Theory Of Computation

Assignment 2

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(BE Third Year)

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ToC (Assign 2)

Ans1. (i) Prove $L = \{a^n \mid n \text{ is a perfect square}\}$ is not regular.

Assuming L is regular, then as per Pumping Lemma; we will prove that L is not regular.

Case I : $n = 4$, so $S = \underbrace{aa}_{u}\underbrace{a}_{v}\underbrace{a}_{w}$

(i) $|uv| \le n \Rightarrow 3 \le 4 \Rightarrow$ satisfy

(ii) $|v| \ge 1 \Rightarrow 1 \ge 1 \Rightarrow$ satisfy

(iii) $uv^iw \in L \Rightarrow$ not satisfy.

let $i = 2$

$\Rightarrow uv^2w = aaaaa \Rightarrow a^5 \notin L$

Case II : $S = \underbrace{a}_{u}\underbrace{aa}_{v}\underbrace{a}_{w}$

(i) $|uv| \le n \Rightarrow 3 \le 4 \Rightarrow$ satisfy

(ii) $|v| \ge 1 \Rightarrow 2 \ge 1 \Rightarrow$ satisfy

(iii) $uv^iw \in L \Rightarrow$ not satisfy.

let $i = 2$

$\Rightarrow uv^2w \Rightarrow aaaaaa \Rightarrow a^6 \notin L$

so, our assumption is being contradicted.

$\therefore$ L is not regular.

(ii) Prove $L = \{n_a(w) < n_b(w) \mid w \in \Sigma^*\}$ is not regular.

we can observe that no. of a's in w is less than b's.

so, let $w = a^n b^{2n}$ & assume $n = 2$.

Case I : $S = \underbrace{aa}_{u}\underbrace{bb}_{v}\underbrace{bb}_{w}$

(i) $|uv| \le n \Rightarrow 4 \not\le 2 \Rightarrow$ not satisfy

$\therefore$ no need to check further in this case.

Case II : $S = \underbrace{a}_{u}\underbrace{a}_{v}\underbrace{bbbb}_{w}$

(i) $|uv| \le n \Rightarrow 2 \le 2 \Rightarrow$ satisfy

(ii) $|v| \geq 1 \Rightarrow 1 \geq 1 \Rightarrow$ satisfy

(iii) $uv^i w \in L$

let $i = 3$

$\Rightarrow uv^3w \Rightarrow aaaabbbb \Rightarrow a^4 b^4 \notin L$

$\therefore$ L is not regular

(iii) Prove $L = \{a^n b^{j^2} \mid n, j \geq 0\}$ is not context-free.

let's assume that L is context-free.

let $S = a^n b^{j^2}$

Case I : $j = n = 3$, so $S = \underbrace{aa}_{u}\underbrace{ab}_{v}\underbrace{bbb}_{w}\underbrace{bb}_{x}\underbrace{bb}_{y}$

(i) $|vwx| \le n \Rightarrow 7 \nleq 3 \Rightarrow$ not satisfy

$\therefore$ no need to check further in this case.

Case II : $S = \underbrace{aaab}_{u}\underbrace{b}_{v}\underbrace{b}_{w}\underbrace{b}_{x}\underbrace{bbbbbb}_{y}$

(i) $|vwx| \le n \Rightarrow 3 \le 3 \Rightarrow$ satisfy

(ii) $|vx| \geq 1 \Rightarrow 2 \geq 1 \Rightarrow$ satisfy

(iii) $uv^iwx^iy \in L \Rightarrow$ not satisfy.

let $i = 2$

$\Rightarrow uv^2wx^2y \Rightarrow aaabbbbbbbbbbb \Rightarrow a^3 b^{11} \notin L$

so, our assumption is being contradicted.

$\therefore$ L is not context-free.

(iv) Prove $L = \{a^n b^m c^k \mid n * m = k\}$ is not context-free.

Let's assume that L is context-free.

Let $S = a^n b^{2n} c^{2n^2}$, & assume $n = 3$.

Case I : $S = \underbrace{aaabbb}_{u}\underbrace{bbb}_{v}\underbrace{cc}_{w}\underbrace{cccc}_{x}\underbrace{cccccccccccc}_{y}$

(i) $|vwx| \leq n \Rightarrow 10 \nleq 3 \Rightarrow$ not satisfy.

$\therefore$ no need to check further in this case.

