Manual – SSPlot : A Plotter and Numerical Solver for Dynamical Systems

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1. Introduction

To help with visualization of dynamical systems in 2d and 3d, a simple plotting utility and dynamical system simulator software has been written by me (Subhraman Sarkar). It was written in the Java programming language. Java has been choosen for the following reasons :

1. Java is highly *object-oriented*. So it is possible to break down large programs into small modules (class) where each module does a small portion of the total job.
2. Java standard library has very good graphics libraries as well as libraries for writing complex graphical user interfaces (gui) which is a must for any kind of simulation. In Java we do not need any external libraries for graphics or gui building. Also libraries are easy to use after learning some basics.
3. Java is cross-platform. Once a Java program has been compiled, it can run on any OS for which a Java Runtime environment is available. (With some more effort it can be ported to Android as well, since Android uses Java for its applications development language.)

2. Installation and Running

Homepage : SSPlot Repository

It is an open source software and is available under the terms of LGPL 2.1 license. It works on any platform with a Java JRE installed, though the exact steps required for installation might vary depending on the OS.

General Instructions :

1. Install Java JRE or JDK. Version 14 preferred as it includes a parsing engine which is used for parsing the equations in the System parameters window. Versions > 14 require some additional work.
2. Install the Apache Commons Math library, version 3 in your CLASSPATH.
3. If you are using Java versions > 14, an external script engine such as Jython is required. (To use Jython, install its jar file in your CLASSPATH. Then setup the SSPLOT_ENGINE environment variable with the value `python.`)
4. Download and install the SSPlot.jar file in your CLASSPATH from the homepage of the SSPlot project given above.

5. Run using the command `java math.plot.PlotterFrame`.

Much more detailed information about installing, running and compilation of the software in given in its homepage given above as well as with the releases obtained from the homepage.

3. The Main Window

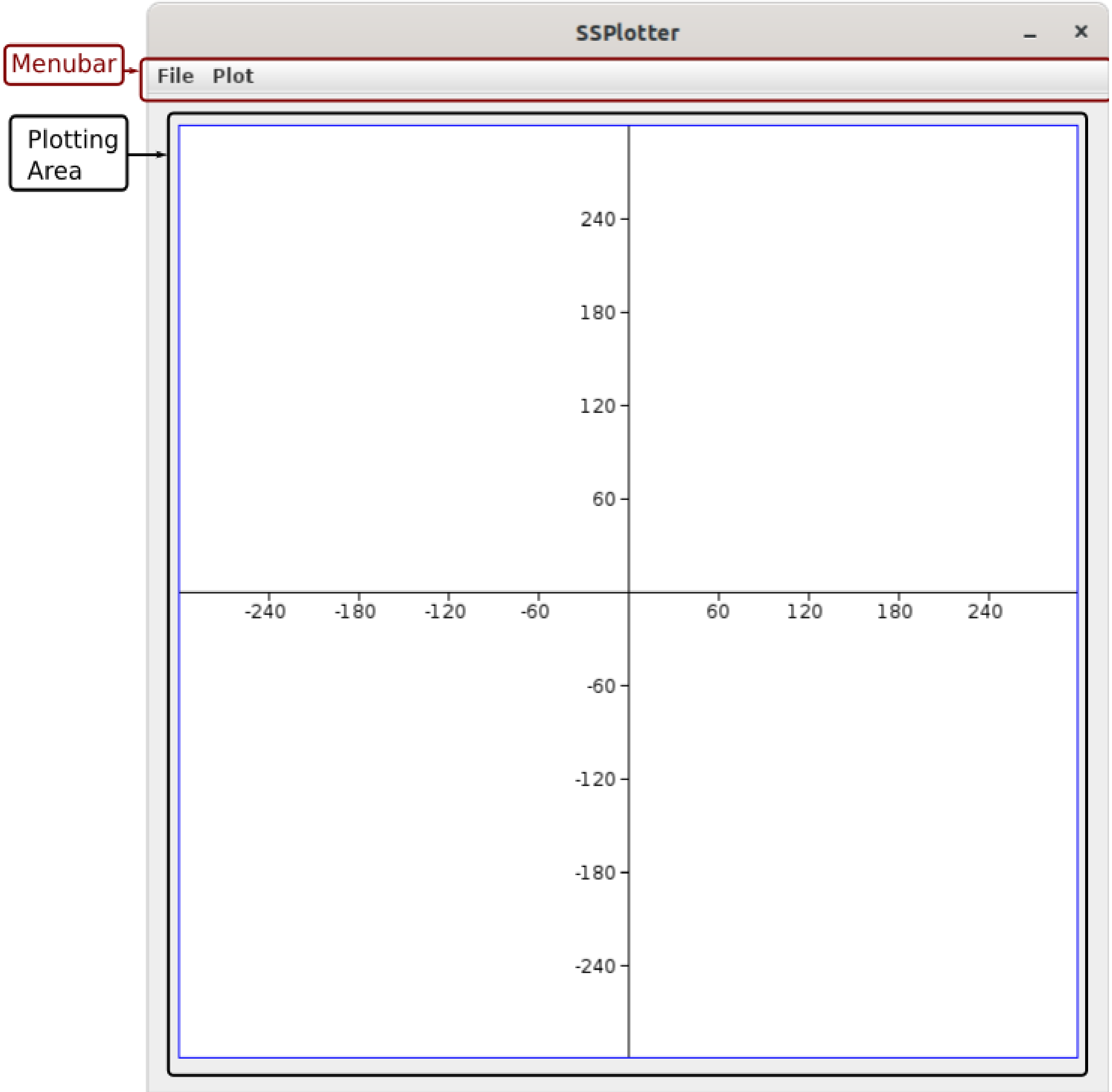

Figure 1: Main Window

The main window consists of two parts,

A) The Menubar

It contains two menus,

1. The File Menu

2. The Plot Menu

B) The Plotting Canvas

This is the area where the Plot and the Axes are shown.

