

# CORE-SATELLITE PORTFOLIO DESIGN UNDER RARE CHANCE CONSTRAINTS: An Application to an AI-Focused Portfolio

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December 29, 2025

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## Abstract

This paper studies how a core–satellite portfolio architecture can be used to control extreme tail risk under rare chance constraints. Focusing on a thematic satellite representing the artificial intelligence ecosystem, we analyze how increasing satellite exposure affects the feasibility and stability of tail-risk constraints applied exclusively to the core portfolio.

Using a Gaussian reference model combined with first-order large deviations approximations and Monte Carlo simulation, we characterize the evolution of rare-event probabilities, tail severity (VaR and CVaR), and dominant-point geometry as a function of the satellite weight. Rather than optimizing the satellite allocation endogenously, we treat it as a policy-controlled exposure parameter, allowing us to identify distinct risk regimes.

Our results reveal a regime-dependent structure of tail risk. While probability- and CVaR-based constraints identify admissible exposure thresholds, large-deviation diagnostics provide early warning signals of structural fragility, capturing sharp changes in tail geometry before conventional constraints become binding. Sensitivity analysis over the loss threshold further shows that probability-based rules lose robustness as tail events become more severe, whereas severity- and geometry-based diagnostics remain stable.

These findings support a regime-based approach to portfolio design, in which satellite exposure is managed through policy rules rather than pointwise optimization. The proposed framework offers a transparent and forward-looking tool for controlling concentrated thematic exposure under rare but economically meaningful extreme risks.

# 1 INTRODUCTION

Recent advances in portfolio optimization have highlighted the importance of explicitly accounting for extreme downside risk, particularly in environments characterized by concentration, structural breaks, and tail dependence. Classical mean–variance frameworks, while effective in controlling overall volatility, often fail to adequately capture the impact of rare but severe losses. To address this limitation, optimization under rare chance constraints has emerged as a principled framework for controlling tail risk beyond standard risk measures.

In parallel, core–satellite portfolio structures have become widely adopted in practice as a way to balance stability and return enhancement. By combining a diversified core with a concentrated satellite exposure, such designs intentionally introduce asymmetry in return profiles, which may amplify tail risk despite improved expected performance. Understanding how this structural choice interacts with rare chance constraints is therefore of central importance.

This paper studies the impact of a core–satellite portfolio design on the feasibility and tightness of rare chance constraints. Using an AI-focused portfolio as a case study, we examine how the inclusion of a concentrated satellite affects extreme loss probabilities and optimal asset allocations. The analysis is conducted within a simplified Gaussian framework, allowing for a transparent assessment of the trade-offs between expected return, diversification, and tail risk control.

Rather than treating tail-risk constraints as purely technical feasibility conditions, this paper adopts a policy-oriented perspective. The satellite weight  $\lambda$  is not optimized endogenously but explored as a controllable exposure parameter, allowing us to characterize distinct risk regimes as tail constraints tighten. By combining rare-event simulation, FORM approximations, and large-deviation geometry, we show that tail risk evolves in a structured, regime-dependent manner as satellite exposure increases.

This regime-based view leads naturally to a set of interpretable policy rules that balance diversification benefits against tail fragility. In particular, we identify thresholds beyond which probability-based constraints lose informativeness, while severity-based and geometric diagnostics remain robust. This framework provides a practical decision tool for managing concentrated thematic exposure under rare but structurally meaningful tail risk.

## 2 OBJECTIVES AND EXPECTED OUTCOMES

### 2.1 Objectives

This study aims to adapt the rare chance constraint optimization framework to a simplified core–satellite portfolio setting, with a focus on extreme downside risk. The main objectives are as follows:

- Model a portfolio composed of:
  - a diversified *core*, and
  - a concentrated *AI satellite* built from major industry leaders (e.g., NVIDIA, Microsoft, AMD, Meta, Broadcom, Oracle).
- Formulate a *rare chance constraint* representing the occurrence of a severe portfolio crash.
- Approximate the probability of extreme losses using a *first-order reliability method (FORM /  $P_1$ )*, consistent with rare-event regimes.
- Identify the *dominating point* of the portfolio, corresponding to the most likely extreme-loss scenario.
- Compare the rare-event approximation with a *Monte Carlo benchmark* to assess accuracy in the tail region.

## 2.2 Expected Outcomes

The expected outcomes of the analysis are summarized below:

- The inclusion of a concentrated AI satellite is expected to:
  - increase expected portfolio returns,
  - but significantly amplify tail risk.
- Rare chance constraints are expected to become more *binding* in the presence of satellite concentration.
- The diversified core is expected to:
  - partially mitigate extreme losses,
  - but not fully neutralize tail risk induced by thematic concentration.
- The dominating point analysis is expected to provide:
  - a clear interpretation of the portfolio's most probable crash scenario,
  - and an economic link between optimization results and extreme-risk behavior.
- The FORM-based approximation is expected to perform well relative to Monte Carlo methods in the *rare-event regime*, while being computationally more efficient.

## 3 CORE–SATELLITE STRUCTURE, BARBEL STRATEGY, AND EXTREME RISK

The core–satellite portfolio structure is a widely used design in asset management, aiming to balance stability and return enhancement. In this framework, the *core* component is typically composed of diversified assets intended to provide robustness, risk control, and long-term stability, while the *satellite* component consists of concentrated exposures targeting higher expected returns or specific investment themes.

This structure is closely related to the notion of a *barbell strategy*, where portfolio risk is intentionally split between low-risk assets and highly convex or high-risk positions. While such a design may improve expected performance, it also introduces structural asymmetry in the return distribution. In particular, the concentration of the satellite can amplify tail risk, even when overall portfolio volatility remains moderate.

From an extreme-risk perspective, diversification primarily acts on second-order moments, reducing variance under normal market conditions. However, rare and severe losses are often driven by joint extreme movements, during which correlations increase and diversification benefits deteriorate. As a result, the presence of a concentrated satellite may significantly increase the probability of extreme portfolio losses, despite the stabilizing role of the core.

In this context, rare chance constraints provide a natural framework to assess the robustness of core–satellite designs. Rather than controlling average risk measures, they explicitly restrict the probability of catastrophic losses. This allows for a transparent evaluation of whether the diversification provided by the core is sufficient to offset the tail risk introduced by the satellite, or whether the portfolio structure inherently tightens the feasibility of extreme-risk constraints.

## 4 MODELING FRAMEWORK AND OPTIMIZATION UNDER RARE CHANCE CONSTRAINTS

### 4.1 Portfolio Return Model

Let  $\mathbf{R} \in \mathbb{R}^d$  denote the vector of asset returns, assumed to follow a multivariate normal distribution,

$$\mathbf{R} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}),$$

where  $\mu$  is the vector of expected returns and  $\Sigma$  the covariance matrix. Portfolio weights are denoted by  $\mathbf{w} \in \mathbb{R}^d$ , satisfying standard budget and feasibility constraints.

