14. Convex functions and problems

- affine sets
- convex sets
- examples

Previous lecture

Euclidean ball and ellipsoid

Euclidean ball centred at x^* with radius r:

$$\mathcal{B}(x^*, r) = \{x | \|x - x^*\|_2 \leq r\}$$

Ellipsoid: Let $P \succ 0$. The set:

$$\{x| \ (x - x^*)P(x - x^*) \leq 1\}$$

Norm balls

Norm: A function $\|\cdot\|$ is a norm if it satisfies:

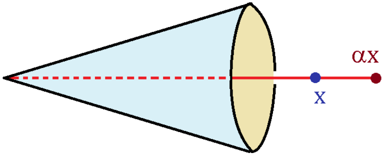
- $\|x\| \geq 0$ and $\|x\| = 0$ if and only if $x = 0$
- $\|\alpha x\| = |\alpha|\|x\|$ for any $\alpha \in \mathbb{R}$
- $\|x + y\| \leq \|x\| + \|y\|$

Any norm ball with centre x^* and radius r:

$$B(x^*, r) = \{x \,|\|x - x^*\| \leq r\}$$

Convex cones

A set $\mathcal{S} \subseteq \mathbf{R}^n$ is a **cone** if $x \in \mathcal{S} \iff \alpha x \in \mathcal{S} \; \forall \alpha \geq 0$



A set $\mathcal{S}$ is a **convex cone** if

A convex cone contains **conic combinations** of its elements

$$x, y \in \mathcal{S} \iff \theta_1 x + \theta_2 y \in \mathcal{S}, \; \forall \theta_1, \theta_2 \geq 0$$

Examples of convex cone

Operations that preserve convexity

Linear combinations

Intersections

Convex Polytopes

$\mathcal{S}$ is a **convex polytope** if it is the intersection of halfspaces

Separating hyperplane theorem