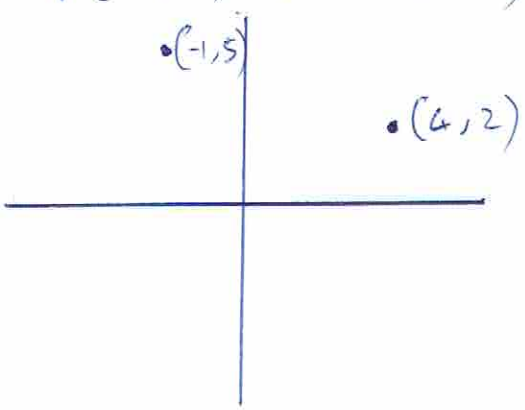
Find the distance between points:

a) P(-1, 5) and Q(4, 2)



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(4 - (-1))^2 + (2 - 5)^2}$$

$$d = \sqrt{25 + 9}$$

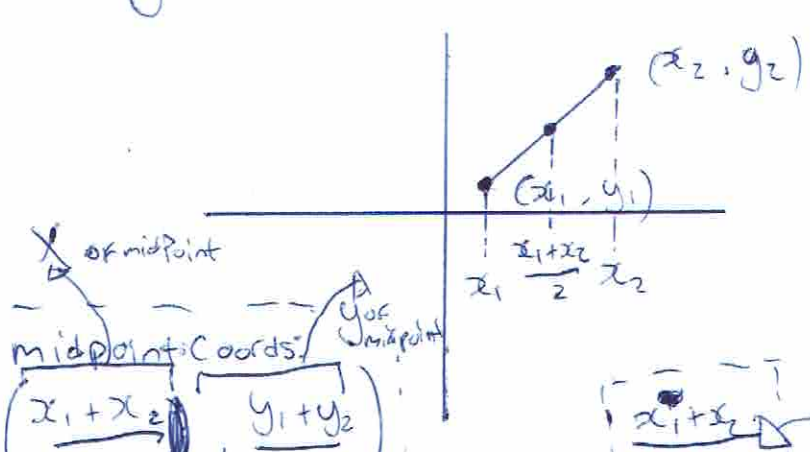
$$d = \sqrt{34}$$

Midpoint Formula

- Helps us find the coordinates of the midpoint of a line segment, as long as we know the coordinates of the endpoints

- The y-Coord of the midpoint will be exactly halfway in between the y-Coords of endpoints
  - i.e the average of the y-coords of endpoints

$$\frac{y_1 + y_2}{2}$$

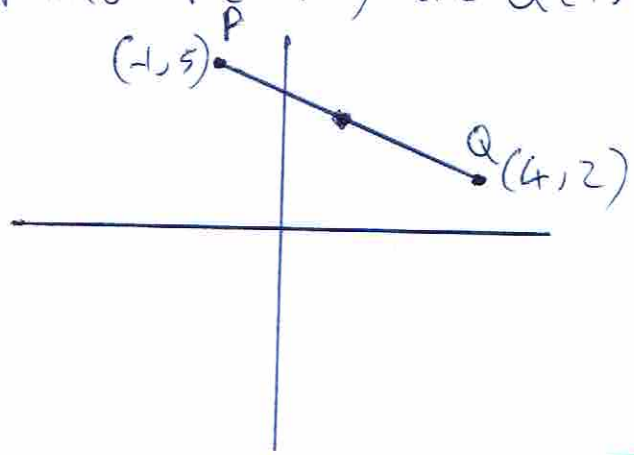


- The x-Coord of the midpoint will be exactly halfway in between the x-coords of the endpoints

$$\frac{x_1 + x_2}{2}$$

- we can get this by taking the average of endpoints

Find the midpoint of the segment between the points: $P(-1,5)$ and $Q(4,2)$



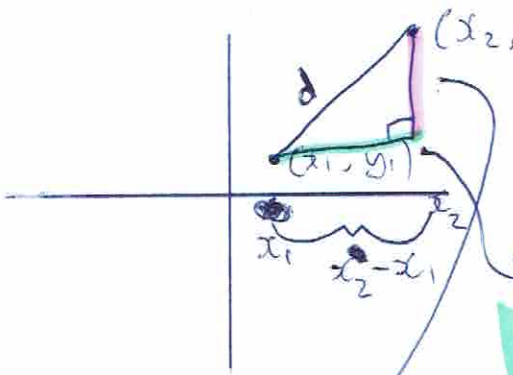
Midpoint formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\therefore \left(\frac{-1+4}{2}, \frac{5+2}{2}\right)$

