Violations of Linear Regression Classic Assumptions and Their Remedies

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Today's plan

- Reviewing the Gauss-Markov Assumptions (Classic OLS assumptions)
- OLS diagnostics
- Omitted variable problem
- Serial correlation vs. auto correlation
- Clustering and robust standard errors
- Does Multicollinearity violate the classic assumptions?

The Gauss-Markov Assumptions

1. **Linearity assumption:** $y = \beta_0 + \beta_1 x + \epsilon$
2. X **is a full rank matrix.**
3. $E(\epsilon|X) = 0$
4. $E(\epsilon\epsilon'|X) = \sigma^2 I$
5. X **and** ϵ **are orthogonal** $X \perp \epsilon$
6. $\epsilon|X \sim N(0, \sigma^2 I)$

Th assumption 6 is not actually required for the Gauss-Markov Theorem. However, we often assume it to make hypothesis testing easier.

Therefore, if assumptions 1-5 are violated, they mess up OLS *BLUEness*:

$$\hat{\beta} = (X^T X)^{-1} X^T y = \beta + (X^T X)^{-1} X\epsilon \tag{1}$$

OLS diagnostics

- How can we test if our estimated model does not violate the OLS assumptions?
- $\mathcal{R}$ offer nice visualization tools to check the possibility of OLS assumptions violations.

Let's get back to our Acemoglu et al. (2001) example:

```
## MODEL INFO:
## Observations: 57
## Dependent Variable: logpgp95
## Type: OLS linear regression
##
## MODEL FIT:
## F(1,55) = 72.32, p = 0.00
## R² = 0.57
## Adj. R² = 0.56
##
## Standard errors: OLS
## ------------------------------------------------
##                      Est.   S.E.   t val.      p
## ---------------- ------ ------ -------- ------
## (Intercept)          4.89   0.38    12.79   0.00
## avexpr               0.49   0.06     8.50   0.00
## ------------------------------------------------
```

Fitted values and residuals

- We need fitted values and residuals values of our estimated model to diagnose possible issues: $e = y - \hat{y}$.
- There are different methods to do this; here I use `broom` package.

```
library(broom)

OLS1_diag_metrics <- augment(OLS1_results)
```

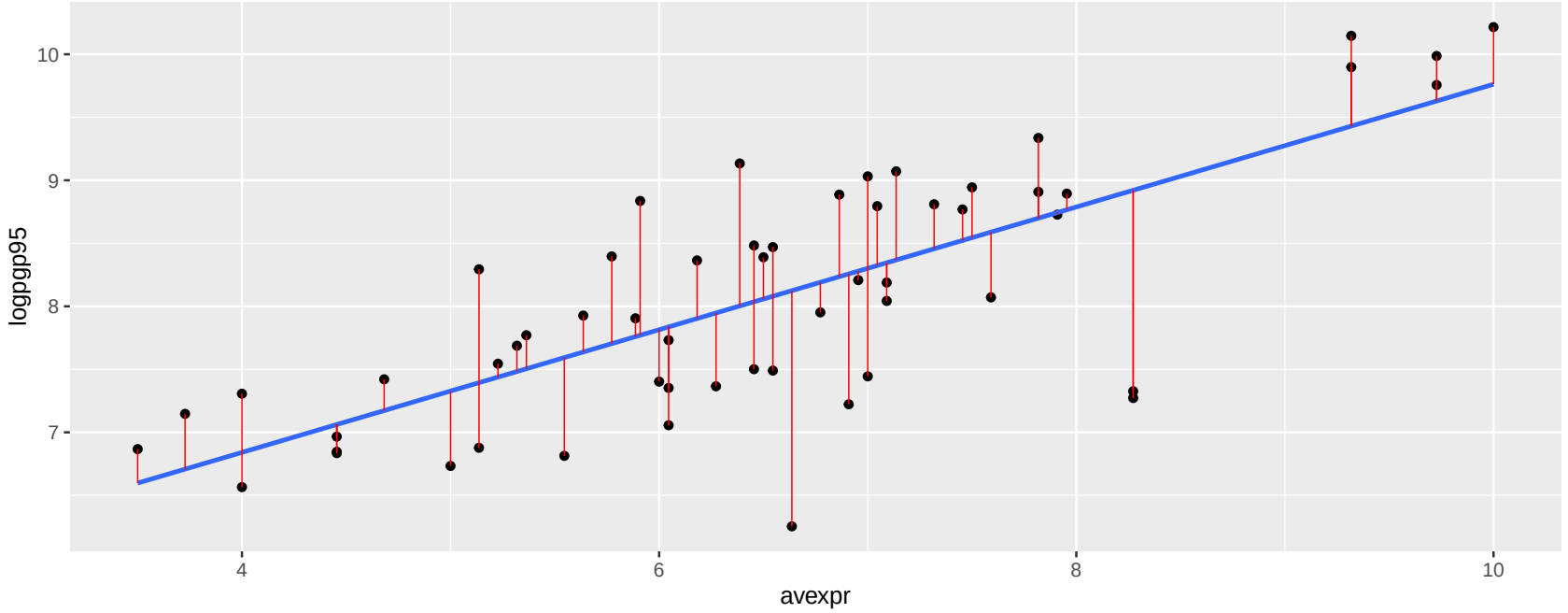
Diagonstics metrics

```
head(OLS1_diag_metrics)
```

```
## # A tibble: 6 x 10
##   .rownames logpgp95 avexpr .fitted .se.fit .resid   .hat .sigma .cooksd
##   <chr>        <dbl>  <dbl>   <dbl>   <dbl>  <dbl>  <dbl>  <dbl>   <dbl>
## 1 372           6.87   3.5     6.60   0.193  0.270 0.0882  0.653 0.00916
## 2 219           7.15   3.73    6.71   0.181  0.439 0.0780  0.652 0.0210
## 3 271           6.57   4       6.84   0.168 -0.276 0.0667  0.653 0.00691
## 4 323           7.31   4       6.84   0.168  0.466 0.0667  0.651 0.0197
## 5 141           6.85   4.45    7.06   0.146 -0.216 0.0505  0.654 0.00311
## 6 264           6.84   4.45    7.06   0.146 -0.227 0.0505  0.654 0.00343
## # ... with 1 more variable: .std.resid <dbl>
```

