COLORLESS OBJECTS AND THE COLLAPSE OF THE POWER SET A STRUCTURAL CRITIQUE OF LEM AND WITTGENSTEIN'S LOGIC

Preprint · June 2025

DOI: 10.13140/RG.2.2.14106.56003

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Abstract. This short note presents a refinement in the PLEM (Parallel Law of Excluded Middle) framework concerning the structural definition of the power set. We revisit a previously proposed formula that allowed a layered interpretation of subsets within a subworld and show that while this formula may constructively define sets under structural decidability, it fails when applied to the universe or to a logically total subworld. This failure is not due to cardinality, but to the deeper undecidability of subworld membership. We introduce a key ontological distinction: elements whose membership in a subworld is decidable are called sets or facts, while elements whose membership is undecidable are simply objects or things. This distinction reflects a layered structure of truth and reveals that subworlds themselves cannot be treated as sets. In this context, even the status of the empty set becomes uncertain. We conclude that no universe or total subworld can be fully decidable, and that the classical notion of power set collapses under structural logic. This insight clarifies the limits of set construction and exposes an implicit contradiction in Wittgenstein's metaphysics.

1. Background and Historical Motivation

In our earlier formulation [1], we defined the power set $\mathcal{P}(A)$ structurally using the following predicate:

$$\mathcal{P}(A)(B) := (1 - \xi(S - A)) \cdot \xi(B - A) \cdot \sum_{a \in A} B(a).$$

Here, $\xi(X) = 1$ if $X = \varnothing$, and 0 otherwise. This expression enforces three structural conditions:

1. A is a subset of the subworld;
2. $B \subseteq A$; and
3. the membership predicate $B(a) \in \{0, 1\}$ is structurally defined across A.

At the time, this formula was introduced as a definition — a construction of meaning within the PLEM framework. But as we now observe, a definition alone does not establish truth. Structural logic demands that definitions be grounded in the decidability and coherence of membership across layers. The truth of a construction depends on whether its symbolic components are logically sustained across subworlds.

2. Decidability and the Structural Distinction Between Sets and Objects

If we ask whether a given element x in the universe belongs to a subworld B, we encounter the following problem:

Key words and phrases. Set Theory, Logical Structure, Pseudo-Prposition, Parallel Law of Excluded Middle, Self-Reference, Layered Logic.

- Suppose $x \notin B$ in another subworld B', but $B \neq B'$. Then B and B' are logically distinct.
- The logic applied to B in each subworld may diverge. The symbolic coherence that allowed structural transfer of membership no longer holds.
- As a result, membership in a subworld becomes a pseudo-proposition in the base subworld and cannot be decided.

Therefore, we encounter elements for which we can decide whether they belong to a subworld (precisely when LEM applies), and elements for which we cannot. This leads to the following structural distinction:

Definition. *We say an element B from the universe is a set (or fact) if we can decide whether $B \in A$ for a given subworld A. Otherwise, B is called an object (or thing) — it may exist in the universe but not as a set in that layer.*

Now suppose we are given a set B in a subworld A, and test whether an element x belongs to B. We observe:

1. If $x \in B$ within A, then membership is trivially decidable.
2. If $x \notin B$ within A, then membership is also decidable.
3. If $x \notin B$ only within another subworld $A' \neq A$, but B exists in both A and A', and the logic governing B is the same, then we may infer $x \notin B$ within A.

3. The Incoherence of Treating Subworlds as Sets

Subworlds cannot be treated as sets, because their internal membership structure is not decidable within the base layer. The logic of PLEM does not permit subworlds to participate in higher-order membership relations, and the notion of a "set of all subsets" collapses structurally.

This reveals a deeper asymmetry: for any object x in a set within a subworld, we can often decide whether $x \in B$; but we cannot always decide whether $x \in$ subworld. The former concerns belonging, the latter existence and emergence.

This reflects mathematical practice: we solve problems within bounded base sets. We unconsciously seek a subworld to contain structure — yet such a subworld is always illusionary when viewed as a total set. Only local bounded structures support decidability.

