Discrete Wavelet Transform

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INTRODUCTION

A wavelet is any signal that moves to and fro around the amplitude zero such that oscillations are produced. Sound, earthquake frequencies and video signals are its real-world examples. When the study was going on seismology, it was a need to provide time dilation for seismic signals hence it was necessary to have a tool that can deal with time variance data [1]. Back then Fourier analysis was only tool to study signals but it lacked time dimension i-e it is very much ideal if we are having study of stationary data whose statistical properties are not changing over time. This tool is not well suited for variating statistical data over time that is which does not hold the property of transientness or nonstationary data. For such events that can't be statistically predicted form data sparse, wavelet analysis comes into play.

A wavelet is a rapidly decaying oscillation that has zero mean. Dissimilar to sinusoids, which reach out to endlessness, wavelets are characterized by having what's called compact support which just means that the signal exists for a limited time. One more attribute of wavelet is that the region under the curve should be zero, such that the energy is equally distributed in a positive and negative directions. Wavelets come in various sizes and shapes. A wavelet transform divides a signal or wavelet into number of basis functions such that different scale components are obtained. For the wavelet transform, we multiply the signal by a wavelet analyzing function, it transforms the signal which is a function of time and outputs a 2 by 2 matrix of coefficients which are identified by their scale and translation. This results in a three-dimensional plot of amplitude as a function of scale and translation. The equation is given below.

$$X(a,b) = \int_{-\infty}^{\infty} x(t)\, \psi^{*}{}_{a,b}(t)dt$$

Where $x(t)$ is signal or function of time, $\psi^{*}{}_{a,b}(t)$ is wavelet analyzing function while a and b are scale and translation coefficients respectively. Translation is simply shifting the wavelet forward in time as the entire signal is analyzed, shifting a wavelet essentially implies delaying or propelling the beginning of the wavelet along the length of the signal. wavelet needs to be moved to line up with the features we are looking for in a signal. Scaling factor is appositive value and shows how much the signal is scaled through time. It is inversely proportional to frequency, where a higher scale normally refers to a wavelet that is stretched out while a low scale refers to a more compressed wavelet which in turn means a high frequency wavelet, thus we can conclude that an extended wavelet helps in capturing the gradually differing changes in a signal while a compacted wavelet helps in catching abrupt changes. Wavelets are used to tackle the problem of frequency and time domain resolution [2]. Wavelet transform works in such a way that by taking a random signal in time we can take wavelet analyzing function and translate the wavelet in time so that it encompasses the entire signal, multiplying each point of signal in the region with a point on the wavelet; wavelet can be scaled after doing this such that it becomes wider and once again it would be translated throughout the entire signal multiplying at each point. Wavelets provide a way for analyzing waveforms in both frequency and time duration and functions having discontinuities and sharp peaks can also be represented (good use case in image edge filtering). It can also accurately construct or deconstruct finite, non-periodic and/or non-stationary signals which is main advantage in comparison to Fourier transform and it also allows signals to be stored more efficiently than by fourier transform. Generally, wavelets can be applied for many different purposes such as audio compression, speech recognition, image and video compression, signal denoising and motion detecting and tracking. Wavelet transform is divided into two types based on how it treats scaling and shifting known as Continuous wavelet transform and discrete wavelet transform. Continuous wavelet transform decomposes a signal or wavelet into non-orthogonal set of wavelets which means data is highly correlated whereas discrete wavelet transform decomposes a signal into orthogonal set of wavelets to the scaling and shifting [3].

A discrete wavelet transform (DWT) is any wavelet transform that decomposes a digital signal into set of wavelets such that they are discretely sampled. One of main characteristics of DWT is temporal resolution that is it not only captures the frequency of input signal but also the temporal information, i.e the times at which these frequencies occur. A digital signal is decomposed into different subbands using DWT in such a way that the lower frequency subbands have relatively infrequent frequency resolution and coarser time resolution compared to the higher frequency subbands. It uses shift and scaling versions of original signal where scaling is used for frequency domain analysis and operations while shifting is used for time domain analysis. For image processing Discrete wavelet transform can be used to decompose any signal or image, it gives an incredibly adaptable multi-resolution image and can decompose an original picture into various subband images including low-and high-frequencies. Accordingly specific subband image or specific resolution data can be picked based upon application and need. (Fig 1) shows transformation of an image where G and H are high and low pass filters respectively [4]. First, they'll be applied along the rows of an image and next both G and H will be applied along the columns of processed image after first step which will obtain four subband regions with high, low and intermediate frequency and written as HH (for high frequency), LL (for low frequency) and HL & LH (for intermediate frequencies where mainly HL preserves vertical while LH preserves horizontal features)

