Cubes and cube roots

Cube

Cube of a number is defined as the number obtained when it is multiplied by itself 3 times.

If **n** is a natural number then its cube will be expressed as $\mathbf{n^3}$ which is nothing but **n x n x n**

Suppose we want to find the cube of 2. So that will be expressed as 2^3 = 2 x 2 x 2 = **8**.

Then **8** will be defined as the cube of 2.

Similarly cube of 3 will be defined as 3^3 = 3 x 3 x 3= 27.

27 will be called the cube of 3.

Likewise we can find out the cube of 4, 5 which are 64 (4 x 4 x 4) and 125(5 x 5 x 5) respectively

So will 1 be called a cube?

Of course yes. As previously defined when a number is multiplied by itself 3 times we define it as a cube. So 1^3 = 1 x 1 x 1= 1 is a cube.

Thus 1, 8, 27, 64, 125 are all called cubes or cube numbers.

Perfect Cubes

A natural number is called a perfect cube when that number can be defined as a cube of some natural number. Thus the numbers 1, 8, 27 are also called perfect cubes as they are the cubes of natural numbers 1, 2 and 3 respectively. Also it can be defined in another way. Prime factors of any perfect cubes will be in a pair of 3.

For ex, 64= 4 x 4 x 4 = (2 x 2 x 2) x (2 x 2 x 2)

125=5^3 = 5 x 5 x 5

But are numbers like 18, 48, and 128 perfect cubes?

No.

Factorizing 18 we get = 3 x 3 x 2. We find that no prime factors are in a pair of 3.

Similarly 48= 2 x 2 x 2 x 2 x 3, again no prime factors in a pair of 3.

128= 2 x 2 x 2 x 2 x 2 x 2 x 2 – no prime factors in a pair of 3

Hence 18, 48 and 128 are not perfect cubes.

Properties of cube of a number

- Cube of a negative number is always negative
  Cube of -3 would be = (-3) x (-3) x (-3) = -27, hence negative
- Cubes of even numbers are always even.
  For ex – 2^3=8, 4^3=64, 8^3=512
- Cubes of odd numbers are always odd.
  For ex – 3^3=27, 5^3=125, 7^3=343.
- There are 4 perfect cubes from 1 to 100 and 10 perfect cubes from 1 to 1000.
- The cubes of numbers ending with 0,1,4,5,6,9 also ends with 0,1,4,5,6,9 respectively.
  Let's look at a few examples:
  21^3=9261, 11^3=1331
  14^3=2744, 64^3=262144
  15^3=3375, 25^3=15625
  36^3=46656, 16^3=4096
  19^3=6859, 29^3=24389
- Cubes of numbers ending with 2 end with 8 and cubes of numbers ending with 8 end with 2.
  Ex – 2^3=8, 8^3= 512
- Cubes of numbers ending with 3 end with 7 and likewise cubes of numbers ending with 7 end with 3.
  Ex- 3^3=27, 7^3=343
- If a particular number has n number of zeroes at its end then cube of that number will contain 3n number of zeroes at its end.

Smallest number multiplied to get a perfect cube

This portion will be understood through an example. Here we are trying to find out the smallest number that will be multiplied with the given number in order to get a perfect cube.

Let's take the number – 968

Upon factorizing 968 we get 968= $\underline{2 \times 2 \times 2}$ x $\underline{11 \times 11}$

From the definition of perfect cubes we already know that all the prime factors should occur in pairs of 3 for the number to be a perfect cube. Here we see that 11 occurs twice. Thus in order to achieve the perfect cube 11 should occur once more and satisfy the condition of perfect cube. So the smallest number in this case that should be multiplied would be 11 and the resultant number is

$\underline{2 \times 2 \times 2}$ x $\underline{11 \times 11 \times 11}$ = 10648.

Smallest number divided to get a perfect cube

Let’s understand this concept using the same number used in the previous example. Here we are trying to find out the smallest number which will be divided from the given number in order to make it a perfect cube.

968= 2 x 2 x 2 x 11 x 11

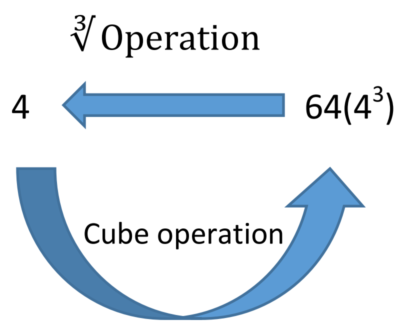
Here we see that in order to make a triplet of prime factors the only way is to divide the number by 11 x 11 that is 121.

Thus the smallest number to be divided from 968 in order to get a perfect cube is 121 and the resultant perfect cube is 2 x 2 x 2 = 8.

Cube Root

Cube root of a number is just the inverse of cube.

If m is a natural number such that m = n^3, then $\sqrt[3]{m}$ = n where $\sqrt[3]{}$ denotes cube root.



Finding cube root by prime factorization

- The first step is to find out all the prime factors of the given cube
- Then groups of 3 need to be formed for all the common digits
- All the groups of 3 now need to be represented by a single digit
- The product of all such single digits will give the cube root of the given number

Illustration:

Let’s consider the number 3375

Prime factorization of 3375 = 3 x 3 x 3 x 5 x 5 x 5

So we see that we get groups of 3 and 5.

Now the next step would be to represent the group of 3 by a single 3 and the group of 5 by a single 5.

Hence the resultant number is 3 x 5 = 15

Thus we get $\sqrt[3]{3375}$ = 15.

Cubes of numbers from 1 to 20

Numbers	Cubes
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000