The portfolio return is given by

$$R_p = \mathbf{w}^\top \mathbf{R}.$$

## 4.2 Core–Satellite Aggregation and Role of $\lambda$

Let  $\mathbf{w}^C$  and  $\mathbf{w}^S$  denote the weight vectors of the Core and Satellite portfolios, respectively. The aggregated portfolio is defined as a convex combination:

$$\mathbf{w}(\lambda) = (1 - \lambda) \mathbf{w}^C + \lambda \mathbf{w}^S, \quad \lambda \in [0, 1].$$

The parameter  $\lambda$  controls the fraction of capital allocated to the Satellite portfolio and is treated as a *strategic decision variable* rather than a statistical parameter to be estimated. Varying  $\lambda$  allows for a controlled exploration of the trade-off between expected return enhancement and tail-risk amplification.

The rare chance constraint is imposed on the aggregated portfolio return,

$$R_p(\lambda) = \mathbf{w}(\lambda)^\top \mathbf{R},$$

so that the probability of extreme losses reflects the joint interaction between the stabilizing role of the Core and the concentrated exposure introduced by the Satellite.

Rather than selecting a single optimal value of  $\lambda$ , the analysis considers a grid of values and evaluates feasibility *ex post* using rare-event probability estimates, Monte Carlo risk measures, and dominant point geometry.

## 4.3 Rare Chance Constraint Formulation

Extreme downside risk is modeled through a rare chance constraint of the form

$$\mathbb{P}(R_p \leq -L) \leq \varepsilon,$$

where  $L > 0$  represents a severe loss threshold and  $\varepsilon \ll 1$  is a small probability level corresponding to a rare event. This constraint explicitly limits the probability of catastrophic portfolio losses rather than controlling average risk measures.

## 4.4 FORM Approximation and Dominating Point

Direct evaluation of rare-event probabilities via Monte Carlo simulation is computationally expensive in the regime where  $\varepsilon$  is very small. To address this issue, we rely on a first-order reliability method (FORM) approximation.

Under the Gaussian assumption, the rare-event probability is approximated by

$$\mathbb{P}(R_p \leq -L) \approx \Phi(-\beta),$$

where  $\Phi(\cdot)$  denotes the standard normal cumulative distribution function and  $\beta$  is the reliability index. The index  $\beta$  is obtained by solving a constrained optimization problem in the standardized space, whose solution defines the *dominating point*. This point corresponds to the most likely realization of asset returns leading to a portfolio crash.

The dominating point provides a geometric and economic interpretation of extreme risk, identifying which assets and directions in return space contribute most to catastrophic losses.

## 4.5 Optimization Problem with Core–Satellite Structure

Let  $\mathbf{w}^C \in \mathbb{R}^d$  and  $\mathbf{w}^S \in \mathbb{R}^d$  denote the Core and Satellite weight vectors, respectively. The aggregated portfolio weights are defined as

$$\mathbf{w}(\lambda) = (1 - \lambda) \mathbf{w}^C + \lambda \mathbf{w}^S, \quad \lambda \in [0, 1].$$

The corresponding portfolio return is

$$R_p(\lambda) = \mathbf{w}(\lambda)^\top \mathbf{R}.$$

For a fixed value of  $\lambda$ , the portfolio optimization problem is formulated as

$$\begin{aligned} \max_{\mathbf{w}^C} \quad & \mathbb{E}[R_p(\lambda)] = (1 - \lambda)(\mathbf{w}^C)^\top \boldsymbol{\mu} + \lambda(\mathbf{w}^S)^\top \boldsymbol{\mu} \\ \text{s.t.} \quad & \mathbb{P}(R_p(\lambda) \leq -L) \leq \varepsilon, \\ & \mathbf{1}^\top \mathbf{w}^C = 1, \\ & \mathbf{w}^C \geq 0, \end{aligned}$$

where  $\mathbf{w}^S$  is treated as fixed and represents the structured AI satellite portfolio.

The rare chance constraint is enforced using a first-order large deviations (FORM) approximation applied to the aggregated portfolio. Varying  $\lambda$  allows for a controlled analysis of how increasing satellite exposure affects feasibility, extreme-loss probabilities, and dominant point geometry.

**Policy constraints and notation.** Throughout the empirical analysis, tail risk is evaluated under a fixed loss threshold  $L$ , while portfolio exposure is governed by the satellite weight  $\lambda \in [0, 1]$ . We consider two classes of tail constraints:

- **Probability-based constraint:**

$$\mathbb{P}(\text{Loss} \geq L) \leq \beta,$$

where  $\beta$  denotes the rare-event probability cap.

- **Severity-based constraint:**

$$\text{CVaR}_\alpha(\text{Loss}) \leq \bar{c},$$

where  $\bar{c}$  is a fixed CVaR cap at confidence level  $\alpha$ .

Monte Carlo estimates are denoted by the subscript  $(\cdot)_{\text{MC}}$ , while FORM-based rare-event approximations are denoted by  $(\cdot)_{\text{FORM}}$ . Large-deviation geometry diagnostics are summarized by the tail index  $t$  and the rate function  $I$ , both evaluated at the dominant point.

**Policy rules.** We evaluate satellite exposure  $\lambda$  under three interpretable policy rules:

- **Rule A (CVaR cap):** the maximum  $\lambda$  such that  $\text{CVaR}_{\alpha, \text{MC}} \leq \bar{c}$ .
- **Rule B (Probability cap):** the maximum  $\lambda$  such that  $\mathbb{P}_{\text{MC}}(\text{Loss} \geq L) \leq \beta$  (with FORM estimates reported for comparison).
- **Rule C (Regime-based):** classification of  $\lambda$  into *Safe*, *Transition*, and *Fragile* regimes based on joint behavior of probability, severity, and large-deviation diagnostics.

## 5 EMPIRICAL ANALYSIS

### 5.1 Portfolio Construction and Asset Selection

This section describes the asset universe and portfolio architecture used in the empirical analysis. The portfolio follows a core–satellite design in which a diversified Core portfolio is combined with a structured Satellite exposure representing the artificial intelligence (AI) ecosystem.

**AI Satellite Architecture.** The Satellite portfolio is constructed to represent the full AI value chain rather than a single technological segment. Assets are grouped according to their economic role within the AI ecosystem, ensuring broad but concentrated thematic exposure. The Satellite portfolio is treated as fixed throughout the analysis and is composed of the following segments:

- **Compute / GPU (AI computation core):** NVIDIA (NVDA), Advanced Micro Devices (AMD).
- **Advanced Semiconductors (fabrication):** Taiwan Semiconductor Manufacturing Company (TSM), ASML Holding (ASML).
- **Servers and Infrastructure (deployment):** Super Micro Computer (SMCI), Dell Technologies (DELL).

- **Cloud / Hyperscalers (AI usage):** Amazon (AMZN), Alphabet (GOOGL).
- **Data Centers and Networking (backbone):** Equinix (EQIX), Arista Networks (ANET).
- **Software and Platforms (integration):** Microsoft (MSFT), Oracle (ORCL).

This segmentation spans the entire AI production and deployment pipeline, from computation and fabrication to cloud usage and enterprise integration. Due to strong cross-correlations across these segments, the Satellite portfolio exhibits heightened exposure to joint extreme losses during technology-driven market stress.