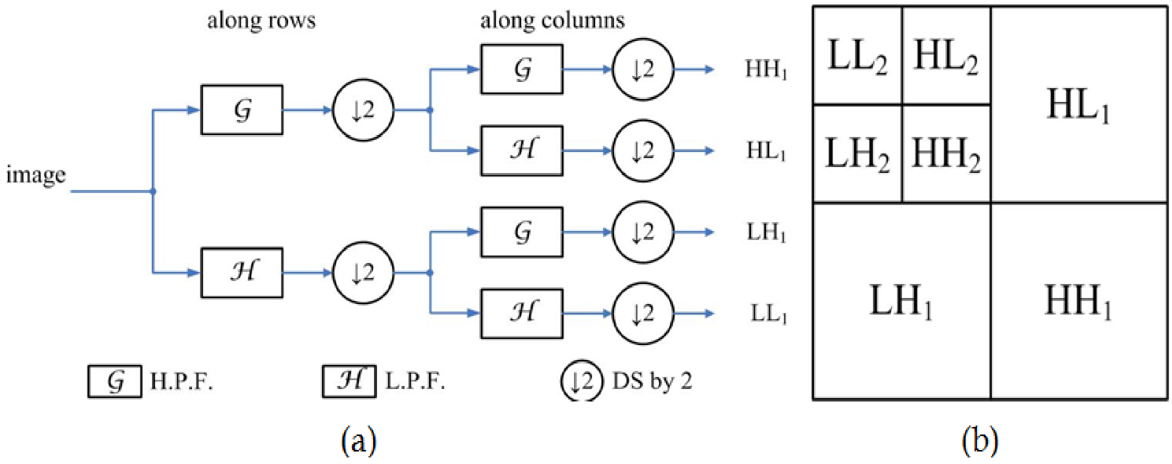

Fig 1

The input signal which is getting processed by discrete wavelet transform is a two-dimensional signal (can also be visualized as image). 1st, 2nd and 3rd level decomposition were carried out with the help of wavelet tool. What is happening is that in 1 level decomposition, whatever the information that can be visualized in the input image, all the visualization details are taken into first or top left cell, any horizontal changes or variations present in image were taken into top right cell, vertical changes in bottom left cell while diagonal changes were taken in bottom right or fourth cell. After performing 1 level decomposition, 2 level decomposition is performed on 1st cell of 1 level decomposition by dividing it into four cells that further contain any visualizing effects, horizontal variations, vertical changes and any diagonal variations respectively. Similarly, 3 level decomposition is carried out on first or top right cell such that it is further divided into four more cells in 3 level decomposition which contains any visualized information, horizontal variation, vertical variation and diagonal variations respectively as shown in (Fig 2). Thus, we can have wavelet coefficients at first, second or third level and so on up to n levels which can be used in performing certain operations or analyze them. Image can also be inverse or reverse transformed if these decomposition coefficients are known.

