16. Linear Constraint

- Reduced gradient
- Optimality conditions

Linear Constrained Optimization

$$\min_{x \in \mathbb{R}^n} \quad f(x) \qquad s.t. \quad Ax = b,$$

Variable reduction

Reformulate the linear constrained problem into an unconstrained problem. (Change of variable).

Feasible set :

Reduced Unconstrained problem

$$\min_{x \in \mathbb{R}^n} f(x) \quad s.t. \quad Ax = b \quad \Longleftrightarrow$$

2nd order sufficient cond.

Optimality conditions.

Reduced objective function

$$f_Z(p) = f(\bar{x} + Zp)$$

The gradient of f_Z at p is:

Stationary:

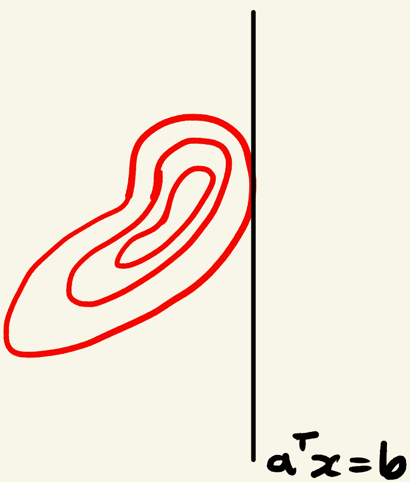
First Order Necessary Cond.

Show $\text{Null}(Z^T) = \text{Range}(A^T)$. $A \in \mathbb{R}^{m \times n}$, $Z \in \mathbb{R}^{n \times (n-m)}$

First order necessary cond.

Integretation

The optimality cond:



Second order necessary cond.

$$f_Z(p) = f(\bar{x} + Zp), \quad \nabla f_Z(p) = Z^T \nabla f(\bar{x} + Zp)$$

The Hessian of f_Z at p is:

Second order necessary cond.

Optimality condition.

Second order sufficient condition.

1. (Feasibility)
2. (Stationary)

3. (Positivity)

Example

Consider

$$\min_{x \in \mathbb{R}^n} \|x\|_2 \quad \text{s.t.} \quad Ax = b.$$

Example contd.