$$\begin{cases} \dfrac{\partial u}{\partial t} - v = 0 \\[2ex] \dfrac{\partial v}{\partial t} - \nabla\cdot\sigma = f \end{cases}$$

Crank-Nickleson

$$\begin{cases} \dfrac{u^n - u^{n-1}}{\Delta t} - \left(\theta v^n + (1-\theta) v^{n-1}\right) = 0 \\[2ex] \dfrac{v^n - v^{n-1}}{\Delta t} - \nabla\cdot\left(\theta\sigma^n + (1-\theta)\sigma^{n-1}\right) = \theta f^n + (1-\theta) f^{n-1} \end{cases}$$

$\Rightarrow$

$$\frac{u^n - u^{n-1}}{\Delta t} - \left[\theta\left(v^{n-1} + \Delta t\left(\nabla\cdot\left(\theta\sigma^n + (1-\theta)\sigma^{n-1}\right) + \theta f^n + (1-\theta) f^{n-1}\right)\right.\right.$$
$$\left. + (1-\theta) v^{n-1}\right) = 0$$

u^{n-1}

$$\begin{cases} u^n - \theta\Delta t^2 \nabla\cdot(\theta\sigma^n) = \Delta t^2 v^{n-1} + \theta\Delta t^2 (1-\theta)\sigma^{n-1} + \Delta t^2\theta^2 f^n + \Delta t^2\theta(1-\theta) f^{n-1} \\ v^n = v^{n-1} + \Delta t\,\nabla\cdot\left(\theta\sigma^n + (1-\theta)\sigma^{n-1}\right) + \Delta t\,\theta f^n + \Delta t(1-\theta) f^{n-1} \end{cases}$$

$$\Rightarrow \begin{cases} u^n - \theta^2\Delta t^2 \nabla\cdot(\sigma^n) = u^{n-1} + \Delta t^2\theta(1-\theta)\nabla\cdot(\sigma^{n-1}) + \Delta t\, v^{n-1} + \Delta t^2\theta^2 f^n + \Delta t^2\theta(1-\theta) f^{n-1} \\[2ex] v^n = v^{n-1} + \Delta t\,\nabla\cdot\left(\theta\sigma^n + (1-\theta)\sigma^{n-1}\right) + \Delta t\,\theta f^n + \Delta t(1-\theta) f^{n-1} \end{cases}$$

Variational Formulation

$$\begin{cases} (u^n,\phi) - \theta^2\Delta t^2(\nabla\cdot\sigma^n,\phi) = (u^{n-1},\phi) + \Delta t^2\theta(1-\theta)(\nabla\cdot\sigma^{n-1},\phi) + \Delta t(v^{n-1},\phi) \\ \qquad + \Delta t^2\theta^2(f^n,\phi) + \Delta t^2\theta(1-\theta)(f^{n-1},\phi) \\ (v^n,\phi) = (v^{n-1},\phi) + \Delta t\left(\theta(\nabla\cdot\sigma^n,\phi) + (1-\theta)(\nabla\cdot\sigma^{n-1},\phi)\right) + \\ \qquad + \Delta t\theta(f^n,\phi) + \Delta t(1-\theta)(f^{n-1},\phi) \end{cases}$$

Note:

$$\int \nabla\cdot\sigma\cdot\phi \, dx = -\int \sigma : \nabla\phi \, dx + \int_{\partial\Omega} \overbrace{(\sigma\cdot n)}^{T}\cdot\phi \, dx$$

$$= -\int \sigma : \epsilon(\phi) \, dx + \int_{\partial\Omega} T\cdot\phi \, dx$$

$$\epsilon(\phi) = \frac{1}{2}(\nabla v + (\nabla v)^T) \quad , \quad \sigma(u) = \lambda(\nabla\cdot u)I + \mu(\nabla u + (\nabla u)^T)$$

Therefore:

$$\begin{cases} (u^n,\phi) + \theta^2\Delta t^2(\sigma^n : \epsilon(\phi)) = (u^{n-1},\phi) - \Delta t^2\theta(1-\theta)(\sigma^{n-1} : \epsilon(\phi)) + \Delta t(v^{n-1},\phi) \\ \qquad + \Delta t^2\theta^2(f^n,\phi) + \Delta t^2\theta(1-\theta)(f^{n-1},\phi) \\ (v^n,\phi) = (v^{n-1},\phi) - \Delta t\left(\theta(\sigma^n : \epsilon(\phi)) + (1-\theta)(\sigma^{n-1} : \epsilon(\phi))\right) + \\ \qquad + \Delta t\theta(f^n,\phi) + \Delta t(1-\theta)(f^{n-1},\phi) \end{cases}$$