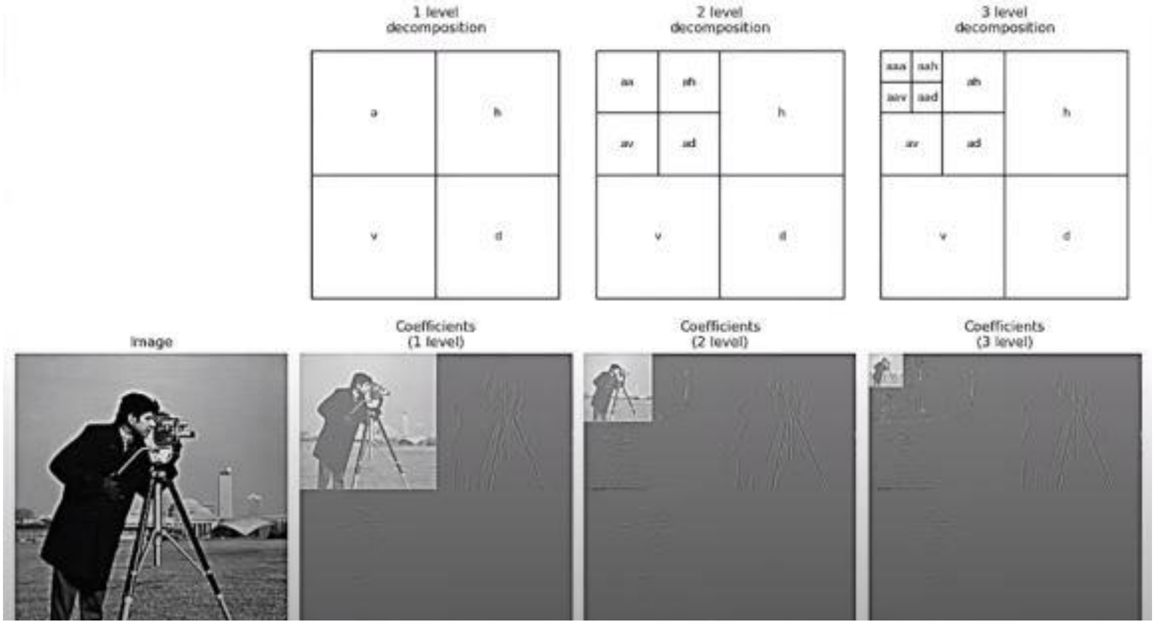

Fig 2

As problem set increased, many enhancements in DWT were made which includes Haar wavelets, Daubechies wavelets and DCWT (dual-tree complex wavelet transform). The discrete wavelet transform has a huge number of applications in scientific field such as numerical analysis, signal coding or data compression, denoising of signal, medical research, earthquake detection, image processing etc.

Literature Review

The Discrete Wavelet Transform (DWT) is used to analyze the temporal and spectral properties of non-stationary signals thus its applications can include decomposition, denoising of signal, analyzing earthquake waves, audio signals, watermarking of digital images etc. Because of the adaptability of wavelet changes for a variety of purposes, various sorts of wavelet transforms are created. There exists a wide range of groups of wavelets. These wavelet families are characterized by their particular filter coefficients which are promptly accessible for the circumstances such as the Daubechies wavelets, Coiflets, Symlets and the Meyer and Haar wavelets. One essential issue to address is concluding which set (or group) of filter coefficients will produce the best outcomes for a specific application. It is feasible to trail various arrangements of filter coefficients and continue with the group of filter coefficients which produces the best outcomes [5]. The Haar wavelet is sequence of square shaped rescale waves which form a wavelet, where the Haar transform cross-multiplies a function against the Haar wavelet with different stretches and shifts but this wavelet is not continuous hence not differentiable [6] [7]. The Haar wavelet is of restricted adequacy in a DWT. It clearly functions well for signals involving transcendently square waves at the same time, doesn't give clear information when handling broader consonant or harmonic waves. Daubechies designed what are called compactly supported orthonormal wavelets and their shape has made discrete wavelet analysis practicable and appealing. The Daubechies wavelets cannot be expressed in a closed form [8]. When Haar and Daubechies wavelets using FPGA technology were implemented to analyze audio signals (as seen in Figure 3 and Figure 4), Daubechies wavelet technique gave lesser bit per error when reconstructing output signal than Haar wavelet [9].

