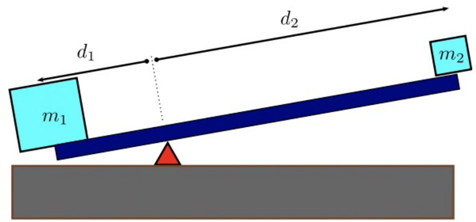
Center of mass.



The see-saw perfectly balances if the lever is at center of mass.

Torque, $T = F \times d$, F is the gravitational force
(rotational force) d is the distance from lever.

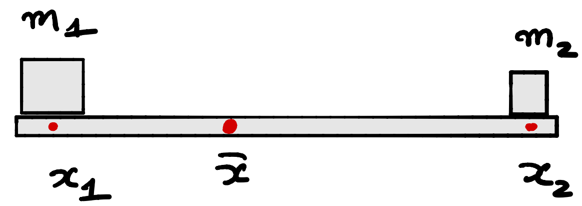
How do we calculate center of mass?

Balance torque exerted by m_1 and m_2 to computed center of mass.

Centre of mass. (1 D and discrete).

Assume mass m_1 is at x_1.

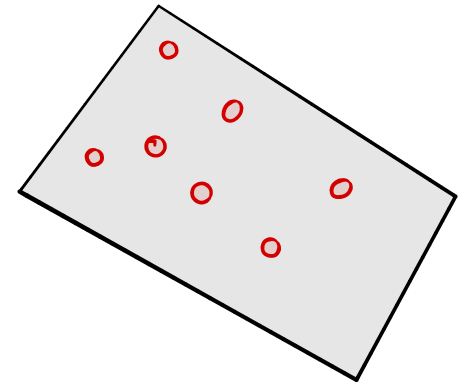
Assume mass m_2 is at x_2.



$$\boxed{\bar{x} = \frac{1}{m} \sum_{i=1}^{N} m_i x_i \quad , \; m = \sum_{i=1}^{N} m_i .}$$

Center of mass (2D and discrete).

In a 2D scenario with point masses $m_1, m_2, \ldots, m_N$ centered at $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$, the center of mass are given by



$$\bar{x} = \frac{1}{m} \sum_{i=1}^{N} m_i x_i \quad , \quad \bar{y} = \frac{1}{m} \sum_{i=1}^{N} m_i y_i$$

1D continuous case.

If a body consists of mass distributed along a straight line with density $\rho(x)$ (kg/m) with $a \leq x \leq b$, the center of mass $\bar{x}$ is:

Example.

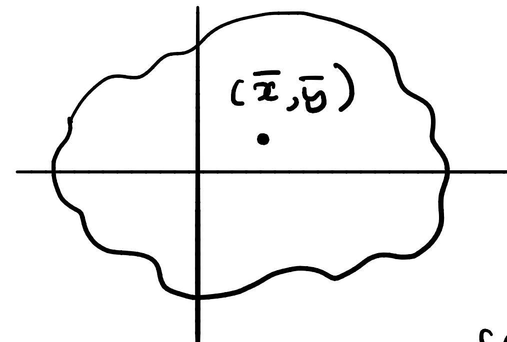
A metal rod is 50 cm long. Its linear density at point x from left end is $\rho(x) = \dfrac{1}{100-x}$ (gm/cm). Find the mass and center of mass of the rod.

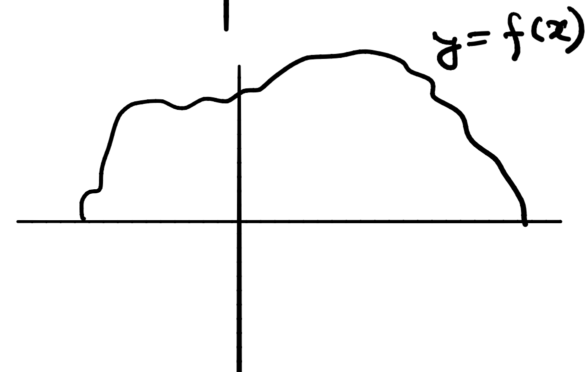
Sol^n

Centroid of a lamina.

Lamina is a thin "plate" which occupies some area in $\mathbb{R}^2$.
We will assume that the density $\rho(x,y)$ is constant.

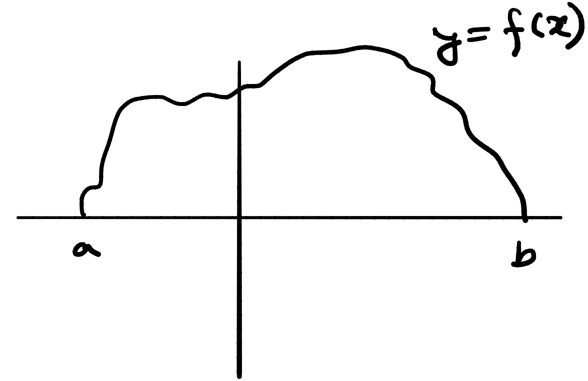
We want to calculate the centroid $(\bar{x},\bar{y})$, i.e.
the center of mass.





