Review
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Source: *The Journal of Philosophy*, Vol. 30, No. 11 (May 25, 1933), pp. 302-306
Published by: Journal of Philosophy, Inc.
Stable URL: http://www.jstor.org/stable/2016345
Accessed: 09-11-2015 09:49 UTC

sumption to be correct when our "expectations," to use Russell's word, of the future are verified. We do not care whether the automobilist "really" sees green or red, but whether he will consistently see the same colors, and drive accordingly.

But it is not the intention of this paper merely to argue that we don't care about material truth. The attempt has been made to show that without transcendent insight we can only discover or approximate the discovery of formal truth in science, in philosophy, and in life. We have tried to show that all rational thinking is within systems. Perhaps there is no real world beyond our systems. Our systems may constitute real worlds. This question falls outside of the scope of this paper. If this is so, formal and material truth coincide; if not, the mystic has his advantage.

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BOOK REVIEWS

Symbolic Logic. C. I. LEWIS AND C. H. LANGFORD. The Century Philosophy Series. New York: The Century Co. 1932. Pp. xi + 506.

The publication of this new *Survey* is an event of first rate importance, for it is the work of two mature minds and it expresses living thought. Where the tradition of science has left behind results of some finality the exposition is sometimes brilliant and always masterful and where matters are in doubt the criticism is free and always searching. One has the pleasure of watching two minds at work, oftentimes as if one were reading "logical confessions" expressed as freshly as in Russell's *Principles*. All this is of the greatest value in a "text," for only by watching minds at work does a beginner come to understand how work is done. Such books are rare. Commonly an author conceals his weaknesses, erases his false starts, and puts down only what he thinks is sound. How much inspiration and how much wisdom must Gauss have buried in his waste-paper basket! Like any technical work this book defies summary, but it will still be possible to touch on certain critical points.

In his *Survey* (University of California, 1918, p. 291) Professor Lewis says: "The systems discussed in the last chapter were all based on material implication, $p \subset q$ meaning exactly 'the statement p is true and q is false is a false statement.' We have already called attention to the fact that this is not the usual meaning of 'implies.' Its divergence from the 'implies' of ordinary inference is exhibited in such theorems as 'A false proposition implies any proposition' and 'A true proposition is implied by any proposition.'" This is one of the characters that distinguishes material implication from strict implication.

Again in *Symbolic Logic* the point is frequently repeated: "By contrast [with material implication] $p \prec q$ . . . is not determined to be either true or false in the other three cases [p true, q true; p false, q true; p false, q false]; whether it holds or not depends upon something else than merely the truth or falsity of p and q (p. 200)." Whether it holds *or not!* This is dangerous ground and it is hard to see why the system should be compromised by introducing it. The intent is clearly to make inference depend on "form," but formal distinctions are constantly lost in special cases or in the process of what Schroeder calls "specialization." No, on this point the *Principia* is as sound as a nut. It has borrowed to its profit from sound tradition. If this is not what ordinary inference means, it is what inference must come to mean if it is to be general. Our author has not here discovered the Achilles' heel of *Principia* which lies precisely in the fallacious conversion of

$$p \subset q \,.\, \subset .\, -p \vee q.$$

It is to be noticed that the system of "strict" implication developed in the *Survey* is very different from that now presented under the same name in *Symbolic Logic*. In the *Survey* we read (p. 293):

$$\begin{array}{ll} 1.02 & p \prec q = \sim(p - q), \\ 1.04 & p \vee q = \sim(-p - q). \end{array}$$

If these are generally true we may substitute in the first $q =$ zero and in the second $p = -p$, $q =$ zero, which would yield,

$$\begin{gathered} p \prec 0 = \sim p, \\ -p = \sim p \quad \text{(by 2.51, p. 297)}, \end{gathered}$$

if zero is the modulus of addition.

This however wipes out the essential distinction, that is, if we press the matter further,

$$\begin{gathered} p \prec q = p \subset q, \\ p \vee q = p + q, \text{ etc.,} \end{gathered}$$

and the system is no longer distinguishable from that of material implication.

In *Symbolic Logic*, however, Professor Lewis takes the essential step to rectify his system. He writes (pp. 141–142): "The definition of material implication has the force of the following pair:

$$\begin{array}{ll} (1) & p \supset q \,.\, \prec .\, \sim(p \sim q), \\ (2) & \sim(p \sim q) \,.\, \prec .\, p \supset q. \end{array}$$

The analogue of (1) for strict implication holds: it is,

$$p \prec q \,.\, \prec \,.\, \sim(p \sim q).$$

. . . But the analogue of (2) which would be

$$\sim(p \sim q) \,.\, \prec \,.\, p \prec q$$

is false. It can be proved that it is not deducible from our postulates."

The equality of the *Survey*,

$$p \prec q = \sim p \vee q,$$

is thus *correctly* abandoned as fallacious and the reviewer can only feel gratified that this character of implication has at last been recognized by some one other than himself. For the last ten years he has repeatedly called attention to it (see, for example, his *Symbolic Logic*, pp. 17–18, 34, 131–132).

