Lemma 2. *Let V be a finite vector space and let $\alpha \in (0,1)$ be a real number. Let A be a random uniformly chosen subset of V of size $|V|(1-\alpha)$ and v be a random uniformly chosen vector of V independent of A. Then*

$$\mathbf{E}(1 - \frac{|A \cup (v+A)|}{|V|}) = \alpha^2.$$

Proof. For a fixed vector $u \in V$, let $X_u = |A \cup (u+A)|$ be a random variable where A is a random uniformly chosen subset of V of size $|V|(1-\alpha)$. Let v be a random uniformly chosen vector $v \in V$. Then

$$\mathbf{E}(1 - \frac{|A \cup (v+A)|}{|V|}) = (1 - \frac{\sum_{u \in V} \mathbf{P}(v=u)\mathbf{E}(X_u)}{|V|}) = \frac{\sum_{u \in V}(1 - \frac{\mathbf{E}(X_u)}{|V|})}{|V|}$$

because $\mathbf{P}(v=u) = \frac{1}{|V|}$.

First we compute $\mathbf{E}(X_u)$ for a fixed vector u. For a random vector v we have that probability $v \in A$ is $|V|(1-\alpha)|V|^{-1} = 1-\alpha$. Thus probability of $v \notin A$ is α and also probability that $v \notin u + A$ is α because $|A| = |(u+A)|$. Hence

$$\begin{aligned}\mathbf{E}(X_u) = &\sum_{v \in V} 1\mathbf{P}((v \in A) \vee (v \in (u+A))) = \\ &\sum_{v \in V} 1 - \mathbf{P}((v \notin A) \wedge (v \notin (u+A))) = \\ &\sum_{v \in V} 1 - \mathbf{P}(v \notin A)\mathbf{P}(v \notin (u+A)) = \\ &\sum_{v \in V} 1 - \alpha^2 = |V|(1-\alpha^2).\end{aligned}$$

If we substitute the value $\mathbf{E}(X_u)$ we obtain

$$\begin{aligned}\mathbf{E}(1 - \frac{|A \cup (v+A)|}{|V|}) = &\frac{\sum_{u \in V}(1 - \frac{\mathbf{E}(X_u)}{|V|})}{|V|} = \frac{\sum_{u \in V}(1 - \frac{|V|(1-\alpha^2)}{|V|})}{|V|} = \\ &\frac{\sum_{u \in V}(1-(1-\alpha^2))}{|V|} = \frac{\sum_{u \in V} \alpha^2}{|V|} = \alpha^2 \quad \square.\end{aligned}$$