Dear Prof. Simos,

I would like to thank you for the giving me opportunity to submit a revised manuscript. I am also grateful to the reviewers for their insightful comments. A detailed response to the reviewers' comments and concerns follows.

## Response Reviewer 1

I have taken the following steps to formalize the ideas presented in the paper,

- 1) The methodology section in the paper is now divided into two sections namely, *Mathematical Framework* (section 2) and *Computation Steps* (section 3) to improve the narrative.
- 2) Section 2.1 has been added to introduce some of the key concepts and definitions that are central to this work. While most of these definitions are standard, presenting them will help with establishing the notation used in the paper.
- 3) Section 2.3 has been added to discuss the existence and uniqueness of a solution to the geodesic equations.
- 4) Section 3.5 has been added to discuss the existence and uniqueness of a solution to the adjoint equations.

## Response Reviewer 2

The issue has been fixed. The correct definition is  $x^{\cdot \wedge}m = (dx^{\wedge}m)/dt$ .

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