Differential equation review.

Solve the differential equation $\frac{dy}{dx} = -xy^3$, $y(0) = -\frac{1}{4}$

Sequences.

limit of a sequence (not so rigorous).

Definition: A sequence $\{a_n\}_{n=1}^{\infty}$ is said to converge to the limit L if a_n approaches A as $n \to \infty$. We write

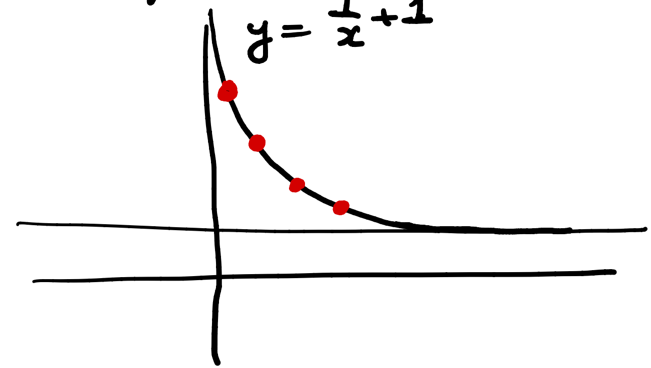
$$\lim_{n\to\infty} a_n = L \quad \text{or} \quad a_n \to L \text{ as } n \to \infty \quad \text{or} \quad a_n \to L.$$

Examples: $\left\{ a_n = \frac{1}{n} \right\}_{n=1}^{\infty}$ converges to zero since $\lim_{n\to\infty} \frac{1}{n} = 0$.

$\left\{ a_n = (-1) \right\}_{n=1}^{\infty}$ diverges since a_n oscillates between -1 and 1. for all n.

limit of a sequence. (rigorous)

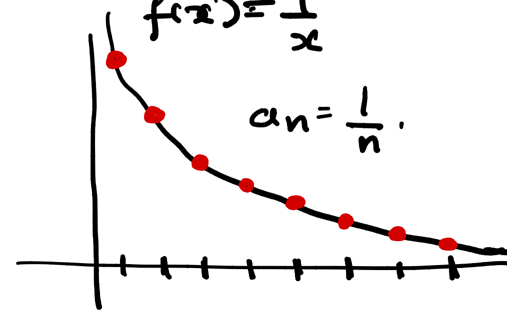
$a_n = f(n)$.



limit of sequence.

Thm: If $\lim_{x\to\infty} f(x) = L$ and $a_n = f(n)$ for $n = 1, 2, 3, \ldots$

then $\lim_{n\to\infty} a_n = L$.



Limit laws.

Thm Let $\{a_n\}$ and $\{b_n\}$ be convergent sequences with $a_n \to A$ and $b_n \to B$. Further let c be a constant. Then.

Example.

Let's use these rules to compute limit of sequences.

Compute $\displaystyle\lim_{n\to\infty} \frac{3n}{2n+7}$

Example

Now, you compute $\lim_{n \to \infty} \frac{3n^2+4}{5n^2+n+7}$.

limit of sequence.

What about $\lim_{n\to\infty} \sin(\pi/n)$?

we know $\pi/n \to 0$. Does $\sin(\pi/n) \to \sin(0)$?

Squeeze theorem.

Some sequences are very "messy" and may be challenging to analyze. We can squeeze it between two sequences.

Squeeze theorem.

Theorem If $a_n \le b_n \le c_n$ and $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} c_n = L$

then $\lim\limits_{b_n\to\infty} = L$.

Example.

- Compute $\lim_{n\to\infty} \frac{n!}{n^n}$

- Let $a_n \le b_n \le c_n$ with $a_n \to L$ and $c_n \to L+1$.
  Is b_n convergent?

- Given that $|a_n| \to 0$, does a_n converge?
  If yes, what is $\lim_{n\to\infty} a_n$?