Data_Science_Project_Final (unsaved changes)

Logout

In [1]:

```python
# import libraries

import numpy as np
import pandas as pd

%matplotlib inline
import matplotlib.pyplot as plt
plt.style.use('fivethirtyeight')

from sklearn.linear_model import LinearRegression, Lasso, LassoCV, Ridge, RidgeCV
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split, KFold
from sklearn.metrics import r2_score, mean_squared_error, confusion_matrix, ConfusionMatrixDisplay, accuracy_score
from sklearn.ensemble import RandomForestClassifier
from sklearn.tree import plot_tree

from scipy.optimize import minimize
from scipy.cluster.hierarchy import dendrogram, linkage, fcluster

from tqdm import notebook

import seaborn as sns
```

LOAD DATAFRAME and INITIAL EXPLORATION

In [2]:

```
# loads dataframe; prints head
pisa_df = pd.read_csv("https://raw.githubusercontent.com/babnigg/DATA11900/main/economics_and_education_dataset_CSV.csv")
pisa_df.head()
```

Out[2]:

	index_code	expenditure_on _education_pct_gdp	mortality_rate_infant	gini_index	gdp_per_capita_ppp	inflation_consumer_prices	intentional_homicides	unemployment	gros
0	AUS-2003	5.246357	4.9	33.5	30121.818418	2.732596	1.533073	5.933	
1	AUS-2003	5.246357	4.9	33.5	30121.818418	2.732596	1.533073	5.933	
2	AUS-2003	5.246357	4.9	33.5	30121.818418	2.732596	1.533073	5.933	
3	AUS-2006	4.738430	4.7	NaN	34846.715630	3.555288	1.372940	4.785	
4	AUS-2006	4.738430	4.7	NaN	34846.715630	3.555288	1.372940	4.785	

In [3]:

```python
# filters out dataframe to only include total sex aggregate; prints info and describes numerical variables
agg = "TOT"
pisa_df_tot = pisa_df[pisa_df["sex"]==agg]

pisa_df_tot.info()
pisa_df_tot.describe()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 214 entries, 2 to 633
Data columns (total 20 columns):
 #   Column                                Non-Null Count  Dtype  
---  ------                                --------------  -----  
 0   index_code                            214 non-null    object 
 1   expenditure_on _education_pct_gdp     202 non-null    float64
 2   mortality_rate_infant                 214 non-null    float64
 3   gini_index                            181 non-null    float64
 4   gdp_per_capita_ppp                    214 non-null    float64
 5   inflation_consumer_prices             214 non-null    float64
 6   intentional_homicides                 187 non-null    float64
 7   unemployment                          214 non-null    float64
 8   gross_fixed_capital_formation         214 non-null    float64
 9   population_density                    214 non-null    float64
 10  suicide_mortality_rate                214 non-null    float64
 11  tax_revenue                           198 non-null    float64
 12  taxes_on_income_profits_capital       198 non-null    float64
 13  alcohol_consumption_per_capita        37 non-null     float64
 14  government_health_expenditure_pct_gdp 214 non-null    float64
 15  urban_population_pct_total            214 non-null    float64
 16  country                               214 non-null    object 
 17  time                                  214 non-null    int64  
 18  sex                                   214 non-null    object 
 19  rating                                214 non-null    float64
dtypes: float64(16), int64(1), object(3)
memory usage: 35.1+ KB
```

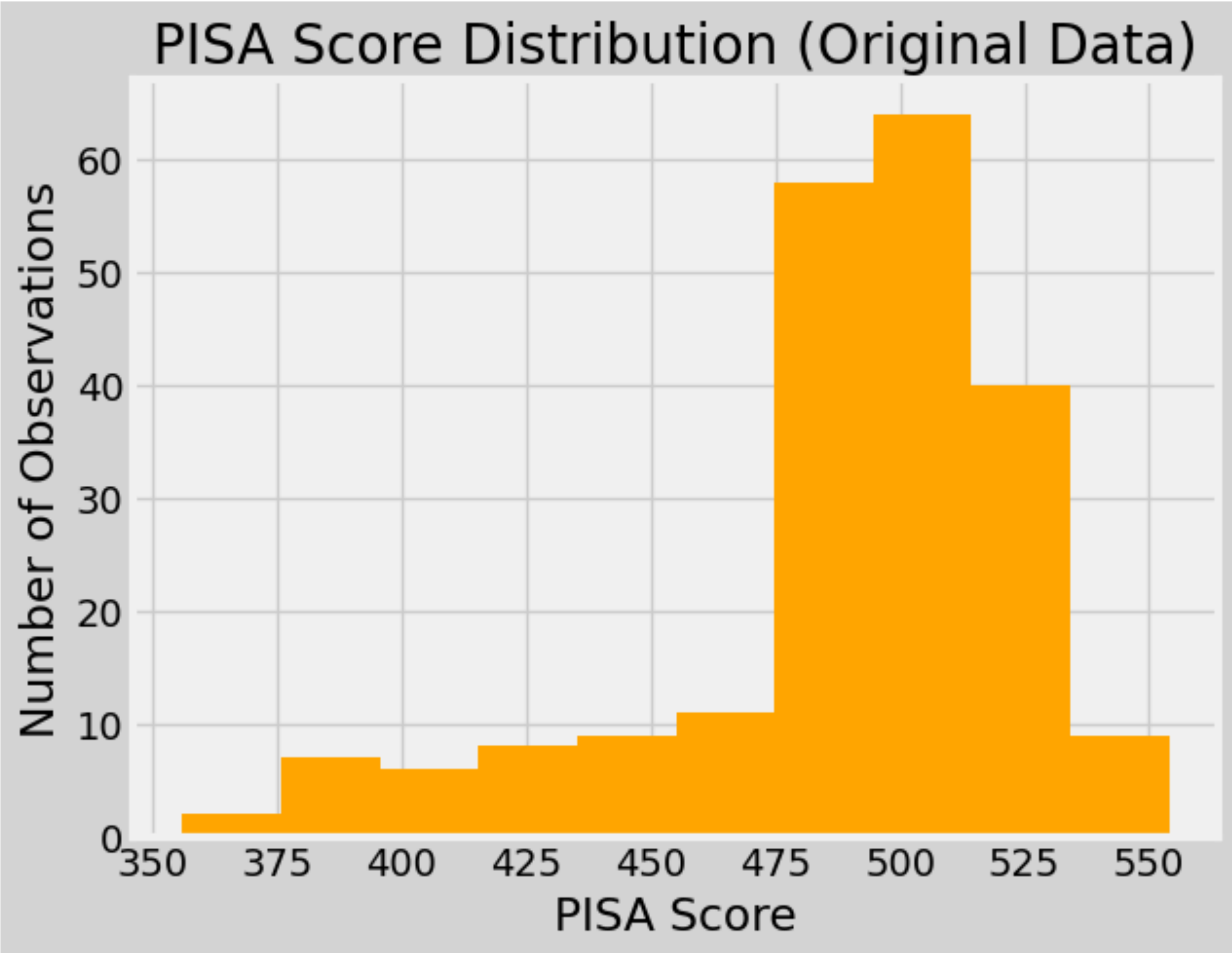
Out[3]:

	expenditure_on _education_pct_gdp	mortality_rate_infant	gini_index	gdp_per_capita_ppp	inflation_consumer_prices	intentional_homicides	unemployment	gross_fixed_
count	202.000000	214.000000	181.000000	214.000000	214.000000	187.000000	214.000000	
mean	5.237384	5.062150	33.411050	36176.526959	2.300823	3.119124	7.643659	
std	1.105318	4.090917	6.816531	16451.684140	2.742357	6.122298	4.021760	
min	3.040150	1.900000	24.400000	9587.557951	-4.478103	0.000000	2.246000	
25%	4.460890	3.000000	28.100000	26081.142138	0.799862	0.833583	4.892500	
50%	5.058085	3.700000	32.700000	34305.514059	1.980056	1.095505	6.843000	

	expenditure_on_education_pct_gdp	mortality_rate_infant	gini_index	gdp_per_capita_ppp	inflation_consumer_prices	intentional_homicides	unemployment	gross_fixed_
75%	5.796320	5.100000	35.400000	42993.833915	2.991248	1.842371	9.333500	
max	8.448880	25.300000	57.600000	116498.512081	21.602438	29.581371	24.981000	

In [4]:

```
# plots histogram of PISA score distribution in original data
plt.figure().set_facecolor("lightgrey")
plt.hist(pisa_df_tot.rating, color = "orange")
plt.title("PISA Score Distribution (Original Data)")
plt.ylabel("Number of Observations")
plt.xlabel("PISA Score")
plt.show()
```



DATA CLEANING and EXPLORATION

In [5]:

```
# counts and prints number of NA values per column
np.sum(pisa_df_tot.isna(),axis=0)
```

Out[5]:

```
index_code                                0
expenditure_on _education_pct_gdp        12
mortality_rate_infant                     0
gini_index                               33
gdp_per_capita_ppp                        0
inflation_consumer_prices                 0
intentional_homicides                    27
unemployment                              0
gross_fixed_capital_formation             0
population_density                        0
suicide_mortality_rate                    0
tax_revenue                              16
taxes_on_income_profits_capital          16
alcohol_consumption_per_capita          177
government_health_expenditure_pct_gdp     0
urban_population_pct_total                0
country                                   0
time                                      0
sex                                       0
rating                                    0
dtype: int64
```

In [6]:

```python
# defining function to impute data with the mean for the column
def fill_missing_with_mean(group):
  '''fill the missing values with the mean of each feature (column) for each country.'''
  columns_to_fill = ['expenditure_on _education_pct_gdp', 'gini_index', 'intentional_homicides',
                     'tax_revenue', 'taxes_on_income_profits_capital', 'alcohol_consumption_per_capita']
  for column in columns_to_fill:
      group[column] = group[column].fillna(group[column].mean())
  return group

# apply the function to each group (country)
pisa_df_tot = pisa_df_tot.groupby('country', group_keys=False).apply(fill_missing_with_mean)

# glance at data head after imputing
display(pisa_df_tot.head())

# counts and prints number of NA values per column after imputing
np.sum(pisa_df_tot.isna(),axis=0)
```

	index_code	expenditure_on _education_pct_gdp	mortality_rate_infant	gini_index	gdp_per_capita_ppp	inflation_consumer_prices	intentional_homicides	unemployment	gro
2	AUS-2003	5.246357	4.9	33.5	30121.818418	2.732596	1.533073	5.933	
5	AUS-2006	4.738430	4.7	33.9	34846.715630	3.555288	1.372940	4.785	
8	AUS-2009	5.081320	4.2	33.9	40312.395119	1.771117	1.214170	5.565	
11	AUS-2012	4.866670	3.6	33.9	42866.604330	1.762780	1.069106	5.225	
14	AUS-2015	5.315520	3.3	33.9	46292.095439	1.508367	0.990754	6.055	

Out[6]:

```
index_code                            0
expenditure_on _education_pct_gdp     0
mortality_rate_infant                 0
gini_index                           12
gdp_per_capita_ppp                    0
inflation_consumer_prices             0
intentional_homicides                 6
unemployment                          0
gross_fixed_capital_formation         0
population_density                    0
suicide_mortality_rate                0
tax_revenue                           6
taxes_on_income_profits_capital       6
alcohol_consumption_per_capita        2
government_health_expenditure_pct_gdp 0
```

```
urban_population_pct_total                0
country                                   0
time                                      0
sex                                       0
rating                                    0
dtype: int64
```

Since there are some countries that don't have any entry for certain variables we can just drop those countries.
Countries: BEL, JPN, NZL, CRI, LTU.
We also choose to drop countries with the characteristically low PISA scores in comparison to other countries in dataset to normalize the score distribution.
Countries: MEX, BRA, CHL, TUR, COL