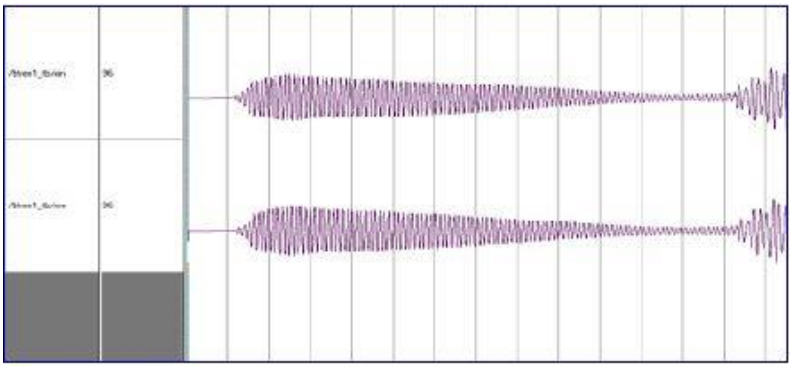

Figure 3: Error rate using Daubechies wavelet

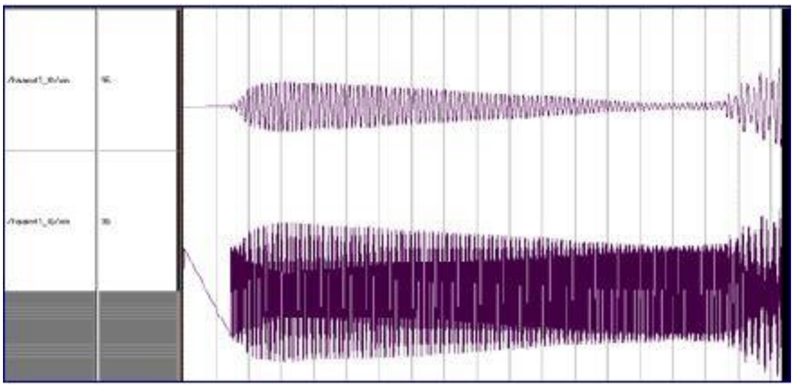

Figure 4: Error rate using Haar wavelet

One of the most important applications of discrete wavelet transform is signal denoising; in cardiac pacemakers, selective noise filtering can be done [10], ECG denoising is very critical and important to detect any abnormality and dwt with its different forms like double-density, dual-tree and double-density dual-tree discrete wavelet transform were used to test the denoising of signal [11], A nonlinear noise reduction method which uses shift-invariant and nonorthogonal wavelet transform is discussed in [12] but computation and storage cost increased, behavior of redundant Discrete Wavelet transform is analyzed under additive noise as most of the signals contain it [13]. Ultra-wideband radio technology has made good progress using discrete wavelet transform, which ensures data throughput with low-power utilization and immunity against fading and electromagnetic obstruction [10]. Digital image watermarking is becoming an important application of discrete wavelet transform where because of its spatio-frequency localization properties, area where watermark should be embedded but computational cost is really high [14]. DWT's probably biggest application is Audio analysis; where analysis of music and audio through wavelets is done in [15], while [16] explores the use of DWT in beats extraction from music and nonspeech audio data classification using statistical pattern recognition. [2] discusses the signal coding and compressing techniques through subband decomposition and attentions needed to improve the signal decomposition problems. Image coding using dual-tree discrete wavelet transform is explored where noise shaping, matching pursuit and basis pursuit were used as coefficients out of which noise shaping performed the best [17]. Vibration analysis for detecting ball bearing race faults by decomposing the wavelet using discrete wavelet transformation is shown in [18]. As discrete wavelet transform deals with decomposing a signal into different frequency subbands and can also reconstruct it through inverse dwt, hence it needs proper architecture to do computations on, [19] proposes architectures for computing dwt in VLSI. Similarly, A hardware signal processing unit for 1-d discrete wavelet transform is proposed such that a low-cost and general-purpose dwt core for embedded applications can be developed [20]. Signal or Image decomposition based on multiresolution decomposition is an important application of dwt due to which user can choose intended resolution that needs to be picked [21] [22]. Detection of high impedance fault using discrete wavelet transform combined with support vector machine in power distribution networks is explored in [23], where dwt is used for feature extraction and SVM is used to train on those features, similarly same combination is used in [24] to locate faults in distribution grid. Brain magnetic resonance image needs to follow region-based coding where selective parts of image needs to be encoded at different bit rates, [25] evaluates performance of dwt variant called Shape adaptive discrete wavelet transform (SA-DWT) on brains magnetic resonance images. [26] Introduces a new framework for complex wavelet transforms which overcomes the poor directionality, shift sensitivity, and lack of phase information disadvantages of traditional discrete wavelet transform. Object detection in video surveillance systems can become difficult due to problems like fake motion, background noise, and night detection, [27] proposes a dwt based system to detect objects in video that deals with these problems.

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