Marking errors in your scatter-fit plot

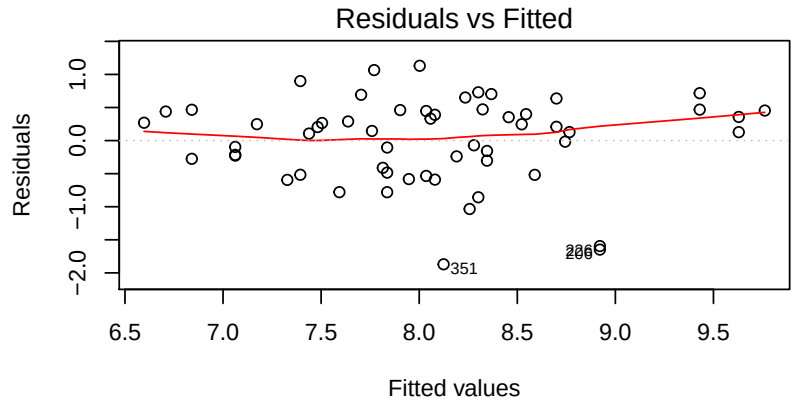
```
library(ggplot2)
ggplot(OLS1_diag_metrics, aes(avexpr, logpgp95)) +
  geom_point() +
  stat_smooth(method = lm, se = FALSE) +
  geom_segment(aes(xend = avexpr, yend = .fitted), color = "red", size = 0.3)
```

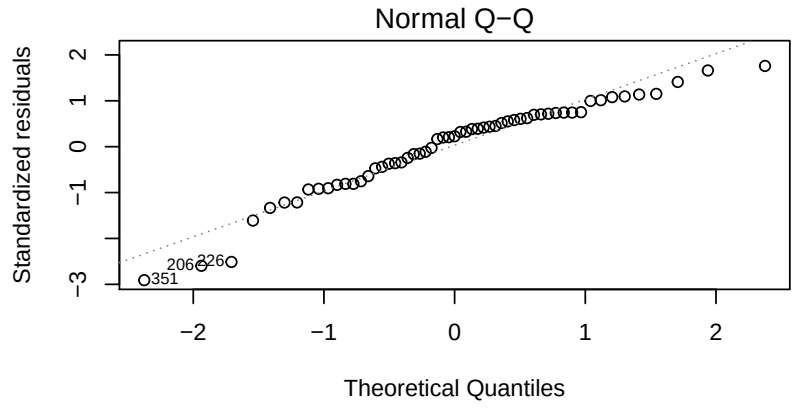


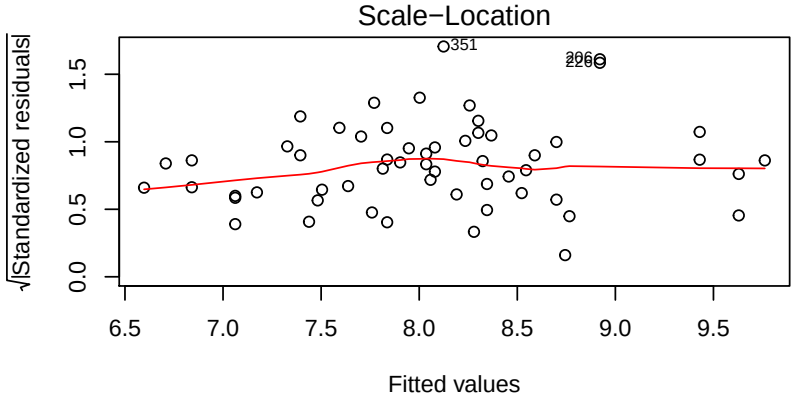
- If you switch to a multivariate regression model, then the above plot would be a little bit different because OLS should fit the data across more than two variables!