**Core Portfolio Construction.** The Core portfolio is designed to provide diversification and partial robustness against severe market shocks. It is optimized under rare chance constraints and consists of a compact set of assets selected for their heterogeneous risk characteristics:

- **U.S. Large-Cap Equities:** Vanguard S&P 500 ETF (VFV).
- **International Equities:** iShares Core MSCI EAFE ETF (XEF).
- **Emerging Markets Equities:** Vanguard FTSE Emerging Markets ETF (VEE).
- **Defensive Equity:** Procter & Gamble (PG).
- **Long-Duration Government Bonds:** iShares 20+ Year Treasury Bond ETF (XTLT).
- **Gold:** Gold exposure (GC).
- **Cash / Money Market:** ZMMK or cash-equivalent asset.

These assets provide diversified exposure across equity risk premia, interest-rate sensitivity, defensive characteristics, and systemic hedging channels, without relying on explicit derivative-based tail-risk hedges.

**Core–Satellite Aggregation and Role of  $\lambda$ .** The overall portfolio is obtained by aggregating the Core and Satellite portfolios according to

$$\mathbf{w}(\lambda) = (1 - \lambda) \mathbf{w}^C + \lambda \mathbf{w}^S, \quad \lambda \in [0, 1],$$

where  $\mathbf{w}^C$  is optimized under a rare chance constraint and  $\mathbf{w}^S$  is fixed.

The parameter  $\lambda$  controls the level of exposure to the AI Satellite and is treated as a strategic decision variable rather than an optimization variable. The empirical analysis considers a grid of  $\lambda$  values in order to examine how increasing Satellite exposure affects feasibility, extreme-loss probabilities, and the geometry of dominant points.

## 5.2 Descriptive Statistics of the Asset Universe

Before analyzing rare-event behavior and tail-risk dynamics, we report basic descriptive statistics for all assets included in the Core–Satellite portfolio. Table 1 summarizes weekly mean returns and volatilities, together with their annualized approximations, computed over the sample period.

The statistics are reported separately for the AI Satellite and the Core portfolio components. This distinction highlights the structural heterogeneity between the two sub-portfolios and motivates the Core–Satellite architecture adopted in this study.

The AI Satellite assets exhibit substantially higher volatility levels and more dispersed return profiles, reflecting their exposure to innovation-driven growth, technological cycles, and strong cross-sectional correlations within the AI ecosystem. In contrast, the Core assets display lower volatility and more stable return characteristics, consistent with their role as diversification anchors and partial hedges against systemic market stress.

These differences in first- and second-moment characteristics underscore the trade-off inherent in combining a concentrated thematic satellite with a diversified core. While the Core contributes stability

and risk absorption, the Satellite introduces amplified tail exposure, which becomes critical under extreme market conditions. This contrast provides the empirical foundation for the rare chance-constrained optimization and regime analysis developed in the subsequent sections.

Table 1: Descriptive statistics of Core and AI Satellite assets

Asset	$\mu_w$	$\sigma_w$	$\mu_{\text{ann}}$	$\sigma_{\text{ann}}$
<b>AI Satellite Portfolio</b>				
NVDA	0.01919	0.06346	0.99789	0.45764
AMD	0.00458	0.06449	0.23830	0.46503
TSM	0.00858	0.04984	0.44637	0.35938
ASML	0.00084	0.05266	0.04360	0.37976
SMCI	0.01242	0.15043	0.64578	1.08475
DELL	0.01063	0.06919	0.55263	0.49894
AMZN	0.00848	0.03384	0.44106	0.24404
GOOGL	0.00732	0.04008	0.38069	0.28905
EQIX	0.00316	0.03193	0.16439	0.23027
ANET	0.01207	0.06110	0.62773	0.44061
MSFT	0.00528	0.02863	0.27462	0.20645
ORCL	0.00693	0.04247	0.36015	0.30622
<b>Core Portfolio</b>				
VFV	0.00470	0.01546	0.24422	0.11149
XEF	0.00188	0.01592	0.09764	0.11478
VEE	0.00204	0.01723	0.10628	0.12426
PG	0.00227	0.01888	0.11806	0.13615
XTLT	-0.00020	0.02195	-0.01062	0.15825
GC	0.00359	0.01940	0.18649	0.13991
ZMMK	0.00091	0.00030	0.04756	0.00219

### 5.3 Impact of Satellite Allocation on Rare-Event Risk

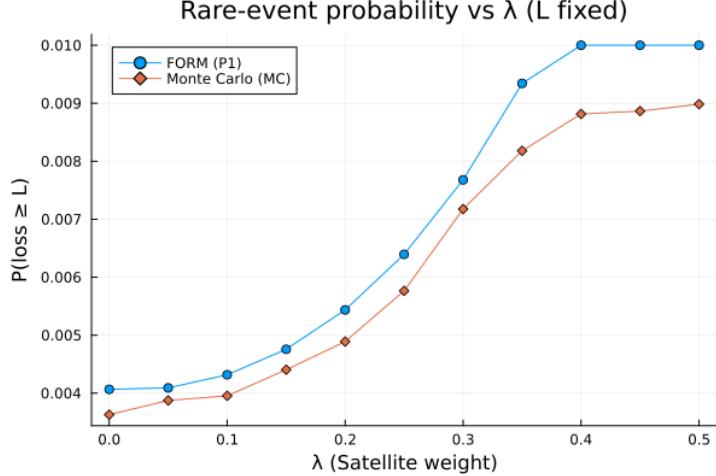
This subsection investigates how increasing the Satellite allocation parameter  $\lambda$  affects the tail risk profile of the aggregated Core–Satellite portfolio. The loss threshold is fixed at  $L = 0.05$ , corresponding to a severe weekly loss event, and all risk measures are evaluated under both first-order large deviations approximations and Monte Carlo simulations.

Table 2 reports, for each value of  $\lambda$ , the probability  $\mathbb{P}(\text{Loss} \geq L)$  computed via the first-order FORM approximation ( $P_1$ ) and Monte Carlo simulation (MC), together with the number of exceedances (*hits*) and a 95% binomial confidence interval for MC. The table also reports Monte Carlo tail severity metrics (VaR and CVaR at level 0.99), as well as large deviations geometry diagnostics through the tail index  $t(\lambda)$ .

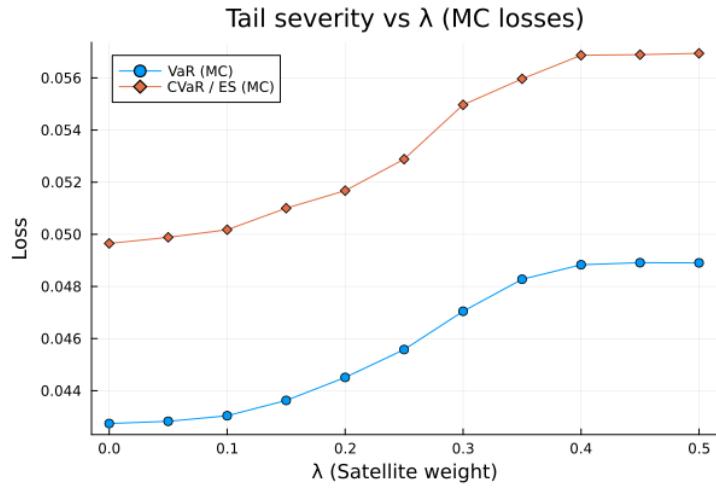
Table 2: Rare-event probability and tail diagnostics as a function of satellite weight  $\lambda$  (loss threshold  $L = 0.05$ ).