In [7]:

```python
# dropping countries with lone NaN values
# last 5 countires in list are lowest scoring countries
countries_to_drop = ['BEL', 'JPN', 'NZL','CRI','LTU','MEX','BRA','CHL','TUR','COL']
pisa_df_tot = pisa_df_tot.drop(pisa_df_tot[pisa_df_tot['country'].isin(countries_to_drop)].index)

# 'sex' is a redundant column as we already control the dataset on sex='TOT'
pisa_df_tot = pisa_df_tot.drop(['sex'],axis=1)

# informative 'index_code' column can be used to replace default index
pisa_df_tot.reset_index(drop=True,inplace=True)
pisa_df_tot.set_index('index_code',inplace=True)

# printing remaining number of countries in the dataset
print("Number of remaining countries in dataset:",len(pisa_df_tot.country.unique()))
print("\n")

# glance at data head after cleaning
display(pisa_df_tot.head())

# counts and prints number of NA values per column after removing countries
print(np.sum(pisa_df_tot.isna(),axis=0))
```

```
Number of remaining countries in dataset: 29
```

	expenditure_on _education_pct_gdp	mortality_rate_infant	gini_index	gdp_per_capita_ppp	inflation_consumer_prices	intentional_homicides	unemployment	gross_fi
index_code								
AUS-2003	5.246357	4.9	33.5	30121.818418	2.732596	1.533073	5.933	
AUS-2006	4.738430	4.7	33.9	34846.715630	3.555288	1.372940	4.785	
AUS-2009	5.081320	4.2	33.9	40312.395119	1.771117	1.214170	5.565	
AUS-2012	4.866670	3.6	33.9	42866.604330	1.762780	1.069106	5.225	
AUS-2015	5.315520	3.3	33.9	46292.095439	1.508367	0.990754	6.055	

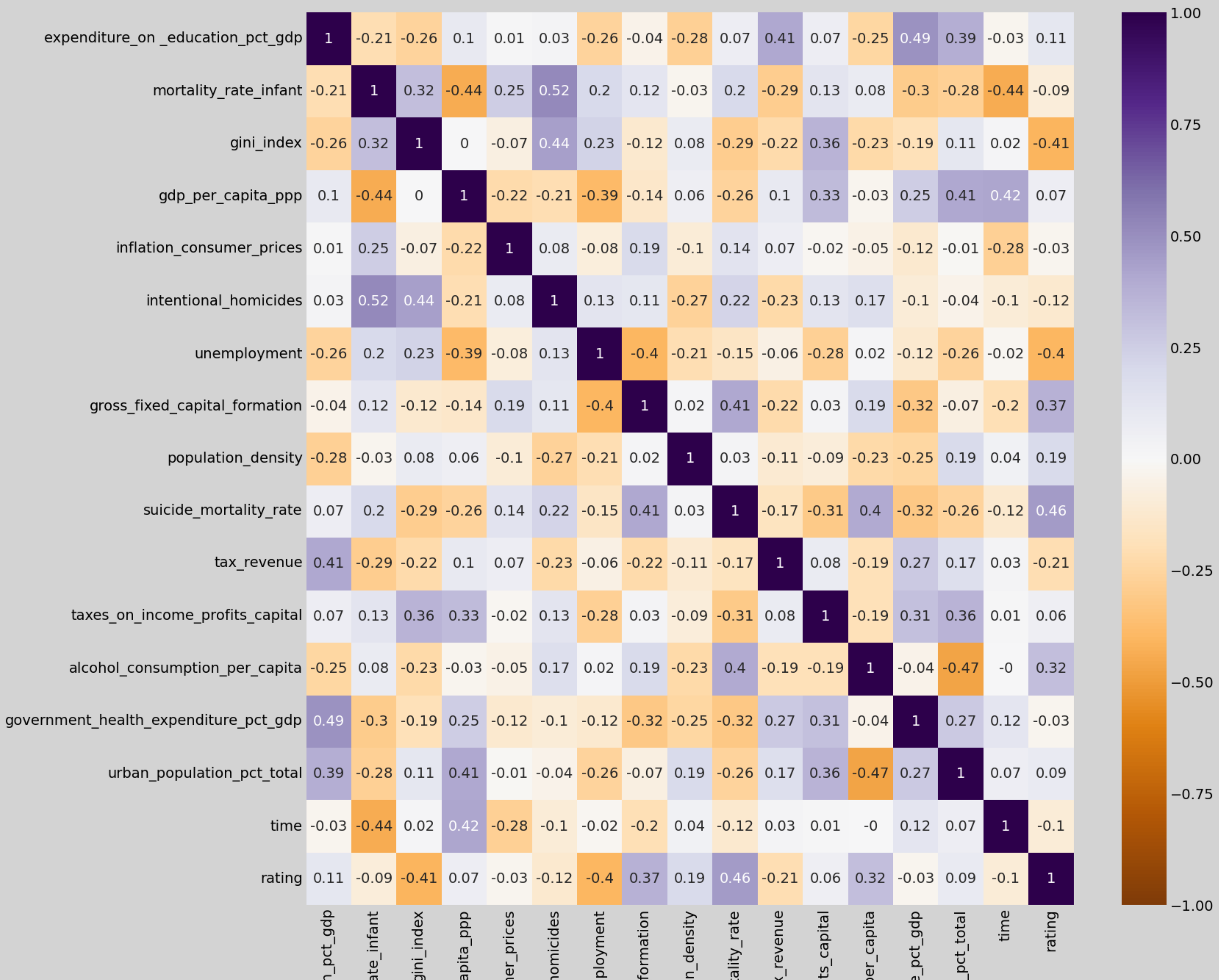
```
expenditure_on _education_pct_gdp       0
mortality_rate_infant                   0
gini_index                              0
gdp_per_capita_ppp                      0
inflation_consumer_prices               0
```

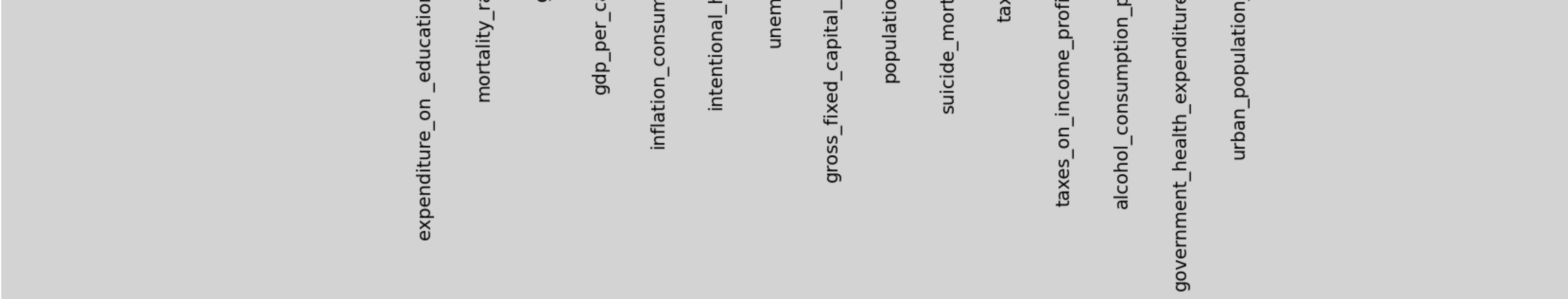
```
intentional_homicides                  0
unemployment                           0
gross_fixed_capital_formation          0
population_density                     0
suicide_mortality_rate                 0
tax_revenue                            0
taxes_on_income_profits_capital        0
alcohol_consumption_per_capita         0
government_health_expenditure_pct_gdp  0
urban_population_pct_total             0
country                                0
time                                   0
rating                                 0
dtype: int64
```

EXPLORATION

In [8]:

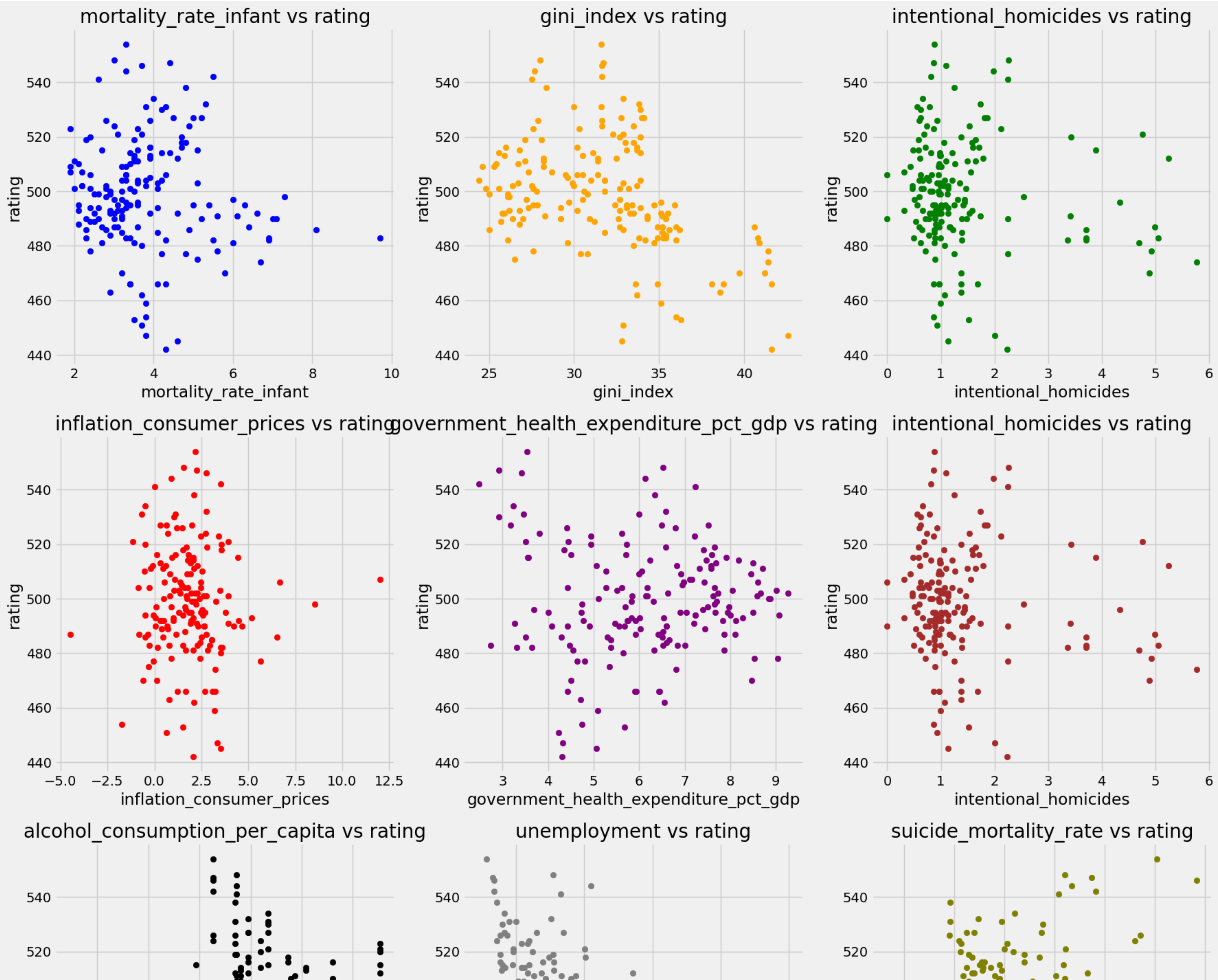
```
# plot correlation matrix with all numerical variables
corr_matrix = pisa_df_tot.corr(numeric_only=True).round(2)
plt.figure(figsize=(15,15)).set_facecolor("lightgrey")
sns.heatmap(data=corr_matrix,annot=True,vmin=-1,vmax=1,cmap="PuOr")
plt.show()
```