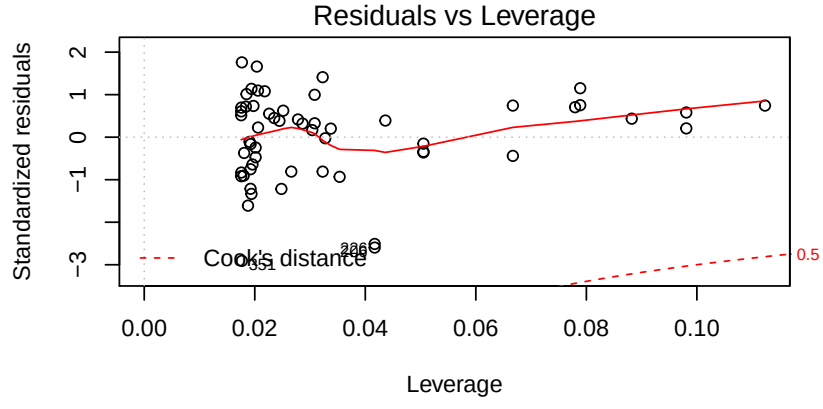
Regression diagnostics plots

```
par(mfrow = c(2, 2))

plot(OLS1_results)
```









Regression diagnostics plots (2)

1. Residuals vs Fitted: Checking the linear relationship assumptions. The trend line should be close to a horizontal line to show a linear relationship.

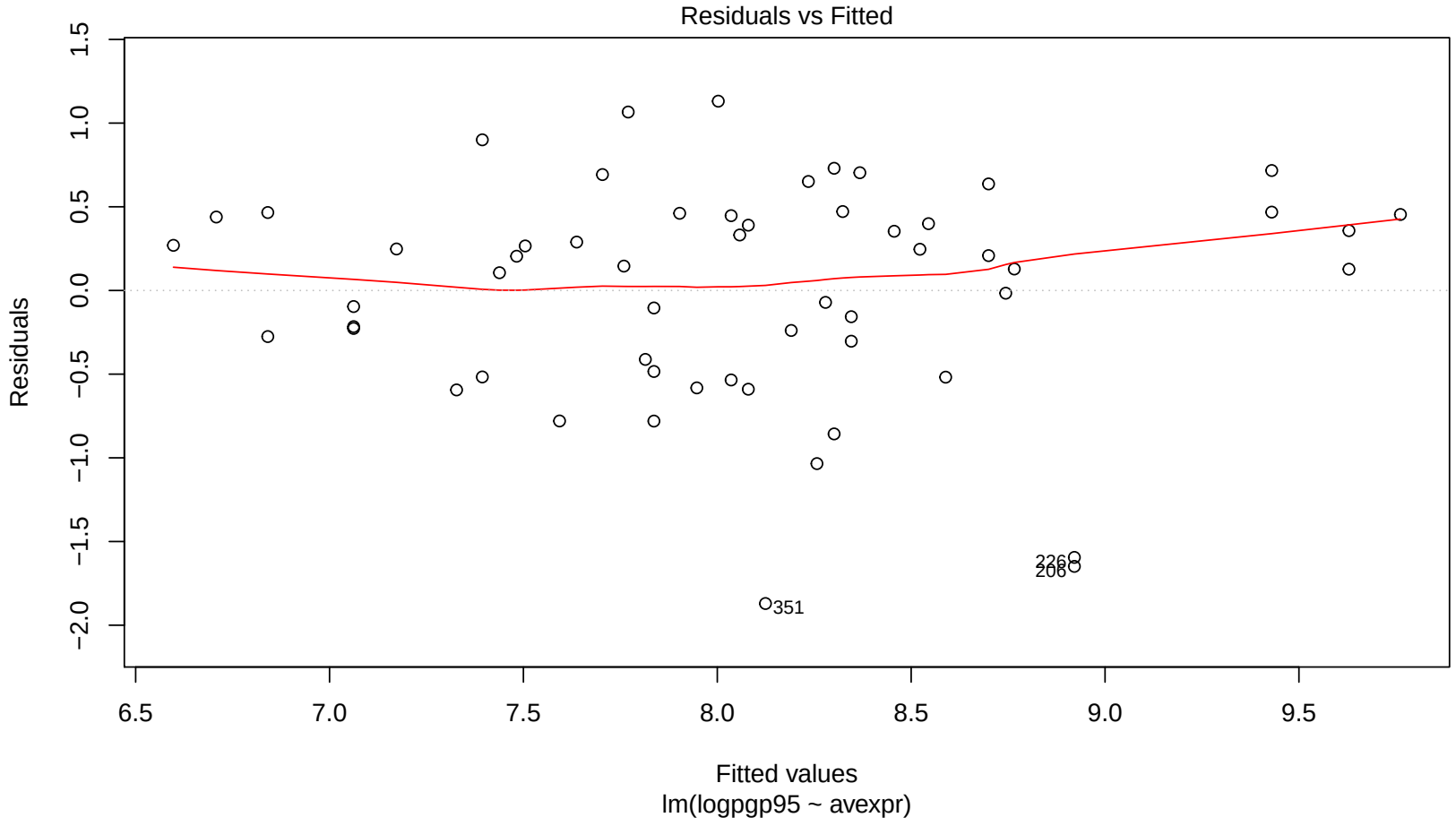
2.Normal Q-Q: Checking if the residuals are normally distributed. The trend of residuals points around the 45°(dashed) line is a good sign.

3.Scale-Location (or Spread-Location): Checking the homogeneity of variance of the residuals (homoscedasticity). Equal spread of the points aroudn a horizontal line is a good indication of homoscedasticity. In our example, there is a small sign of heteroskedasticity.

4.Residuals vs Leverage: Checkig if there are extreme values (outliers) that might influence the regression results.