$\lambda$	$L$	FORM ( $P_1$ )	MC	hits	MC <sub>lo</sub>	MC <sub>hi</sub>	VaR <sub>0.99</sub>	CVaR <sub>0.99</sub>	$t$
0.00	0.05	0.004066	0.003630	726	0.003376	0.003903	0.042742	0.049650	2.683
0.05	0.05	0.004092	0.003875	775	0.003612	0.004157	0.042826	0.049884	2.681
0.10	0.05	0.004318	0.003955	791	0.003689	0.004240	0.043040	0.050176	2.663
0.15	0.05	0.004757	0.004405	881	0.004124	0.004705	0.043628	0.051003	2.631
0.20	0.05	0.005437	0.004890	978	0.004594	0.005205	0.044510	0.051675	2.587
0.25	0.05	0.006395	0.005765	1153	0.005443	0.006106	0.045579	0.052883	2.532
0.30	0.05	0.007679	0.007175	1435	0.006814	0.007554	0.047048	0.054969	2.468
0.35	0.05	0.009342	0.008180	1636	0.007795	0.008584	0.048275	0.055965	2.400
0.40	0.05	0.010000	0.008815	1763	0.008415	0.009234	0.048832	0.056869	2.375
0.45	0.05	0.010000	0.008865	1773	0.008464	0.009285	0.048911	0.056892	2.375
0.50	0.05	0.010000	0.008985	1797	0.008581	0.009408	0.048904	0.056941	2.375

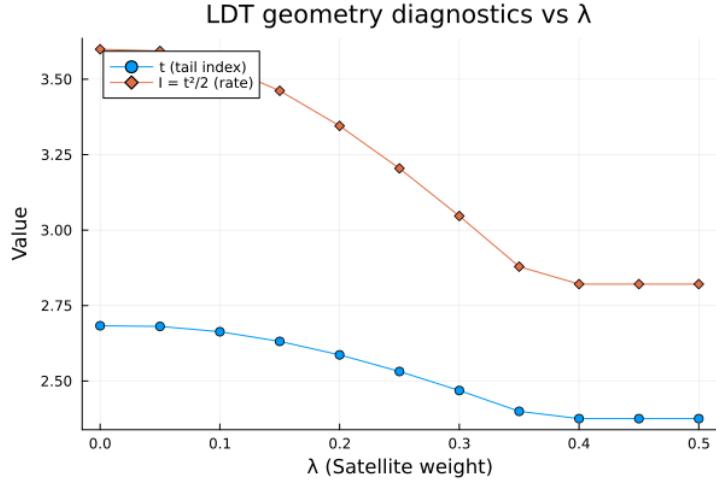
Figure 1 summarizes the same experiment in a visual form. Panel (a) shows the rare-event probability under FORM and Monte Carlo, panel (b) reports tail severity (VaR and CVaR), and panel (c) provides large deviations geometry diagnostics via  $t(\lambda)$  and  $I(\lambda) = t(\lambda)^2/2$ .



(a) Rare-event probability  $\mathbb{P}(\text{Loss} \geq L)$  versus  $\lambda$ . Comparison between FORM ( $P_1$ ) and Monte Carlo (MC).



(b) Tail severity versus  $\lambda$ , measured by Monte Carlo VaR and CVaR (Expected Shortfall) at level 0.99.



(c) Large deviations geometry diagnostics versus  $\lambda$ : tail index  $t$  and rate  $I = t^2/2$ .

Figure 1: Impact of satellite allocation  $\lambda$  on rare-event probability, tail severity, and large deviations geometry (loss threshold  $L = 0.05$ ).

**Rare-event probability dynamics.** Figure 1a shows that for small Satellite allocations ( $\lambda \leq 0.10$ ), the rare-event probability remains close to its Core-only level, with Monte Carlo estimates below the target probability budget. As  $\lambda$  increases beyond approximately 0.15, the probability rises at an accelerating rate and approaches the constraint boundary for  $\lambda \gtrsim 0.35$ . Across the full range, FORM closely tracks Monte Carlo and behaves conservatively, providing a slight upper bound.

**Tail severity.** Figure 1b complements the probability analysis by reporting VaR and CVaR (Expected Shortfall) computed from Monte Carlo losses. Both metrics increase monotonically with  $\lambda$ , with CVaR rising more sharply than VaR. This indicates that increasing Satellite exposure not only makes extreme losses more frequent, but also increases their expected magnitude conditional on being in the tail.

**Large deviations geometry.** Figure 1c reports the large deviations diagnostics. Both the tail index  $t(\lambda)$  and the rate function  $I(\lambda) = t(\lambda)^2/2$  decrease monotonically with  $\lambda$ , confirming that the exponential decay rate of rare-event probabilities weakens as Satellite exposure increases. Geometrically, this corresponds to the dominant point moving closer to the mean return configuration, making extreme-loss events increasingly “accessible” under smaller deviations.

**Implications.** Taken together, Table 2 and Figure 1 reveal a smooth but strongly nonlinear deterioration of tail robustness as  $\lambda$  increases. This motivates treating  $\lambda$  as a policy control variable rather than an optimizable parameter, a perspective developed in the next subsection through explicit regime classification and tail-risk budgeting.

## 5.4 Dominant Point Decomposition and Shock Attribution

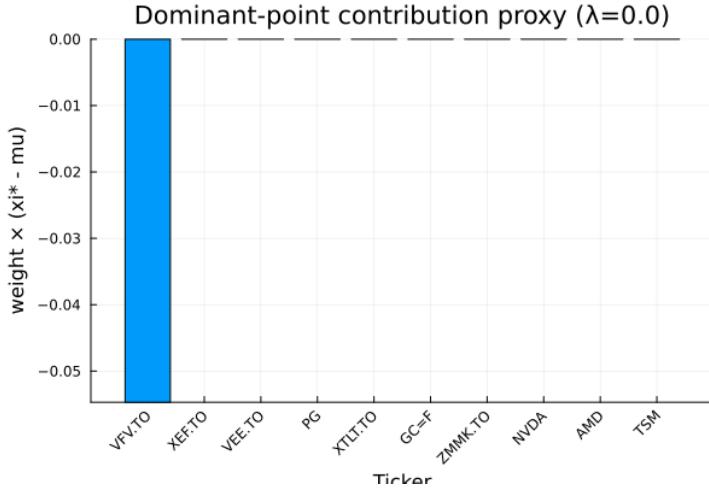
This subsection analyzes how extreme losses materialize across individual assets through the geometry of the dominant point associated with the rare-event constraint. While Section 5.3 focused on aggregate tail probabilities and severities, the present analysis decomposes the extreme-loss event at the asset level in order to identify which components of the Core–Satellite portfolio drive systemic tail risk as the satellite allocation  $\lambda$  increases.

