var, x			
i			
$typ,\ A,\ B,\ C$	::=		type
	|	Int	
	|	$A \to B$	
	|	$A \vee B$	
	|	(A)	S
$exp,\ e$	::=		expression
	|	x	
	|	i	
	|	$e : A$	
	|	$\lambda x.e$	$\mathsf{bind}\ x\ \mathsf{in}\ e$
	|	$e_1\ e_2$	
	|	(e)	S
	|	$e_1[x \rightsquigarrow e_2]$	M
$ctx,\ \Gamma$	::=		typing context
	|	$\cdot$	
	|	$\Gamma, x : A$	
$dirflag$	::=		typing direction
	|	$\Rightarrow$	
	|	$\Leftarrow$	
$terminals$	::=		
	|	λ	
	|	$\longrightarrow$	
	|	$\rightsquigarrow$	
	|	$\to$	
	|	$\vee$	
	|	$\vdash$	
	|	$\mathbf{Int}$	
	|	$\in$	
	|	;	
	|	fv	
$formula$	::=		
	|	$judgement$	
	|		
	|	$x : A \in \Gamma$	
	|	$x\ \mathbf{notin}\ \mathsf{fv}\, e$	
	|	$x \notin \Gamma$	
$Value$	::=		
	|	$\mathbf{value}\ e$	Values
$Infterm$	::=		
	|	$\mathbf{infterm}\ e$	Inferable terms
$Typing$	::=		

	$\Gamma \vdash e\ \mathit{dirflag}\ A$	Typing rules

Steping	::=	
	$e \longrightarrow e'$	Small-step operational semantics

judgement	::=	
	Value	
	Infterm	
	Typing	
	Steping	

user_syntax	::=	
	var	
	i	
	typ	
	exp	
	ctx	
	dirflag	
	terminals	
	formula	

$\boxed{\mathbf{value}\ e}$ Values

$$\frac{}{\mathbf{value}\ i}\ \text{VALUE\_LIT}$$

$$\frac{}{\mathbf{value}\ (\lambda x.e) : A \to B}\ \text{VALUE\_ANN}$$

$\boxed{\mathbf{infterm}\ e}$ Inferable terms

$$\frac{}{\mathbf{infterm}\ i}\ \text{INFT\_LIT}$$

$$\frac{}{\mathbf{infterm}\ e_1\ e_2}\ \text{INFT\_APP}$$

$\boxed{\Gamma \vdash e\ \mathit{dirflag}\ A}$ Typing rules

$$\frac{}{\Gamma \vdash i \Rightarrow \mathsf{Int}}\ \text{TYP\_LIT}$$

$$\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A}\ \text{TYP\_VAR}$$

$$\frac{\Gamma \vdash e \Leftarrow A}{\Gamma \vdash e : A \Rightarrow A}\ \text{TYP\_ANN}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow A \to B \quad \Gamma \vdash e_2 \Leftarrow A}{\Gamma \vdash e_1\ e_2 \Rightarrow B}\ \text{TYP\_APP}$$

$$\frac{\Gamma \vdash e \Rightarrow A}{\Gamma \vdash e \Leftarrow A}\ \text{TYP\_CHK}$$

$$\frac{\Gamma, x : A \vdash e \Leftarrow B}{\Gamma \vdash \lambda x.e \Leftarrow A \to B}\ \text{TYP\_ABS}$$

$\boxed{e \longrightarrow e'}$ Small-step operational semantics

$$\frac{}{(\lambda x.e) : A \to B\ i \longrightarrow (e[x \rightsquigarrow i]) : B}\ \text{STEP\_BETA\_LIT}$$

$$\frac{}{(\lambda x.e) : A \to B\ (e_2 : C) \longrightarrow (\lambda x.e) : A \to B\ e_2}\ \text{STEP\_BETA\_ANN}$$

$$\frac{}{(\lambda x.e) : A \to B\ (\lambda x.e_1) \longrightarrow (e[x \rightsquigarrow (\lambda x.e_1) : A]) : B}\ \text{STEP\_BETA\_ABS}$$

$$\frac{e_1 \longrightarrow e_1'}{e_1\ e_2 \longrightarrow e_1'\ e_2}\ \text{STEP\_APPL}$$

$$\frac{\begin{array}{l}\textbf{value}\ e_1 \\ \textbf{infterm}\ e_2 \\ e_2 \longrightarrow e_2'\end{array}}{e_1\ e_2 \longrightarrow e_1\ e_2'}\ \text{STEP\_APPR}$$

$$\frac{}{e : A : B \longrightarrow e : B}\ \text{STEP\_ANN\_ANN}$$

$$\frac{\textbf{infterm}\ e}{e : A \longrightarrow e}\ \text{STEP\_ANN\_INF}$$

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Definition rules:          17 good    0 bad
Definition rule clauses:   30 good    0 bad
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