Case II : $S = \underbrace{aaabbb}_{u}\underbrace{b}_{v}\underbrace{b}_{w}\underbrace{b}_{x}\underbrace{bcccccccccccccccccc}_{y}$

(i) $|vwx| \leq n \Rightarrow 3 \leq 3 \Rightarrow$ satisfy

(ii) $|vx| \geq 1 \Rightarrow 2 \geq 1 \Rightarrow$ satisfy

(iii) $uv^iwx^iy \in L$

Let $i = 2$

$\Rightarrow uv^2wx^2y \Rightarrow aaabbbbbbbb\,cccccccccccccccccc$

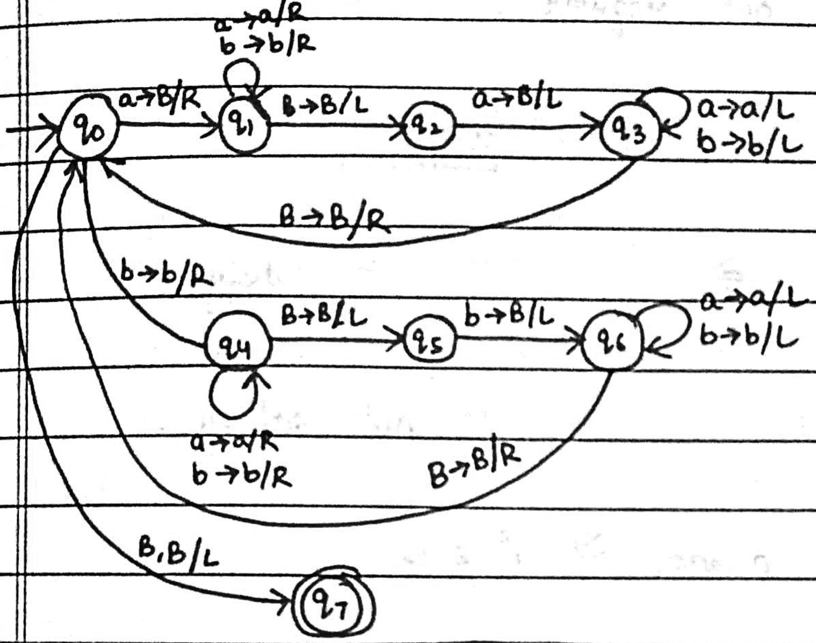
$\Rightarrow a^3 b^8 c^{18} \notin L$

so, our assumption is being contradicted.

$\therefore$ L is not context-free.

