Constrained optimization

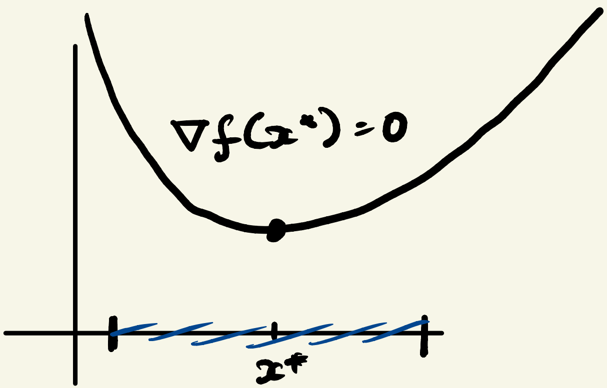
$$\min_{x \in \mathbb{R}^n} f(x) \quad s.t. \quad x \in S \quad \text{---}(1)$$

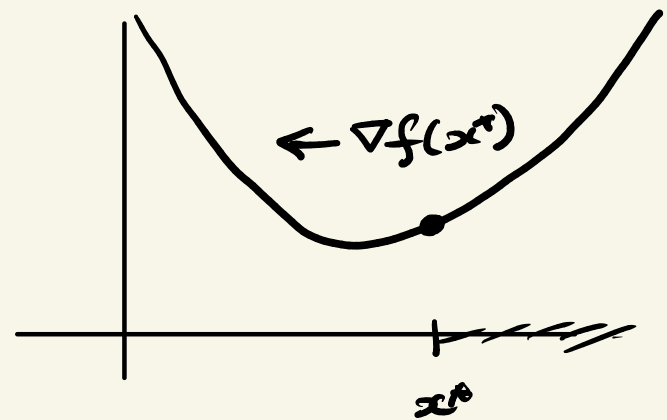
Def^n: (Stationary point)

Stationary point in $\mathbb{R}^n$.

Necessary Condition

Thm:





Example

$$\min_{x \in \mathbb{R}^n} \quad f(x) \qquad \text{s.t.} \qquad x_i \geq 0 \qquad i = 1, \dots, n$$

Characterize the stationary points: Assume x^* is statio-

.

Normal cone.

We can express necessary condition in terms of normal cone.

Example

1. Let $S = \{z\}$.

2. Let $S = \{x \mid \|x\|_2 \leq 1\}$

Stationary point and normal cone

Thm:

Example: x is in interior of S.

Claim For any $x \in int(S)$

$$N_S(x) = \{0\}.$$

proof:

Example: Normal cone to affine set

$$N_S(x) = \{ g \in \mathbb{R}^n \mid g^T(z-x) \leq 0, \forall z \in S \}$$

Affine set $S = \{ x \mid Ax = b \}$.

- Trick: shift the set.

Example: Normal cone to affine halfspace.

Affine halfspace: $S = \{x \mid Ax \le b\} = \{x : a_i^T x \le b_i,\ i = 1, \dots, m\}$

What is the normal cone of S?