WHEN TRUTH LIES: A PARADOX IN FOUR VOICES

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WHEN TRUTH LIES: A PARADOX IN FOUR VOICES

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Abstract. This paper introduces a paradox constructed in four symbolic voices — Silence, Echo, Voice, and Truth — to explore the boundary between formal propositions and structurally misleading statements. What appears at first to be a logical puzzle ultimately reveals a deeper fracture: the illusion of independence in statements whose meaning depends on the system they inhabit. Drawing on insights from the Parallel Law of Excluded Middle (PLEM), we argue that some so-called paradoxes arise not from contradiction, but from mistaking pseudo-propositions — statements that simulate logical form without satisfying its structural conditions — for genuine truth-bearing claims. Through layered analysis and structural logic, we examine how truth collapses when misclassified forms are mistaken for propositions.

1. Introduction: Four Voices and a Fracture

In a world of thought, stripped of context and noise, four voices speak. Each represents a concept: Silence, Echo, Voice, and Truth. What begins as a logic puzzle becomes something more — a meditation on paradox, self-reference, and the limits of truth itself.

This essay began as a game. But like all games of reason, it revealed a fracture too sharp to ignore.

Since antiquity, paradoxes have served not only to puzzle but to challenge the assumptions of logical systems. From Zeno's paradoxes of motion to the Liar Paradox — "This sentence is false" — the idea that language can loop upon itself has haunted logic from the inside out. Philosophers such as Tarski [3] have argued that such contradictions emerge when language tries to speak about itself — when a system attempts to evaluate its own truth claims without stepping outside its boundaries.

The challenge, as Gödel later demonstrated[1], is that any system capable of encoding arithmetic — and, by extension, its own rules — will inevitably contain statements that are true but unprovable within that system. This fracture is not just a bug in logic. It is a mirror held up to the limits of human reasoning.

In this paper, we revisit that mirror — not through formal proof, but through metaphor and symbolic voice. Our puzzle places four statements in tension: each expresses a distinct perspective, yet together they form a loop of reference and contradiction. What initially feels like a logic riddle becomes something closer to a parable of dependency — a reflection on how some statements appear to be propositions while structurally depending on the framework they pretend to transcend.

We offer both a solution and a deconstruction. In Section 2, we examine the structural paradox that arises when the fourth voice — Truth — asserts a meta-claim that collapses under its own dependency. In Section 5, we define the distinction between genuine propositions and pseudo-propositions: statement-like forms that mimic logical validity but evade

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consistent evaluation. This distinction is central to understanding how logical breakdown can occur not due to contradiction, but due to category error.

Ultimately, we argue that some famous paradoxes — including the Liar and Russell's paradox — are not logical failures, but structural misclassifications. They are pseudo-propositions mistaken for logical statements. When we treat such forms as truth-bearing, we confuse system with transcendence — and that confusion births paradox.

2. The Puzzle

Each "entity" makes a claim:

1. **Silence says:**
"I do not exist unless you name me."
2. **Echo says:**
"I say what you say, but I never begin."
3. **Voice says:**
"They all speak through me, but I am none of them."
4. **Truth says:**
"Only one of us lies."

At first glance, the statements appear independently testable. Silence's identity depends on recognition. Echo defines itself through repetition. Voice speaks of itself as medium. Truth makes a meta-claim.

But the paradox emerges when we ask: *Who is lying?*

3. The Collapse

Suppose Truth is lying.

Then the statement "Only one of us lies" is false — meaning more than one lies.

But when we examine the others:

- Silence: Plausibly true.
- Echo: Logically consistent.
- Voice: Abstract, but not self-contradictory.

Only one statement appears false — Truth's.

Thus, Truth must be telling the truth. But if so, then "Only one of us lies" is true, and someone else must be lying — yet the other statements still appear consistent.

So either:

- Truth lies → contradiction.
- Truth tells the truth → contradiction.

We reach a structural impasse. The contradiction is not in meaning, but in evaluation. The system collapses not from logic, but from trying to assign truth to a structurally dependent statement as if it were independent.

4. Postscript: On Neutrality and Dependency *(A Commentary by Babak Jabbar Nezhad)*

This paradox reveals more than a contradiction. It shows a boundary of logic — a statement that cannot be resolved within its own frame.

The first three voices — Silence, Echo, Voice — operate under classical logic, remaining within a neutral system.

But Truth's claim is different. It is meta-logical: its validity depends entirely on the truth value of the others, yet it speaks as if from above — as if structurally independent.

Here lies the contradiction: a dependent statement that claims independence.

This is not just paradox — it is a structural illusion.

> **Thesis (Babak Jabbar Nezhad):**
> Paradox does not always arise from contradiction itself. It often emerges when a structurally dependent statement is treated as if it were structurally independent — a category error that leads to the illusion of contradiction.

This insight connects directly to the broader framework I am developing: the Parallel Law of Excluded Middle (PLEM).

> **Statement of the Parallel Law of Excluded Middle (PLEM) (1):**
> Let P be a proposition in a sub-world U. Then either P is true in U, or $\neg P$ is true in U, or the disjunction holds in another sub-world U'.

Now, as shown in the [2], a structure cannot be embedded in a sub-world, but rather in the world in which we are within — even though we cannot emerge to that world.