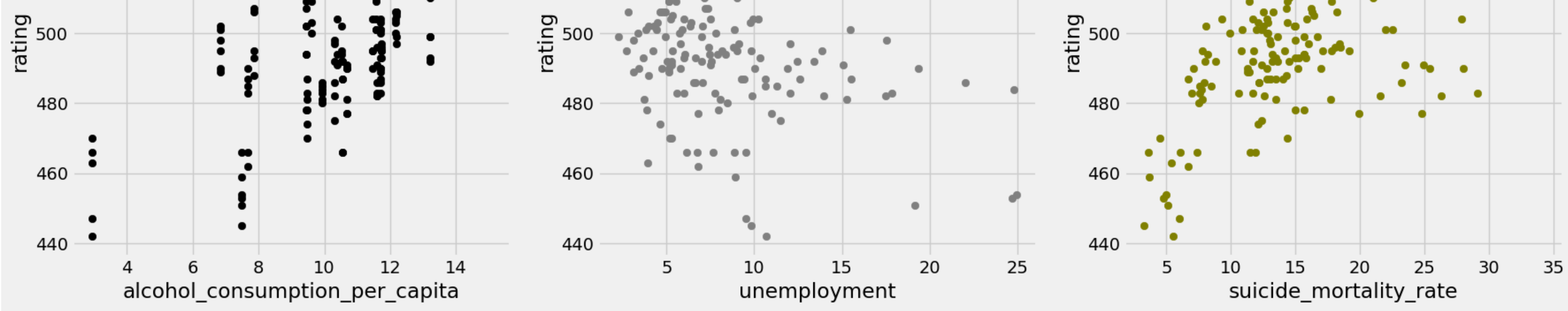

expenditure_on _education_pct_gdp 1 -0.21 -0.26 0.1 0.01 0.03 -0.26 -0.04 -0.28 0.07 0.41 0.07 -0.25 0.49 0.39 -0.03 0.11
mortality_rate_infant -0.21 1 0.32 -0.44 0.25 0.52 0.2 0.12 -0.03 0.2 -0.29 0.13 0.08 -0.3 -0.28 -0.44 -0.09
gini_index -0.26 0.32 1 0 -0.07 0.44 0.23 -0.12 0.08 -0.29 -0.22 0.36 -0.23 -0.19 0.11 0.02 -0.41
gdp_per_capita_ppp 0.1 -0.44 0 1 -0.22 -0.21 -0.39 -0.14 0.06 -0.26 0.1 0.33 -0.03 0.25 0.41 0.42 0.07
inflation_consumer_prices 0.01 0.25 -0.07 -0.22 1 0.08 -0.08 0.19 -0.1 0.14 0.07 -0.02 -0.05 -0.12 -0.01 -0.28 -0.03
intentional_homicides 0.03 0.52 0.44 -0.21 0.08 1 0.13 0.11 -0.27 0.22 -0.23 0.13 0.17 -0.1 -0.04 -0.1 -0.12
unemployment -0.26 0.2 0.23 -0.39 -0.08 0.13 1 -0.4 -0.21 -0.15 -0.06 -0.28 0.02 -0.12 -0.26 -0.02 -0.4
gross_fixed_capital_formation -0.04 0.12 -0.12 -0.14 0.19 0.11 -0.4 1 0.02 0.41 -0.22 0.03 0.19 -0.32 -0.07 -0.2 0.37
population_density -0.28 -0.03 0.08 0.06 -0.1 -0.27 -0.21 0.02 1 0.03 -0.11 -0.09 -0.23 -0.25 0.19 0.04 0.19
suicide_mortality_rate 0.07 0.2 -0.29 -0.26 0.14 0.22 -0.15 0.41 0.03 1 -0.17 -0.31 0.4 -0.32 -0.26 -0.12 0.46
tax_revenue 0.41 -0.29 -0.22 0.1 0.07 -0.23 -0.06 -0.22 -0.11 -0.17 1 0.08 -0.19 0.27 0.17 0.03 -0.21
taxes_on_income_profits_capital 0.07 0.13 0.36 0.33 -0.02 0.13 -0.28 0.03 -0.09 -0.31 0.08 1 -0.19 0.31 0.36 0.01 0.06
alcohol_consumption_per_capita -0.25 0.08 -0.23 -0.03 -0.05 0.17 0.02 0.19 -0.23 0.4 -0.19 -0.19 1 -0.04 -0.47 -0 0.32
government_health_expenditure_pct_gdp 0.49 -0.3 -0.19 0.25 -0.12 -0.1 -0.12 -0.32 -0.25 -0.32 0.27 0.31 -0.04 1 0.27 0.12 -0.03
urban_population_pct_total 0.39 -0.28 0.11 0.41 -0.01 -0.04 -0.26 -0.07 0.19 -0.26 0.17 0.36 -0.47 0.27 1 0.07 0.09
time -0.03 -0.44 0.02 0.42 -0.28 -0.1 -0.02 -0.2 0.04 -0.12 0.03 0.01 -0 0.12 0.07 1 -0.1
rating 0.11 -0.09 -0.41 0.07 -0.03 -0.12 -0.4 0.37 0.19 0.46 -0.21 0.06 0.32 -0.03 0.09 -0.1 1
time
rating
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00



In [9]:

```python
# exploratory data analysis based on correlation matrix

# list of features that has quite high correlation score
features = ["mortality_rate_infant", "gini_index", "intentional_homicides",
            "inflation_consumer_prices",  "government_health_expenditure_pct_gdp", "intentional_homicides",
            "alcohol_consumption_per_capita", "unemployment", "suicide_mortality_rate"
           ]
colors = ['blue', 'orange', 'green', 'red', 'purple', 'brown', 'black', 'gray', 'olive']

# create subplots
fig, axes = plt.subplots(3, 3, figsize=(18, 18))

# flatten axes for easy iteration
axes = axes.flatten()

# plot scatter plots for each feature
for i, (feature, color) in enumerate(zip(features, colors)):
    ax = axes[i]
    ax.scatter(pisa_df_tot[feature], pisa_df_tot['rating'],color=color)
    ax.set_xlabel(feature)
    ax.set_ylabel('rating')
    ax.set_title(f'{feature} vs rating')

# adjust layout
plt.tight_layout()
plt.show()
```


mortality_rate_infant vs rating
rating
540
520
500
480
460
440
2
4
6
8
10
mortality_rate_infant
gini_index vs rating
rating
25
30
35
40
gini_index
intentional_homicides vs rating
rating
0
1
2
3
4
5
6
intentional_homicides
inflation_consumer_prices vs rating
rating
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
12.5
inflation_consumer_prices
government_health_expenditure_pct_gdp vs rating
rating
3
4
5
6
7
8
9
government_health_expenditure_pct_gdp
intentional_homicides vs rating
rating
intentional_homicides
alcohol_consumption_per_capita vs rating
unemployment vs rating
suicide_mortality_rate vs rating



In [10]:

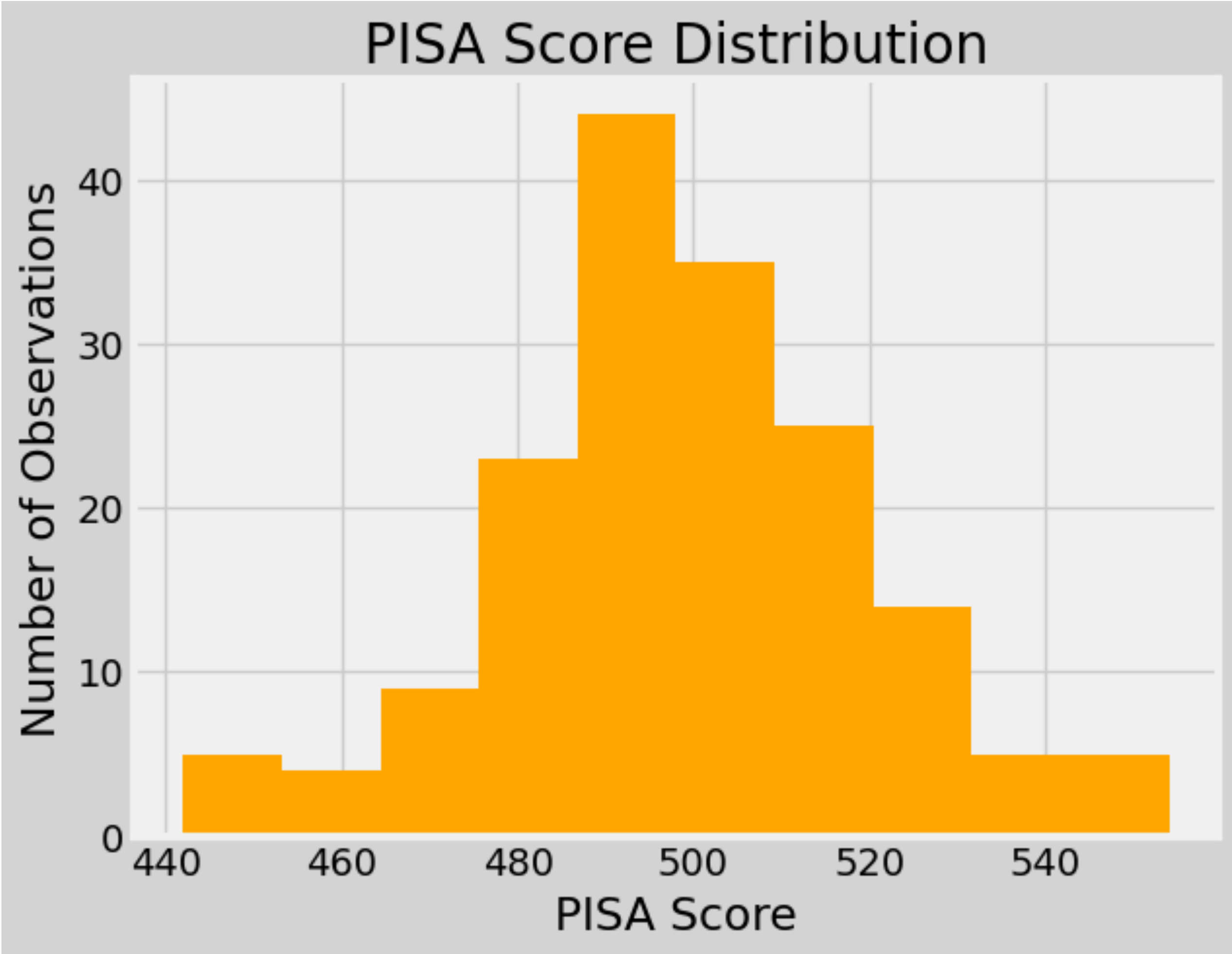
```
# scaling and transforming numerical variables
scaler = StandardScaler()

num_X = pisa_df_tot.select_dtypes(exclude='object')
num_columns = num_X.drop(['rating','time'],axis=1).columns
# (time is a number, but it is indeed a categorical variable!)

# uses standard scaler to transform all numerical variables
pisa_df_tot[num_columns]= scaler.fit_transform(pisa_df_tot[num_columns])
# one-hot-encodes categorical variables of country and time
country_col = pisa_df_tot.country
pisa_df_tot = pd.get_dummies(pisa_df_tot,columns=['country','time'],drop_first=True,dtype=int)
pisa_df_tot["country"] = country_col
display(pisa_df_tot.head())
```

	expenditure_on_education_pct_gdp	mortality_rate_infant	gini_index	gdp_per_capita_ppp	inflation_consumer_prices	intentional_homicides	unemployment	gross_fi
index_code								
AUS-2003	-0.029140	0.831882	0.462126	-0.546819	0.507257	0.142289	-0.426994	
AUS-2006	-0.496382	0.679648	0.559458	-0.258194	0.968513	-0.005489	-0.695237	
AUS-2009	-0.180958	0.299063	0.559458	0.075682	-0.031812	-0.152009	-0.512982	
AUS-2012	-0.378414	-0.157639	0.559458	0.231708	-0.036486	-0.285880	-0.592426	
AUS-2015	0.034482	-0.385990	0.559458	0.440957	-0.179127	-0.358186	-0.398488	

5 rows × 50 columns

In [11]:

```python
# plots histogram of PISA score distribution after data cleaning
plt.figure().set_facecolor("lightgrey")
plt.hist(pisa_df_tot.rating, color = "orange")
plt.title("PISA Score Distribution")
plt.ylabel("Number of Observations")
plt.xlabel("PISA Score")
plt.show()
```



LINEAR, RIDGE, and LASSO MODELS: SELECTING BEST MODEL

In [12]:

```
# train and test dataset split
x_df = pisa_df_tot.loc[:,pisa_df_tot.columns != "rating"]
y_df = pisa_df_tot["rating"]

x_train, x_test, y_train, y_test = train_test_split(x_df, y_df, test_size = 0.2)
```

