WORK

ENERGY EXPENDED ACTING AGAINST A FORCE, i.e. ENERGY EXPENDED MOVING A WEIGHT AGAINST GRAVITY.

- t : SECONDS
- MASS $m \to$ Kg
- position $s \to$ metres
- NEWTON'S SECOND LAW : FORCE = MASS x Acceleration $\quad F = m \dfrac{d^2 s}{dt^2}$

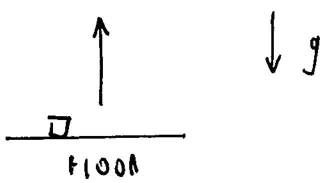
  NEWTONS $\to$ Kg - m/SEC^2

- WORK AT CONSTANT FORCE IS $W =$ FORCE x DISPLACEMENT $= F \times d$

  FORCE MEASURED IN NEWTONS. WORK IN Joules (Newton-metre

QUESTION (CONSTANT FORCE)

HOW MUCH WORK DONE MOVING A 1Kg BOOK FROM FLOOR TO TOP OF A 2 m HEIGH shelF?



DISPLACEMENT = 2 m

FORCE : 9.8 x 1 = 9.8 NewTONS

WORK = 19.6 Joules

FOR A VARIABLE FORCE $\quad W = \int_a^b F(x)\,dx.$

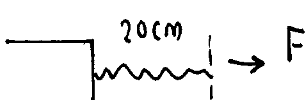
HOOKE'S LAW

$F = KX \qquad$ K : SPRING CONSTANT

X : AMOUNT OF STRETCHING.

A SPRING HAS A NATURAL LENGTH OF 20 cm. IF A 25N FORCE IS REQUIRED TO KEEP IT STRETCHED AT A LENGTH OF 30 cm HOW MUCH WORK IS REQUIRED TO STRETCH IT FROM 20cm TO 25cm.

(i) FIRST WORK OUT SPRING CONSTANT

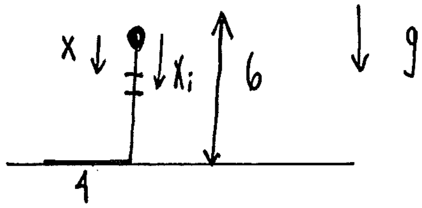


$F = K(.30 - .20) = 25\,N \to K = 250\,N/m$

(ii) NOW X RANGES FROM $X=0$ TO $X=.05$. WE HAVE

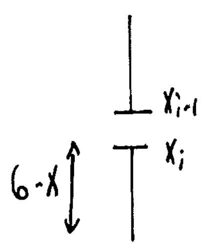
$$W=\int_0^{0.05} F(x)\,dx=\int_0^{0.05} 250x\,dx = 125x^2\Big|_0^{0.05} = 125\times .0025 = .3125 J$$

EXAMPLE A CHAIN LYING ON THE GROUND IS 10 m LONG AND WEIGHS 80 kg. HOW MUCH WORK IS REQUIRED TO RAISE ONE END OF THE CHAIN TO A HEIGHT OF 6 m? THE CONSTANT DENSITY OF THE CHAIN IS 8 kg/m.



$0< X<6$.

SPLIT CHAIN INTO SEGMENTS $[X_{i-1}, X_i]$ AND FIGURE OUT HOW MUCH WORK IS DONE IN LIFTING EACH SEGMENT.



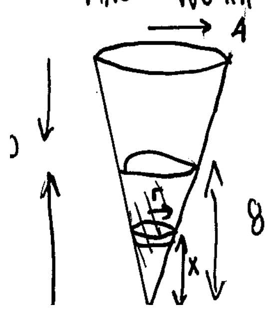
CONSIDER SEGMENT $[X_{i-1}, X_i]$

- LIFTED $6-X$ m AGAINST FORCE OF GRAVITY
- MASS $=$ DENSITY $\cdot\, \Delta X = 8\,\Delta X$
- SEGMENT WEIGHT MASS $\cdot\, g = 8(9.8)\,\Delta X = \Delta F$
- $\Delta W = (6-X)\,8(9.8)\,\Delta X$

SO $W=\int_0^6 78.4(6-x)\,dx = 78.4\left(6x-x^2/2\right)\Big|_0^6 = 18\times 78.4 = 1411.2$

EXAMPLE (WORK DONE IN PUMPING WATER OUT OF A TANK)

TANK IS SHAPED LIKE INVERTED CONE HEIGHT = 10 m RADIUS = 4 m FILLED TO HEIGHT OF 8 m. ASSUME DENSITY OF WATER IS 1000 kg/m^3. FIND WORK INVOLVED IN PUMPING ALL WATER OUT OF TANK



HOW MUCH WORK FOR EACH SLICE OF WATER.