4. The Menus

1. *The File Menu*

The File Menu is show in Figure 2. It is the most import menu as it contains most of the tools we need.

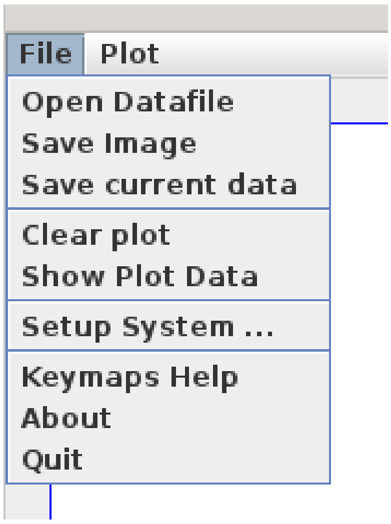

Figure 2: File Menu

1. *Open Datafile*

   This launches a file selection window, from which you can select a datafile with your data, which then gets plotted in the Plotting Area. A datafile is a plain-text file with numbers separated by spaces, like this :

```
0.00 1.0000
1.00 0.5403
2.00 -0.4161
3.00 -0.9900
4.00 -0.6536
5.00 0.2837
6.00 0.9602
7.00 0.7539
8.00 -0.1455
9.00 -0.9111
10.00 -0.8391
11.00 0.0044
```

   Each row is one of the data points. There is no restriction on the number of rows. However, the number of columns is restricted by the current Plot Type. It is explained in section 5.1 on different plot types.

   If you open multiple datafiles, each file will be plotted *over* the previous plot.

2. *Save Image*

   This saves the current plot as a png graphic file. (Please enter the filename with the extension. (ie. like *plot1.png*))

3. *Save current data*

    Saves the datapoints associated with the current plot (usually the last) to a datafile.

4. *Clear plot*

    Clears all plots from the Plotting Area as well as their data from memory, and resets the axes (and any zoom level changes).

5. *Show plot data*

    Shows the datapoints of the current plot in a separate data viewer window. Explained in section 5.2.

6. *Setup System…*

    Opens the *System Parameters* window, which allows you to solve systems of ODEs or systems of Difference equations and plot them. How to use this window to solve systems and plot them is explained in detail later in section 7

7. *Keymaps Help*

    There are some keys on the keyboard which are mapped to special functions inside the plotting area. For example, the **J** and **B** keys are mapped to Zoom in and Zoom out functions, so you can use them to zoom in and out the plot. The various keybindings are explained in detail in section 6.

8. *About*

    Shows information about the software.

9. *Quit*

    Stops and exits the application.

2. *The Plot Menu*

   Shown in [Figure 3](). This menu changes the properties of the current plot.

   1. *Show/hide axes*

       Toggles the axis visibility.

   2. *Set line width*

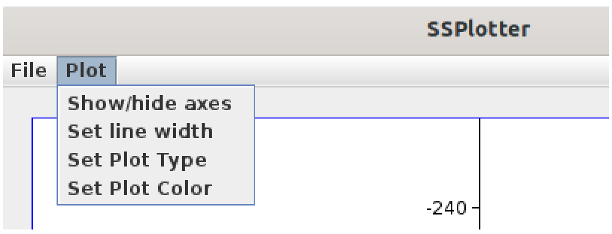

Figure 3: Plot Menu

Sets the thickness of the line or size of points. (both are same)

3. *Set Plot Type*

   This is an extremely important menu item. It sets the type of the plot, which changes the interpretation of the datafile. For example, a two column datafile is valid in *Lines* or *Points* modes, but is *illegal* in *3d Lines*, *3d Points* or *Vector Field* modes, which require a datafile with at least 3 columns, 3 columns or 4 columns respectively. (The program will show an error message if run from command line.)

   Various plot types are explained in section 5.1.

4. *Set Plot Color*

   Opens a color chooser dialog, from which you can change the color of the current plot.

5. Working with Data

5.1 Plot Types

For now, there are five plot modes,

1. *Lines*

   This mode requires a datafile with at least two columns. Each row of the datafile contains two values separated by whitespace. The first value specifies the x coordinate, while the second value specifies the y coordinate of one data point. Each row thus represents one single datum. The software then draws a plot by joining each point (specified by a row) with the next point in the datafile by a line segment.

   For a datafile with more than two columns, you can select which column to plot along the x axis and which column to plot along the y axis from the Data Viewer window. See the next sub-section (§5.2).

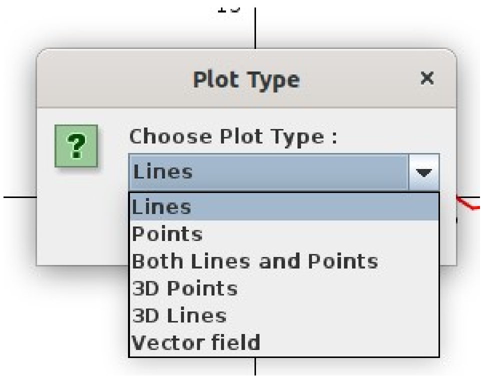

Figure 4: Plot Type Dialog

2. *Points*

Similar to the lines mode, except that it just draws a point on the plot corresponding to each datum. Changing the line color from the plot menu changes the color of the points.

