

Warm-up

- Definition of antiderivative of $f(x)$?
- What is an anti derivative of x^n ?
- What is another anti derivative of x^n ?

Warm-up

- Let f be a continuous function.
Find an anti derivative of f .

Fundamental theorem of calculus (part 1)

Let $F(x) = \int_a^x f(t) dt$. dummy variable t

Then $\frac{d}{dx} F(x) = f(x)$

Chain rule

Compute let $\bar{F}(x) = \int_0^{x^2} \sin(u^5) du$. find $\bar{F}'(x)$.

$$F(x) = \tilde{F}(v(x)) \text{ where } \tilde{F}(v) = \int_0^v \sin(u^5) du$$
$$v(x) = x^2$$

$$\begin{aligned} \text{so, } \frac{d}{dx} F(x) &= \frac{d}{dx} \tilde{F}(v(x)) = \frac{d\tilde{F}}{dv} \frac{dv}{dx} \\ &= \sin(v^5) \cdot 2x \\ &= \sin(x^{10}) \cdot 2x. \end{aligned}$$

Application of FTC - 1

Sketch the graph of $\int_0^x \frac{t^3+1}{t^2+1} dt$

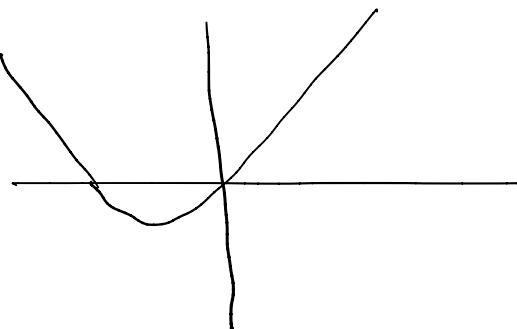
graphing requires derivatives.

$$\frac{d}{dx} \int_0^x \frac{t^3}{t^2+1} dt = \frac{x^3+1}{x^2+1}$$

so, critical points is $x = -1$

increasing on $[-1, \infty)$

decreasing on $[-\infty, -1]$



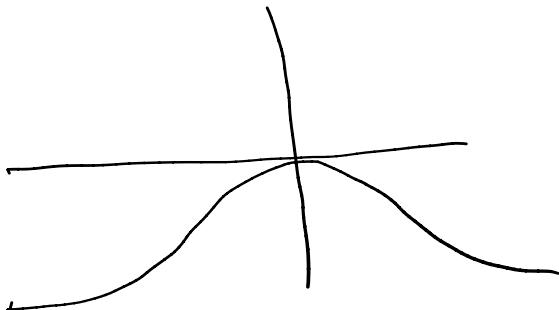
Application of FTC - 1

Sketch the graph of $\int_0^x -2t e^{-t^2} dt$

$$\frac{d}{dx} \int_0^x -2t e^{-t^2} dt = -2x e^{-x^2} = 0 \quad \text{if } x = 0 .$$

$$u = -t^2 \\ du = -2t dt \\ \int_0^{-t^2} e^u du =$$

$$e^u \Big|_0^{-t^2} = e^{-t^2} - 1$$



Fundamental theorem of calculus part 2.

Let f be a continuous function and

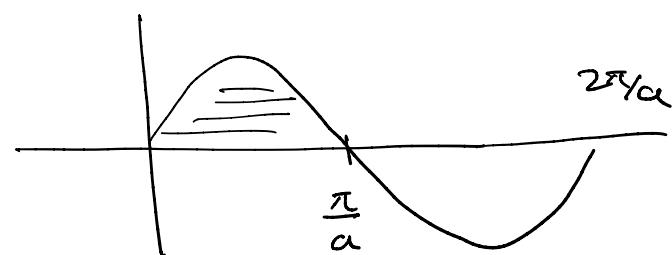
let $F'(x) = f(x)$.

Then $\int_a^b f(x) dx = F(b) - F(a).$

Q: $\int_0^{\pi/a} \sin ax dx = -\frac{1}{a} \cos(ax) \Big|_0^{\pi/a}$

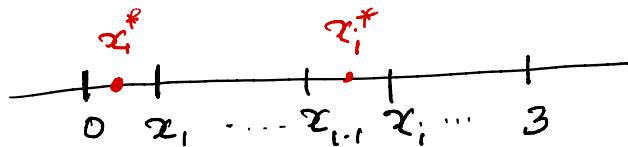
$$= \frac{2}{a}$$

period of $\sin ax$ is $\frac{2\pi}{a}$



Q. Given a rod of length 3m and density of $\delta(x) = (2x^2 + 1) \text{ g/m}$ find the mass of the rod.

First build Riemann sum:



$$\text{mass of rod on } [x_{i-1}, x_i] = \Delta x \cdot f(x_i^*)$$

$$R_n = \sum_{i=1}^n \Delta x f(x_i^*) \approx M$$

$$M = \int_0^3 (2x^2 + 1) dx = \frac{2}{3}x^3 + x \Big|_0^3 = 18 + 3 = 21 \text{ g}$$

Alternate proof of FTC-2.

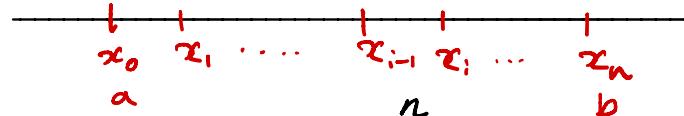
Let $F(t)$ be the position of mass moving along a line.

$F'(t) = f(t)$ is the velocity.

want to show: $\underbrace{F(b) - F(a)}_{\text{displacement}} = \int_a^b f(t) dt$

i.e. integral of velocity is displacement.

Building Riemann sum:



$$\text{Approximate displacement} = \sum_{i=1}^n f(x_i^+) \Delta x$$

$$\text{Exact displacement} = \int_a^b f(x_i) dx.$$

$$\text{Exact displacement} = F(b) - F(a) \text{ (also)}$$

Important integrals

See them 1.3.16 in CLP-2 book.

| $f(x)$ | $F(x)$ (anti derivative) |
|---------------|--|
| 1 | $x + C$ |
| x^n | $\frac{x^{n+1}}{n+1} + C, \quad n \neq -1$ |
| $\frac{1}{x}$ | $\ln x + C$ |
| e^x | $e^x + C$ |
| \vdots | \vdots |

Indefinite integrals-

$$\int_a^b f(x) dx \quad \text{vs.} \quad \int f(x) dx.$$

- Indefinite integral doesn't have terminals.
- Solution of indefinite integral contain constant of integration.
- Indefinite integral is a general anti derivative of the integrand.

Net change theorem

- o If f is continuous on $[a, b]$
- o and F is any antiderivative of f , then

$$\int_a^b f(x) dx = F(b) - F(a)$$

$$\text{or} \quad \int_a^b F'(x) dx = F(b) - F(a)$$

This is net change theorem. Integral of rate of change give net-change.

Approximate sum.

consider $f(x) = x(50-x)$

(Q) Find the total length of

support beams.

i.e. $\sum_{i=1}^n f(x_i) \Delta x$

so,
$$\begin{aligned}\sum_{i=1}^n f(x_i) &\approx \int_0^{50} f(x) dx \\ &= \int_0^{50} 50x - x^2 dx \\ &= \left[\frac{50x^2}{2} - \frac{x^3}{3} \right]_0^{50} \\ &= 20,833.3\end{aligned}$$

exact = 20825