For a fixed loss threshold  $L = 0.05$ , the dominant point  $\xi^*(\lambda)$  is computed under the Gaussian large deviations framework. Recall that  $\xi^*$  represents the most likely joint return configuration conditional on the rare loss event  $\{-u^\top \xi \geq L\}$ . The quantity

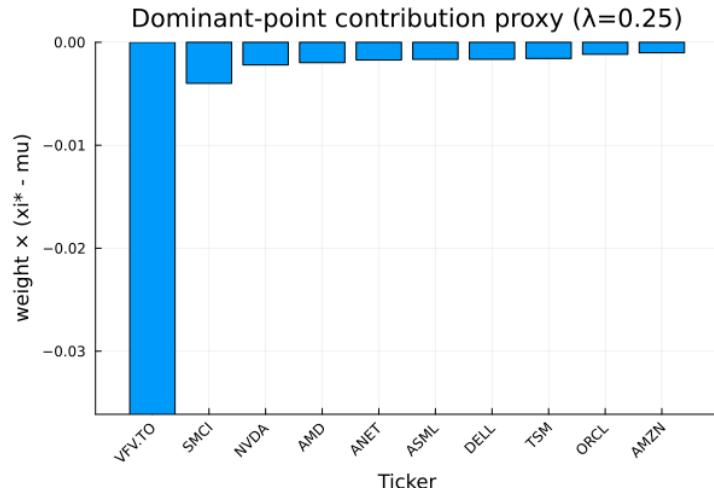
$$\Delta_i(\lambda) = \xi_i^*(\lambda) - \mu_i$$

captures the asset-level shock relative to the mean, while the product  $u_i \Delta_i$  provides a proxy for each asset’s contribution to the portfolio-level extreme loss.

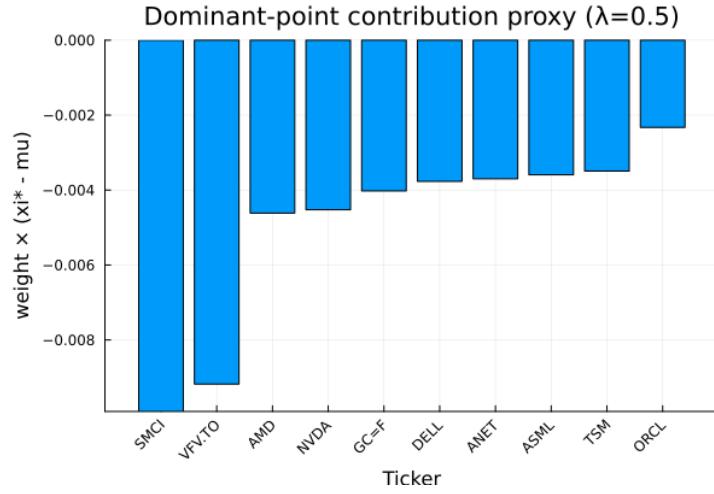
Figure 2 visualizes the dominant-point contribution proxy  $u_i(\xi_i^* - \mu_i)$  for three representative satellite allocations:  $\lambda = 0$ ,  $\lambda = 0.25$ , and  $\lambda = 0.5$ .



(a) Dominant-point contribution proxy for  $\lambda = 0$  (Core-only portfolio).



(b) Dominant-point contribution proxy for  $\lambda = 0.25$  (mixed Core-Satellite portfolio).



(c) Dominant-point contribution proxy for  $\lambda = 0.5$  (Satellite-heavy portfolio).

Figure 2: Asset-level contribution to extreme losses via dominant-point geometry for increasing satellite exposure  $\lambda$ .

To complement the graphical analysis, Table 3 reports the top contributors ranked by weighted shock magnitude  $|u_i(\xi_i^* - \mu_i)|$  for the same values of  $\lambda$ . This tabular representation facilitates a direct comparison of the dominant drivers of tail risk across portfolio regimes.

Table 3: Top contributors to extreme losses at the dominant point for selected satellite allocations  $\lambda$  (loss threshold  $L = 0.05$ ).

Asset	Weight $u_i$	Mean $\mu_i$	Shock $\Delta_i$	Contribution $u_i\Delta_i$
$\lambda = 0$ ( <b>Core-only</b> )				
VFV	0.9999	0.0047	-0.0547	-0.0547
XEF	$\approx 0$	0.0019	-0.0208	$\approx 0$
VEE	$\approx 0$	0.0020	-0.0190	$\approx 0$
$\lambda = 0.25$ ( <b>Balanced Core–Satellite</b> )				
VFV	0.7500	0.0047	-0.0482	-0.0361
SMCI	0.0208	0.0124	-0.1914	-0.0040
NVDA	0.0208	0.0192	-0.1055	-0.0022
$\lambda = 0.5$ ( <b>Satellite-heavy</b> )				
SMCI	0.0417	0.0124	-0.2378	-0.0099
VFV	0.2779	0.0047	-0.0330	-0.0092
AMD	0.0417	0.0046	-0.1107	-0.0046

**Interpretation.** For  $\lambda = 0$ , the dominant point is almost entirely driven by the Core equity exposure, with the S&P 500 ETF (VFV) accounting for nearly the entire extreme-loss realization. This reflects a concentrated but diversified market-wide shock.

As satellite exposure increases to  $\lambda = 0.25$ , tail risk becomes distributed across multiple assets. While the Core component remains dominant, high-volatility AI-related stocks such as SMCI and NVDA begin to contribute meaningfully to the extreme-loss event, indicating the emergence of sector-specific tail amplification.

At  $\lambda = 0.5$ , the dominant point shifts markedly toward Satellite assets. High-beta technology stocks (SMCI, NVDA, AMD) now dominate the extreme-loss configuration, while the Core still contributes through defensive and hedging channels (equities and gold). This regime corresponds to a fundamentally different tail-risk structure in which extreme losses are no longer driven by broad market movements alone, but by concentrated sectoral shocks.

**Implications.** This dominant-point decomposition highlights that tail risk is not only a question of magnitude but also of *structure*. Increasing  $\lambda$  alters the identity of the assets responsible for extreme losses, reinforcing the need for policy-based constraints on Satellite exposure rather than reliance on diversification arguments alone.

## 5.5 Policy Rules and Regime Classification

This subsection translates the quantitative results obtained in Sections 5.3 and 5.4 into explicit decision rules for selecting the satellite allocation parameter  $\lambda$ . Rather than treating  $\lambda$  as an optimizable variable, we interpret it as a policy control reflecting tolerance to extreme tail risk. Three complementary policy rules are considered.