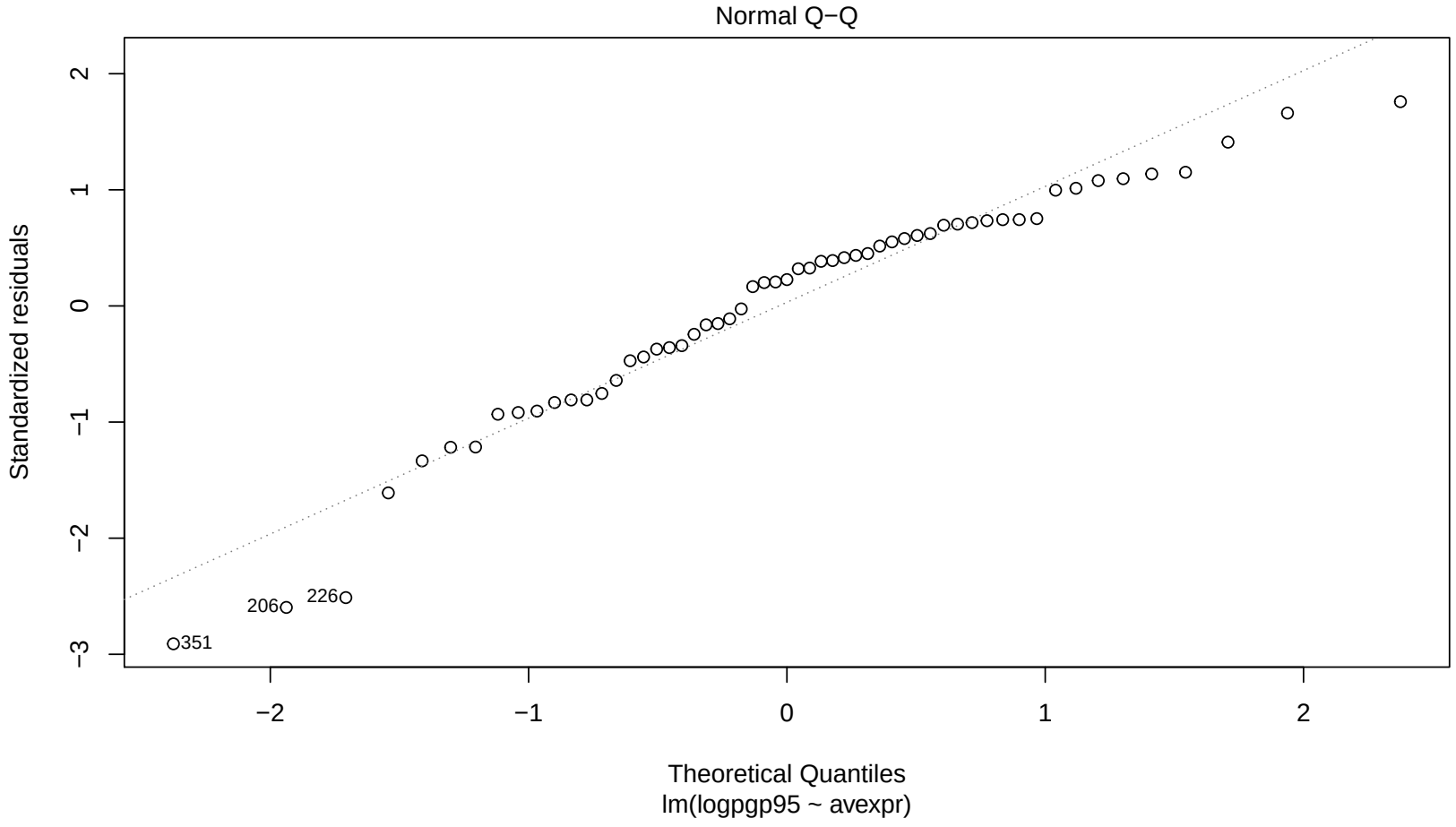
Regression diagnostics individual plots

```
plot(OLS1_results, 1)
```



