# Task Splitting on Multiprocessors

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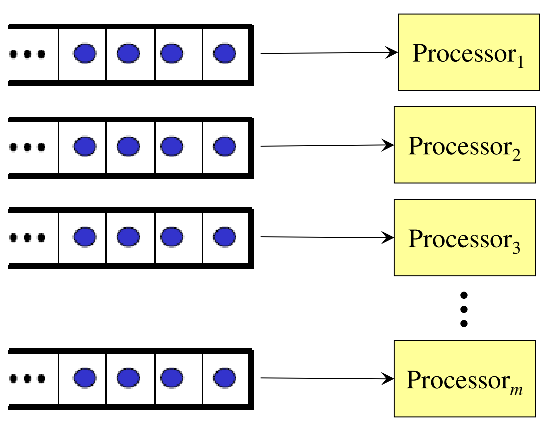
Lecture #14



## Partitioned Scheduling for Multiprocessors

### One Ready Queue Per Processor

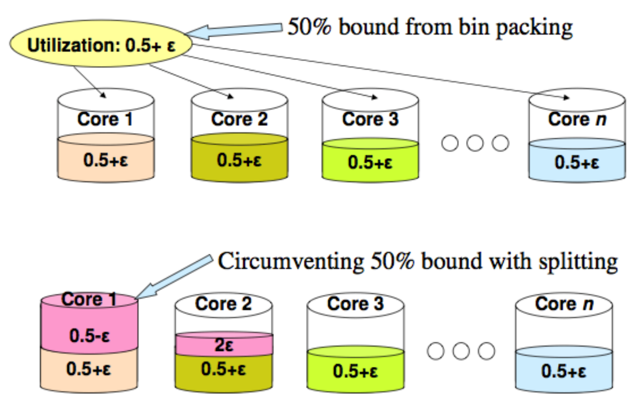


## Remember: The Bad Case of Bin Packing

- Given a list of objects of size 0.51, 0.51, 0.51, 0.51, 0.51
- Pack these objects into bins of capacity 1.0
- How many bins are required?
- With objects of size 0.50+ε, 0.50+ε, 0.50+ε, 0.50+ε, 0.50+ε,
  - What is the utilization of the system?
- Large objects can be a problem!
  - Think *rocks*…

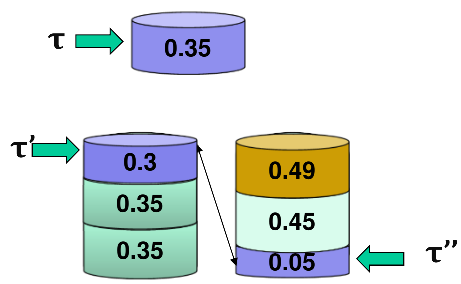


## Task Splitting



# Splitting: Illustration

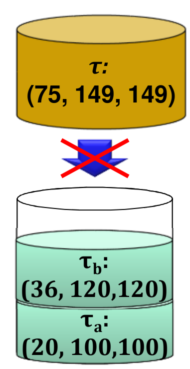
- Keep allocating tasks to a core ($M_p$) if they "fit"
- If a task does not fit
  - Split the task $\tau$ into **$\tau'$ and $\tau''$**
  - Assign **$\tau'$ to** $M_p$ and assign remainder **$\tau''$** to next core ($M_{p+1}$)



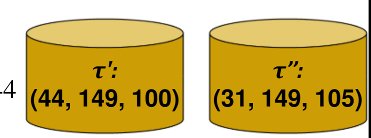



# How to Split a Task?

- If task $\tau$ (C, T, D) does not fit into core $M_p$
  - And $M_p$ is non-full
  - Split task $\tau$ into $\tau'$ and $\tau''$ such that
    - $\tau'$ has C' units of execution on core $M_p$
    - $\tau''$ has (C-C') units of execution on core $M_{p+1}$
    - Let $R'$ be the response time of task $\tau'$
- Deadline of $\tau'$ = deadline of the highest priority task
- $\tau'$ given the highest priority
- $\tau''$ should start after R' but finish by original D
- *$\tau''$ will be the highest priority task on $M_{p+1}$*




Will complete at 44



## Design Considerations

- When to split tasks?
  - Scenarios with "chunky" tasks
    - Too big to fit in one core
    - Too much utilization wasted in core otherwise
  - Task splitting is done to further improve efficiency
  - Tasks with soft deadlines are ideal to split
- Task splitting reduces processor count
  - Leads to savings in cost
  - Reduction in baseline power, and
  - Savings in physical space



