A Correctness Proof of a Topology Information Maintenance Protocol for a Distributed Computer Network

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Assumptions

- bi-directional links
- all nodes are similar
- no requirements for relative speeds of nodes or transmission rates of links
- unit link cost

Data Structures (for each node)

- Distance Table (DT)
  - column for every neighbor of a node
  - row for each node in the network
  - entries are distances to destination node (row label) via a node's neighbor.
- Route Table (RT)
  - entry for every node in the network
  - entry has (n,Y) : shortest distance from this node to a destination node (row label) is n via neighbor Y.

Specification of Netchange Protocol

[TID OID , SD] where TID is This-node ID (Sender), OID is Other-node ID and SD is the shortest distance between TID and OID.

Node B carrying out the algorithm

Node C is a neighbor of node B.

Node Y is some other node (not B nor C)

NN is the total number of nodes in the network (include nodes that are down).

DT: entry (y,bc) is Dy,bc, which stands for distance from node b to node y via c.

RT: Sp,Np where Sp is the shortest distance via Np

Events at node B

- Link BC comes up or adjacent node C comes up.
- Adjacent link BC goes down or neighbor goes down.
- A message [CY,D] is received (from neighbor).

Algorithm 1 (A link comes up)

- Entry (c,bc) in DT set to 1
- Route table entry set to 1,c
- Send message [bc,1] to all neighbors of B
- If entries in B's RT are numbered p,q,....,t. send messages [bp, Sp], [bq,Sq], ...,[bt, St] to node C.

- Note that the number NN is used to indicate the lack of a path, since the longest route without loops in a network of NN nodes can be no longer than NN-1 links. When a link goes down, the corresponding columns in DT of the two adjacent nodes are filled with NNs.

Algorithm 2 (A link goes down)

- All entries in B's DT are set to NN in column bc.
- For each row, p, of DT and RT do

  Compute the minimum of Dp,bf ; ...., Dp,bg ; .....;Dp,bk . Let this minimum Dp,bg

  If (Dp,bg = Sp)

  then Np is set to g

  else {Sp is set Dp,bg; Np is set to g and send messages [bp,Sp] to all neighbors}.

Algorithm 3 (A message [CY,D] is received by node B)

If Y=B, message is ignored by node B; else

- Dy,bc is set to min[D+1,NN]
- Compute the minimum of Dy,bf ; ....; Dy,bg ;.....;Dy,bk Let the minimum be Dy,bg.
- If (Dy,bg = Sy),
  - then Ny is set to *g*;
  - else { Sy is set to Dy,bg ; Ny is set to *g* and send [by,Sy] to all neighbors}.

Theorems

Theorem 1: If and when all message activity has ceased (no messages on queues, or in the process of being transmitted), all entries in all Distance Tables (DT) are correct.

Corollary 1: If and when all message activity has ceased, all entries in all Route Table (RT) will be correct.

Theorems (Cont)

Theorem 2: If a node b receives a message cb(i) with $SD[cb(i)]=d$, then if a message bx(i) is sent out by b as a result of cb(i) either $SD[bx(i)] = d+1$ and/or $SD[bx(i)] > SD[bx(i-1)]$.

Corollary 2: If a message bx(i) is sent out by node b as a result of receiving a message cb(i), then: If $SD[bx(i)]<SD[bx(i-1)]$ then $SD[bx(i)]=SD[cb(i)]+1$

Theorem 3: Consider a network in which an arbitrary series of topology changes occur between time 0 and t ; no changes occur after time t. The network will generate only a finite number of messages with $SD=1$.

Theorem (Cont)

Theorem 4: Consider a network in which an arbitrary but finite series of topology changes occur between time 0 and t ; no changes occur after time t. Then a finite time after t, all message activity will cease and all the entries in all the Distance Tables (DT) and Route Tables (RT) in all the nodes will be correct.