Quiz 3 solutions

Solution 2: False Let $g(x) = -\frac{1}{x^2}$ and $f(x) = -\frac{1}{x}$. Notice that $f(x) \leq g(x)$ and $\int_a^\infty g(x)\ dx$ converges. However, $\int_a^\infty f(x)\ dx$ is not convergent. **Note:** The statement was incorrectly set as true in WeBWorK. Full mark was given to all students.

Solution 3: False Consider two rectangles with base on the x-axis of different dimensions but same area.

Solution 4: The integral

$$I = \int_1^\infty \frac{x^{82}}{x^{np+1}}\ dx = \int_1^\infty \frac{1}{x^{np+1-82}}\ dx$$

converges if $np + 1 - 82 > 1$ and diverges if $np + 1 - 82 \leq 1$. So, the statement integral diverges if $p > \frac{41}{n}$ is false. **Note:** There is an error in WeBWorK version of this question. Full mark was given to all students.

Solution 5: The general partial fraction decomposition of

$$f(x) = \frac{x^3 + 1}{(x^2 - 16)(x^2 + 16)}$$

is

$$\frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x+4} + \frac{D}{(x+4)^2} + \frac{Ex + F}{x^2 + 16}.$$

Solution 6: WeBWorK solution of the question: The average value of the function $v(x) = 2/x^2$ on the interval $[1, c]$ is equal to 1. Find the value of c.

Solution: First of all, the function $\dfrac{1}{(4x+4)^3}$ is positive and decreasing for all $x \geq 1$, because its denominator is positive and increasing for $x \geq 1$. Also, the function goes to 0 as $x \to \infty$ because its denominator goes to infinity. Therefore, the Integral Test, and its Remainder Estimate, can be applied.

For part (A), the Remainder Estimate is

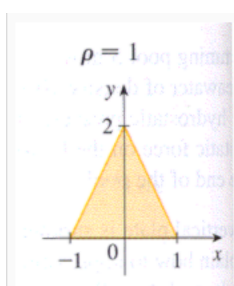
$$
\begin{aligned}
|s - s_n| &\leq \int_n^\infty \frac{dx}{(4x+4)^3} \\
&= \lim_{t\to\infty} \frac{-1}{4(2)(4x+4)^2}\Bigg]_n^t \qquad = \frac{1}{4(2)(4n+4)^2} .
\end{aligned}
$$

For part (B), we need to find the smallest positive integer n for which the above expression is less than 0.00002. So we set up the relevant inequality and solve it:

$$
\begin{aligned}
\frac{1}{4(2)(4n+4)^2} &< 0.00002 \\
(4n+4)^2 &> \frac{1}{4(2)0.00002} \\
(4n+4) &> \frac{1}{\sqrt[2]{4(2)0.00002}} \\
n &> \frac{\frac{1}{\sqrt[2]{4(2)0.00002}} - 4}{4} \approx 18.7642
\end{aligned}
$$

Rounding this up to the next largest integer gives $n = 19$.

Solution 7:



By symmetry, the moment about y-axis, M_y, is 0. The moment about the x-axis is,

$$
\begin{aligned}
M_x &= \int_{-1}^{1} \rho x \text{ Top function } dx \\
&= \int_{-1}^{0} x(2+2x)\ dx + \int_{0}^{1} x(2-2x)\ dx \\
&= \left[x^2 + \frac{2x^3}{3}\right]_{-1}^{0} + \left[x^2 + \frac{2x^3}{3}\right]_{0}^{1} \\
&= -\left(1 - \frac{2}{3}\right) + \left(1 + \frac{2}{3}\right) \\
&= \frac{4}{3}
\end{aligned}
$$

Since the mass of the enclosed is 2 (product of area and density), the center of mass is $(\bar{x}, \bar{y}) = (\frac{M_y}{\text{Mass}}, \frac{M_x}{\text{Mass}}) = (0, \frac{4}{6})$.