## Period Transformation

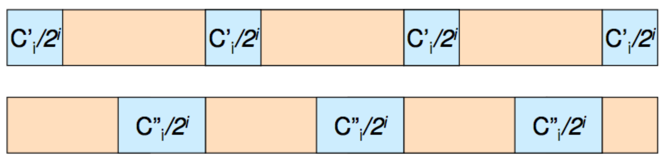
- A technique to reconcile the scheduling priority and semantic importance of a task.
- **Recall**: In case of an overload, only the highest priority tasks can meet their deadlines under fixed-priority preemptive scheduling.
- A task (*C, T*) can be transformed into a task (C/2, T/2) i.e. with two virtual pieces, the first piece executing the first half of the task and the second piece executing the second piece of the task
  - With a shorter period, the task can be assigned a higher priority
- A task (*C, T*) can be transformed into *a task* (*C/k*, *T/k*) with *k* virtual pieces
  - The task has a much shorter period, and can be assigned a higher priority

# Task Splitting for Harmonics

- Consider tasks,
- Assume **period transformation** of split tasks
- How to split $\frac{C_i}{2^i T}$ among processors $P_1$ and $P_2$
- Periodic servers ($C'_i/2^i$, T) and ($C''_i/2^i$, T) :

$$\frac{C_0}{2^0 T}, \frac{C_1}{2^1 T}, \frac{C_2}{2^2 T}, \ldots, \frac{C_n}{2^n T}$$

$C''_i = C - C'_i$



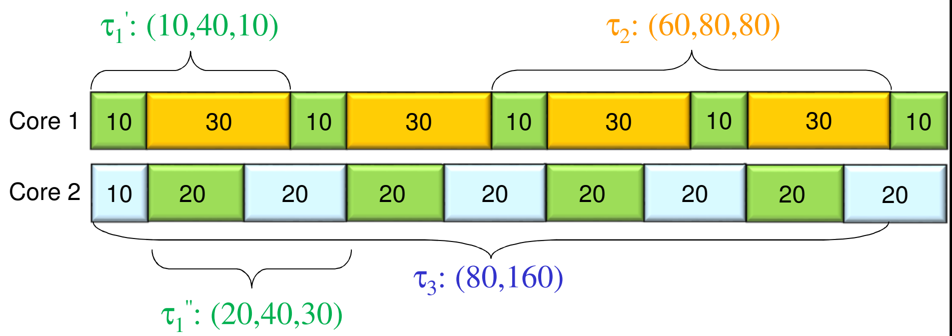



# Example A (1 of 2)

- Consider the task set $\tau_1$: (30, 40), $\tau_2$: (60, 80), $\tau_3$: (80, 160).
  – The utilization values are 0.75, 0.75 and 0.5 for a total utilization of 2.0.
  – But, no two tasks fit together → need 3 processors!
- Use BF**D**:
  – Per RMS, priority sequence: $\tau_1$, $\tau_2$ and $\tau_3$
  – These are harmonic tasks: hence, schedulability bound = 100% for RMS.
  – Bin-packing sequence: $\tau_1$, $\tau_2$ and $\tau_3$.
  1. $\tau_1$ → core 1
  2. $\tau_2$ → cannot fit into core 1. So, we decide to split.
  – Split which task? Task $\tau_1$ has the highest priority on core 1. So, we split task $\tau_1$.
    - $U(\tau_1') = 0.25$ (1.0 – 0.75 of $\tau_2$) for schedulability
    - $C_1' = 0.25 * T_1 = 0.25 * 40 = 10$
    - $D_1' = C_1' = 10$
    - $C_1'' = C_1 - C_1' = 30 - 10 = 20$
    - $D_1'' = D_1 - D_1' = 40 - 10 = 30$.
  3. $\tau_1''$ (20, 40, 30) → core 2.
  4. task $\tau_3$ → core 2.

## Example A (2 of 2)

- Core 1: $\tau_1'$: (10, 40, 10), $\tau_2$: (60, 80, 80).
  - Is this taskset schedulable? **Yes.**
- Core 2: $\tau_1''$: (20, 40, 30), $\tau_3$: (80, 160, 160).
  - Is this taskset schedulable?
  - Yes!






## Example B (1 of 2)

- Consider the task set $\tau_1$: (10, 20), $\tau_2$: (30, 40), $\tau_3$: (60, 80).
  - The utilization values are 0.5, 0.75 and 0.75 for a total utilization of 2.0.
  - Again, no two tasks fit together.
- Use BF**D**:
  - Per RMS, priority sequence: $\tau_1$, $\tau_2$ and $\tau_3$
  - These are harmonic tasks: hence, schedulability bound = 100% for RMS.
  - Bin-packing sequence: $\tau_2$, $\tau_3$ and $\tau_1$.

