PROBLEM (integration by parts)

DEFINE $I_n \equiv \int_0^\infty x^n e^{-x}\,dx$ FOR n integer WITH $n = 1, 2, \ldots$

INTEGRATE BY PARTS ONCE:

let $u = x^n$, $du = n x^{n-1} dx$

$dv = e^{-x} dx$ $\quad v = -e^{-x}$

SO $I_n = -x^n e^{-x} \Big|_0^\infty + n \int_0^\infty x^{n-1} e^{-x}\, dx$ FOR $n \geq 1$

$\longrightarrow$ $I_n = n I_{n-1}$, $n \geq 1$ (RECURSION relation)

WE CALCULATE $I_0 = \int_0^\infty e^{-x} dx = 1$ SO

$I_1 = (1) I_0$

$I_2 = 2 I_1$

$I_3 = 3 I_2 = 3 \cdot 2\, I_1 = 3 \cdot 2 \cdot 1 \cdot I_0 = 3!$

$I_4 = 4 I_3 = 4 \cdot 3 \cdot 2 \cdot 1 = 4!$

Proceding .. we get $I_n = n!$, $n \geq 1$ (don't prove induction step).

SO $\int_0^\infty x^{20} e^{-x}\, dx = 20!$

OTHER POSSIBLE IBP PROBLEMS ARE:

(i) $\int x \sin(3x)\, dx$

(ii) $\int x \ln x\, dx$

(iii) $\int_0^{2\pi} x^2 \cos(x)\, dx$

(iv) $I = \int_0^1 \sin(\ln x)\, dx$

(HARD: SUBSTITUTION + IBP let $u = -\ln x$, $du/dx = \frac{1}{-x}$ $\quad dx = -x\, du = -e^{-u} du$

so $I = \int_{+\infty}^0 (-e^{-u} \sin(-u))\, du = -\int_0^\infty e^{-u} \sin(u)\, du$)

SINCE $\sin(-u) = -\sin(u)$ $\quad$ THEN USE IBP TO GET $I = -\frac{1}{2}$ USING (V)

(V) $I = \int_0^\infty e^{-ax} \sin(bx)\, dx$ WITH $a > 0$, $b \neq$

IBP TWICE AND SOLVE FOR I

(this is needed below with $a = 1$, $b = 1$)