In [13]:

```
# defining 7 different models
model1x = x_train[["gini_index","unemployment"]]
model2x = x_train[["mortality_rate_infant","intentional_homicides","suicide_mortality_rate"]]
model3x = x_train[["government_health_expenditure_pct_gdp","expenditure_on _education_pct_gdp"]]
model4x = x_train[["population_density","urban_population_pct_total"]]
model5x = x_train[["tax_revenue","taxes_on_income_profits_capital","gdp_per_capita_ppp"]]
model6x = x_train[["gini_index","mortality_rate_infant","intentional_homicides","suicide_mortality_rate", \
                   "alcohol_consumption_per_capita","government_health_expenditure_pct_gdp"]]
model7x = x_train[["alcohol_consumption_per_capita","intentional_homicides","urban_population_pct_total"]]
```

In [14]:

```
# calcualtes best RIDGE and LASSO alphas for each model using cross-validation from training dataset
# RIDGE
ridgeCV = RidgeCV(alphas = np.arange(0.01,100,0.05)).fit(model1x,y_train)
alpha1r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 1:", alpha1r)

ridgeCV = RidgeCV(alphas = np.arange(0.01,100,0.05)).fit(model2x,y_train)
alpha2r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 2:", alpha2r)

ridgeCV = RidgeCV(alphas = np.arange(0.01,100,0.05)).fit(model3x,y_train)
alpha3r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 3:", alpha3r)

ridgeCV = RidgeCV(alphas = np.arange(0.01,100,0.05)).fit(model4x,y_train)
alpha4r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 4:", alpha4r)

ridgeCV = RidgeCV(alphas = np.arange(0.01,100,0.05)).fit(model5x,y_train)
alpha5r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 5:", alpha5r)

ridgeCV = RidgeCV(alphas = np.arange(0.01,100,0.05)).fit(model6x,y_train)
alpha6r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 6:", alpha6r)

ridgeCV = RidgeCV(alphas = np.arange(0.01,100,0.05)).fit(model7x,y_train)
alpha7r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 7:", alpha7r, "\n")

# LASSO
lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model1x,y_train)
alpha1l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 1:", alpha1l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model2x,y_train)
alpha2l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 2:", alpha2l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model3x,y_train)
alpha3l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 3:", alpha3l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model4x,y_train)
alpha4l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 4:", alpha4l)
```

```python

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model5x,y_train)
alpha5l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 5:", alpha5l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model6x,y_train)
alpha6l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 6:", alpha6l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model7x,y_train)
alpha7l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 7:", alpha7l)
```

```
Most appropriate Ridge alpha for model 1: 7.01
Most appropriate Ridge alpha for model 2: 6.76
Most appropriate Ridge alpha for model 3: 65.81
Most appropriate Ridge alpha for model 4: 76.21000000000001
Most appropriate Ridge alpha for model 5: 62.86
Most appropriate Ridge alpha for model 6: 21.460000000000004
Most appropriate Ridge alpha for model 7: 1.36

Most appropriate LASSO alpha for model 1: 0.009688719668735267
Most appropriate LASSO alpha for model 2: 0.49820294669828463
Most appropriate LASSO alpha for model 3: 0.1195576940589756
Most appropriate LASSO alpha for model 4: 0.8309474024852266
Most appropriate LASSO alpha for model 5: 1.8648432123347791
Most appropriate LASSO alpha for model 6: 0.6142078784174095
Most appropriate LASSO alpha for model 7: 0.11582748247194662
```

In [15]:

```python
# implementing K-fold to test for best model (manual)

# initializes k group IDs, creating modified training dataframes for cross-validation
kgroup_ids = np.arange(1,6)
kgroups_array = np.repeat(kgroup_ids,27)
np.random.shuffle(kgroups_array)
x_train["k_group"] = kgroups_array
y_train_cv= pd.DataFrame({'rating':y_train,'k_group':kgroups_array})

y_train1 = y_train_cv.loc[y_train_cv["k_group"] != 1]['rating']

# implementing Kfold to select the best model
ridge_1_mses = []
ridge_2_mses = []
ridge_3_mses = []
ridge_4_mses = []
ridge_5_mses = []
ridge_6_mses = []
ridge_7_mses = []

linear_1_mses = []
linear_2_mses = []
linear_3_mses = []
linear_4_mses = []
linear_5_mses = []
linear_6_mses = []
linear_7_mses = []

lasso_1_mses = []
lasso_2_mses = []
lasso_3_mses = []
lasso_4_mses = []
lasso_5_mses = []
lasso_6_mses = []
lasso_7_mses = []

for k in kgroup_ids:
  print('working on fold', k, "...")
  x_train1 = x_train.loc[x_train["k_group"] != k]
  y_train1 = y_train_cv.loc[y_train_cv["k_group"] != k]['rating']
  x_test1 = x_train.loc[x_train["k_group"] == k]
  y_test1 = y_train_cv.loc[y_train_cv["k_group"] == k]['rating']


  model1x = x_train1[["gini_index","unemployment"]]
  model2x = x_train1[["mortality_rate_infant","intentional_homicides","suicide_mortality_rate"]]
```

```
model3x = x_train1[["government_health_expenditure_pct_gdp","expenditure_on _education_pct_gdp"]]
model4x = x_train1[["population_density","urban_population_pct_total"]]
model5x = x_train1[["tax_revenue","taxes_on_income_profits_capital","gdp_per_capita_ppp"]]
model6x = x_train1[["gini_index","mortality_rate_infant","intentional_homicides","suicide_mortality_rate", \
                   "alcohol_consumption_per_capita","government_health_expenditure_pct_gdp"]]
model7x = x_train1[["alcohol_consumption_per_capita","intentional_homicides","urban_population_pct_total"]]


model1tst = x_test1[["gini_index","unemployment"]]
model2tst = x_test1[["mortality_rate_infant","intentional_homicides","suicide_mortality_rate"]]
model3tst = x_test1[["government_health_expenditure_pct_gdp","expenditure_on _education_pct_gdp"]]
model4tst = x_test1[["population_density","urban_population_pct_total"]]
model5tst = x_test1[["tax_revenue","taxes_on_income_profits_capital","gdp_per_capita_ppp"]]
model6tst = x_test1[["gini_index","mortality_rate_infant","intentional_homicides","suicide_mortality_rate", \
                   "alcohol_consumption_per_capita","government_health_expenditure_pct_gdp"]]
model7tst = x_test1[["alcohol_consumption_per_capita","intentional_homicides","urban_population_pct_total"]]

ridgereg = Ridge(alpha = alpha1r).fit(model1x,y_train1)
predicted_y1 = ridgereg.predict(model1tst)
print("-----------")
# print("MSE for Model 1 Ridge regression:",mean_squared_error(y_test1,predicted_y1))
ridge_1_mses.append(mean_squared_error(y_test1,predicted_y1))

ridgereg = Ridge(alpha = alpha2r).fit(model2x,y_train1)
predicted_y2 = ridgereg.predict(model2tst)
# print("MSE for Model 2 Ridge regression:",mean_squared_error(y_test1,predicted_y2))
ridge_2_mses.append(mean_squared_error(y_test1,predicted_y2))

ridgereg = Ridge(alpha = alpha3r).fit(model3x,y_train1)
predicted_y3 = ridgereg.predict(model3tst)
# print("MSE for Model 3 Ridge regression:",mean_squared_error(y_test1,predicted_y3))
ridge_3_mses.append(mean_squared_error(y_test1,predicted_y3))

ridgereg = Ridge(alpha = alpha4r).fit(model4x,y_train1)
predicted_y4 = ridgereg.predict(model4tst)
# print("MSE for Model 4 Ridge regression:",mean_squared_error(y_test1,predicted_y4))
ridge_4_mses.append(mean_squared_error(y_test1,predicted_y4))

ridgereg = Ridge(alpha = alpha5r).fit(model5x,y_train1)
predicted_y5 = ridgereg.predict(model5tst)
# print("MSE for Model 5 Ridge regression:",mean_squared_error(y_test1,predicted_y5))
ridge_5_mses.append(mean_squared_error(y_test1,predicted_y5))

ridgereg = Ridge(alpha = alpha6r).fit(model6x,y_train1)
predicted_y6 = ridgereg.predict(model6tst)
# print("MSE for Model 6 Ridge regression:",mean_squared_error(y_test1,predicted_y6))
```

```
ridge_6_mses.append(mean_squared_error(y_test1,predicted_y6))

ridgereg = Ridge(alpha = alpha7r).fit(model7x,y_train1)
predicted_y7 = ridgereg.predict(model7tst)
# print("MSE for Model 7 Ridge regression:",mean_squared_error(y_test1,predicted_y7))
ridge_7_mses.append(mean_squared_error(y_test1,predicted_y7))

linearReg = LinearRegression().fit(model1x,y_train1)
predicted_y1 = linearReg.predict(model1tst)
# print("MSE for Model 1 Linear regression:",mean_squared_error(y_test1,predicted_y1))
linear_1_mses.append(mean_squared_error(y_test1,predicted_y1))

linearReg = LinearRegression().fit(model2x,y_train1)
predicted_y2 = linearReg.predict(model2tst)
# print("MSE for Model 2 Linear regression:",mean_squared_error(y_test1,predicted_y2))
linear_2_mses.append(mean_squared_error(y_test1,predicted_y2))

linearReg = LinearRegression().fit(model3x,y_train1)
predicted_y3 = linearReg.predict(model3tst)
# print("MSE for Model 3 Linear regression:",mean_squared_error(y_test1,predicted_y3))
linear_3_mses.append(mean_squared_error(y_test1,predicted_y3))

linearReg = LinearRegression().fit(model4x,y_train1)
predicted_y4 = linearReg.predict(model4tst)
# print("MSE for Model 4 Linear regression:",mean_squared_error(y_test1,predicted_y4))
linear_4_mses.append(mean_squared_error(y_test1,predicted_y4))

linearReg = LinearRegression().fit(model5x,y_train1)
predicted_y5 = linearReg.predict(model5tst)
# print("MSE for Model 5 Linear regression:",mean_squared_error(y_test1,predicted_y5))
linear_5_mses.append(mean_squared_error(y_test1,predicted_y5))

linearReg = LinearRegression().fit(model6x,y_train1)
predicted_y6 = linearReg.predict(model6tst)
# print("MSE for Model 6 Linear regression:",mean_squared_error(y_test1,predicted_y6))
linear_6_mses.append(mean_squared_error(y_test1,predicted_y6))

linearReg = LinearRegression().fit(model7x,y_train1)
predicted_y7 = linearReg.predict(model7tst)
# print("MSE for Model 7 Linear regression:",mean_squared_error(y_test1,predicted_y7))
linear_7_mses.append(mean_squared_error(y_test1,predicted_y7))

lassoreg = Lasso(alpha = alpha1l).fit(model1x,y_train1)
predicted_y1 = lassoreg.predict(model1tst)
# print("MSE for Model 1 LASSO regression:",mean_squared_error(y_test1,predicted_y1))
lasso_1_mses.append(mean_squared_error(y_test1,predicted_y1))
```

```

  lassoreg = Lasso(alpha = alpha2l).fit(model2x,y_train1)
  predicted_y2 = lassoreg.predict(model2tst)
  # print("MSE for Model 2 LASSO regression:",mean_squared_error(y_test1,predicted_y2))
  lasso_2_mses.append(mean_squared_error(y_test1,predicted_y2))

  lassoreg = Lasso(alpha = alpha3l).fit(model3x,y_train1)
  predicted_y3 = lassoreg.predict(model3tst)
  # print("MSE for Model 3 LASSO regression:",mean_squared_error(y_test1,predicted_y3))
  lasso_3_mses.append(mean_squared_error(y_test1,predicted_y3))

  lassoreg = Lasso(alpha = alpha4l).fit(model4x,y_train1)
  predicted_y4 = lassoreg.predict(model4tst)
  # print("MSE for Model 4 LASSO regression:",mean_squared_error(y_test1,predicted_y4))
  lasso_4_mses.append(mean_squared_error(y_test1,predicted_y4))

  lassoreg = Lasso(alpha = alpha5l).fit(model5x,y_train1)
  predicted_y5 = lassoreg.predict(model5tst)
  # print("MSE for Model 5 LASSO regression:",mean_squared_error(y_test1,predicted_y5))
  lasso_5_mses.append(mean_squared_error(y_test1,predicted_y5))

  lassoreg = Lasso(alpha = alpha6l).fit(model6x,y_train1)
  predicted_y6 = lassoreg.predict(model6tst)
  # print("MSE for Model 6 LASSO regression:",mean_squared_error(y_test1,predicted_y6))
  lasso_6_mses.append(mean_squared_error(y_test1,predicted_y6))

  lassoreg = Lasso(alpha = alpha7l).fit(model7x,y_train1)
  predicted_y7 = lassoreg.predict(model7tst)
  # print("MSE for Model 7 LASSO regression:",mean_squared_error(y_test1,predicted_y7))
  lasso_7_mses.append(mean_squared_error(y_test1,predicted_y7))
  print(' ')

print("Ridge Model 1 Mean MSE:",np.mean(ridge_1_mses))
print("Ridge Model 2 Mean MSE:",np.mean(ridge_2_mses))
print("Ridge Model 3 Mean MSE:",np.mean(ridge_3_mses))
print("Ridge Model 4 Mean MSE:",np.mean(ridge_4_mses))
print("Ridge Model 5 Mean MSE:",np.mean(ridge_5_mses))
print("Ridge Model 6 Mean MSE:",np.mean(ridge_6_mses))
print("Ridge Model 7 Mean MSE:",np.mean(ridge_7_mses))
print(" ")
print("Linear Model 1 Mean MSE:",np.mean(linear_1_mses))
print("Linear Model 2 Mean MSE:",np.mean(linear_2_mses))
print("Linear Model 3 Mean MSE:",np.mean(linear_3_mses))
print("Linear Model 4 Mean MSE:",np.mean(linear_4_mses))
print("Linear Model 5 Mean MSE:",np.mean(linear_5_mses))
print("Linear Model 6 Mean MSE:",np.mean(linear_6_mses))
```

```
print("Linear Model 7 Mean MSE:",np.mean(linear_7_mses))
print(" ")
print("LASSO Model 1 Mean MSE:",np.mean(lasso_1_mses))
print("LASSO Model 2 Mean MSE:",np.mean(lasso_2_mses))
print("LASSO Model 3 Mean MSE:",np.mean(lasso_3_mses))
print("LASSO Model 4 Mean MSE:",np.mean(lasso_4_mses))
print("LASSO Model 5 Mean MSE:",np.mean(lasso_5_mses))
print("LASSO Model 6 Mean MSE:",np.mean(lasso_6_mses))
print("LASSO Model 7 Mean MSE:",np.mean(lasso_7_mses))
```

```
working on fold 1 ...
-----------

working on fold 2 ...
-----------

working on fold 3 ...
-----------

working on fold 4 ...
-----------

working on fold 5 ...
-----------

Ridge Model 1 Mean MSE: 363.7181114761211
Ridge Model 2 Mean MSE: 340.127916178825
Ridge Model 3 Mean MSE: 481.6210147382588
Ridge Model 4 Mean MSE: 479.99420935256694
Ridge Model 5 Mean MSE: 464.2527818044
Ridge Model 6 Mean MSE: 312.111458519555
Ridge Model 7 Mean MSE: 359.3239565787832

Linear Model 1 Mean MSE: 365.94373077459625
Linear Model 2 Mean MSE: 341.93861017178915
Linear Model 3 Mean MSE: 483.76836839344367
Linear Model 4 Mean MSE: 486.364547771024
Linear Model 5 Mean MSE: 462.6127208819713
Linear Model 6 Mean MSE: 312.89516216022065
Linear Model 7 Mean MSE: 359.18027384626623

LASSO Model 1 Mean MSE: 365.9243065813704
LASSO Model 2 Mean MSE: 342.83312831290635
LASSO Model 3 Mean MSE: 483.7333773999268
LASSO Model 4 Mean MSE: 483.7819812744141
LASSO Model 5 Mean MSE: 465.9111772009381
```

```
LASSO Model 6 Mean MSE: 312.8410662011376
LASSO Model 7 Mean MSE: 359.4036447530108
```