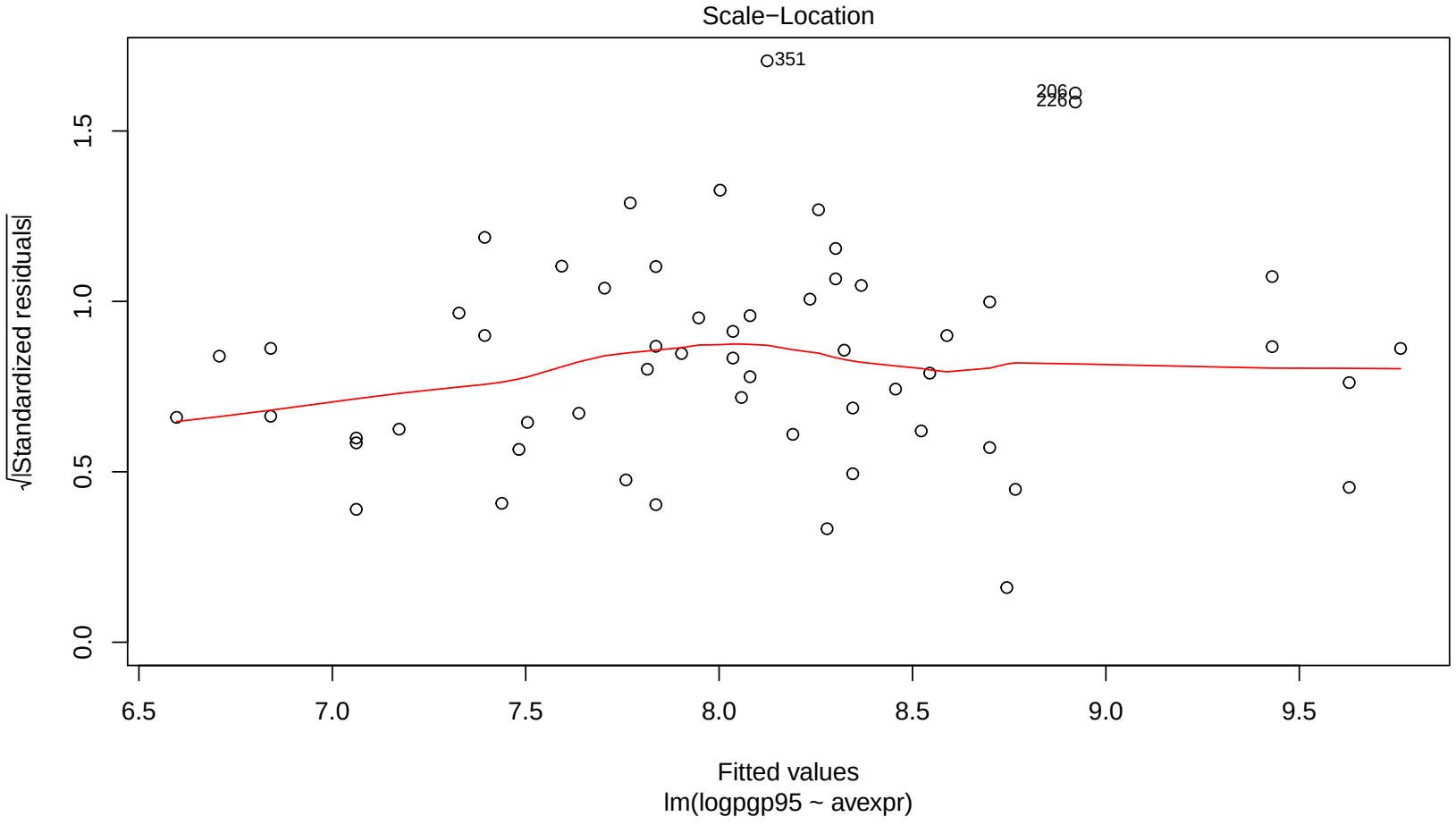
Regression diagnostics individual plots

```
plot(OLS1_results, 2)
```