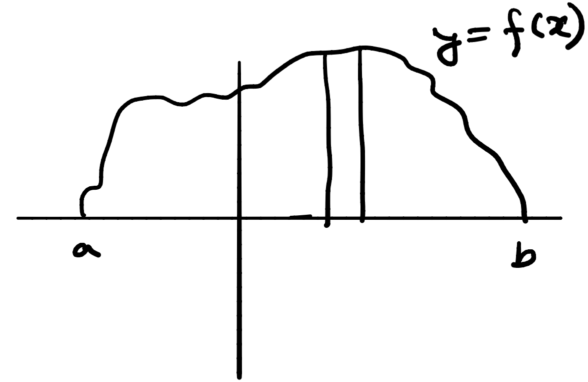
Case 1 lower bound is x-axis.

We first find moment about y-axis:



Case 1 lower bound is x-axis (contd.)

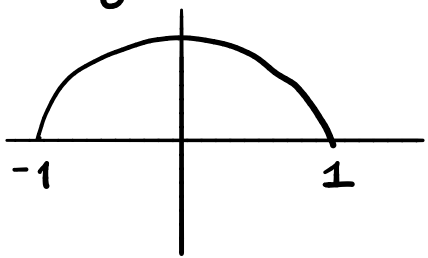
Now, we find moment about x-axis.



Example:

Find the center of mass of a parabolic plate $y = 1 - x^2$ above $y = 0$ and $-1 \le x \le 1$. Assume constant density.

Solⁿ:



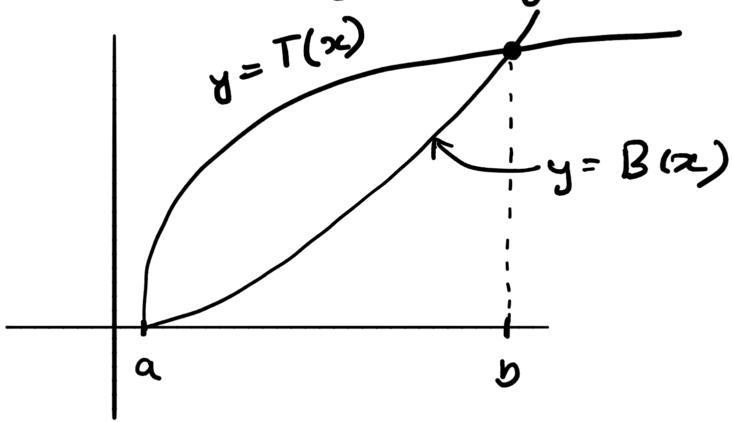
Example contd.

Now, $M_x = \frac{1}{2}\int_{-1}^{1} \rho f(x)^2 dx$

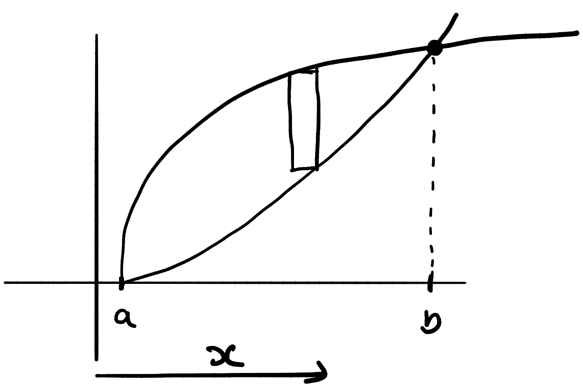
Case 2 centroid of general lamina.

Next we develop the centroid of lamina defined by

$$a \le x \le b\,, \quad B(x) \le y \le T(x)\,.$$



Thus, x-coordinate of centroid $\bar{x}$ is:



So, moment about x-axis of a slice is:

$$\Delta M_x = \left(\frac{T(x) + B(x)}{2}\right)\left(T(x) - B(x)\right)\rho\,\Delta x$$

Summing over all slices to get moment about x-axis.

Key Result:

$$\boxed{\bar{y} = \frac{M_x}{M} = \frac{\frac{1}{2}\int_a^b \left(T(x)^2 - B(x)^2\right) dx}{\int_a^b \left(T(x) - B(x)\right) dx}}$$

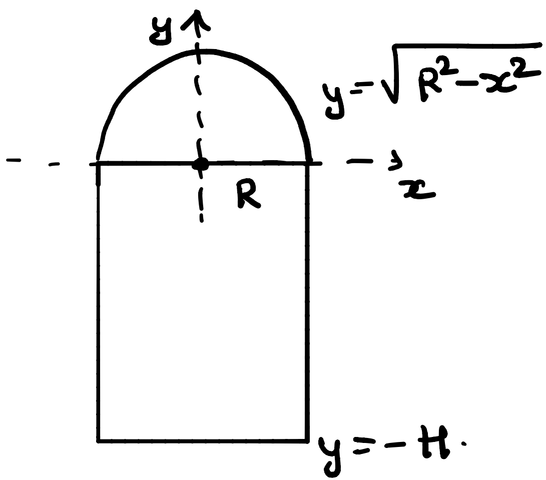
Moment about x-axis.

$$\boxed{\bar{x} = \frac{M_y}{M} = \frac{\int_a^b x\left(T(x) - B(x)\right) dx}{\int_a^b \left(T(x) - B(x)\right) dx}}$$

Moment about y-axis.

Example

Find the centroid of a region consisting of a rectangle of width $2R$ and height H which has a semicircle of radius R on one end. The picture is



Example (contd.)

Question :

Find the centroid of a semi circle of radius R as shown.