Ans2 (i) Design TM for $L = \{ww^r \mid w \in (a,b)^*\}$

Considering w = aab



B	~~a~~ B	a	b	b	a	a	B

B	B	a	b	b	a	~~a~~ B	B

B	B	~~a~~ B	b	b	a	B	B

.... & so on until

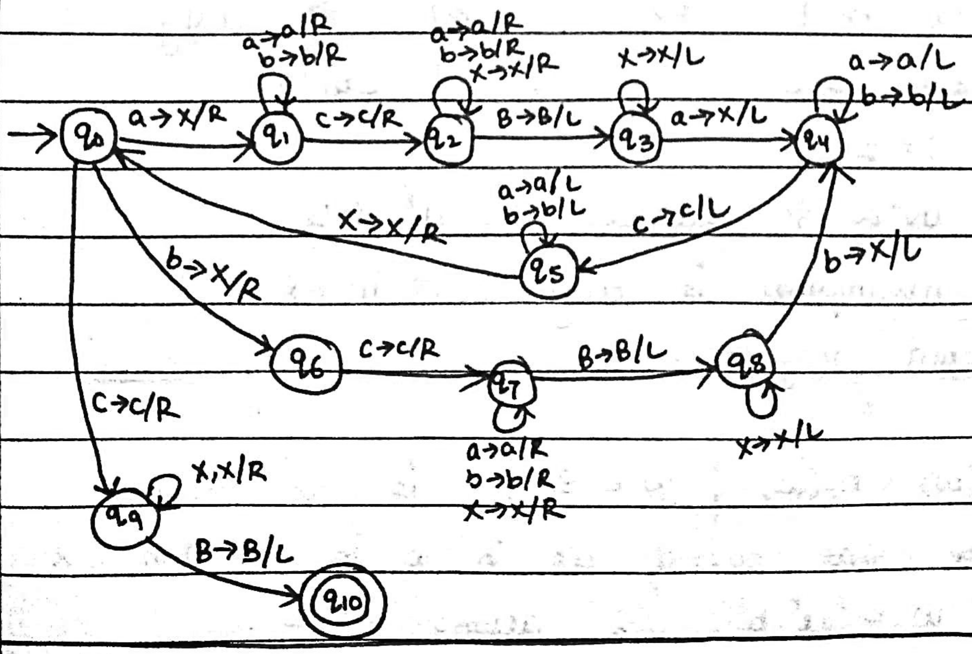
B	B	B	B	B	B	B	B

(Accepted String)

(ii) $L = \{wcw^r \mid w \in (a,b)^*\}$

Consider w = aabbab

B	a	a	b	b	a	b	c	b	a	b	b	a	a	B



Ans3. (i) $L = \{ xww^r \mid x, w \in (0,1)^* \}$

$S \rightarrow S_1 S_2$

$S_1 \rightarrow 0S_1 \mid 1S_1 \mid \epsilon$

$S_2 \rightarrow 0S_20 \mid 1S1 \mid \epsilon$

(ii) $L = \{ wa^n ba^m w^r \mid n \geq 1, \; m \geq 0 \}$

$S \rightarrow aSa \mid bSb \mid A$

$A \rightarrow aA \mid abA \mid \epsilon$

Ans4.

$S \to abc \mid aAbc$

$Ab \to bA$

$Ac \to Bbcc$

$bB \to Bb$

$aB \to aa \mid aaA$

Lets check what language can be generated by given context sensitive grammar.

$\{abc,\ a^2b^2c^2,\ \ldots\}$

$S \to aAbc$

$\to abAc$

$\to abBbcc$

$\to aBbbcc$

$\to aabbcc = a^2b^2c^2$

$S \to aAbc$

$\to abAc$

$\to abBbcc$

$\to aBbbcc$

$\to aaAbbcc$

$\to aabbAcc$

$\to aabbBbccc$

$\to aabBbbccc$

$\to aaBbbbccc$

$\to aaabbbccc = a^3b^3c^3$

Hence, we can say this grammar generates $a^n b^n c^n$. So, it must generate also $a^3b^3c^3$.

$\therefore$ we can say this grammar generates $a^3b^3c^3$.

Anbs Convert the following grammar to CNF

$$S \to ABa$$

$$A \to aab$$

$$B \to Ac$$

Step1 : introducing variables for terminals.

$$S \to ABT_a$$

$$A \to T_aT_aT_b$$

$$B \to AT_c$$

$$T_a \to a$$

$$T_b \to b$$

$$T_c \to c$$

Step 2 : introducing intermediate variables.

$S \rightarrow AV_1$

$V_1 \rightarrow BT_a$

$A \rightarrow T_aT_aT_b$

$B \rightarrow AT_c$

$T_a \rightarrow a$

$T_b \rightarrow b$

$T_c \rightarrow c$

Step 3 : introducing intermediate variables.

$S \rightarrow AV_1$

$V_1 \rightarrow BT_a$

$A \rightarrow T_aV_2$

$V_2 \rightarrow T_aT_b$

$B \rightarrow AT_c$

$T_a \rightarrow a$

$T_b \rightarrow b$

$T_c \rightarrow c$

$\therefore$ above obtained result is in CNF.

Ans6 Consider the grammar $S \rightarrow S+S \mid S-S \mid S*S \mid S/S \mid (S) \mid x \mid y \mid z$ is the productions of G. Check for ambiguity : $\omega \rightarrow (x+y)*x-z*y/(x+x)$

