MATH 104 Workshop 4

The Derivative and Rules of Differentiation

1. Carefully state the definition of the derivative of a function $f(x)$ at a point $x = a$. Use this definition to compute $f'(2)$ for $f(x) = \sqrt{4x+1}$. (Do not use the rules of differentiation to do this calculation.)

2. Find the equation of the straight line that is tangent to $y = 2^x$ and passes through the point $(1, 0)$.

3. Suppose two curves $y = f(x)$ and $y = g(x)$ intersect at a point (x_0, y_0). How might you define the angle between these two curves at this intersection point?

4. Differentiate:

   (a) $f(x) = \frac{x^2+\sqrt{x}+5}{2-x}$

   (b) $f(x) = e^{\cos(x^2)}$

   (c) $f(x) = xe^{2x}\cos(4x)$

5. Find all values of a and b for which

$$f(x) = \begin{cases} \sin(x) & x \leq 0 \\ ax + b & x > 0 \end{cases}$$

   is differentiable everywhere.