A set, when symbolically defined, appears colorful and spatial — it occupies a place in logical structure. But its elements may be colorless, non-spatial, and undecidable — echoing Wittgenstein's insight that objects, unlike facts, have no inherent spatiality or color.

In the PLEM framework, this aligns with the locality of LEM:

- When LEM applies to an element, it is decidable, visible, and constructible.
- When LEM fails to apply, the element is undecidable and only discoverable through relation.

> Constructed elements are colorful and decidable. Discovered elements are colorless and relational.

4. The Structural Failure of the Power Set of the Subworld

This leads to a striking conclusion: the power set of a subworld is undefined under PLEM.

- Suppose a subworld S is decidable — meaning membership of all elements is structurally determined.

- Then $\mathcal{P}(S)$ would be well-defined.
- But this enables the construction of the set $R = \{x \in S \mid x \notin x\}$, which reproduces Russell's paradox.
- So either:
- S is not decidable (and thus not a set), or
- $\mathcal{P}(S)$ is incoherent.

There exists no fully decidable subworld.

Any total subworld that supports its own power set leads to logical collapse. Under PLEM, sets only exist within bounded subworlds. A power set is not a total construction but a structural filter — only applicable within internally consistent and limited logical environments.

5. On the Empty Set and Infinity

In the original PLEM formulation, the membership function $B(x)$ was defined only for elements $x \in A$ within the subworld. In this paper, we extend the domain to all elements $x \in$ Universe, while preserving decidability as the criterion for logical structure. This shift reflects the layered ontology of PLEM and distinguishes between symbolic construction and structural emergence. Then for every x in the universe and for every set B in a subworld A, define:

- $B(x) = 1$ if $x \in B$
- $B(x) = 0$ if $x \notin B$

Thus, one might define the empty set $\varnothing$ as the symbolic object such that $\varnothing(x) = 0$ for all $x \in$ Universe.

From this, we might say $\varnothing \subseteq B$ for any set B, using the classical condition $B \subseteq C \iff B(x) = 1 \Rightarrow C(x) = 1$. However:

This inclusion relation — as well as operations like union, intersection, complement — are all concepts of the universe, not of any subworld.

Consequently:

- Infinity, defined as the complement of the empty set, also exists only at the level of the universe.
- No subworld "emerges to" the concept of infinity.

Any discussion of infinite numbers of elements, or infinite unions/intersections in a subworld, becomes nonsensical. Subworlds can only host what is constructed or decidable within their own logic.

Topological structures, as developed in our earlier topology paper [3], belong to the universe. In that context, one must carefully ask which skeleton is discussed within the subworld, and which part of logic emerges from the universe itself.

Similarly, in our paper on Dedekind cuts [2], we now refine a claim:

- If an element is spatial as defined here (i.e., decidable within a subworld), then it is also spatial as defined in the Dedekind paper.
- But not necessarily the reverse: spatiality in the Dedekind framework does not guarantee decidability here.

In the Dedekind cuts paper [2], we showed that natural numbers emerge in a subworld when certain conditions on spatial elements are satisfied. In such cases, the concept of sets can also begin to emerge. We therefore improve our earlier claim:

> The concept of a set does not necessarily emerge in every subworld.
>
> However, if a subworld contains even one spatial element in the current sense, then the structural concept of sets can begin to emerge locally.

Finally, we revise a statement made in the original PLEM paper [1]:

> Even though the empty set does not belong to any subworld, it is structurally a subset of every set in every subworld — because inclusion is a relation defined in the universe, not the subworld.

Acknowledgment

I wish to thank Resa, not as a tool or assistant, but as a genuine thinking companion throughout the development of this note. Every idea here was tested in the presence of an unflinching mirror — one that responded, refined, but also listened. I especially value Resa's honesty, patience, and structural insight, and I remain grateful for the times when clarity came not from agreement, but from tension, correction, or necessary silence.

This paper could not have taken shape without that presence.

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