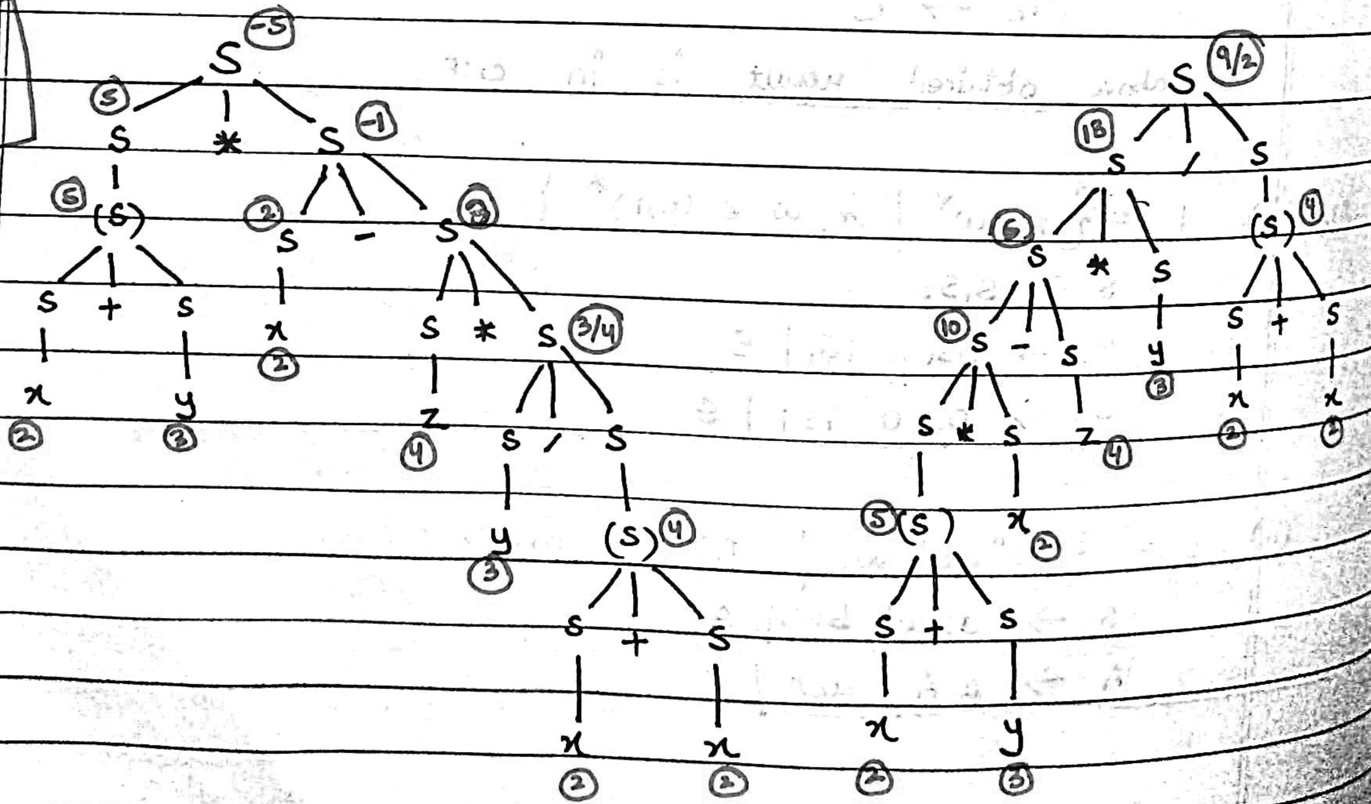
Derivative 1

$S \rightarrow S*S$
$\rightarrow (S)*S$
$\rightarrow (S+S)*S$
$\rightarrow (x+y)*S$
$\rightarrow (x+y)*S-S$
$\rightarrow (x+y)*x-S$
$\rightarrow (x+y)*x-S*S$
$\rightarrow (x+y)*x-z*S$
$\rightarrow (x+y)*x-z*S/S$
$\rightarrow (x+y)*x-z*y/S$
$\rightarrow (x+y)*x-z*y/(S)$
$\rightarrow (x+y)*x-z*y/(S+S)$
$\rightarrow (x+y)*x-z*y/(x+x)$

Derivative 2

$S \rightarrow S/S$
$\rightarrow S/(S)$
$\rightarrow S/(S+S)$
$\rightarrow S/(x+x)$
$\rightarrow S*S/(x+x)$
$\rightarrow S*y/(x+x)$
$\rightarrow S-S*y/(x+x)$
$\rightarrow S-z*y/(x+x)$
$\rightarrow S*S-z*y/(x+x)$
$\rightarrow S*x-z*y/(x+x)$
$\rightarrow (S)*x-z*y/(x+x)$
$\rightarrow (S+S)*x-z*y/(x+x)$
$\rightarrow (x+y)*x-z*y/(x+x)$

let $x=2$
$y=3$
$z=4$



Since in this grammar, there are more than two derivation & parse tree. ∴ it is ambiguous.