3. *Lines and Points*

   A combination of the previous two modes. Both the datapoints and the line segments joining them are drawn. Changing line color does not changes the color of the point only in this mode. The defaut point color is *black*.

4. *3d Points*

   This mode requires a datafile with at least three columns. Everything is the same as the Points mode. The first three columns are used as the x, y and z data and the rest of the columns are ignored. You can use the 3d rotation keys on the keyboard to rotate the view. (See §6) The default view shows the XY plane.

5. *3d Lines*

   Same as 3d points, except this mode joins each point on the plot with the point following it by a straight line segment. (just like Lines mode, but in 3d) You can use the 3d rotation keys similarly.

6. *Vector field*

   Requires a datafile with at least 4 columns (all columns except the first four are ignored). The first two values in a row indicate a point (say A) and the last two values indicate another point (say B). The program then draws an arrow from the first point to the second point.

5.2 Data Viewer window

This window is accessed by clicking the *Show Plot Data* in the *File* menu. (shown in the Figure on the side)

The area marked by 1 allows you to set the columns which will be used as the X data and Y data respectively. Just enter the column number in the respective boxes and click *Apply*.

The area marked by 2 shows the data in your datafile.

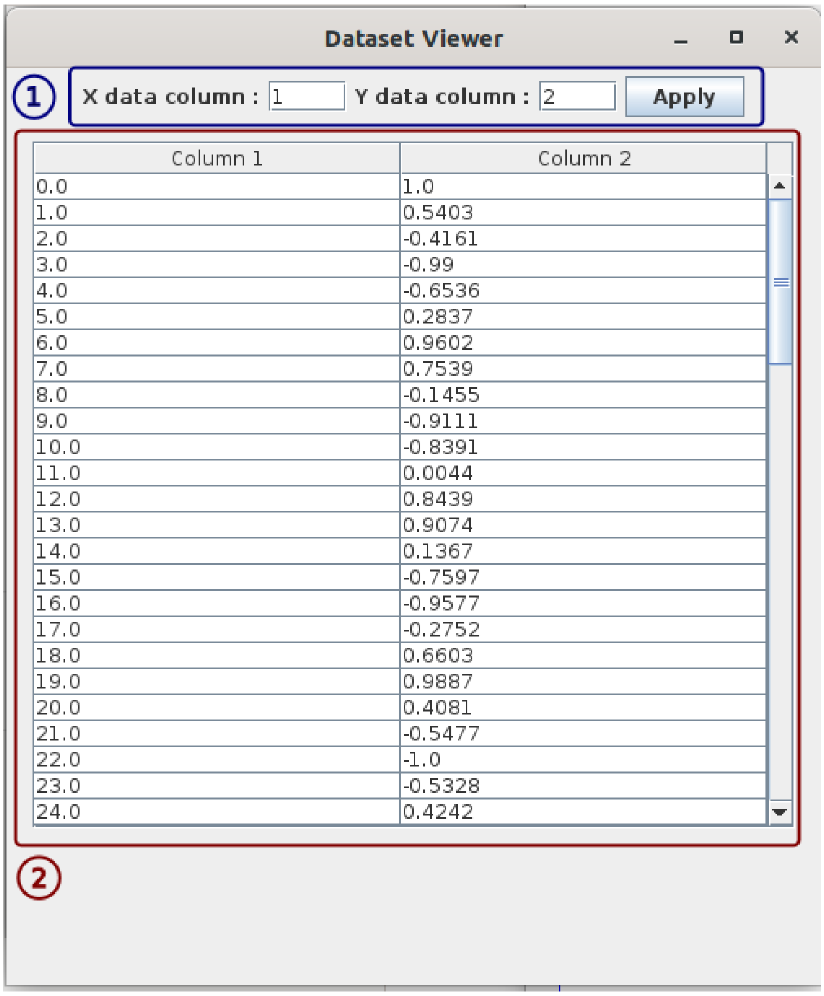

Figure 5: The Data Viewer Window

6. Keybindings

As mentioned earlier, various keys in the keyboard have been mapped to various plot related functions. These keys are given the table below.

Key	Function
J	Big Zoom. Double the Zoom level (i.e., if the zoom was 2x, then after pressing it will be 4x)
F	Big Zoom. Halve the Zoom level (i.e., if the zoom was 2x, then after pressing it will be 1x)
H	Small Zoom. Increase the zoom by 1x. (i.e., if the zoom was 3x, then after pressing it will be 4x)
G	Small Zoom. Decrease the zoom by 1x. (i.e., if the zoom was 3x, then after pressing it will be 2x)
Arrow keys	Shift the plot in the direction of the arrow key pressed.
Q	Rotate the view by 10° counterclockwise about the x axis.
A	Rotate the view by 10° clockwise about the x axis.
W	Rotate the view by 10° counterclockwise about the y axis.
S	Rotate the view by 10° clockwise about the y axis.
E	Rotate the view by 10° counterclockwise about the z axis.
D	Rotate the view by 10° clockwise about the z axis.

Whenever these keys are pressed, the current position, zoom or viewing angle in printed to the standard output stream (the terminal if you are running it from the terminal).