The next step in the development of the system of inference (which Professor Lewis does not take, and may, indeed, not be willing to take) is to introduce a fundamental expansion formula which enables us to express the product of any number of variables implying zero in terms of a sum of products of functions of the single variables. This formula, which we hope we express correctly in Professor Lewis's symbolism, would read:

$$\Diamond(pq) = pq \vee (\sim p \sim q) \,.\, \Diamond p \,.\, \Diamond q \vee \Diamond p \,.\, \sim\Diamond(\sim q) \vee \Diamond q \,.\, \sim\Diamond(\sim p),$$

from which follows by the ordinary laws of the calculus the theorem,

$$\sim\Diamond(pq) = \sim\Diamond p \vee \sim\Diamond q \vee \sim qp \; \Diamond(\sim p) \vee \sim pq \; \Diamond(\sim q),$$

and the formulas for the expansion of functions of higher order and degree can now be progressively developed.

The importance of these formulas lies in the fact that they reduce the solution of the most general problem of logic to the solution of a comparatively simple case. These matters once carried out, implication loses all its adjectives and becomes not "strict" but implication simply.

However, the theory of implication as Professor Lewis and his collaborators are working it out, is on most points far from settled in their own minds (cf. the extraordinary proposal C 13., along with a number of false postulates by Becner, p. 497). For example, in the second Appendix there appears, on the authority of Dr. Wajsberg and Dr. Parry, the theorem,

$$p \prec q \,.\, \prec : q \prec r \,.\, \prec \,.\, p \prec r,$$

which is said to result from the revised *Survey* postulates (this

JOURNAL, Vol. XVII (1920), pp. 300–302), and Professor Lewis observes: "I doubt whether this proposition should be regarded as a valid principle of deduction," etc.

Now if Professor Lewis and his collaborators will express this in terms of their modal function, then by a repeated application of the expansion formulas given above all doubts will be removed. The work is somewhat long, but the result can be depended upon, if my formulas are correctly expressed in the author's symbolism.

One matter which caused the reviewer no little surprise was the treatment of traditional logic as contained in the chapter on the logic of terms. This criticism which begins with Leibnitz and extends to our own day has never been more admirably put, but one would not have expected the chapter to close without its having met its answer.

The whole fallacy of this criticism consists in the assumption that the "all, is" relation of traditional logic must be identified with the relation of Boolean inclusion, whereas it is actually to be represented as a rather complicated function of that relation. The device of extending the meaning of a relation symbol to overcome some obstacle of interpretation is a very common mathematical procedure and could be illustrated in many ways. Following long established method, then, we extend the meaning of the "all, is" relation and we write not

$$\text{all } a \text{ is } b = a \supset b,$$

which would be an oversimplification of the actual state of affairs, but

$$\text{all } a \text{ is } b = (a \supset b) \,.\, \{(b \supset a) + -(a \supset -b) \,.\, -(-b \supset a)\}.$$

On this interpretation, which coincides with the other for all "empirical" meanings of a and b and differs only in the limiting cases, all the postulates of Aristotle's logic hold true in a perfectly general sense, that is, without any restriction whatever as to what the terms may mean. To get "no a is b" we have simply to change b into $-b$ in the formula and it is then easy to see that the product of A and E vanishes for all meanings of the terms, so that subalternation holds true unexceptionally.

All the "valid" moods of the syllogism hold. For example, to prove the syllogism EAO in the third figure (fallaciously stated on p. 50 to be fallacious) we simply multiply together,

$$\begin{aligned} E(ba) &= (b \supset -a) \,.\, \{(-a \supset b) + -(b \supset a) \,.\, -(a \supset b)\}, \\ A(bc) &= (b \supset c) \,.\, \{(c \supset b) + -(b \supset -c) \,.\, -(-c \supset b)\}, \\ A(ca) &= (c \supset a) \,.\, \{(a \supset c) + -(c \supset -a) \,.\, -(-a \supset c)\}, \end{aligned}$$

and observe that the terms in the product vanish identically.

Nothing could be more masterful than Professor Langford's exposition of the logical paradoxes and his manner of classifying them. For the reviewer a great many of these turn on the familiar case of the confusion of the use of "all" in the collective and in the distributive sense. In order to control the meaning of "This proposition is false," for example, we must define the parts that enter into the assertion. If we write: $f(p) = p$ is false and $\phi(p)$ is the form of the assertion about p, ϕ standing for what the authors would call some modal function of p, then we define "is false" as any f which satisfies $f(f) = \phi(f)$ and we define "this proposition" as the roots of the equation,

$$\{f \supset f(f)\}' \cdot \{f(f) \supset f\}' = 0.$$

This makes "this proposition" a two-valued function, ambiguously zero or one, so that $p = f(p)$ for the two values taken collectively, but not for the two values taken distributively.

We began by saying that the new logic is a work of first-rate importance and we should like to end on the same note. One can be thankful for an undertaking that is ripe and thorough even when one does not "go along" with every argument that turns on a critical point.

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