Regression diagnostics individual plots

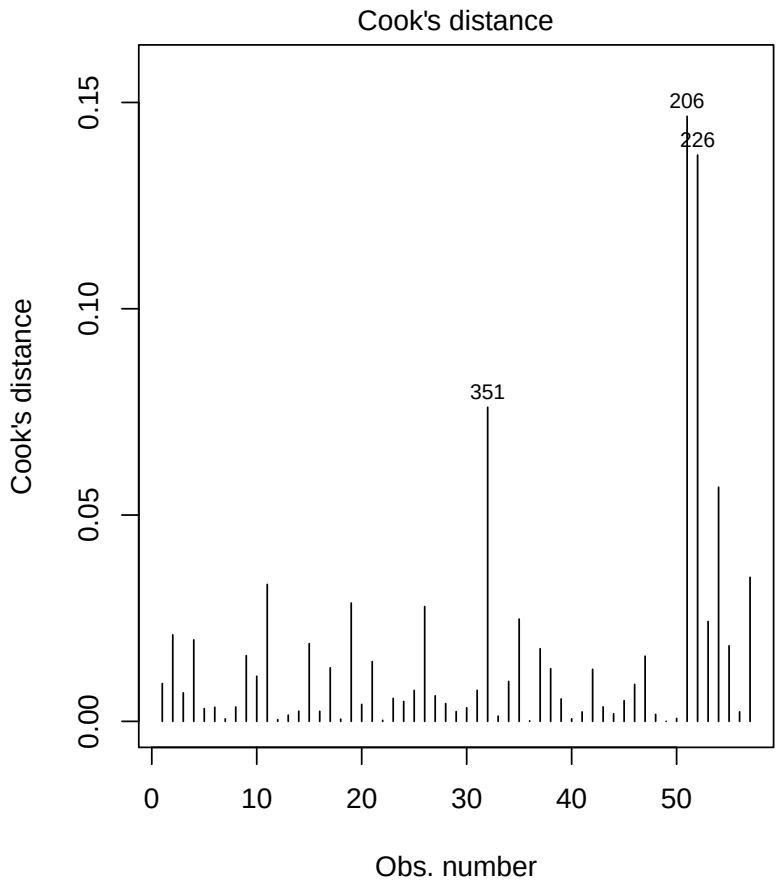
```
plot(OLS1_results, 3)
```

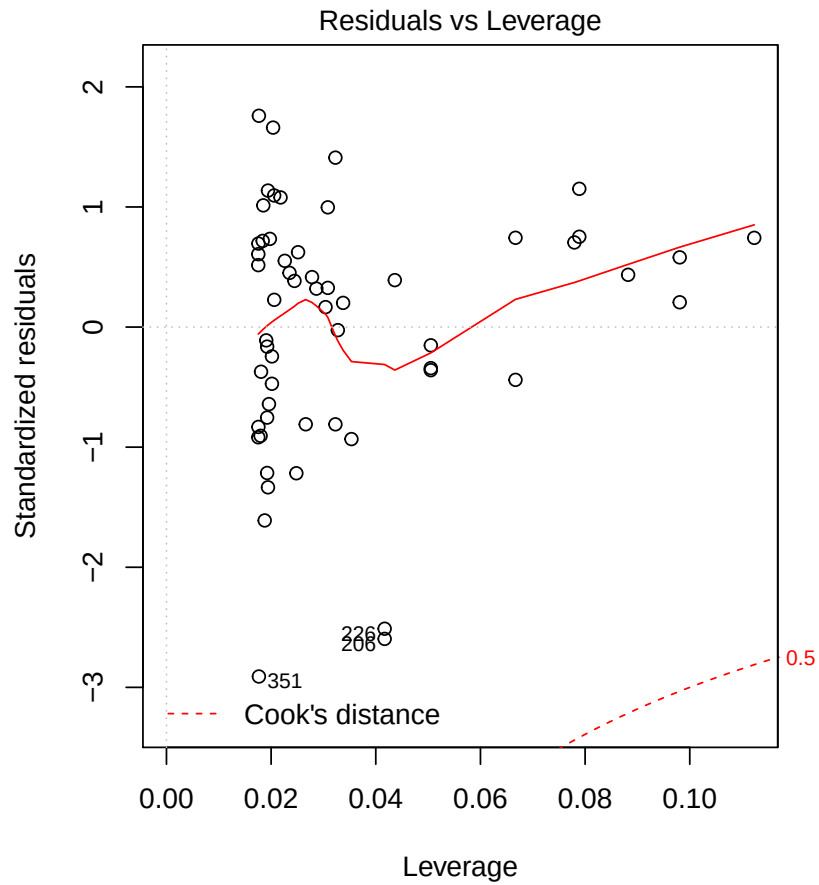


Identifying outliers/influencial variables

There variables are called influencial because adding or removing to the sample can change the results substantially. To measure the influence of observatiaons, we can use the *Cook's distance*.

```
par(mfrow = c(1, 2))
plot(OLS1_results, 4)
plot(OLS1_results, 5)
```



