

Disjoint Union Types

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1 Rules



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XX:2 Disjoint Union Types

Types	A, B	$::=$	$\perp \mid \text{Int} \mid A \rightarrow B \mid A \vee B$
Terms	e	$::=$	$x \mid n \mid \lambda x. e : A \rightarrow B \mid e_1 e_2 \mid e : A \mid \text{typeof } e \text{ as } \{x : A \rightarrow e_1, x : B \rightarrow e_2\}$
Values	v	$::=$	$n \mid \lambda x : A. e$
Context	Γ	$::=$	$\bullet \mid \Gamma, x : A$

DisSpec	$A * B$	$::=$	$\forall C, C <: A \wedge C <: B \rightarrow \rfloor C \rfloor$
---------	---------	-------	-----------------------------------------------------------------

$\rfloor C \rfloor$ (Bottom Like)

$$\frac{}{\rfloor \perp \rfloor} \text{BL-BTM} \qquad \frac{\rfloor A \rfloor \quad \rfloor B \rfloor}{\rfloor A \vee B \rfloor} \text{BL-OR}$$

$A \star_i B$ (Algorithmic Disjointness)

$$\frac{}{\perp \star_i A} \text{AD-BTML} \qquad \frac{}{A \star_i \perp} \text{AD-BTMR} \qquad \frac{}{\text{Int} \star_i A \rightarrow B} \text{AD-INTL} \qquad \frac{}{A \rightarrow B \star_i \text{Int}} \text{AD-INTR}$$

$$\frac{A \star_i C \quad A \star_i C}{A \vee B \star_i C} \text{AD-ORL} \qquad \frac{A \star_i C \quad A \star_i C}{C \star_i A \vee B} \text{AD-ORR}$$

$A <: B$ (Subtyping)

$$\frac{}{\perp <: A} \text{TS-BTM} \qquad \frac{}{\text{Int} <: \text{Int}} \text{TS-INT} \qquad \frac{B_1 <: A_1 \quad A_2 <: B_2}{A_1 \rightarrow A_2 <: B_1 \rightarrow B_2} \text{TS-ARROW}$$

$$\frac{A_1 <: A \quad A_2 <: A}{A_1 \vee A_2 <: A} \text{TS-ORA} \qquad \frac{A <: A_1}{A <: A_1 \vee A_2} \text{TS-ORB} \qquad \frac{A <: A_2}{A <: A_1 \vee A_2} \text{TS-ORC}$$

$A <<: B$ (Subsub)

$$\frac{}{A <<: A} \text{TSS-REFL} \qquad \frac{B_1 <: A_1 \quad A_2 <<: B_2}{A_1 \rightarrow A_2 <<: B_1 \rightarrow B_2} \text{TSS-ARROW} \qquad \frac{A <<: A_1}{A <<: A_1 \vee A_2} \text{TSS-ORA}$$

$$\frac{A <<: A_2}{A <<: A_1 \vee A_2} \text{TSS-ORB}$$

■ **Figure 1** Disjoint Union Type : Syntax

$\Gamma \vdash e \Rightarrow A$

(Bidirectional Typing)

$$\begin{array}{c}
\frac{x : A \in \Gamma}{\Gamma \vdash x \Rightarrow A} \text{ T-VAR} \quad \frac{}{\Gamma \vdash n \Rightarrow \text{Int}} \text{ T-INT} \quad \frac{\Gamma, x : A_1 \vdash e \Leftarrow A_2}{\Gamma \vdash \lambda x. e : A_1 \rightarrow A_2 \Rightarrow A_1 \rightarrow A_2} \text{ T-ABS} \\
\\
\frac{\Gamma \vdash e_1 \Rightarrow A_1 \rightarrow A_2 \quad \Gamma \vdash e_2 \Leftarrow A_1}{\Gamma \vdash e_1 e_2 \Rightarrow A_2} \text{ T-APP} \quad \frac{\Gamma \vdash e \Rightarrow B \quad B <: A}{\Gamma \vdash e \Leftarrow A} \text{ T-SUB} \\
\\
\frac{\Gamma \vdash e \Leftarrow A}{\Gamma \vdash (e : A) \Rightarrow A} \text{ T-ANNO} \\
\\
\frac{\Gamma \vdash e \Leftarrow A \vee B \quad \Gamma, x : A \vdash e_1 \Leftarrow C \quad \Gamma, x : B \vdash e_2 \Leftarrow C \quad A \star_i B}{\Gamma \vdash \text{typeof } e \text{ as } \{x : A \rightarrow e_1, x : B \rightarrow e_2\} \Leftarrow C} \text{ T-TYPEOF}
\end{array}$$

$e \longrightarrow e'$

(Reduction)

$$\begin{array}{c}
\frac{}{(\lambda x. e : A \rightarrow B) v \longrightarrow [x \mapsto v] e : B} \text{ RED-BETA} \quad \frac{e_1 \longrightarrow e'_1}{e_1 e_2 \longrightarrow e'_1 e_2} \text{ RED-APPL} \\
\\
\frac{e_2 \longrightarrow e'_2}{v e_2 \longrightarrow v e'_2} \text{ RED-APPR} \quad \frac{e \longrightarrow e'}{e : A \longrightarrow e' : A} \text{ RED-ANNO} \quad \frac{}{v : A \longrightarrow v} \text{ RED-ANNOV} \\
\\
\frac{e \longrightarrow e'}{\text{typeof } e \text{ as } \{x : A \rightarrow e_1, x : B \rightarrow e_2\} \longrightarrow \text{typeof } e' \text{ as } \{x : A \rightarrow e_1, x : B \rightarrow e_2\}} \text{ RED-TYPEOF} \\
\\
\frac{v :: A}{\text{typeof } v \text{ as } \{x : A \rightarrow e_1, x : B \rightarrow e_2\} \longrightarrow [x \mapsto v] e_1} \text{ RED-TYPEOFVL} \\
\\
\frac{v :: B}{\text{typeof } v \text{ as } \{x : A \rightarrow e_1, x : B \rightarrow e_2\} \longrightarrow [x \mapsto v] e_2} \text{ RED-TYPEOFVR}
\end{array}$$

$v :: A$

(Type Reduction)

$$\begin{array}{c}
\frac{}{n :: \text{Int}} \text{ TRED-INT} \quad \frac{A_1 <: B_1 \quad B_2 <: A_2}{(\lambda x. e_1) : B_1 \rightarrow B_2 :: A_1 \rightarrow A_2} \text{ TRED-BETA} \quad \frac{v :: A}{v :: A \vee B} \text{ TRED-ORL} \\
\\
\frac{v :: B}{v :: A \vee B} \text{ TRED-ORR}
\end{array}$$

■ **Figure 2** Disjoint Union Type : Semantics