$= \left(\frac{3}{2}, \frac{7}{2}\right)$

Distance Formula

- We can use this to find the distance between two points, if we know their coordinates



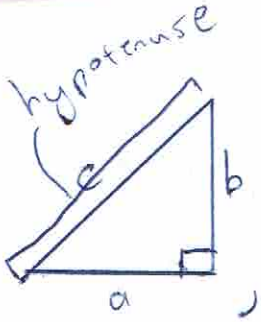
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Length of the horizontal side is the difference between the two x-coordinates

Length of the vertical side is the difference between the two y coordinates $(y_2 - y_1)$

Pythagorean Theorem

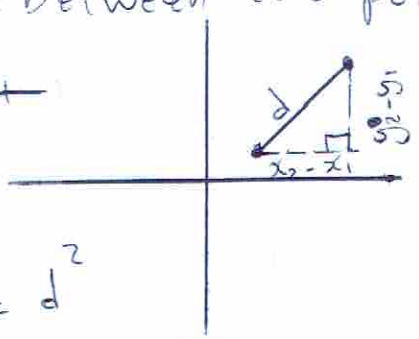
- For any right triangle, given



$$a^2 + b^2 = c^2$$

* Since distance is always positive, we don't need to consider the square root of two points

- We can say that the distance between two points is an hypotenuse ~~if we construct~~



$$a^2 + b^2 = c^2$$

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$$

$$\therefore \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\frac{x^2+6x+9}{x-1} \leq 0$$

① ✓

② Solve

③ Find where rational expression does not exist

$$\frac{x^2+6x+9}{x-1} = 0$$

$$x^2+6x+9 = 0$$

$$x^2+3x+3x+9 = 0$$

$$x(x+3)+3(x+3)$$

$$(x+3)(x+3)$$

$$x = -3$$

③ If the denominator is $x-1$, then to make the expression irrational, $x = 1$ since $1-1=0$, and you can't have 0 as a denominator

$x < 1$

$(-\infty, 1)$

$$\frac{(-4)^2+6(-4)+9}{-4-1}$$

$$= \frac{16-24+9}{-5} = \frac{1}{-5} = -\frac{1}{5}$$

$$\frac{(0)^2+6(0)+9}{0-1}$$

$$= -9$$

$$\frac{(2)^2+6(2)+9}{2-1}$$

$$\frac{4+12+9}{1} = \frac{25}{1} = 25$$

Different Types of Acceleration

- Free Fall

↳ When the force of gravity which is acting on an object makes the object accelerate at a rate of $9.81\ m/s^2$

↳ we can abbreviate this as 'g'

The Displacement Curve

- takes acceleration, starting velocity, time duration of motion as variables

↳ uses these variables to ascertain Displacement value

Displacement Curve Equation

$$X - X_0 = V_0 t + \frac{1}{2} a t^2$$

- X: Position
- X_0: initial Position
- V_0: initial velocity
- t: time
- a: acceleration
- t^2: time squared

- If acceleration is the change in velocity, and the velocity is the change in position, then there should be some way to unify acceleration and velocity theorems in such a way that we can understand the relationship between Position and Acceleration

$$V^2 = V_0^2 + 2a(x - x_0) \qquad x - x_0 = \left(\frac{1}{2}\right)(V_0 + V)t$$

Dealing with dynamic (changing) velocity

- If we wanted to know our average velocity for a specific period of time, we only need to know the quotient of the change in position (Δx) & the change in time (Δt)

  ↳ To understand the rate of change of something, we only need understand the ~~starting~~ final value subtracted by the starting value, divided by some amount of time elapsed

- We abbreviate 'change' with Δ ← the delta symbol

Constant Acceleration

- Acceleration which takes place at a constant rate

  ↳ This allows us to describe Average Velocity as:

$$\bar{v} = v_0 + at$$

(v_0: initial velocity; t: time; a: acceleration; $\bar{v}$: average velocity)

  ↳ This equation is called

The Definition of Acceleration

  ↳ Constant acceleration is equal to the change in velocity divided by the change in time

  ∴ Average velocity is equal to initial velocity + the product of acceleration and time.