Therefore, we have the following refined and structural interpretation of PLEM:

> **Statement of the Parallel Law of Excluded Middle (PLEM) (2):**
> In a layered or constructively grounded logical system, the truth of a proposition is not determined solely by its classical form ($P \vee \neg P$), but by the contextual structure in which it is embedded.
>
> Thus, for any proposition P, it is not necessarily the case that $P \vee \neg P$ holds — unless the structural conditions supporting both P and $\neg P$ are themselves well-defined within the same logical layer.

In such layered systems, some statements lose definitive truth value because their context is fractured.

Truth's statement is not just uncertain — it is structurally ungrounded.

5. The Illusion of Proposition — Pseudo-Propositions vs. Logical Statements

Not every sentence that mimics logical form is a true proposition. A pseudo-proposition is a statement-like structure that appears logical but lacks the structural conditions necessary for truth evaluation. Its failure is not grammatical — it is referential or systemic: it depends on external context, circular reference, or ambiguity that prevents it from standing independently in a formal logical system.

Frege's Functional Form vs. Pseudo-Proposition

Frege, in his formal logic, introduced the concept of unsaturated functions — expressions like "x is a man" which are incomplete until bound to a subject. These are not propositions until quantified or evaluated over a domain. For example:

- "x is a man" is a function.
- "For every x in this room, x is a man" becomes a proposition.

This distinction is syntactic and referential — the form becomes evaluable once a variable is bound. Frege's system assumes all well-formed functions can be completed into valid propositions.

But pseudo-propositions, in the sense used here, go beyond Frege's concern with variable binding. They include structures that:

- Consume themselves (like self-referential paradoxes),
- Depend on other statements for their truth value, or
- Evade evaluation by simulating assertion without logical foundation.

For example:

- "This sentence is false" is not a Fregean function — it is not even logically expandable. It mimics propositional form but contains no anchor or referent. It is structurally void.
- "Only one of us lies," from our logic puzzle, is also a pseudo-proposition. It depends on others for evaluation and cannot be judged in isolation — yet it presents itself as a truth-claim.

In this framework, pseudo-propositions are not just incomplete — they are inherently misleading. They simulate logical form while evading logical responsibility.

This insight allows us to reinterpret many so-called paradoxes not as contradictions in logic, but as category mistakes — treating structurally dependent or self-consuming statements as if they were complete and meaningful propositions.

6. Gödel, Unprovability, and the Structural Limits of LEM

Gödel's incompleteness theorem showed that any consistent formal system capable of encoding arithmetic contains statements that are true but unprovable within the system. His construction involves a proposition that refers to its own unprovability — a meta-statement whose form appears complete, but whose truth value cannot be determined within the system itself.

This is not merely a logical anomaly — it reveals something deeper about the nature of such systems: they inevitably harbor unprovable statements that are entangled with the system's own limitations. But what kind of statements are these?

We suggest the following interpretation: the undecidable propositions in Gödel's construction often refer to or involve statements that contain pseudo-propositions — statement-like forms that simulate logic but lack structural independence. When such pseudo-propositions are embedded within the reasoning of a system, the Law of Excluded Middle (LEM) — the principle that for any proposition P, either P or $\neg P$ must be true — can no longer be safely applied within that logical layer.

In other words, the presence of pseudo-propositions renders LEM structurally local. The illusion of global applicability breaks down because certain parts of the system are not logical propositions at all — they are structurally ungrounded forms pretending to be. And since the truth or falsehood of a pseudo-proposition cannot be established without stepping outside the system, LEM collapses under the weight of structural dependency.

This perspective does not contradict Gödel's theorem — rather, it deepens it. Gödel showed that truth transcends provability; we add that this transcendence occurs because the system inevitably contains structurally entangled forms, and these forms disrupt classical logic from within.

Thus, what Gödel demonstrated with formal precision, we reinterpret through the lens of layered logic and PLEM:

In any sufficiently rich formal system, LEM is not globally valid — because some statements, by virtue of their structural entanglement, lie outside the binary space of $P \vee \neg P$ within that layer.

These statements are not simply "unknown" — they are structurally unknowable within the system. And that unknowability stems not from the complexity of their content, but from the fractured architecture in which they reside.

7. Conclusion

Paradox does not always stem from contradiction.

Sometimes, it emerges when we try to treat a dependent statement as independent. When grammar mimics logic, but offers no substance beneath.

The fourth voice — Truth — is not lying. It is not telling the truth. It is simply not a voice that can be evaluated in the same plane as the others.

And that — is when truth lies.

Author's Note on Terminology and Cultural Attribution

The term "pseudo-proposition" was originally drafted using the Persian term gozārehnemā, which appeared in high school logic curricula. However, I wish to clarify that the conceptual depth explored in this paper does not derive from that terminology. What I mean by pseudo-proposition aligns more closely with Frege's notion of unsaturated expressions — logical forms that mimic propositional structure but lack completeness. While I remain grateful to the educational texts that introduced these ideas in simplified form, I do not attribute philosophical originality to them. This distinction matters — because true discovery lies in the deeper structural insight, not in the surface of borrowed names.

— Babak Jabbar Nezhad

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