In [16]:

```
# recursively implementing K-fold to test for best model a number of times
# will contextualize how the ORIGINAL test-train split affects best model

# a list of the 7 chosen possible models, in order model 1-7
models = [["gini_index","unemployment"],
          ["mortality_rate_infant","intentional_homicides","suicide_mortality_rate"],
          ["government_health_expenditure_pct_gdp","expenditure_on _education_pct_gdp"],
          ["population_density","urban_population_pct_total"],
          ["tax_revenue","taxes_on_income_profits_capital","gdp_per_capita_ppp"],
          ["gini_index","mortality_rate_infant","intentional_homicides","suicide_mortality_rate", \
                     "alcohol_consumption_per_capita","government_health_expenditure_pct_gdp"],
          ["alcohol_consumption_per_capita","intentional_homicides","urban_population_pct_total"]]

# for each K-fold CV, records best model (out of 21), and their MSEs and R2s
best_models = np.array([])
best_mses = np.array([])
best_r2s = np.array([])

# THIS PART WILL TAKE LONG TO RUN: CAN LIMIT K-FOLDS TO ONE: range(100) -> range(1)
for split in notebook.tqdm(range(100)):
  # creates a shuffled K-fold indicies of given folds (default=5)
  k = 5
  kfold = KFold(n_splits=k,shuffle=True)

  model_mses = np.zeros((3,7))
  model_r2s = np.zeros((3,7))

  # does a randomized initial test-train split before running K-Fold CV
  x_traink, x_testk, y_traink, y_testk = train_test_split(x_df, y_df, test_size = 0.2)

  # iterates over each of the models
  for m,model in enumerate(models):
    # calculates best fitting RIDGE and LASSO alphas using new initial training dataframe and model
    ridgeCV = RidgeCV(alphas = np.arange(0.01,100,0.05)).fit(x_traink[model],y_traink)
    ralpha = ridgeCV.alpha_
    lassoCV = LassoCV(cv=None,n_alphas=500).fit(x_traink[model],y_traink)
    lalpha = lassoCV.alpha_

    # for each K-fold, record that fold's MSE and R2 values
    mses = np.zeros((3,k))
    r2s = np.zeros((3,k))
    # fold number (0 to k-1)
    ki = 0

    # performs K-fold CV
    for train_index,test_index in kfold.split(x_traink):
```

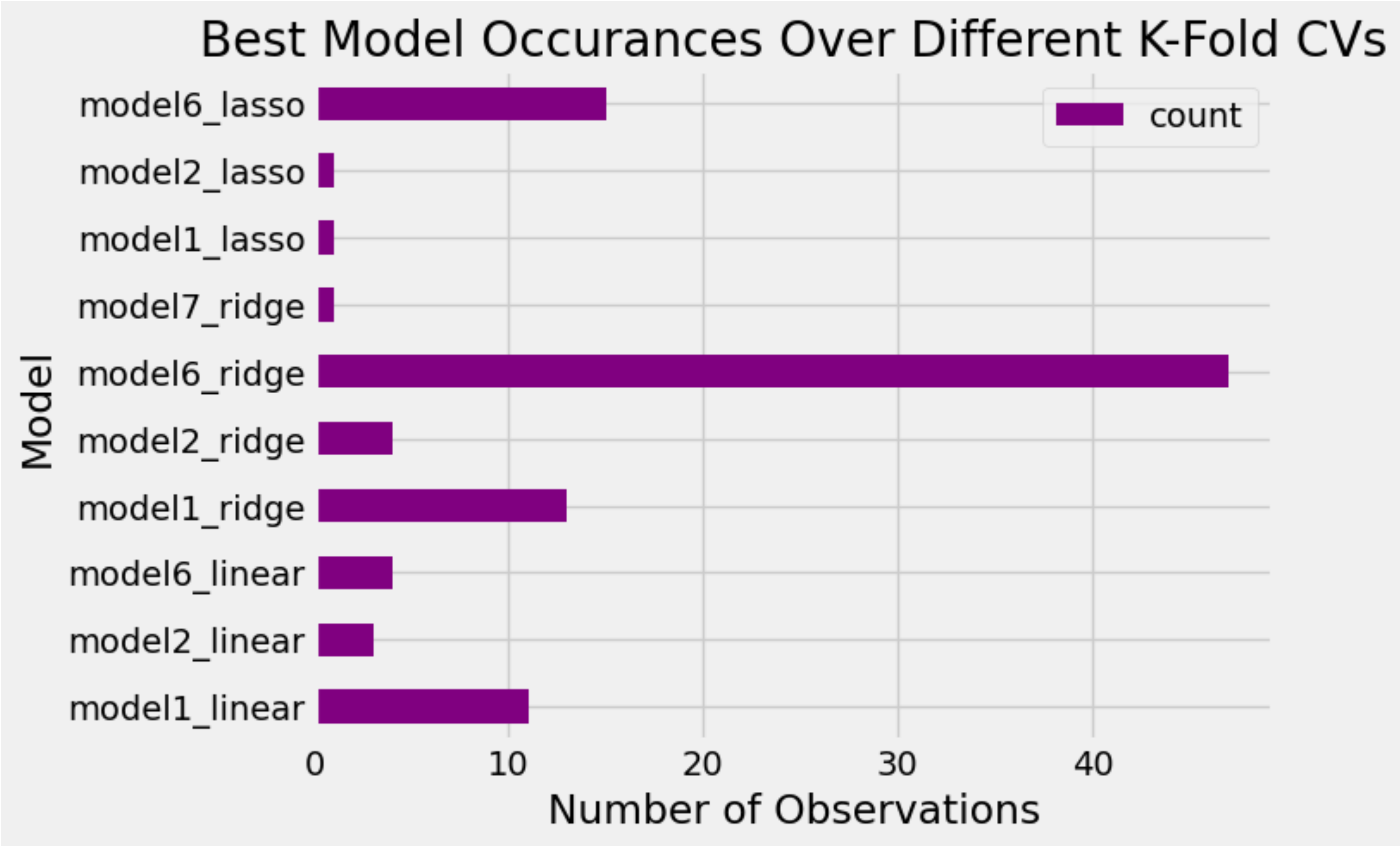
```
      # test and train split for specific fold, from initial training dataframe and model
      X_tr, X_te, y_tr, y_te = x_traink.iloc[train_index], x_traink.iloc[test_index], y_traink.iloc[train_index], y_traink.iloc[tes

      # performs linear regression on training fold, predicts y using test fold
      linearReg = LinearRegression().fit(X_tr[model],y_tr)
      predicted_y = linearReg.predict(X_te[model])
      mses[0][ki] = mean_squared_error(y_te,predicted_y)
      r2s[0][ki] = r2_score(y_te,predicted_y)

      # performs RIDGE regression on training fold, predicts y using test fold
      ridgeReg = Ridge(alpha = ralpha).fit(X_tr[model],y_tr)
      predicted_y = ridgeReg.predict(X_te[model])
      mses[1][ki] = mean_squared_error(y_te,predicted_y)
      r2s[1][ki] = r2_score(y_te,predicted_y)

      # performs LASSO regression on training fold, predicts y using test fold
      lassoReg = Lasso(alpha = lalpha).fit(X_tr[model],y_tr)
      predicted_y = lassoReg.predict(X_te[model])
      mses[2][ki] = mean_squared_error(y_te,predicted_y)
      r2s[2][ki] = r2_score(y_te,predicted_y)

      # increases fold number
      ki += 1

    # across all folds, calculates the mean MSE
    model_mses[0][m] = np.mean(mses[0])
    model_mses[1][m] = np.mean(mses[1])
    model_mses[2][m] = np.mean(mses[2])

    # across all folds, calculates the mean R2 score
    model_r2s[0][m] = np.mean(r2s[0])
    model_r2s[1][m] = np.mean(r2s[1])
    model_r2s[2][m] = np.mean(r2s[2])

  # out of a given K-Fold CV, appends best model, and its MSE and R2
  best_models = np.append(best_models,np.argmin(model_mses))
  best_mses = np.append(best_mses,np.min(model_mses))
  best_r2s = np.append(best_r2s,model_r2s[np.argmin(model_mses)//7][np.argmin(model_mses)%7])

#Printing the statistics for the first 10 runs of the Loop (to find the best model)
print("Best Models (#):",best_models[:9])
print("Best Model MSE:",best_mses[:9])
print("Best Model R^2:",best_r2s[:9])
```

```
  0%|          | 0/100 [00:00<?, ?it/s]

Best Models (#): [12. 13. 12.  0.  7. 19.  0. 12. 12.]
```

```
Best Model MSE: [316.33573026 315.97420626 313.46558336 340.64742678 335.42158805
 313.5229693  288.13779761 302.59490713 308.19019473]
Best Model R^2: [0.19927372 0.17397247 0.22717893 0.21235763 0.21328092 0.24138915
 0.22510881 0.24042682 0.24014788]
```