MASS = VOLUME $\times$ density.

Δm = MASS IN SLICE $= \left(\pi (r(x))^2\,\Delta X\right) 1000$.

NOW WEIGHT $\Delta F = 9.8\,\Delta m$ AND $r = \frac{2}{5}X$ BY SIMILAR TRIANGLES

WE HAVE TO MOVE IT $10-X$ METRES

$$\Delta W = (10-x)\,9.8\,\Delta m = (10-x)(9.8)\,\pi\left(\frac{4}{25}x^2\right)\Delta X\,(1000)$$

SO WORK

$$W = (1000)\left(\frac{4}{25}\pi\right)(9.8)\int_0^8 (10-x)\,x^2\,dx$$

$$W = 1568\,\pi\int_0^8 (10-x)\,x^2\,dx$$

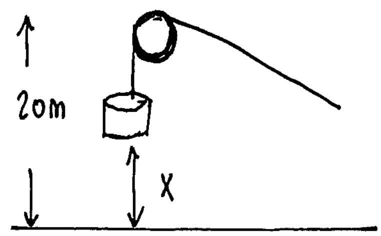
$$W = 1568\,\pi\left[\frac{10}{3}(8)^3 - \frac{1}{4}(8)^4\right]$$

$$W = (1568\,\pi)(8^3)\left(\frac{10}{3} - 2\right)$$

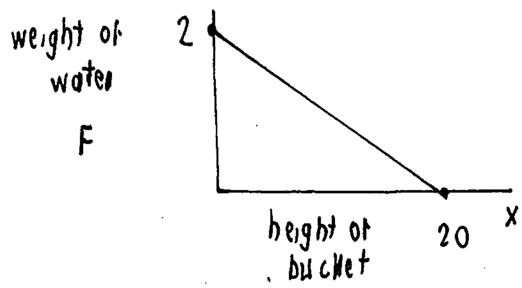
$$= (1568)\,\pi\,(512)\left(\frac{4}{3}\right) \doteq 3.36 \times 10^6 \text{ Joules.}$$

EXAMPLE A LEAKY BUCKET WEIGHING 5 N IS LIFTED 20 m INTO THE AIR AT CONSTANT SPEED. THE BUCKET STARTS WITH 2N OF WATER AND LEAKS AT A CONSTANT RATE. IT FINISHES DRAINING JUST AS IT REACHES THE TOP. HOW MUCH WORK WAS DONE LIFTING THE WATER ALONE?

SOLUTION



SINCE WATER DRAINS OUT AT A CONSTANT RATE WE HAVE



THUS $F = -\frac{1}{10}x + 2$ FOR WEIGHT OF WATER.

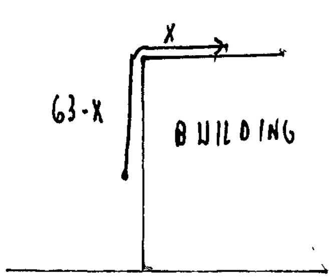
THE TOTAL WORK NEEDED IS

$$W = \int_0^{20} F(x)\,dx = \int_0^{20} \left(2 - x/10\right) dx$$

SO $W = 40 - \frac{1}{20}x^2 \Big|_0^{20} = 20$ JOULES.

EXAMPLE A CHAIN 63 metres LONG WHOSE MASS IS 27 kilograms IS HANGING OVER THE EDGE OF A TALL BUILDING AND DOES NOT TOUCH THE GROUND. HOW MUCH WORK IS REQUIRED TO LIFT THE TOP 11 METRES OF THE CHAIN TO THE TOP OF THE BUILDING. (HINT: DON'T FORGET THAT WHEN YOU LIFT THE TOP 11 METRES OF THE CABLE YOU ARE ALSO LIFTING THE BOTTOM 52 metres OF THE CABLE, JUST NOT ALL THE WAY TO THE TOP).

SOLUTION



AFTER X METRES OF CHAIN PULLED UP, 63 - X METRES REMAIN THAT ARE SUBJECT TO GRAVITY.