**Policy Rule A (CVaR-based cap).** The first rule imposes an upper bound on tail severity by requiring that the Monte Carlo Conditional Value-at-Risk remains below a predefined threshold:

$$\text{CVaR}_{0.99}(\lambda) \leq \text{CVaR}_{\max}.$$

Using  $\text{CVaR}_{\max} = 0.056$ , this rule yields an admissible satellite allocation

$$\lambda_{\text{CVaR}}^* = 0.35.$$

**Policy Rule B (probability-based cap).** The second rule constrains the probability of extreme losses:

$$\mathbb{P}(\text{Loss} \geq L) \leq \beta,$$

with  $\beta = 0.008$ . This rule is evaluated using both Monte Carlo estimates and the first-order large deviations approximation (FORM). Both approaches yield the same admissible allocation:

$$\lambda_{\text{Prob}}^* = 0.30.$$

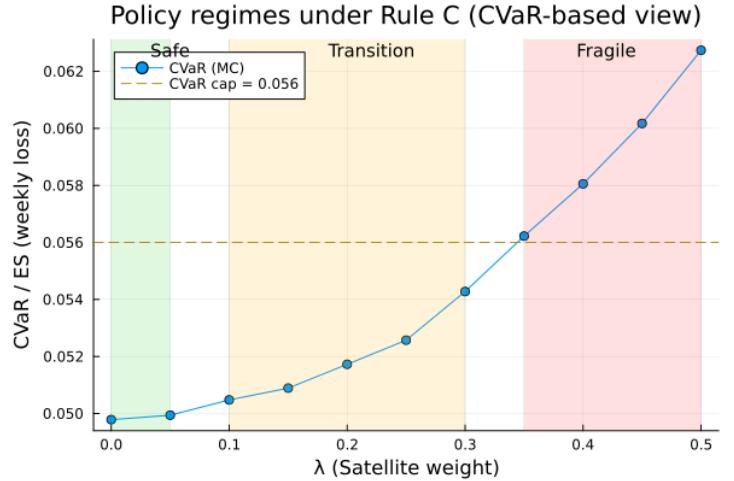
**Policy Rule C (regime classification).** The third rule adopts a regime-based interpretation of tail risk dynamics. Based on the joint behavior of rare-event probability, tail severity, and dominant-point geometry, the  $\lambda$ -axis is partitioned into three regimes:

- **Safe:** low tail probability and mild tail severity,
- **Transition:** rapidly increasing tail exposure,
- **Fragile:** binding tail constraints and structural tail amplification.

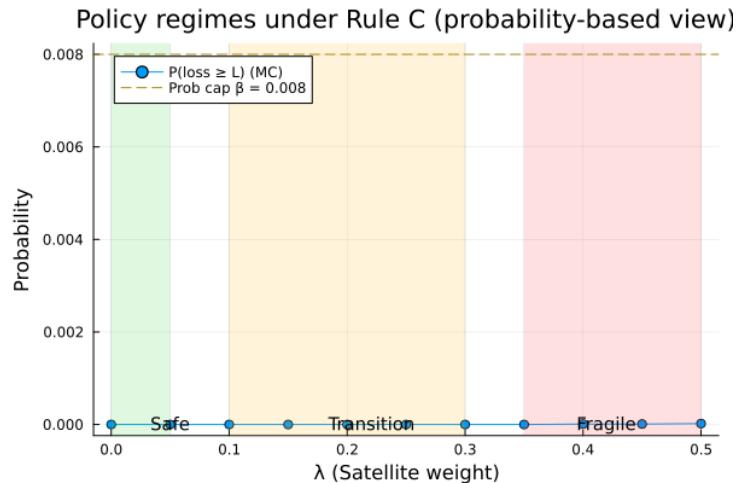
Under this classification, the maximum admissible allocations are:

$$\lambda_{\max}^{\text{Safe}} = 0.10, \quad \lambda_{\max}^{\text{Non-Fragile}} = 0.30.$$

Figure 3 visualizes the regime structure under Rule C using both CVaR-based and probability-based perspectives.



(a) Policy regimes under Rule C based on tail severity (CVaR).



(b) Policy regimes under Rule C based on rare-event probability.

Figure 3: Regime classification under Policy Rule C. Green: Safe region. Yellow: Transition region. Red: Fragile region.

Table 4 summarizes the admissible satellite allocations implied by each policy rule.

Table 4: Summary of policy rules and admissible satellite allocations.

Policy rule	Admissible $\lambda$
Rule A: CVaR cap ( $\text{CVaR}_{0.99} \leq 0.056$ )	$\lambda \leq 0.35$
Rule B: Probability cap ( $\mathbb{P}(\text{Loss} \geq L) \leq 0.008$ )	$\lambda \leq 0.30$
Rule C: Safe regime	$\lambda \leq 0.10$
Rule C: Non-fragile regime	$\lambda \leq 0.30$

**Discussion.** While Rule A allows for the largest satellite allocation, it tolerates configurations that already exhibit fragile tail dynamics. Rule B is more conservative and aligns closely with the onset of rapid probability escalation observed in Section 5.3. Rule C provides the most informative framework by explicitly distinguishing between stable, transitional, and fragile regimes. Importantly, the boundary between the Transition and Fragile regimes coincides with a sharp deterioration in dominant-point geometry and tail amplification.

**Implications.** These results confirm that  $\lambda$  should be treated as a policy variable rather than an optimizable parameter. Regime-based rules offer a transparent and interpretable framework for managing extreme tail risk in Core–Satellite portfolio structures, especially in environments dominated by sectoral concentration and systemic technological risk.

## 5.6 Sensitivity Analysis over the Loss Threshold

To assess the robustness of the proposed policy rules, we conduct a sensitivity analysis with respect to the loss threshold  $L$  defining the rare-event region. While the baseline analysis focuses on a weekly loss threshold of  $L = 0.05$ , we now consider higher thresholds  $L \in \{0.08, 0.10\}$ , corresponding to increasingly severe tail events.

This sensitivity analysis serves two complementary purposes. First, it evaluates whether the classification into *Safe*, *Transition*, and *Fragile* regimes under Rule C remains stable as the definition of a rare loss becomes more stringent. Second, it highlights the limitations of probability-based constraints in the extreme tail and contrasts them with CVaR-based diagnostics grounded in tail severity.

**CVaR-based sensitivity.** Figures 4–6 report the CVaR-based policy regimes under Rule C for  $L = 0.05$ ,  $L = 0.08$ , and  $L = 0.10$ , respectively. Across all thresholds, the CVaR curve increases monotonically with the satellite weight  $\lambda$ , reflecting the progressive amplification of tail losses induced by greater exposure to the AI satellite portfolio.

Despite the upward shift of the CVaR level as  $L$  increases, the qualitative regime boundaries remain remarkably stable. The *Safe* region consistently corresponds to low satellite exposure (approximately  $\lambda \leq 0.1$ ), while the *Transition* region extends up to  $\lambda \approx 0.3$ . Beyond this point, the CVaR constraint becomes binding and the portfolio enters a *Fragile* regime characterized by excessive tail severity.

This stability indicates that the regime structure is not an artifact of a specific loss threshold but rather reflects a structural change in the geometry of the loss distribution as satellite exposure increases.

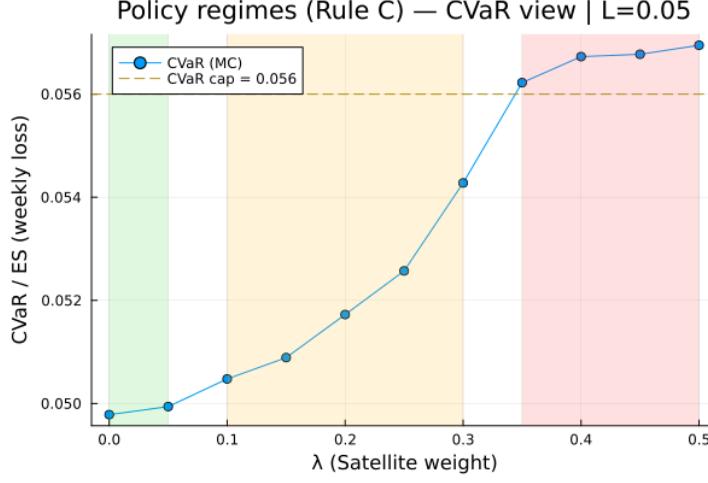


Figure 4: CVaR-based policy regimes under Rule C for  $L = 0.05$ .

