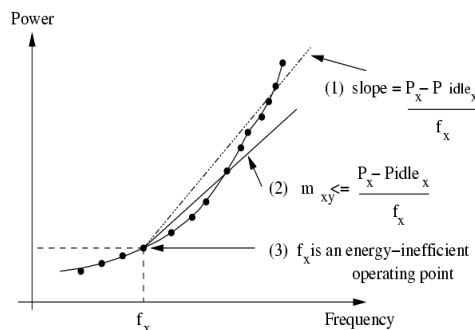
- An Ideal Voltage-Scaling Processor consumes less energy at any given smaller operating frequency
- Some commercially available processors do not!
  - Clock Frequency vs. voltage of Transmeta Crusoe processor

Frequency $f$ (MHz)	Voltage $V_{DD}$ (V)	Relative power (%)
600	1.60	100.00
525	1.50	70.00
450	1.35	45.00
375	1.22	33.33
300	1.20	26.67
225	1.10	23.33

$$\begin{aligned}
 - E_{225} &= P_{225} * t_{225} = 0.233 * t_{225} * P_{max} \\
 - E_{300} &= P_{300} * t_{300} + P_{idle} * (t_{225} - t_{300}) \\
 &= (0.267 * 225/300 * t_{225} + 0.05 * 75/300 * t_{225}) * P_{max} \\
 &= 0.21275 * t_{225} * P_{max} < E_{225}
 \end{aligned}$$

## Energy-Efficient Operating Frequency

- Running Crusoe processor at 300 MHz consumes **less** energy than at 225 MHz.
- The frequency 225 MHz is an *energy-inefficient* operating frequency



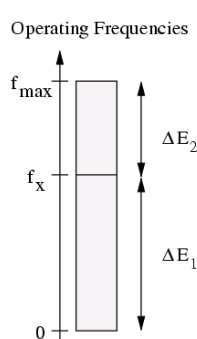
### Complexity of finding inefficient frequencies:

- $O(\log n)$  for a convex non-decreasing power-frequency processor
- $O(n^2)$  for the most general case
- $n$  is the number of available operating frequencies

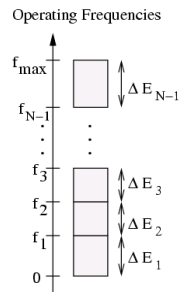
## The Effect of Finite Operating Frequencies

- CPU needs to operate at higher clock frequencies to satisfy the timing constraints if the expected freq. is not available.
- More energy is consumed: *Energy loss* due to *energy quantization* error
- Goal: Investigate an operating frequency grid which minimizes the worst-case energy quantization error

## 2-Point Operating Frequencies

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- Let  $\Delta E_1$  and  $\Delta E_2$  be the worst-case energy quantization error when  $f_{opt} \in (0, f_x]$  and  $(f_x, f_{max}]$
  - The worst  $\Delta E_1$  occurs when  $f_{opt} = \varepsilon$  and  $\varepsilon \rightarrow 0$  to compute the task with  $\lambda$  cycles
    - $\Delta E_1 = E_{2-point} - E_{ideal}$   
 $= [k\lambda f_x^2 + P_{idle}(1/\varepsilon - 1/f_x)] - k\lambda \varepsilon^2$
  - The worst  $\Delta E_2$  occurs when  $f_{opt} = f_x + \varepsilon$  and  $\varepsilon \rightarrow 0$  to compute the task with  $\lambda$  cycles
    - $\Delta E_2 = E_{2-point} - E_{ideal}$   
 $= [k\lambda f_{max}^2 + P_{idle}(1/(f_x + \varepsilon) - 1/f_{max})] - k\lambda (f_x + \varepsilon)^2$
  - Solving  $\Delta E_1 = \Delta E_2$ 
    - $f_x = f_{max} / \sqrt{2}$  assuming that  $P_{idle}$  is negligible

## N-Point Operating Frequency



In the worst case, having  $N$  operating points is  $1/N$  away from the ideal case (of infinite operating points)

$$\forall i \in [1, N-1] : f_i = \sqrt{\frac{i}{i+1}} f_{i+1}$$

$$f_N = f_{max}$$

$$\Delta E = \frac{k\lambda f_{max}^2}{N} = \frac{E_{noDVS}}{N}$$

## Summary

- **Sys-Clock** is the optimal system clock frequency assignment with complexity  $O(Mn^2)$  at the admission control
- **PM-Clock** is a task clock frequency assignment with same complexity at the admission control
  - The clock frequency assigned to a task is greater than or equal to that assigned to a lower priority task
- **Dynamic PM-Clock** with addition  $O(1)$  at every context switch
- **PM-Clock** and **Dynamic PM-Clock** reduce energy by 71% at 50% system utilization
- Determine **Energy-inefficient operating frequencies** which should be excluded from voltage-scaling algorithm
- Determine the **optimal clock frequency grid** to minimize energy quantization error when the # of operating frequencies is limited