Also, if you Right Click on a point on the plotting area with the mouse, a message will appear showing the coordinates of the point. [v1.3]

You can now big zoom (zoom */ 2) using the mouse wheel and set the zoom center using middle click.[v1.4]

7. The System Parameters Window

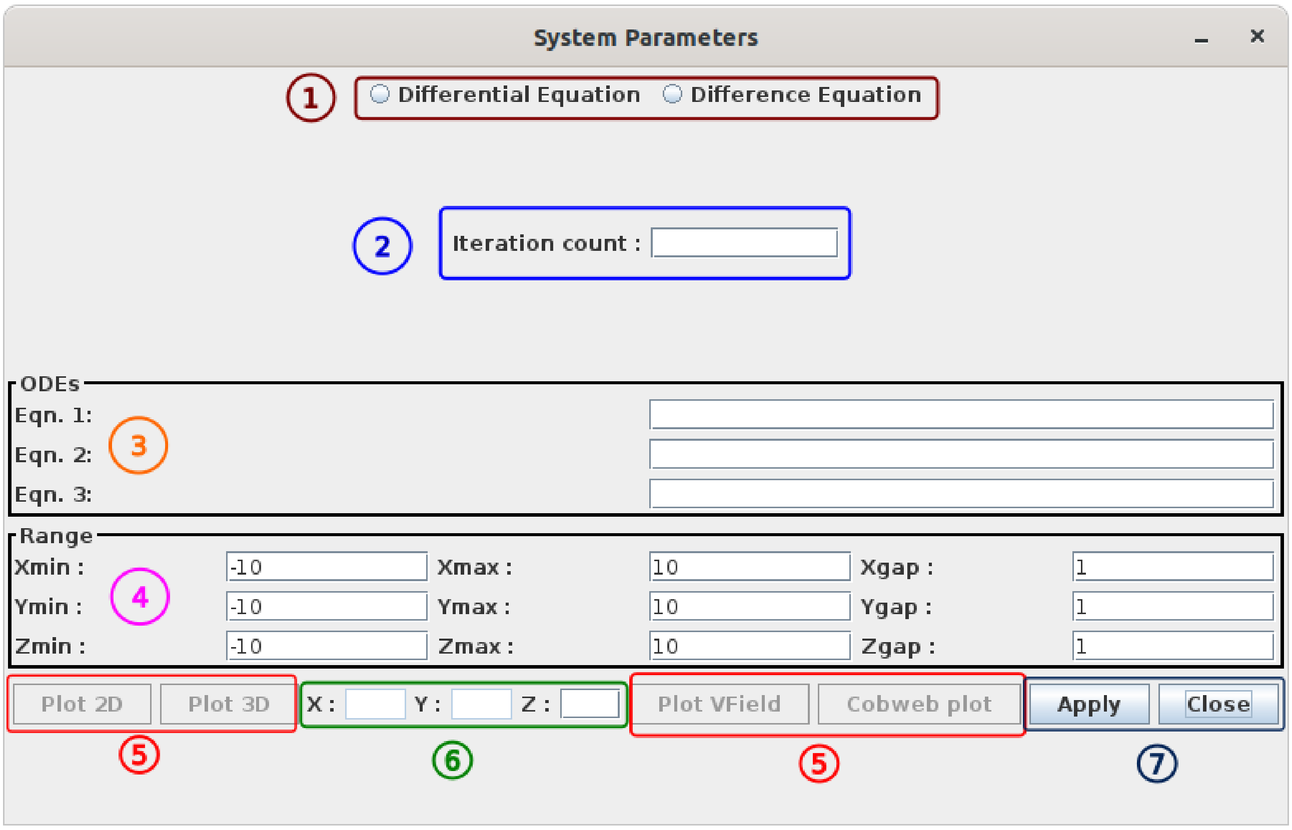

Figure 6: The System Parameters Window

This window can be accessed by clicking the *System Parameters…* item from the *File* menu. It allows you to numerically solve and plot systems of ordinary differential equations (in 2 and 3 dimensions) as well as systems of difference equations or iterated maps (in 1, 2 and 3 dimensions). Let's divide this window into some sections to better explain how it works.

From the top,

1. The area marked by ① allows you to choose whether you want to solve a system of differential equations or difference equations.

2. In the area marked by ②, you have to put how many times the system is going to be iterated. (Values like 5000 or 10000 are good for ODEs, while 100 is pretty high for some cobwebs plot, because it makes the plot a blob of black lines!)

3. Next is the area called *ODEs*, marked ③. You will enter the system of equations, either differential equations or difference equations here. The first, second and third textboxes are for the x, y and z equations respectively. Note that you must use x, y, z as the variables. The program cannot recognize any other variables.

Also, you need to enter <u>*the right hand side of the equations only*</u>. The left hand side will be automatically added. Also note that you will have to enter the equations in the notation of whatever script engine the program is using. For Java 14, this engine is a JavaScript parser, so you have to enter the equations in JavaScript notation for that case, i.e., $x^3 + \sin(x)$ becomes `Math.pow(x,3) + Math.sin(x)`.

4. You can enter the range as well as the spacing between arrows for vector field plot in area ④.
5. After you have entered the equations, setup the system by pressing the *Apply* button in area ⑦. Or, you can close the window by pressing *Close* if you don't need it.
6. As soon as you have setup the system, you can plot it by using the buttons *Plot 2D*, *Plot 3D*, *Plot Vfield* or *Cobweb Plot* in area (5). Except for the *Plot Vfield* button, you must enter the appropriate initial conditions in area (6) before pressing the buttons. This will now solve the system and plot the solution or vector field in the Plotting Area in the main window. As soon as the plot appears, you will be able to view the solutions in the *Data Viewer* window (File → Show Plot Data) and save it as a datafile using *Save Current Data* item in the *File* menu.
7. The various plotting buttons in the area (5) are :
    1. *Plot 2D* : Computes the numerical solution for the 2D system entered in the area (6). You need not enter the initial condition for N. You also don't need to enter the z equation in area (3). Uses RK 4^{th} order method for ODEs for the numerical solution. For difference equations it just iterates the system.
    2. *Plot 3D* : Same as *Plot 2D* button, except it needs all three initial conditions as well as all three equations. Initially the xy view is shown. You can rotate the view using the 3d rotation keys described in section 6.
    3. *Plot Vfield* : Plots a 2D vector field. You need to enter the x and y equations only.
    4. *Cobweb plot* : Requires only the x equation as well as the x initial condition. If you need to solve a 1D difference equation, don't use this. Use the Plot 2D button with 0 for the y equation and the y initial conditon instead. Use this button only to visualize the Cobweb plot of a 1D map.