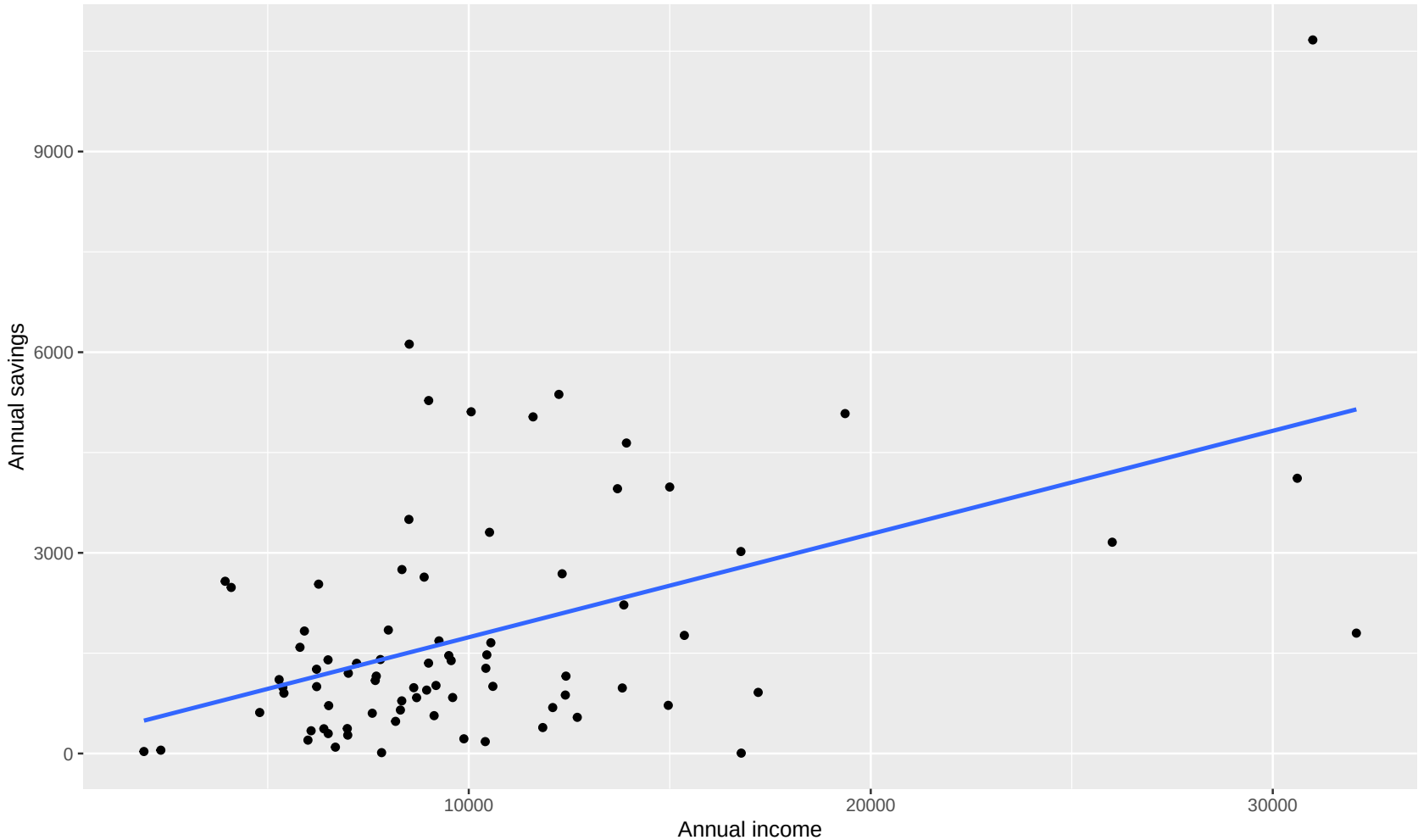


Robust and clustered standard errors

- *heteroskedasticity* does not lead to a biased coefficient, but it can lead to a biased estimation of the variance-covariance matrix.
- This can lead to incorrect t-statitics and confidence intervals. That is rejecting null hypothesis and finding support for our theory incorrectly.
- We learned how to identify this issue using *Scale-Location (or Spread-Location)*, but how can we solve the problem?
- Serial correlation and auto correlation are among the main causes of heterogenous.
- Often, estimating robust and clustered standard errors can help to ease this problem.

A new example: the association between income and saving

$$Saving_i = \beta_0 + \beta_1 Income_i \tag{2}$$



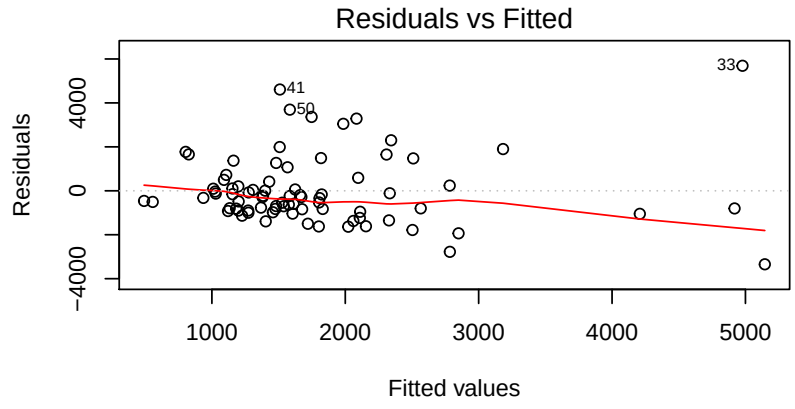
Estimation: non-robust

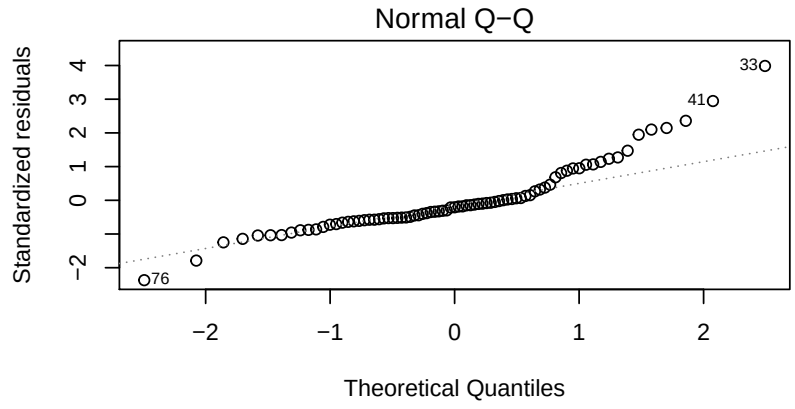
```r
model1 <- lm(sav ~ inc, data = saving)

summary(model1)
```

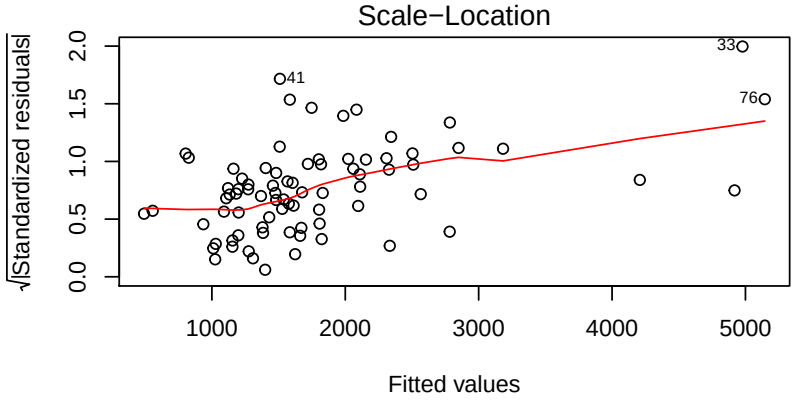
```
##
## Call:
## lm(formula = sav ~ inc, data = saving)
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -3345   -900   -323    457   5690
##
## Coefficients:
##             Estimate Std. Error t value  Pr(>|t|)
## (Intercept) 195.1883   366.6416    0.53       0.6
## inc           0.1543     0.0312    4.95 0.0000042 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1580 on 77 degrees of freedom
## Multiple R-squared:  0.242,  Adjusted R-squared:  0.232
## F-statistic: 24.5 on 1 and 77 DF,  p-value: 0.00000424
```