Velocity $v = \frac{\Delta \text{Position}}{\text{time}}$

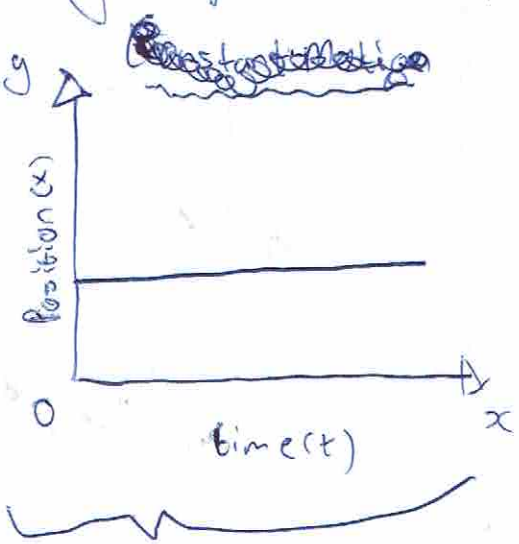
- The change in position over time

- Similar to speed, except that it also describes which direction you're moving in, based on whether displacement is positive or negative

Change in Velocity is Acceleration $a = \Delta \text{velocity}$

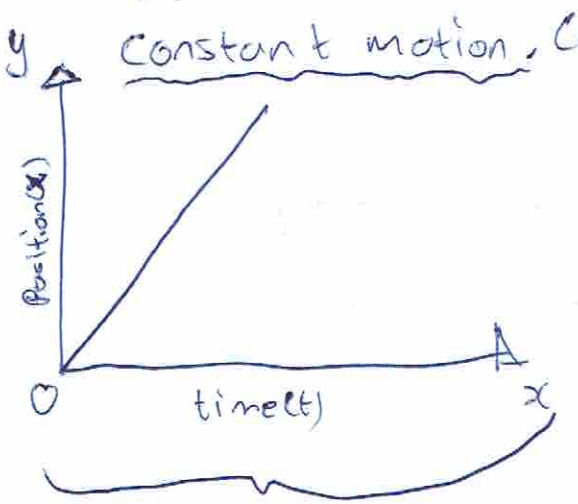
- Acceleration = change in velocity

Using Graphs to illustrate Kinematics



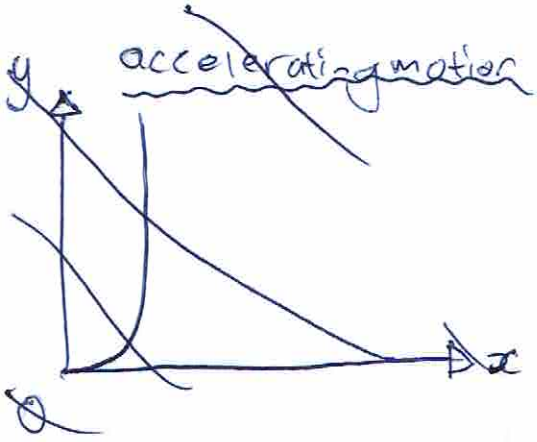
↳ No motion

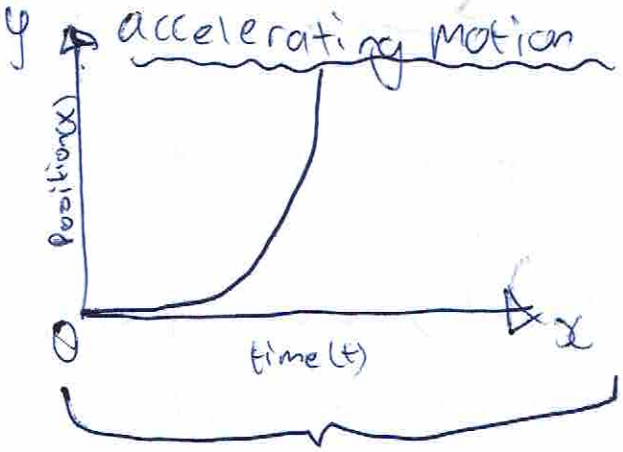
↳ So an object finds at a standstill



↳ Creates a perfectly diagonal curve

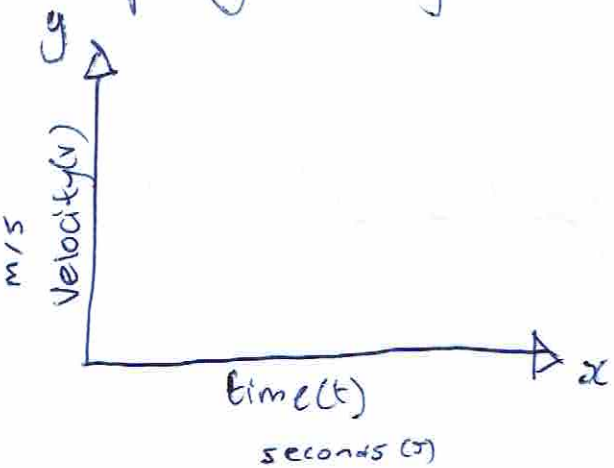
↳ Position and time are directly proportional