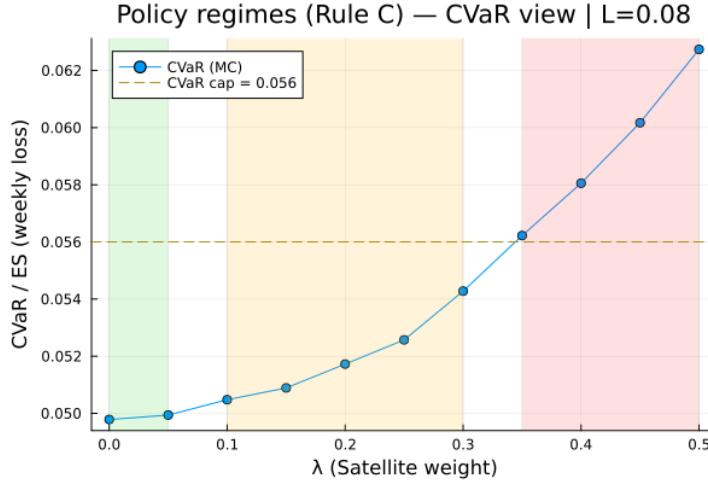


Figure 5: CVaR-based policy regimes under Rule C for  $L = 0.08$ .

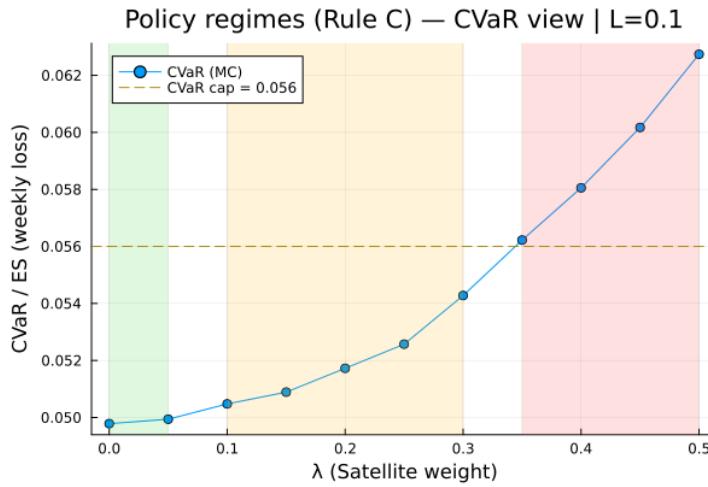


Figure 6: CVaR-based policy regimes under Rule C for  $L = 0.10$ .

**Probability-based sensitivity.** Figures 7–9 display the corresponding probability-based diagnostics based on Monte Carlo estimates of  $P(\ell \geq L)$ . As the loss threshold increases, the estimated probabilities

rapidly collapse toward zero for most values of  $\lambda$ , reflecting the extreme rarity of such tail events.

In particular, for  $L = 0.08$  and  $L = 0.10$ , the probability constraint becomes largely non-informative over a wide range of satellite weights. This behavior illustrates a fundamental limitation of probability-based rules in the far tail: once events become sufficiently rare, probability estimates lose resolution and fail to provide meaningful policy guidance.

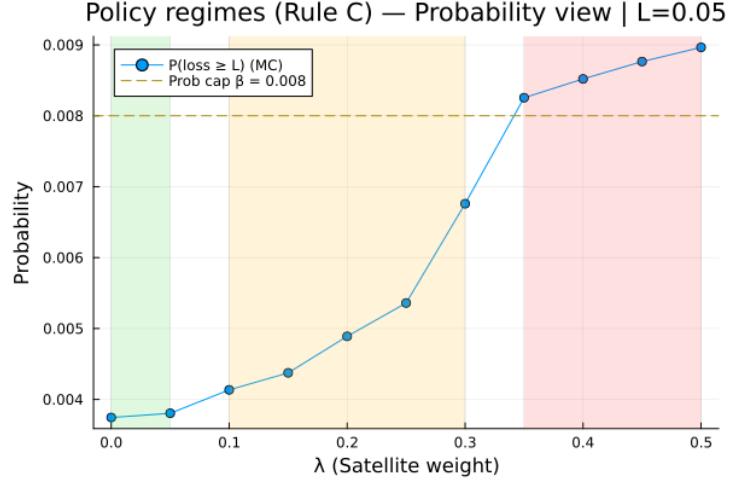


Figure 7: Probability-based policy regimes under Rule C for  $L = 0.05$ .

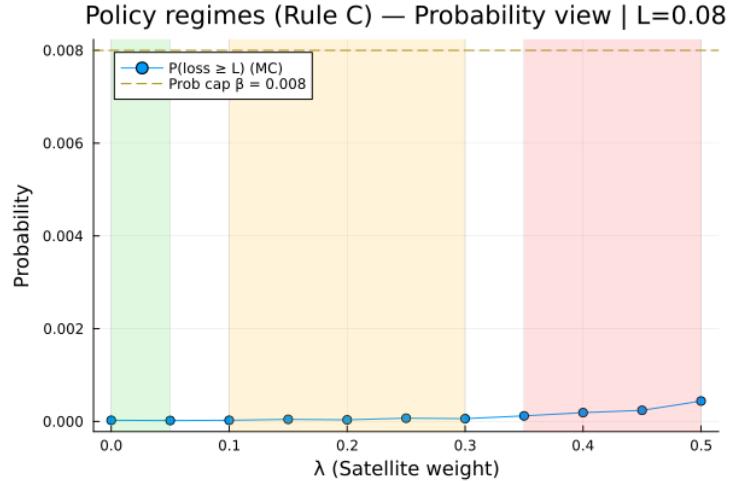


Figure 8: Probability-based policy regimes under Rule C for  $L = 0.08$ .

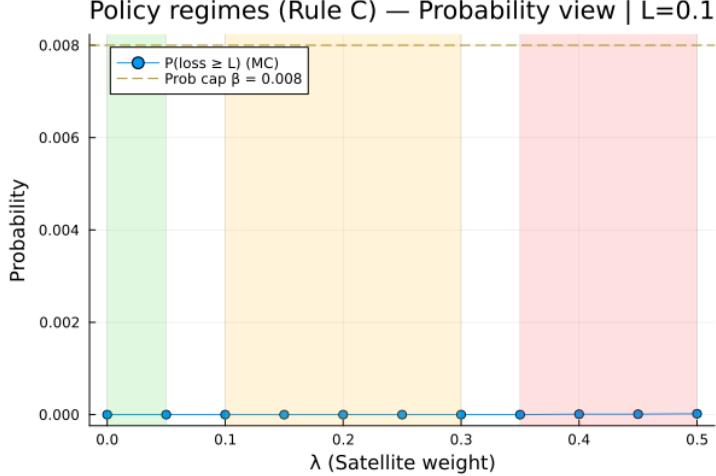


Figure 9: Probability-based policy regimes under Rule C for  $L = 0.10$ .