8. Tutorial

1. Plotting a 2D system given by a system of ODEs

As an example, let us plot the Simple Harmonic Oscillator system, given by :

$$\ddot{x} + \omega^2 x = 0$$

This is a 2^{nd} order ordinary differential equation (ODE). To plot this, we first need to convert it to a system of equations. Since this is a 2^{nd} order equation, this can be converted to a system of two first order ODEs. Let's introduce $\dot{x} = y$. Then this equation becomes a system of two equations :

$$\dot{x} = y \qquad (1)$$
$$\dot{y} = -\omega^2 x \qquad (2)$$

Now launch SSPlot and open the System Parameters window by clicking File menu → System Parameters…

Since this is a system of differential equations, we choose the differential equations option. We want to iterate the system 5000 times, so we enter 5000 in the iteration count box. Next, we enter the right hand sides in the ODEs section in the System Parameters window. Enter the RHS of equation (1), i.e. the RHS of the $\dot{x}$ equation in the Eqn. 1 field, which is y. Similary, enter the RHS of the $\dot{y}$ equation in the Eqn. 2 window. You can choose any value of ω. For simplicity, we will set $\omega = 1$ here. So let's set Eqn. 2 field to $-x$. Leave the Range parameters to their default values. The window should look like this :

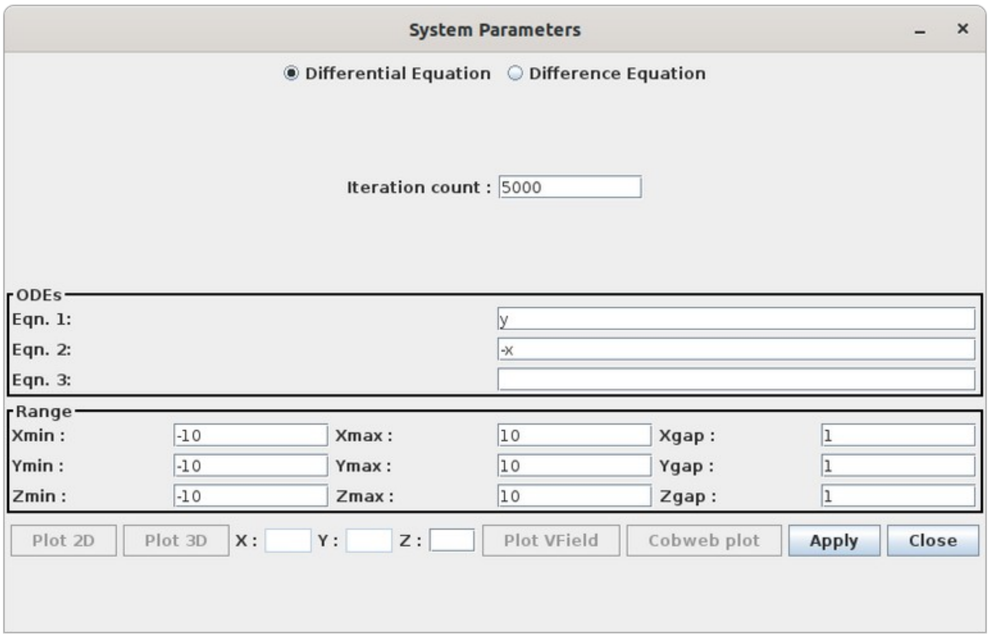

Figure 7: System Parameters for Example 1

Now click the *Apply* button. The *Plot VField*, *Plot 2D*, *Plot 3D* button, and the X, Y, Z textboxes on the bottom of the window will become active. Click on the *Plot VField* button, and you will see the main window like Figure 8.