- Results in a curved graph as opposed to the straight line of no motion, and directly proportional diagonal line of constant motion

Graphing Velocity & Acceleration



- Since we describe Velocity as the change in position over time, we measure it in meters per second, meters/second, m/s

- Since acceleration can be described as the change in meters per second, we can measure it in $\frac{\text{meters per second}}{\text{second}}$, $\frac{m/s}{s} = m/s \times \frac{1}{s}$

- Velocity will usually be dynamic
- Time, position, velocity & acceleration all relate to each other

v = average velocity = $\frac{\Delta x \text{ (Position)}}{\Delta t \text{ (time)}}$ → Change in Position

a = average acceleration = $\frac{\Delta v \text{ (velocity)}}{\Delta t \text{ (time)}}$ → Change in velocity; → Change in time

Long Term Debt

- Loans longer than 12 months

- mortgage
  - When real estate stands as collateral for a long term loan

Owners Equity Capital

- money that owners invest in a business

*Note that owners are only residual owners until creditors are paid.

- Can't retrieve equity until creditors are paid
- If the company is liquidated, creditors are paid before owners

Retained Earnings

- Earnings reinvested into business

Owners Equity Capital Formula

$OEC = i + CP + w$

$OEC = i + er$

OEC = Owners Equity Capital
i = Owners Investment
CP = Cumulative Profits
w = Owners Cash withdrawals
er = Earnings Retained within business

Accrual-Basis Accounting

- When income is recorded when it's earned regardless of whether it's been received in cash
- when expenses are recorded as they are incurred regardless of whether the money has been paid out.

- Income Statement is Accrual-Based
  - does not consider the purchase of equipment that will last for more than a year as an expense

Depreciation Expense

- When the total amount of purchased equipment is recorded as an asset, and the cost is depreciated over the equipments lifespan
  - it's useful life
- We use depreciation to match an asset with the sales generated from its use

- * Not considered a cash flow

Earnings before Interest, Taxes, Depreciation and Amortization Formula (EBITDA)

$EBITDA = oi + d$

oi = Operating Income
d = depreciation

After-Tax Cash flows from Operations Formula (ATCFO)

$ATCFO = EBITDA - CTP$

CTP = Cash Tax Payments

Cash Taxes Formula (CT)

$CT = it - \Delta adt$

CT = Cash Taxes
it = Income Taxes
Δadt Changes in Accrued or Deferred Taxes

Change in Net Operating Working Capital Formula ($\Delta NOWC$)

$\Delta NOWC = \Delta ca - \Delta nibcol$

Δca = Change in Current Assets
$\Delta nibcol$ = Change in non-interest-bearing current operating liabilities

Change in Long-Term Assets Formula (ΔLTA)

$\Delta LTA = gpfa + nci$

gpfa = Gross Purchase Price of Fixed Assets
nci = Net Cash used for Investments

Cash-Basis Accounting

- Income Recorded when cash is received
- Expenses recorded when cash is paid

Firms Cash Flows = Financing Cash Flows

Firms Cash Flows

Cash Flow formula

$CF = atcf - ic - ia$

atcf = After Tax Cash Flows
ic = Investments in Net Operating working capital
ia = Investments in fixed assets and other assets
cf = Cash Flow

Three steps to Calculating Cash Flows

1. Convert Income Statement from Accrual Basis to Cash-Basis
   a) Compute after tax cash-flows from operations
2. Calculate change in net operating working capital
3. Calculate change in fixed assets and other assets