MASS HANGING BELOW $= (63 - x)\cdot \underbrace{\frac{27}{63}}_{\text{DENSITY}}$

WEIGHT HANGING BELOW IS

$9.8\,(63 - x)\,27/63$

THUS WORK $= 9.8 \int_0^{11} \left(27 - \frac{3x}{7}\right) d$

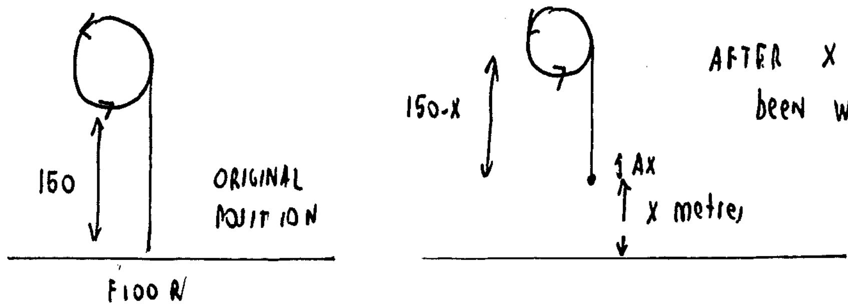
$= 9.8\left[(27)(11) - \frac{3}{14}(11)^2\right]$

WORK $= \frac{3795}{14} \doteq 2656.5$ Joules.

EXAMPLE

WE HAVE A FULLY EXTENDED CABLE OF 150 metres WEIGHING 2 KG/metre. HOW MUCH WORK IS DONE AFTER WINDING 50 m OF CABLE?

SOLUTION



AFTER X metres have been WOUND UP.

AS CABLE IS WOUND UP IT BECOMES SHORTER AND SHORTER WEIGHING LESS AND LESS. THE FORCE IS $\Delta F = \underbrace{(150 - X)}_{\text{}} \; \underbrace{\frac{2\text{ KG}}{\text{METRE}}}_{\text{density}} \times \underbrace{9.8}_{\text{gravity}} \times \Delta X$

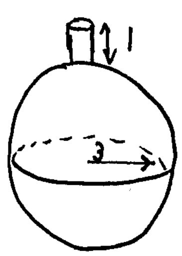
THUS $$W = 9.8 \int_0^{50} 2(150 - X)\, dX = 9.8\left[300(50) - 50^2\right]$$

$$W = (9.8) \times 1.25 \times 10^4 \text{ JOULES}$$

EXAMPLE

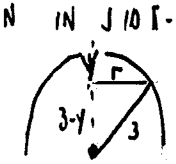
A TANK OF DIMENSIONS SHOWN IS INITIALLY FULL OF WATER THAT HAS DENSITY OF $1000\, kg/m^3$. FIND THE WORK REQUIRED TO PUMP THE WATER OUT OF THE SPOUT.

SOLUTION



THE TANK IS A SPHERE OF RADIUS 3 AND THE SPOUT IS 1 m HIGH. GRAVITY IS ACTING DOWNWARDS.

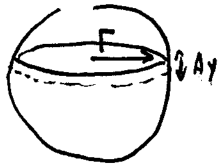
TAKE A HORIZONTAL SLICE AT DEPTH y FROM SPOUT AS SHOWN IN SIDE-VIEW



SO $r^2 = 9 - (3-y)^2 = 6y - y^2$ BY PYTHAGORAS

$r = \sqrt{6y - y^2}$

THE MASS IN VOLUME OF WIDTH Δy IS THEN



$$\Delta V = \pi r^2 \Delta y = \pi (6y - y^2)\Delta y$$

$$\text{MASS} = \Delta V \times \text{density} = 1000\pi (6y - y^2)\Delta y \text{ IN SLICE}$$

$$\text{GRAVITY FORCE} = 9.8(1000\pi)(6y - y^2)\Delta y.$$

NOW SINCE WATER MUST BE PUMPED OUT OF TANK'S SPOUT WHICH IS A DISTANCE OF y+1 (metres), THE WORK DONE IS

(W6)

$$W = (9.8)(1000\pi)\int_0^6 (y+1)(6y-y^2)\,dy$$

$$= 9800\pi \int_0^6 (6y-y^2+6y^2-y^3)\,dy$$

$$= 9800\pi\left(-\frac{6^4}{4}+\frac{5\cdot 6^3}{3}+3\cdot 6^2\right)$$

$$W = 9800\pi\left(\frac{-1296}{4}+\frac{1080}{3}+108\right)$$

OR $W \doteqdot 4.4\times 10^6$ Joules.