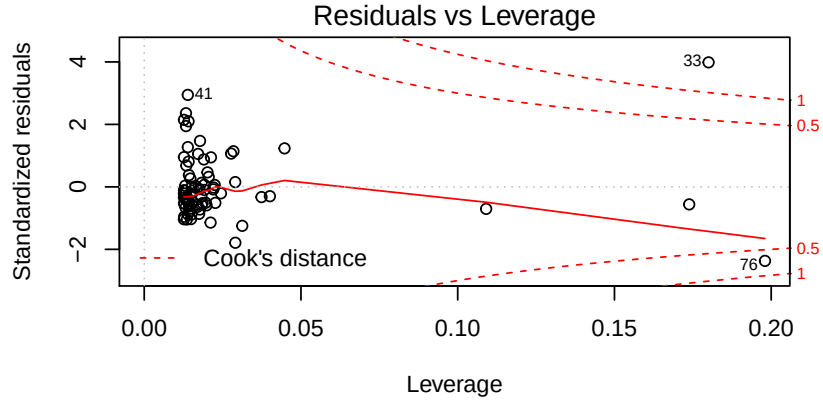
diagnostics plots

```
par(mfrow = c(2, 2))

plot(model1)
```









Estimation: robust

- As always there are different ways to cluster standard errors; I here use `lmtest` and `sandwich` package.

```
library(lmtest)
library(sandwich)

coeftest(model1, vcov = sandwich)
```

```
##
## t test of coefficients:
##
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 195.1883   524.2427    0.37   0.7107
## inc           0.1543     0.0572    2.70   0.0086 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

```
coeftest(model1, vcov = vcovHC(model1, type = "HC0"))
```

```
##
```

Cluster standard errors

- Again different ways possible; I here use `multiwayvcov` package.

```
# FE regression with SE clustered by firm
library(multiwayvcov)
model2<-miceadds::lm.cluster(sav ~ inc, data = saving,
                             cluster="size")
model2
```

```
## $lm_res
##
## Call:
## stats::lm(formula = formula, data = data, weights = wgt__)
##
## Coefficients:
## (Intercept)          inc
##     195.188        0.154
##
##
## $vcov
##             (Intercept)       inc
## (Intercept)     96074.7 -12.37845
## inc               -12.4   0.00207
##
## attr(,"class")
## [1] "lm.cluster"
```

Omitted variable problem

$$y = \beta_0 + \beta_1 x + \epsilon \tag{3}$$

- When we estimate a regression model, how do we know that what variables should be included in the model?
- And, what is the potential problem caused by omitting an important variable?
- Let's check our Acemoglu et al. (2001) example again:

Model 1:

$$logGDP = \beta_0 + \beta_1 Expropriation + \epsilon \tag{4}$$

Model 2:

$$logGDP = \beta_0 + \beta_1 Expropriation + \beta_2 Democracy1900 + \epsilon \tag{5}$$

Results of Model 1

```
library(jtools)
# OLS model
OLS1_results <- lm(logpgp95~avexpr, data=myData)

summ(OLS1_results)
```

```
## MODEL INFO:
## Observations: 57
## Dependent Variable: logpgp95
## Type: OLS linear regression
##
## MODEL FIT:
## F(1,55) = 72.32, p = 0.00
## R² = 0.57
## Adj. R² = 0.56
##
## Standard errors: OLS
## ------------------------------------------------
##                      Est.   S.E.   t val.      p
## ----------------- ------ ------ -------- ------
## (Intercept)          4.89   0.38    12.79   0.00
## avexpr               0.49   0.06     8.50   0.00
## ------------------------------------------------
```

Results of Model 2

```
## MODEL INFO:
## Observations: 57
## Dependent Variable: logpgp95
## Type: OLS linear regression
##
## MODEL FIT:
## F(2,54) = 44.65, p = 0.00
## R² = 0.62
## Adj. R² = 0.61
##
## Standard errors: OLS
## ------------------------------------------------
##                      Est.   S.E.   t val.      p
## ----------------- ------ ------ -------- ------
## (Intercept)          5.33   0.39    13.58   0.00
## avexpr               0.40   0.06     6.37   0.00
## democ00a             0.09   0.03     2.81   0.01
## ------------------------------------------------
```

Confounding effect

- A situation in which the effect or association between an independent variable and outcome is distorted by the presence of another variable.
- Assume that we estimated $y = \beta_0 + \beta_1 x$. There is a variable z that is argued to be added to this model. Not including z in the model can cause a problem, only this *omitted variable* causes a confounding bias.
- An omitted variable problem can lead to a confounding bias if:

1. x is correlated with the omitted variable z.
2. The omitted variable has a causal association with the dependent variable Y.

Confounding effect and biased estimation

- If z is correlated with y, but not included in the model, its effect will show up as part of the residuals. Therefore, residuals will be associated with x (because z is correlated with x).
- This is a violation of 3^{rd} classic assumption: $E(\epsilon|X) = 0$, leading to a biased estimation.

Solutions to omitted variable problem

- This is one of the most challenging problems to address
- Depends on whether the omitted variable is time-variant or invariant
- Adding the lag of dependent variable
- Adding fixed-effects and time effects