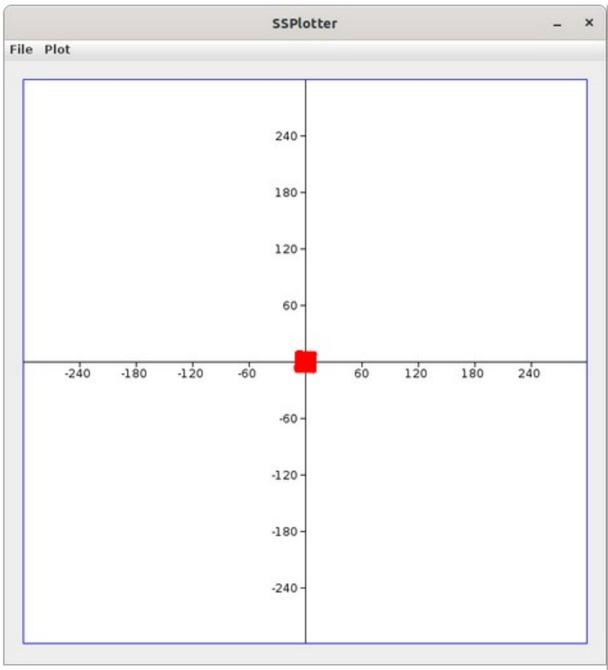

Figure 8: Main Window, not zoomed in

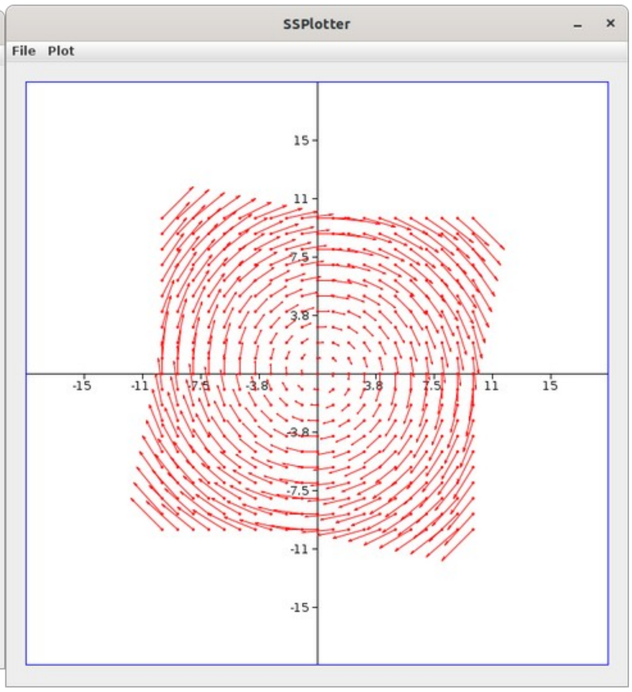

Figure 9: Main Window, zoomed in

The problem is that the default scale is way too big for our choice of ω, so we have to zoom in to see the system. Click on the Main Window, and press the **J** key on the keyboard a couple of times to zoom in, and you will get a vector field like Figure 9.

Now suppose you want to see what happens for large amplitude oscillations. In this case the equation are :

$$\ddot{x} + \omega^2 \sin(x) = 0 \Rightarrow \begin{aligned} \dot{x} &= y \quad (1) \\ \dot{y} &= -\sin(x) \quad (2) \end{aligned}$$

Just change the value of the Eqn. 2 field to `-Math.sin(x)`, in order to match the JavaScript notation. (Change the notation to match whatever parsing engine you are using.). In this case we will also need to tweak the Range parameters to increase the resolution of the Vector Field plot and plotting area in order to see the features of this system. Put Ymin = -5, Ymax = 5 and Xgap = Ygap = 0.5, then press the *Apply* button once more to update the system. Then clear the previous plot using File menu → *Clear Plot*, and plot the new system by clicking the *Plot VField* button on the System Parameters window. Zoom in a couple of times, and you will end up with something like Figure 11.

Now suppose you want to plot the solution over this direction field, say for initial conditions $(-10, 2)$. You can easily do so by entering the initial conditions in the X