Financing Cash Flows

Financing cash flows formula

$CF = i + \Delta dp + div + \Delta s$

i = Interest Payments to Creditors

Δdp = Increase in Debt Principal
or Decrease in Debt Principal
i.e Change in Debt Principal

div = dividends paid to stockholders

Δs = increase or decrease in stock
i.e change in stock

Positive Cash Flows

↳ net cash flows received by investors

Negative Cash Flows

↳ cash paid into the firm by investors
↳ Capital Infusion

Total Debt & Equity Formula

$t = d + o$

d = Debt Capital

o = Owners Equity Capital

t = Total Debt & Equity

Debt Capital Formula

$dc = C + L$

C = Current Debt

L = Long Term Debt

dc = Debt Capital

Current Debt Formula

$cd = a + o + e + st$

a = Accounts Payable

o = other payables

e = Accrued Expenses

st = Short-Term notes

cd = Current Debt

Long-Term Debt Formula

$ltd = Lt + m$

Lt = Long-term Notes

m = Mortgages

ltd = long-term debt

Total Assets Formula

$t = c + f + o$

C = Current Assets
F = Fixed Assets
O = Other Assets
t = Total Assets

Current Assets Formula

$ca = c + a + i + p$

C = Cash
a = accounts receivable
i = inventories
P = Prepaid Expenses
ca = Current Assets

Fixed Assets Formula

$fa = m + b + L$

m = machinery & equipment
b = buildings
L = Land
Fa = Fixed Assets

Other Assets Formula

$oa = i + p$

i = investments
P = Patents
Oa = Other Assets

Owners Equity ~~Formula~~ Capital Formula

oec = O || P || C

O = Owners Networth

P = Partnership Equity

C = Common Stock Equity

oec = Owners Equity Capital

Types of Assets

→ assets that are relatively liquid

1. Current Assets
2. Fixed Assets
3. Other Assets

Liquid Assets

- "Can be converted into cash within a given operating cycle"

Current Assets include:

- cash
- accounts receivable
- inventories
- Prepaid Expenses

* Small business can run into issues when current assets aren't managed properly

Cash Reservoir

- a business should have a cash reservoir
  - ↳ helps compensate for unequal cash flow into and out of the business

- Size of reservoir is comensurate to volume of sales & predictability of cash receipts and cash payments

Accounts Receivable

- Payments due from credit sales

Inventories ↳ is proprietary tech considered inventory

- Raw materials & Products held by the firm
- Seasonality of sales & Production Levels affects the size of an inventory

Prepaid Expenses

- Expenses that are paid in advance
- We record these payments current assets for accounting purposes, but once they are used, they are recorded as operating expenses

Net Income Available To Owners

Keys: - Reinvest
- Distribute

- Income that can be reinvested into firm
- Distributed to owners

Definition:

- All Equity Company

  ↳ A business without Debt

- Operating income indicates profitability

The Balance Sheet

- "Snapshot of a business's financial position at a specific point in time."

- Captures cumulative effects of a business's financial decision making

- Shows:- liabilities
  - Owner Investment

Basic Balance Sheet Formula

$TA = OD + OE$

od = Outstanding Debt
oe = Owners Equity
TA = Total Assets

Earnings Before Interest & Taxes ~~Operating Activities~~ Formula

$e = g - ox$

e = Earnings before interest & taxes (Operating Income)

g = Gross Profit

ox = Operating Expenses

Gross profit

$g = s - c$

g = Gross Profit

s = Sales Revenue

c = Cost of Producing / Acquiring Good or Service

Operating Expenses

$ox = m + d + a$

ox = Operating Expenses

m = Marketing Expense

s = selling Expense

a = administration Expense

Earnings Before Taxes ~~Financing Activities~~ Formula

$e = i - ix$

e = Earnings before taxes

i = Operating Income

ix = Interest Expense on Debt (financing costs)

Net Income Formula

$ni = e - t$

ni = Net Income

e = Earnings Before Taxes

t = Corporate Taxes

The Income Statement

- Profit vs Loss.
- Profits generated over a given time period

Basic Income Equation

$S - X = P$

S = Sales
X = Expenses
P = Profits

Financial Statement gives us 5 key Data Points

1. Revenue
2. Costs
3. Operating Expenses
4. Financing Costs
5. Tax

Keys:

- Sales
- Producing
- acquiring
- marketing
- distribution
- admin
- interest

Revenue:

- Revenue from sales

Costs

- Costs of Producing goods/services
- Costs of acquiring goods/services

Operating Expenses:

- Marketing Expenses
- Distribution Expenses
- Administration Expenses

Financing Costs

- costs of doing business
- interest paid to creditors

What makes a good investment Opportunity?

- Creates Competitive Advantage
- Meets a need
- Profitability
- Investment required needs to be worth Projected Profits
  - ie economically feasible

Keys:
- Competitive
- Needs
- Profitability
- Feasability

Financial Statements

- Used to evaluate performance & financial resources
- Key Financial Statements
  - Income Statement
  - Balance Sheet
  - Cash Flow Statement

Keys:
- Performance
- Requirements

- determine financial requirements
- assess financial soundness of business plan