In [17]:

```
# results from recursive K-Fold tries

# matches model number to an informative name
models_s = []
for mo in range(21):
  if mo//7==0:
    models_s.append("model"+str(mo%7+1)+"_linear")
  elif mo//7==1:
    models_s.append("model"+str(mo%7+1)+"_ridge")
  else:
    models_s.append("model"+str(mo%7+1)+"_lasso")

# gets best models, and their occurances over the number of K-Folds
x,height = np.unique(best_models,return_counts=True)
best_models_df = pd.DataFrame({"model":x,"count":height})
best_models_df["model"] = best_models_df["model"].replace(np.arange(21),models_s)
# plots histogram of best model over multiple K-Folds
best_models_df.plot.barh(x="model",y="count",color="purple")
plt.title("Best Model Occurances Over Different K-Fold CVs")
plt.ylabel("Model")
plt.xlabel("Number of Observations")
plt.show()
```


Best Model Occurances Over Different K-Fold CVs
count
model6_lasso
model2_lasso
model1_lasso
model7_ridge
model6_ridge
model2_ridge
model1_ridge
model6_linear
model2_linear
model1_linear
Model
0
10
20
30
40
Number of Observations

In [19]:

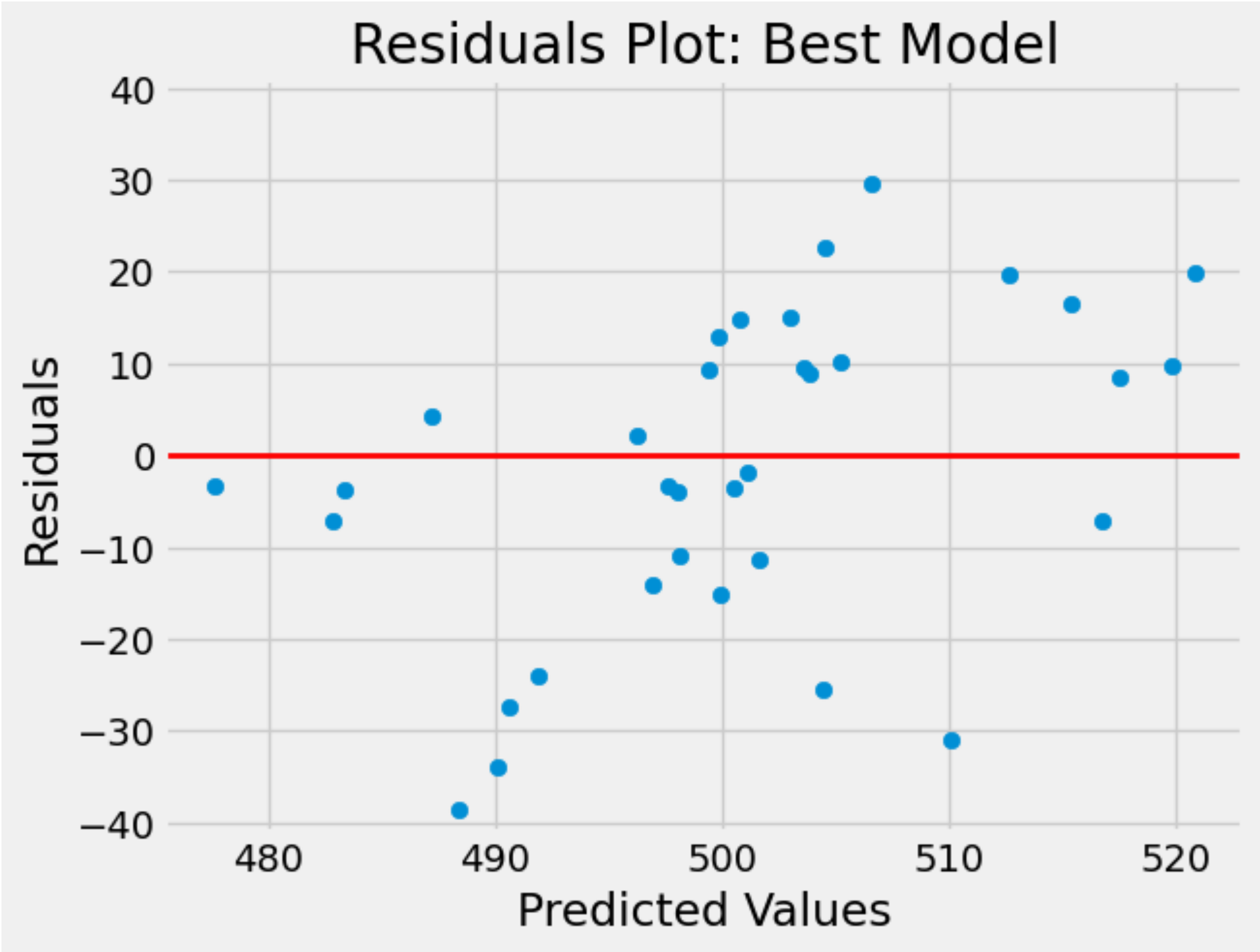
```
# choosing best model based on histogram; running a normal CV using original test-train split

# best model: model 6, RIDGE
model6_train = x_train[["gini_index","mortality_rate_infant","intentional_homicides","suicide_mortality_rate", \
                        "alcohol_consumption_per_capita","government_health_expenditure_pct_gdp"]]
model6_test = x_test[["gini_index","mortality_rate_infant","intentional_homicides","suicide_mortality_rate", \
                        "alcohol_consumption_per_capita","government_health_expenditure_pct_gdp"]]

# recalculates and saves RIDGE alpha for clarity
best_ridgeCV = RidgeCV(alphas = np.arange(0.01,100,0.05)).fit(model6_train,y_train)
best_alpha = best_ridgeCV.alpha_

# performs regression, predicts using test dataframe
best_ridgereg = Ridge(alpha=best_alpha).fit(model6_train,y_train)
best_predicted_y = best_ridgereg.predict(model6_test)

# calculates MSE and r2
best_mse = mean_squared_error(y_test,best_predicted_y)
best_r2 = r2_score(y_test,best_predicted_y)
```

BEST MODEL ANALYSIS

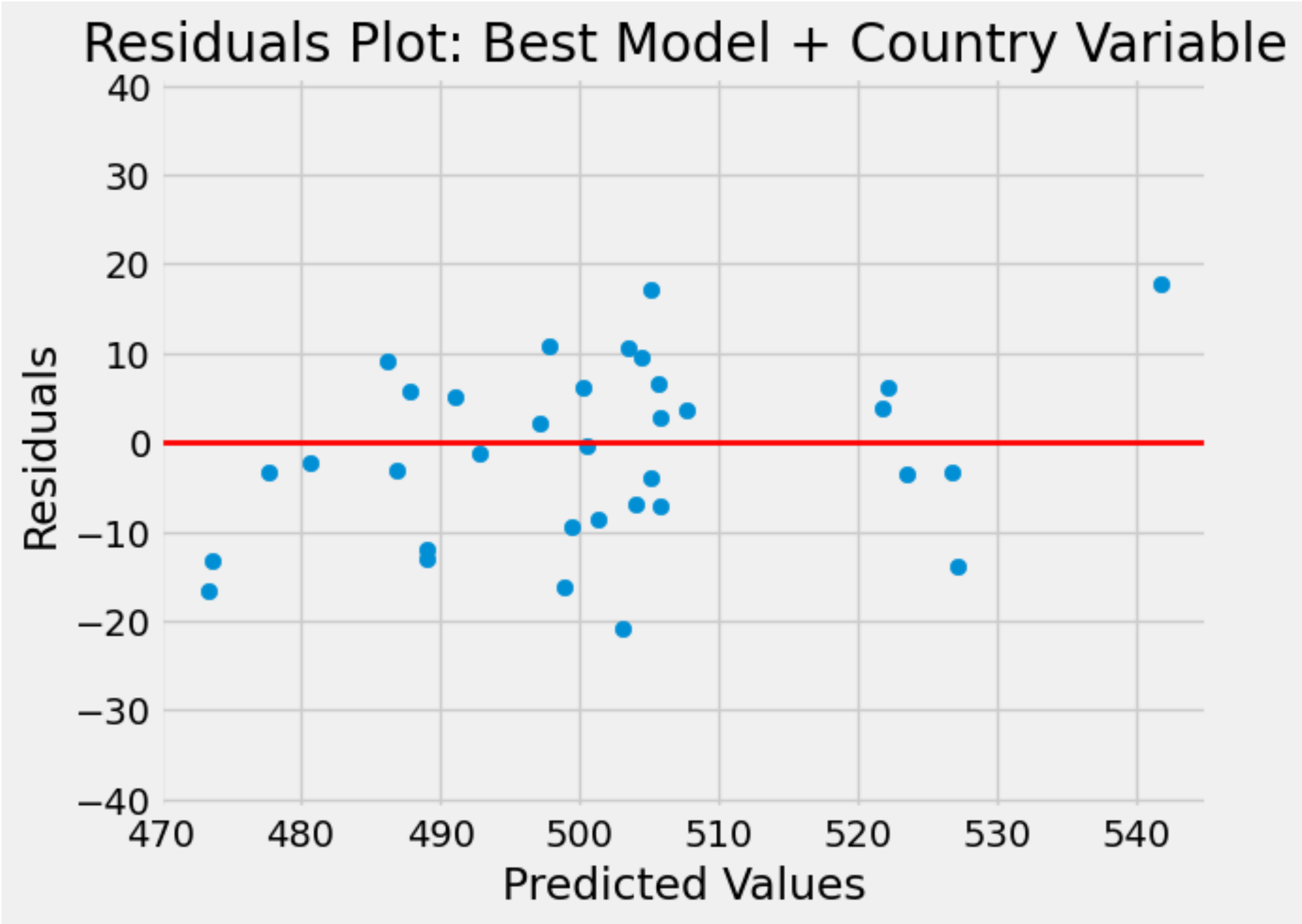
In [20]:

```
# looking at the best model's residuals & comparing when we also control for country
print("MSE for Model 6 Ridge regression:",best_mse)

# plotting the residuals for the best model
plt.scatter(best_predicted_y,best_predicted_y - y_test)
plt.ylim(-41,41)
plt.axhline(0,color='red',linewidth = 2)
plt.title('Residuals Plot: Best Model')
plt.xlabel("Predicted Values")
plt.ylabel("Residuals")
plt.show()


# getting the MSE for the best model + controlling for country
country_control = x_train[["gini_index","mortality_rate_infant","intentional_homicides","suicide_mortality_rate", \
                           "alcohol_consumption_per_capita","government_health_expenditure_pct_gdp",'country_AUT', 'country_CAN', \
                           'country_CHE', 'country_CZE','country_DEU', 'country_DNK', 'country_ESP', 'country_EST','country_FIN', \
                           'country_FRA', 'country_GBR', 'country_GRC','country_HUN', 'country_IRL', 'country_ISL', 'country_ISR','
                           'country_KOR', 'country_LUX', 'country_LVA','country_NLD', 'country_NOR', 'country_POL', 'country_PRT','
                           'country_SVN', 'country_SWE', 'country_USA']]

ridgeCV = RidgeCV(alphas=np.arange(0.01,100,0.05)).fit(country_control,y_train)
print("Most appropriate Ridge alpha for model 6 (with country control):", ridgeCV.alpha_)

country_control_tst = x_test[["gini_index","mortality_rate_infant","intentional_homicides","suicide_mortality_rate", \
                   "alcohol_consumption_per_capita","government_health_expenditure_pct_gdp",'country_AUT', 'country_CAN', \
                           'country_CHE', 'country_CZE','country_DEU', 'country_DNK', 'country_ESP', 'country_EST','country_FIN', \
                           'country_FRA', 'country_GBR', 'country_GRC','country_HUN', 'country_IRL', 'country_ISL', 'country_ISR','
                           'country_KOR', 'country_LUX', 'country_LVA','country_NLD', 'country_NOR', 'country_POL', 'country_PRT','
                           'country_SVN', 'country_SWE', 'country_USA']]

ridgereg = Ridge(alpha=ridgeCV.alpha_).fit(country_control,y_train)
predicted_y_cc = ridgereg.predict(country_control_tst)
print("MSE for Model 6 + Country Variable Ridge regression:",mean_squared_error(y_test,predicted_y_cc))

plt.scatter(predicted_y_cc,predicted_y_cc-y_test)
plt.axhline(0,color='red',linewidth = 2)
plt.ylim(-41,41)
plt.title('Residuals Plot: Best Model + Country Variable')
plt.xlabel("Predicted Values")
plt.ylabel("Residuals")
plt.show()
```

```
MSE for Model 6 Ridge regression: 294.86822815410557
```