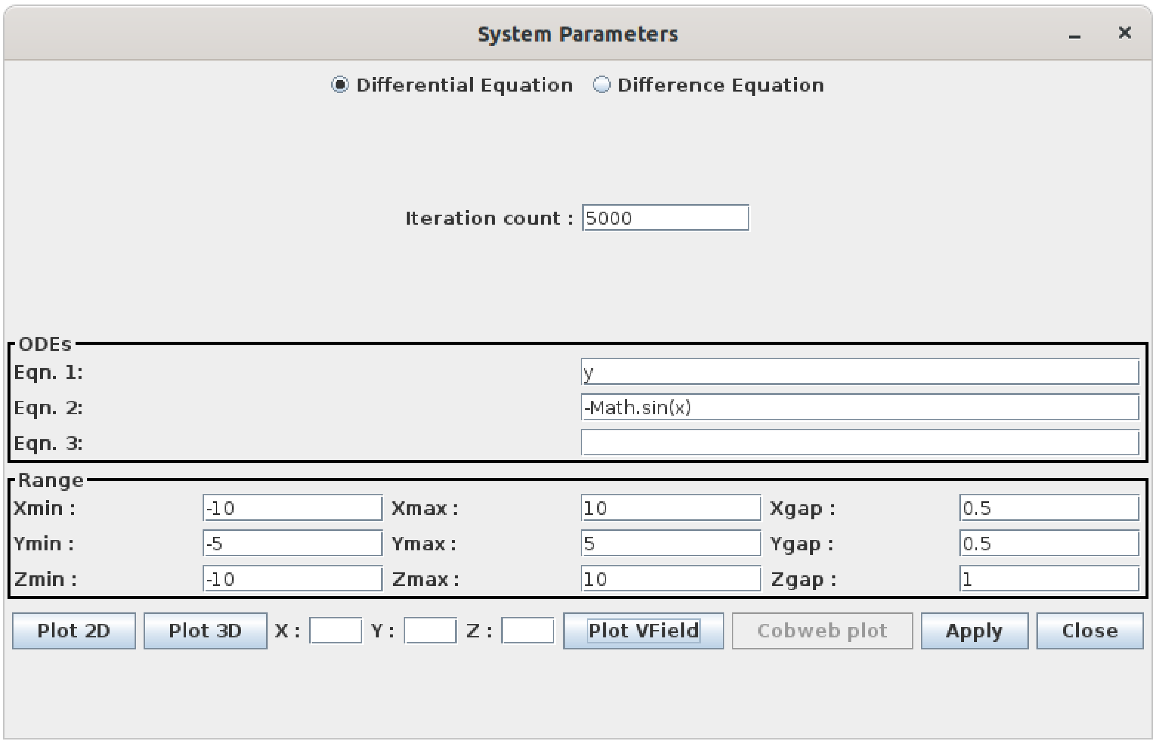

Figure 10: System Parameters for nonlinear SHO

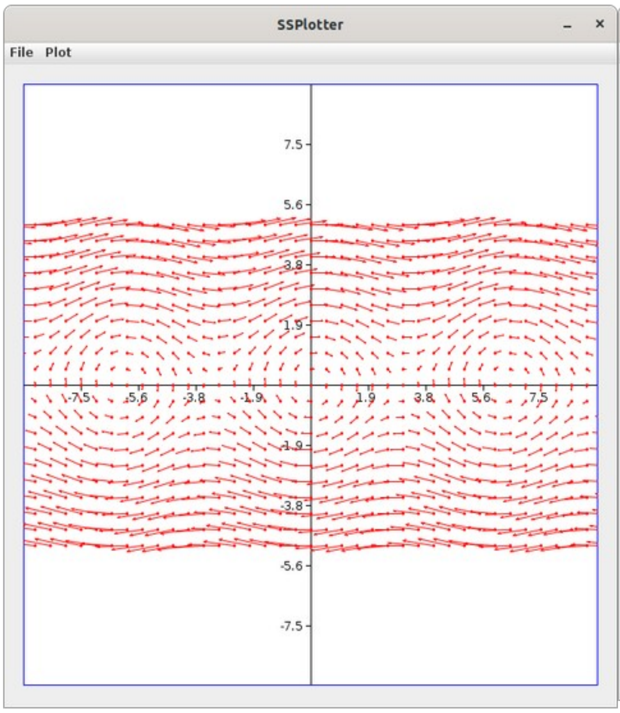

Figure 11: Direction Field for nonlinear SHO

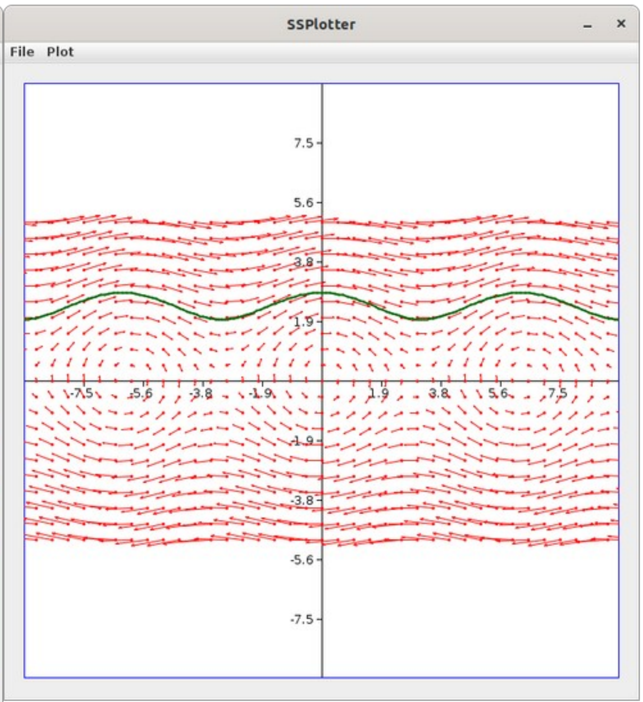

Figure 12: Trajectory for (-10,2)

and Y boxes at the bottom of the System parameters window and clicking *Plot 2D*. The result is something like Figure 12. The color of the trajectory is changed by using the *Set Plot Color* item in the *Plot* menu in the Main Window.

9. Gallery

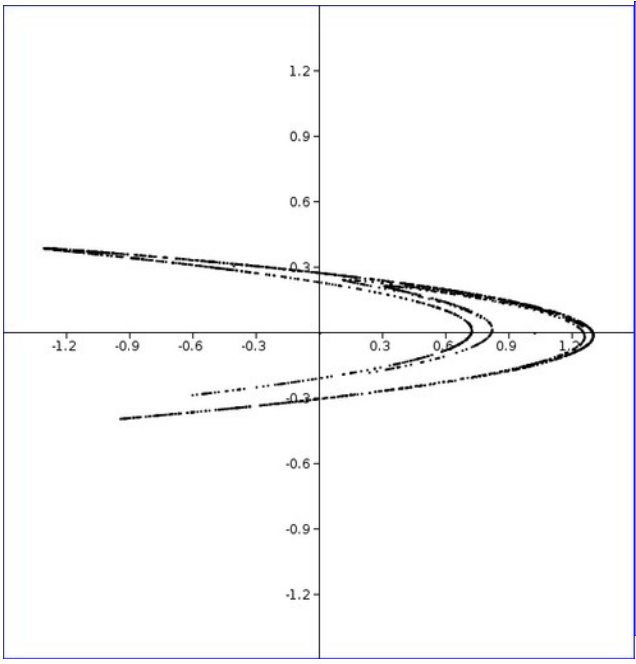

Figure 13: Henon map for parameter values a=1.4, b=0.3, initial condition (0,0)