1. $\tau_2$ → core 1
2. $\tau_3$ → cannot fit into core 1. So, split! Which task? Task $\tau_2$ has the highest priority on core 1. So, we split task $\tau_2$.
   - $U(\tau_2') = 0.25$ (1.0 – 0.75 of $\tau_3$) for schedulability
   - $C_2' = 0.25 * T_2 = 0.25 * 40 = 10$
   - $D_2' = C_2' = 10$
   - $C_2'' = C_2 - C_2' = 30 - 10 = 20$
   - $D_2'' = D_2 - D_2' = 40 - 10 = 30$.
3. $\tau_2''$ (20, 40, 30) → core 2.
4. task $\tau_1$ → core 2.

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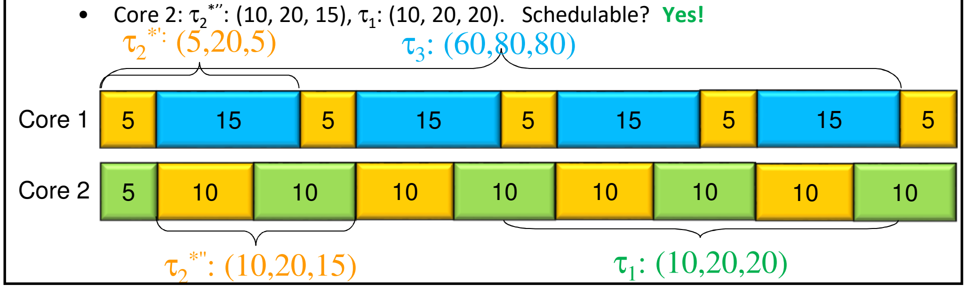
# Example B (2 of 2)

- Core 1: $\tau_2'$: (10, 40, 10), $\tau_3$: (60, 80, 80).
  - Is this taskset schedulable? **Yes.**
- Core 2: $\tau_1$: (10, 20, 20), $\tau_2''$: (20, 40, 30).
  - Is this taskset schedulable?
  - No! Oops – our allocation did not fit – we couldn't do that!
- Solution: Use period transformation to make $\tau_2''$ the highest priority task on core 2?
  - Easier to transform task $\tau_2$ so that it is the highest priority task in the taskset.

Break tie in favor of task $\tau_2^*$

- Original taskset becomes $\tau_1$: (10, 20), $\tau_2^*$: (15, 20), $\tau_3$: (60, 80).
- *Now*, use task splitting.
- Core 1: $\tau_2^{*'}$: (5, 20, 5), $\tau_3$: (60, 80, 80). Schedulable? Yes.
- Core 2: $\tau_2^{*''}$: (10, 20, 15), $\tau_1$: (10, 20, 20). Schedulable? **Yes!**




# Takeaways from Examples A and B

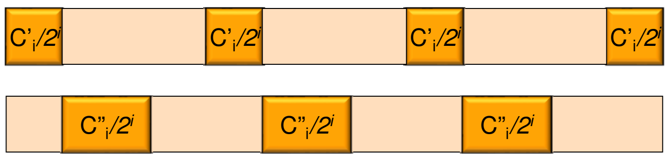
- When we have to split a task, pick the highest priority task on that bin to make sure its $C' = D'$.
- The second split piece has a shorter deadline ($D'' = D - D'$). By using period transformation, we make this second piece be the highest priority task on the other bin as well.
- If you period-transform a split piece, might as well transform the *entire* task.

## Task Splitting For Harmonics

- Consider harmonic tasks
- Assume **period transformation** of split tasks
- How to split $\frac{C_i}{2^i T}$ among processors $P_1$ and $P_2$
- Periodic servers $(C'_i/2^i, T)$ and $(C''_i/2^i, T)$ :

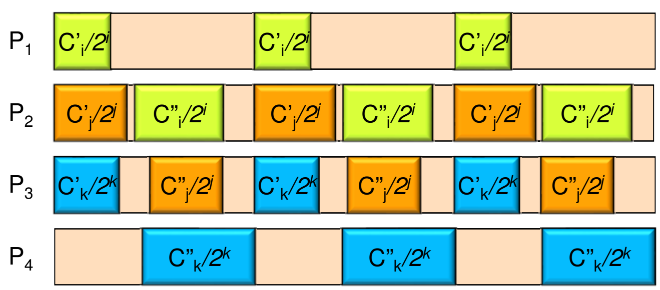
$$\frac{C_0}{2^0 T}, \frac{C_1}{2^1 T}, \frac{C_2}{2^2 T}, \ldots, \frac{C_n}{2^n T}$$