**Discussion.** Overall, the sensitivity analysis confirms the robustness of the regime-based policy framework. While probability-based constraints deteriorate rapidly as the loss threshold increases, the CVaR-based regime classification remains stable, interpretable, and economically meaningful. This robustness underscores the relevance of Rule C as a practical decision framework for managing extreme tail risk in portfolios combining diversified core assets with concentrated thematic satellite exposures.

In particular, the persistence of a well-defined transition region highlights the importance of intermediate satellite allocations, where tail risk is rising but has not yet reached a structurally fragile state. These findings reinforce the value of combining large-deviation geometry with tail severity measures when designing risk-aware portfolio policies under rare but severe loss scenarios.

### 5.6.1 Policy interpretation and risk management implications

The empirical results highlight that tail risk does not increase smoothly with satellite exposure  $\lambda$ , but instead exhibits a regime-dependent structure. While traditional constraints based on probability or CVaR identify admissible exposure levels  $\lambda^*$ , they fail to capture early signals of structural fragility.

Under low satellite exposure (Safe regime), rare-event probabilities, tail severity, and large-deviation diagnostics remain stable. In this region, tail events are driven by diversified, moderate shocks, and the dominant-point geometry remains diffuse.

As  $\lambda$  increases, the system enters a Transition regime in which probability and CVaR constraints are still satisfied, yet large-deviation diagnostics reveal a sharp decline in the rate function  $I$  and a concentration of dominant-point contributions on a subset of assets. This regime corresponds to an amplification phase, where tail losses become increasingly driven by coordinated shocks within the AI satellite.

Finally, in the Fragile regime, all policy constraints are violated simultaneously. Tail risk becomes dominated by a small number of extreme contributors, and both probability-based and severity-based measures escalate rapidly. In this region, marginal increases in  $\lambda$  lead to disproportionate increases in tail exposure.

From a risk-management perspective, these results suggest that optimal satellite allocation should not be determined solely by a binding constraint  $\lambda^*$ , but rather by avoiding entry into the Transition regime altogether. Large-deviation diagnostics thus provide a forward-looking early warning signal, enabling conservative portfolio design before conventional tail-risk measures deteriorate.

## 6 DISCUSSION AND POLICY IMPLICATIONS

This section synthesizes the empirical findings and discusses their implications for portfolio construction under rare chance constraints. The results highlight both the strengths and limitations of classical risk control mechanisms when applied to portfolios combining diversified core assets with concentrated thematic satellite exposures.

## 6.1 From Probability Control to Tail Geometry

A central finding of the empirical analysis is the structural divergence between probability-based and tail-severity-based risk controls. While probability constraints on  $P(\ell \geq L)$  are intuitive and widely used, they rapidly lose discriminative power as the loss threshold increases. For sufficiently large  $L$ , Monte Carlo estimates collapse toward zero across a wide range of satellite weights, rendering probability caps effectively non-binding.

In contrast, CVaR-based diagnostics remain informative even in the far tail. By conditioning on the loss event, CVaR captures the magnitude of extreme outcomes rather than their frequency alone. This distinction proves critical in the present setting, where rare but severe losses dominate the risk profile of the AI satellite portfolio.

The divergence between these two perspectives underscores a key insight: controlling the probability of rare losses is not equivalent to controlling their economic impact. In regimes characterized by extreme tail risk, severity-based measures provide a more robust and policy-relevant signal.

## 6.2 Large-Deviation Geometry as a Structural Diagnostic

Beyond standard risk measures, the large-deviation geometry offers a deeper structural interpretation of tail risk. The evolution of the tail index  $t$  and the rate function  $I$  as functions of the satellite weight  $\lambda$  reveals a progressive deformation of the loss landscape.

As  $\lambda$  increases, the dominant point shifts toward configurations in which shocks are increasingly concentrated in AI-related assets. This geometric transition precedes the binding of conventional risk constraints and provides an early warning signal of systemic fragility. In particular, the monotonic decline of the rate function  $I$  indicates a growing vulnerability to extreme joint losses driven by correlated shocks within the satellite portfolio.

These results demonstrate that large-deviation diagnostics are not merely theoretical constructs but operational tools capable of detecting regime shifts before they materialize in standard risk metrics.

## 6.3 Interpretation of Policy Rules

The three policy rules considered in the empirical analysis play complementary roles:

- **Rule A (CVaR cap)** provides a direct and economically interpretable constraint on tail severity. It yields a conservative upper bound on satellite exposure and remains stable across different loss thresholds.
- **Rule B (probability cap)** offers a simple and intuitive control mechanism but becomes unreliable in the extreme tail, particularly for large  $L$ . Its effectiveness is therefore limited to moderately rare events.
- **Rule C (regime-based rule)** integrates probability, severity, and large-deviation geometry into a unified framework. By distinguishing *Safe*, *Transition*, and *Fragile* regimes, it provides a nuanced policy map rather than a single binding constraint.

Among these, Rule C emerges as the most informative decision tool. It preserves interpretability while explicitly acknowledging the nonlinear and regime-dependent nature of tail risk in core–satellite portfolios.

## 6.4 Implications for Core–Satellite Portfolio Design

From a practical perspective, the results have direct implications for portfolio design. Moderate satellite exposure can enhance returns without immediately compromising tail robustness, but beyond a critical threshold, risk escalates sharply and nonlinearly. Importantly, this transition is not smooth: it is characterized by a geometric shift in the dominant loss scenario rather than a gradual increase in volatility.

This finding challenges naive diversification arguments often applied to thematic investments. Even when the core portfolio remains well diversified, a sufficiently concentrated satellite can dominate tail behavior and induce systemic fragility.

The regime-based framework proposed in this paper provides a disciplined approach to navigating this trade-off. By identifying a non-fragile region of satellite exposure, it allows investors to participate in thematic growth while maintaining explicit control over extreme downside risk.

## 6.5 Limitations and Extensions

Several limitations of the present study suggest directions for future research. First, the analysis relies on a Gaussian approximation of returns, which, while convenient for large-deviation analysis, may underestimate higher-order tail dependencies. Extending the framework to elliptical or mixture models would enhance realism.

Second, the current implementation considers a static portfolio. Incorporating dynamic rebalancing and time-varying correlations could further refine the policy rules, particularly during periods of market stress.

Finally, while the empirical focus is on the AI ecosystem, the methodology is generic and can be applied to other thematic satellites such as climate transition assets, commodities, or geopolitical risk exposures.

## 6.6 Concluding Remarks

Overall, the empirical results demonstrate that rare chance constraints, when combined with large-deviation geometry, offer a powerful lens for understanding and managing tail risk in modern portfolio architectures. Rather than replacing classical risk measures, the proposed framework complements them by revealing structural fragilities invisible to variance-based or probability-only controls.

In this sense, the core–satellite portfolio is not merely an allocation problem but a geometric object whose tail behavior must be understood holistically. The regime-based policy rules developed in this paper provide a principled foundation for such an understanding.