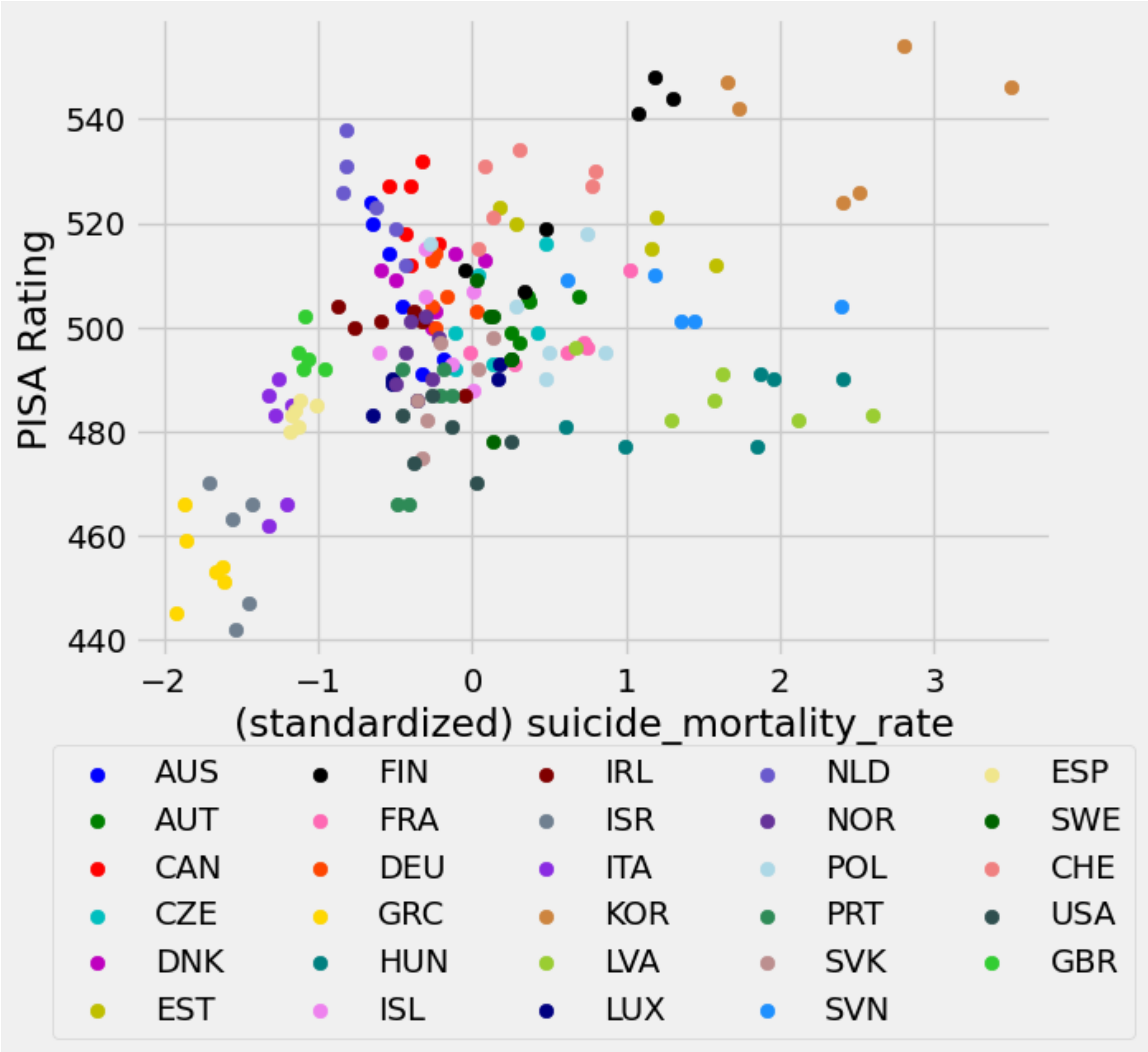

```
Most appropriate Ridge alpha for model 6 (with country control): 0.11
MSE for Model 6 + Country Variable Ridge regression: 94.60608174263879
```

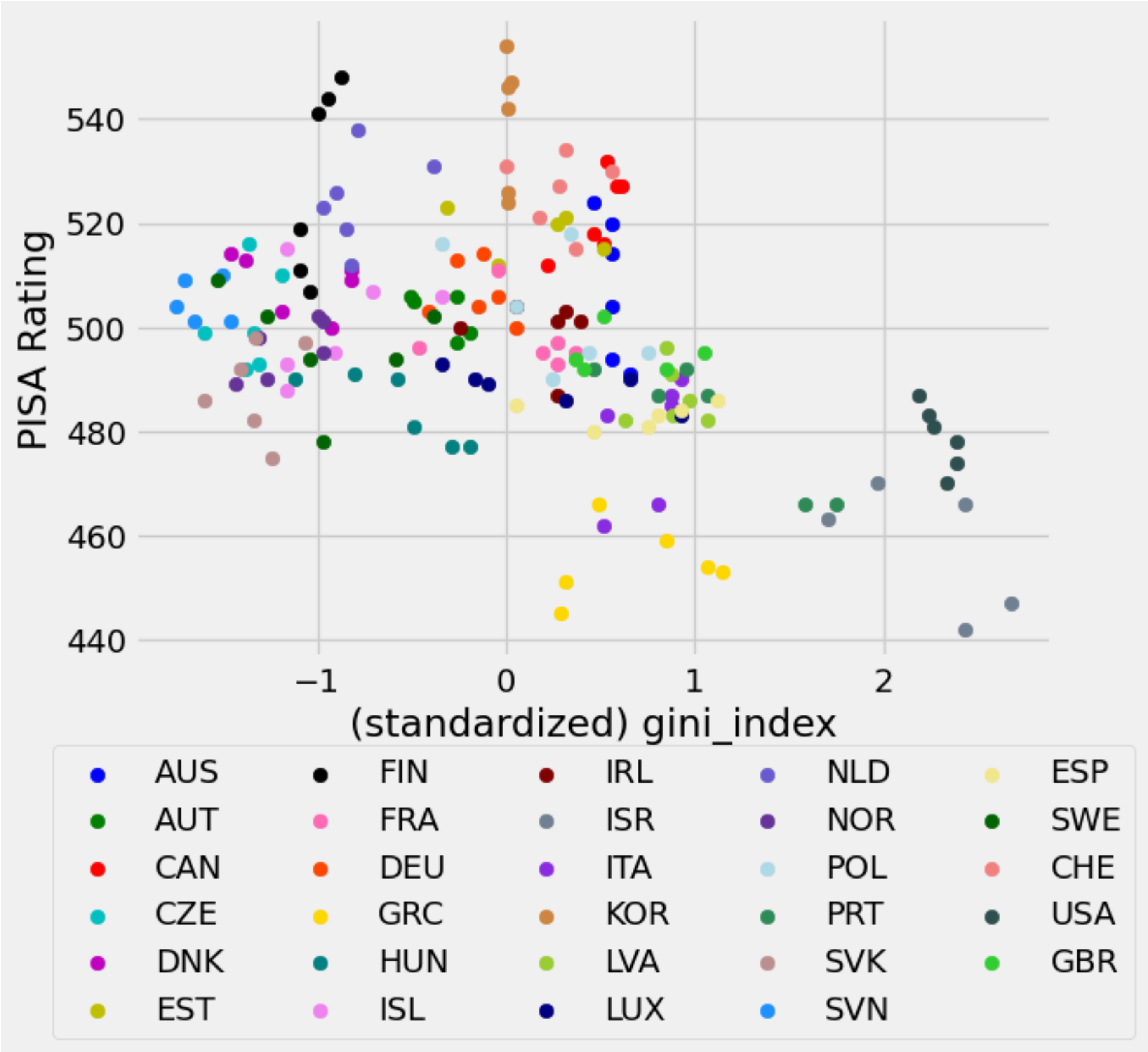


Residuals Plot: Best Model + Country Variable
Residuals
Predicted Values
40
30
20
10
0
−10
−20
−30
−40
470
480
490
500
510
520
530
540

In [21]:

```
# analyze how countries cluster over influential variables and the PISA score

colors = ["b","g","r","c","m","y","k","hotpink","orangered","gold","teal","violet","maroon","slategrey","blueviolet","peru","yellow
          ,"navy","slateblue","rebeccapurple","lightblue","seagreen","rosybrown","dodgerblue","khaki","darkgreen","lightcoral","dar

for i,country in enumerate(pisa_df_tot.country.unique()):
  plt.scatter(pisa_df_tot.loc[pisa_df_tot["country"] == country].suicide_mortality_rate,pisa_df_tot.loc[pisa_df_tot["country"] == c
              label = country,color = colors[i])
  plt.legend(ncols=5,loc='upper center',bbox_to_anchor=(0.5, -0.11))
  plt.xlabel("(standardized) suicide_mortality_rate")
  plt.ylabel("PISA Rating")
plt.show()

print("\n")
for i,country in enumerate(pisa_df_tot.country.unique()):
  plt.scatter(pisa_df_tot.loc[pisa_df_tot["country"] == country].gini_index,pisa_df_tot.loc[pisa_df_tot["country"] == country].rati
              label = country,color = colors[i])
  plt.legend(ncols=5,loc='upper center',bbox_to_anchor=(0.5, -0.11))
  plt.xlabel("(standardized) gini_index")
  plt.ylabel("PISA Rating")
plt.show()
```


PISA Rating
540
520
500
480
460
440
−2
−1
0
1
2
3
(standardized) suicide_mortality_rate
AUS
AUT
CAN
CZE
DNK
EST
FIN
FRA
DEU
GRC
HUN
ISL
IRL
ISR
ITA
KOR
LVA
LUX
NLD
NOR
POL
PRT
SVK
SVN
ESP
SWE
CHE
USA
GBR