$C''_I = C - C'_i$






## Analysis




- Achieves 100% utilization with harmonic tasksets
  - General algorithm also applies to other *harmonic* task sets
- Migration costs:
  - total migrations in the worst-case (per $T_n$)

$$O(m\frac{T_n}{T_0})$$

## Conclusions

- With task splitting, the bad case of bin packing (even for the optimal bin-packing algorithm) is avoided.
- With period transformation, the scheduling priority of a task can be reconciled with the semantic importance of a task.
- By combining period transformation and task splitting, 100% schedulable utilization of each of *m* processors can be obtained for harmonic task sets.
  - Can be applied to non-harmonic tasksets too, but the schedulable utilization levels are lower (as in RMS for non-harmonic tasksets).



## Priority Granularity

## Number of Priority Levels Needed

- Suppose we have a taskset with 1000 unique periods (or deadlines)
- Do we need 1000 priority levels to represent each of them uniquely?
- What about 10000 tasks?

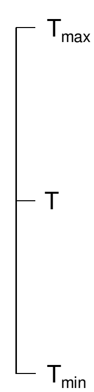


## Priority Spectrum Analysis

- Suppose we have one priority bit. All tasks must be either marked as high or low priority.
  - Consider period spectrum between $T_{min}$ and $T_{max}$
  - Periods above *T* are marked low priority and periods below *T* are marked high priority
- Priority inversion is maximum when a task with period ($T_{min}$ + ε) shares the same priority with a task of period (T - ε).
- We want $(T - \varepsilon) / (T_{min} + \varepsilon) = (T_{max} - \varepsilon) / (T + \varepsilon)$
- As $\varepsilon \rightarrow 0$, $T/T_{min} = T_{max}/T = 1/r$

➔ $T^2 = T_{min}T_{max}$

i.e. $T_{min}$, T and $T_{max}$ form a *geometric progression*.

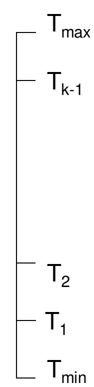


# For *k* priority levels

- Generalizing this notion, for *k* priority levels, we have

$$T_1/T_{min} = T_2/T_1 = T_3/T_2 = \ldots = T_{max}/T_{k-1} = r$$

- That is, the period boundaries for priority assignment form a geometric progression.






# Schedulability Bound with finite '*k*'

- The worst-case scheduling bound with insufficient priority levels is given (approximately) by*

$$U = \begin{cases} MIN_{1 \le i \le K}(1 - G_i + ln(2G_i)) & 1/2 \le G_i \le 1 \\ G_i & 0 \le G_i \le 1/2 \end{cases}$$

where $G_i = (L_{i-1} + 1)/L_i$

Where, $L_i$ corresponds to the discrete boundary of each priority "grid".

If a task's period falls between two grid bound boundaries such that $L_{j-1} < T \le L_j$, priority *j* will be assigned.

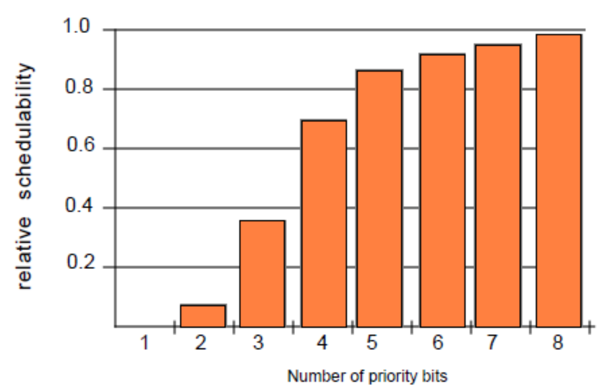
*John Lehoczky and Lui Sha, "Performance of real-time bus scheduling algorithms", Joint International Conference on Measurement and Modeling of Computer Systems, Proceedings of the *1986 ACM SIGMETRICS joint international conference on Computer Performance Modeling, Measurement and Evaluation*, 1986.

# Revisiting $k$ needed

- Suppose we pick $T_{min}$ = 100µ$s$ and $T_{max}$ = 100$s$, and we have $k$ priority levels
- $T_{max} / T_{min} = 10^6 = r^k$



# Relative Schedulability



# Conclusions

- A small number of priority bits (8) is very close to having an infinite number of priority bits in practice.