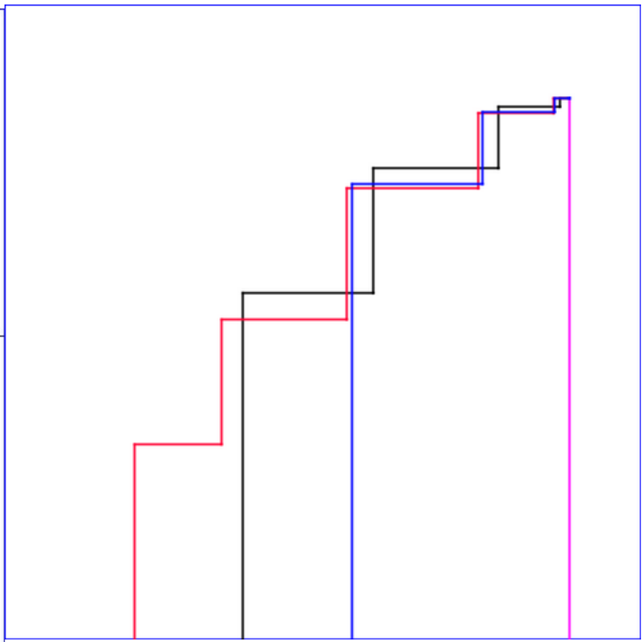

Figure 14: logistic map cobweb plot for parameter value a = 2 and several initial conditions in different colors. All trajectories converge to x = 0.5.

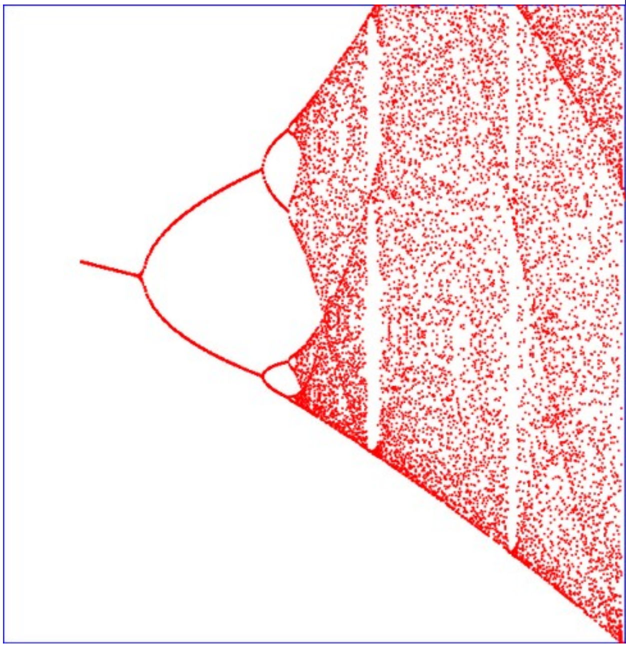

Figure 15: orbit diagram of logistic map, 1213x zoomed, showing self similarity. Plotted from a datafile created by a Fortran90 program

Figure 16: logistic map cobweb plot showing a single chaotic trajectory for parameter value a = 3.9, initial condition x = 0.2.

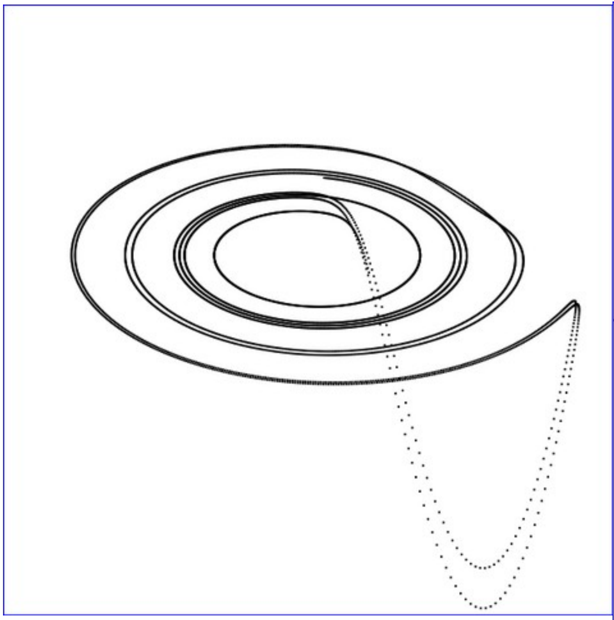

Figure 17: Rossler system, a=0.1, b=0.1, c=14, initial condition (5,5,5), rotated 60º counterclockwise, about x axis.

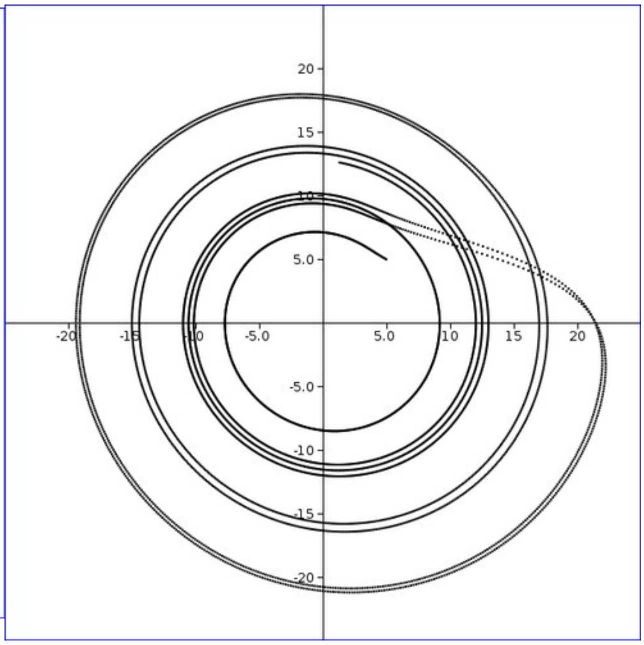

Figure 18: Rossler system, parameters a = 0.1, b= 0.1, c = 14, initial condition (5,5,5), XY view

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