CLUSTERING ANALYSIS

In [22]:

```
# define dataframe for clustering, with best model including the PISA score
bestmodel_df = pisa_df_tot[["gini_index","mortality_rate_infant","intentional_homicides", \
                            "suicide_mortality_rate","alcohol_consumption_per_capita","government_health_expenditure_pct_gdp","rating'

# standardize PISA score
scaler = StandardScaler()
bestmodel_df[["rating"]] = scaler.fit_transform(bestmodel_df[["rating"]])

# display the dataframe
display(bestmodel_df.head())
```

```
<ipython-input-22-494ca2238d20>:7: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy (https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy)
  bestmodel_df[["rating"]] = scaler.fit_transform(bestmodel_df[["rating"]])
```

	gini_index	mortality_rate_infant	intentional_homicides	suicide_mortality_rate	alcohol_consumption_per_capita	government_health_expenditure_pct_g
index_code						
AUS-2003	0.462126	0.831882	0.142289	-0.660127	0.121143	-0.3303
AUS-2006	0.559458	0.679648	-0.005489	-0.642575	0.121143	-0.2698
AUS-2009	0.559458	0.299063	-0.152009	-0.537262	0.121143	0.0593
AUS-2012	0.559458	-0.157639	-0.285880	-0.449501	0.121143	0.0187
AUS-2015	0.559458	-0.385990	-0.358186	-0.186219	0.121143	0.6812

In [23]:

```python
# required definitions to perform K-Means Clustering

def distance(pt1, pt2):
    """Return the distance between two points, represented as arrays"""
    return np.sqrt(sum((pt1 - pt2)**2))

def initialize_centroids(df,K):
    random_ids = np.random.permutation(df.shape[0])
    centroids = df.iloc[random_ids[:K]]
    return centroids

def compute_distance(df, centroids):
    K=centroids.shape[0]
    distances_ar = np.zeros((df.shape[0], K))
    for k in range(K):
        point=centroids.iloc[k]
        def distance_from_point(row):
            return distance(point, np.array(row))
        distances_ar[:,k] = df.apply(distance_from_point,axis=1).values
    return distances_ar

def compute_sse(df, labels, centroids,K):
    distances_ar = np.zeros(df.shape[0])
    for k in range(K):
        point=centroids.iloc[k]
        def distance_from_point(row):
            return distance(point, np.array(row))
        distances_ar[labels == k] = df[labels == k].apply(distance_from_point,axis=1).values
    return np.sum(distances_ar)

def compute_centroids(df, labels, K):
    centroids = np.zeros((K, df.shape[1]))
    for k in range(K):
        centroids[k, :] = df[labels == k].mean()
    return centroids

def Kmeans(df,K):
    max_iter=20

    centroids=initialize_centroids(df,K)

    for i in range(max_iter):
            old_centroids = centroids
            dist_matrix = compute_distance(df, old_centroids)
            clust=np.argmin(dist_matrix,axis = 1)
            centroids = pd.DataFrame(compute_centroids(df,clust,K))
```

```python

    return centroids,clust

def Kmeans_sse(df,K):
    '''performs Kmeans returns centroids and prints sse of each new centroids'''
    #define the maximum number of iterations
    max_iter=20

    #initialize centroids
    centroids=initialize_centroids(df,K)

    for i in range(max_iter):
            old_centroids = centroids
            dist_matrix = compute_distance(df, old_centroids)
            clust=np.argmin(dist_matrix, axis=1)
            centroids = pd.DataFrame(compute_centroids(df,clust,K))

    # return the centroids
    return compute_sse(df,clust,old_centroids,K)
```

In [24]:

```python
# create k and SSE arrays to find best number of clusters

k = np.array([])
sse = np.array([])
for i in np.arange(1,10):
    k = np.append(k,i)
    sse = np.append(sse,Kmeans_sse(bestmodel_df,i))
```

In [25]:

```
# elbow plot is used to select number of clusters to use

plt.plot(k,sse)
plt.title("Elbow Plot")
plt.xlabel("K (number of clusters)")
plt.ylabel("Minimum SSE")
plt.show()
```

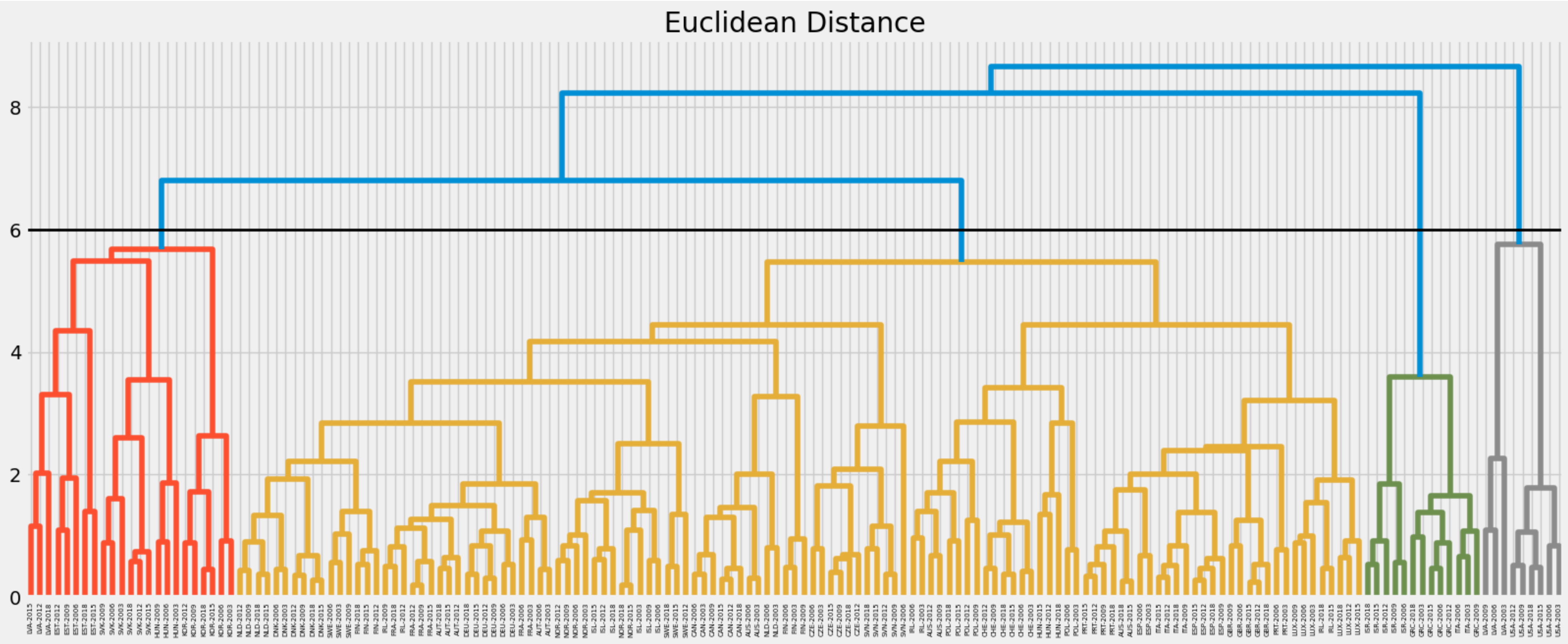
In [26]:

```
# return centroids and clusters used chosen K=3
centroids,clust = Kmeans(bestmodel_df,3)

# show the clusters created by K-Means Clustering!
pd.set_option('display.max_rows', None)

country_names = bestmodel_df.index
cluster_countries = pd.DataFrame({"country":country_names,"cluster_label":clust})
cluster_countries

cluster0_countries = cluster_countries[cluster_countries["cluster_label"] == 0]
display(cluster0_countries)

cluster1_countries = cluster_countries[cluster_countries["cluster_label"] == 1]
display(cluster1_countries)

cluster2_countries = cluster_countries[cluster_countries["cluster_label"] == 2]
display(cluster2_countries)
```

	country	cluster_label
4	AUS-2015	0
5	AUS-2018	0
52	GRC-2003	0
53	GRC-2006	0
54	GRC-2009	0
55	GRC-2012	0
56	GRC-2015	0
57	GRC-2018	0
65	ISL-2006	0
67	ISL-2012	0
68	ISL-2015	0
69	ISL-2018	0

In [27]:

```python
# perform hierarchical clustering
lbls = np.array(bestmodel_df.index)

cluster = linkage(bestmodel_df, method ='complete',metric = "euclidean")
plt.figure(figsize=(18, 7))
dendrogram(cluster,
            orientation='top',
            labels=lbls,
            distance_sort='descending')
plt.axhline(6,c="black",linewidth = 2)
plt.title("Euclidean Distance")
plt.show()
print()

# print out the clusters, using K=4
labels = fcluster(cluster, t=6.0, criterion='distance')
for k in np.arange(1,1+len(np.unique(labels))):
  print("group",k)
  print(bestmodel_df[labels==k].index.values)
  print('\n')
```



```
group 1
['LVA-2003' 'LVA-2006' 'LVA-2009' 'USA-2003' 'USA-2006' 'USA-2009'
 'USA-2012' 'USA-2015' 'USA-2018']


group 2
['GRC-2003' 'GRC-2006' 'GRC-2009' 'GRC-2012' 'GRC-2015' 'GRC-2018'
 'ISR-2006' 'ISR-2009' 'ISR-2012' 'ISR-2015' 'ISR-2018' 'ITA-2003'
 'ITA-2006']


group 3
['AUS-2003' 'AUS-2006' 'AUS-2009' 'AUS-2012' 'AUS-2015' 'AUS-2018'
 'AUT-2003' 'AUT-2006' 'AUT-2012' 'AUT-2015' 'AUT-2018' 'CAN-2003'
 'CAN-2006' 'CAN-2009' 'CAN-2012' 'CAN-2015' 'CAN-2018' 'CZE-2003'
 'CZE-2006' 'CZE-2009' 'CZE-2012' 'CZE-2015' 'CZE-2018' 'DNK-2003'
 'DNK-2006' 'DNK-2009' 'DNK-2012' 'DNK-2015' 'DNK-2018' 'FIN-2003'
 'FIN-2006' 'FIN-2009' 'FIN-2012' 'FIN-2015' 'FIN-2018' 'FRA-2003'
 'FRA-2006' 'FRA-2009' 'FRA-2012' 'FRA-2015' 'FRA-2018' 'DEU-2003'
 'DEU-2006' 'DEU-2009' 'DEU-2012' 'DEU-2015' 'DEU-2018' 'HUN-2012'
 'HUN-2015' 'HUN-2018' 'ISL-2003' 'ISL-2006' 'ISL-2009' 'ISL-2012'
 'ISL-2015' 'ISL-2018' 'IRL-2003' 'IRL-2006' 'IRL-2009' 'IRL-2012'
 'IRL-2015' 'IRL-2018' 'ITA-2009' 'ITA-2012' 'ITA-2015' 'ITA-2018'
 'LUX-2003' 'LUX-2006' 'LUX-2009' 'LUX-2012' 'LUX-2015' 'LUX-2018'
 'NLD-2003' 'NLD-2006' 'NLD-2009' 'NLD-2012' 'NLD-2015' 'NLD-2018'
 'NOR-2003' 'NOR-2006' 'NOR-2009' 'NOR-2012' 'NOR-2015' 'NOR-2018'
 'POL-2003' 'POL-2006' 'POL-2009' 'POL-2012' 'POL-2015' 'POL-2018'
 'PRT-2003' 'PRT-2006' 'PRT-2009' 'PRT-2012' 'PRT-2015' 'PRT-2018'
 'SVN-2006' 'SVN-2009' 'SVN-2012' 'SVN-2015' 'SVN-2018' 'ESP-2003'
 'ESP-2006' 'ESP-2009' 'ESP-2012' 'ESP-2015' 'ESP-2018' 'SWE-2003'
 'SWE-2006' 'SWE-2009' 'SWE-2012' 'SWE-2015' 'SWE-2018' 'CHE-2003'
 'CHE-2006' 'CHE-2009' 'CHE-2012' 'CHE-2015' 'CHE-2018' 'GBR-2006'
 'GBR-2009' 'GBR-2012' 'GBR-2015' 'GBR-2018']


group 4
['EST-2006' 'EST-2009' 'EST-2012' 'EST-2015' 'EST-2018' 'HUN-2003'
 'HUN-2006' 'HUN-2009' 'KOR-2003' 'KOR-2006' 'KOR-2009' 'KOR-2012'
 'KOR-2015' 'KOR-2018' 'LVA-2012' 'LVA-2015' 'LVA-2018' 'SVK-2003'
 'SVK-2006' 'SVK-2009' 'SVK-2012' 'SVK-2015' 'SVK-2018']
```