```
# import libraries

import numpy as np
import pandas as pd

%matplotlib inline
import matplotlib.pyplot as plt
plt.style.use('fivethirtyeight')

from sklearn.linear_model import LinearRegression, Lasso, LassoCV,
Ridge, RidgeCV
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split, KFold
from sklearn.metrics import r2_score, mean_squared_error,
confusion_matrix, ConfusionMatrixDisplay, accuracy_score
from sklearn.ensemble import RandomForestClassifier
from sklearn.tree import plot_tree

from scipy.optimize import minimize
from scipy.cluster.hierarchy import dendrogram, linkage, fcluster

from tqdm import notebook

import seaborn as sns
```

LOAD DATAFRAME and INITIAL EXPLORATION

```
# loads dataframe; prints head
pisa_df =
pd.read_csv("https://raw.githubusercontent.com/babnigg/DATA11900/main/
economics_and_education_dataset_CSV.csv")
pisa_df.head()
```

```
  index_code  expenditure_on _education_pct_gdp  mortality_rate_infant
\
0   AUS-2003                           5.246357                    4.9

1   AUS-2003                           5.246357                    4.9

2   AUS-2003                           5.246357                    4.9

3   AUS-2006                           4.738430                    4.7

4   AUS-2006                           4.738430                    4.7


   gini_index  gdp_per_capita_ppp  inflation_consumer_prices  \
```

```
0        33.5        30121.818418                   2.732596
1        33.5        30121.818418                   2.732596
2        33.5        30121.818418                   2.732596
3         NaN        34846.715630                   3.555288
4         NaN        34846.715630                   3.555288

   intentional_homicides  unemployment  gross_fixed_capital_formation
\
0               1.533073         5.933                      26.050295

1               1.533073         5.933                      26.050295

2               1.533073         5.933                      26.050295

3               1.372940         4.785                      27.789132

4               1.372940         4.785                      27.789132


   population_density  suicide_mortality_rate  tax_revenue  \
0            2.567036                    10.5    24.299970
1            2.567036                    10.5    24.299970
2            2.567036                    10.5    24.299970
3            2.662089                    10.6    24.511772
4            2.662089                    10.6    24.511772

   taxes_on_income_profits_capital  alcohol_consumption_per_capita  \
0                        62.726546                             NaN
1                        62.726546                             NaN
2                        62.726546                             NaN
3                        65.231562                             NaN
4                        65.231562                             NaN

   government_health_expenditure_pct_gdp  urban_population_pct_total
country  \
0                               5.623778                      84.343
AUS
1                               5.623778                      84.343
AUS
2                               5.623778                      84.343
AUS
3                               5.719998                      84.700
AUS
4                               5.719998                      84.700
AUS

   time   sex  rating
0  2003   BOY   527.0
1  2003  GIRL   522.0
2  2003   TOT   524.0
```

```
3  2006   BOY   527.0
4  2006  GIRL   513.0
```

```
# filters out dataframe to only include total sex aggregate; prints
info and describes numerical variables
agg = "TOT"
pisa_df_tot = pisa_df[pisa_df["sex"]==agg]

pisa_df_tot.info()
pisa_df_tot.describe()
```

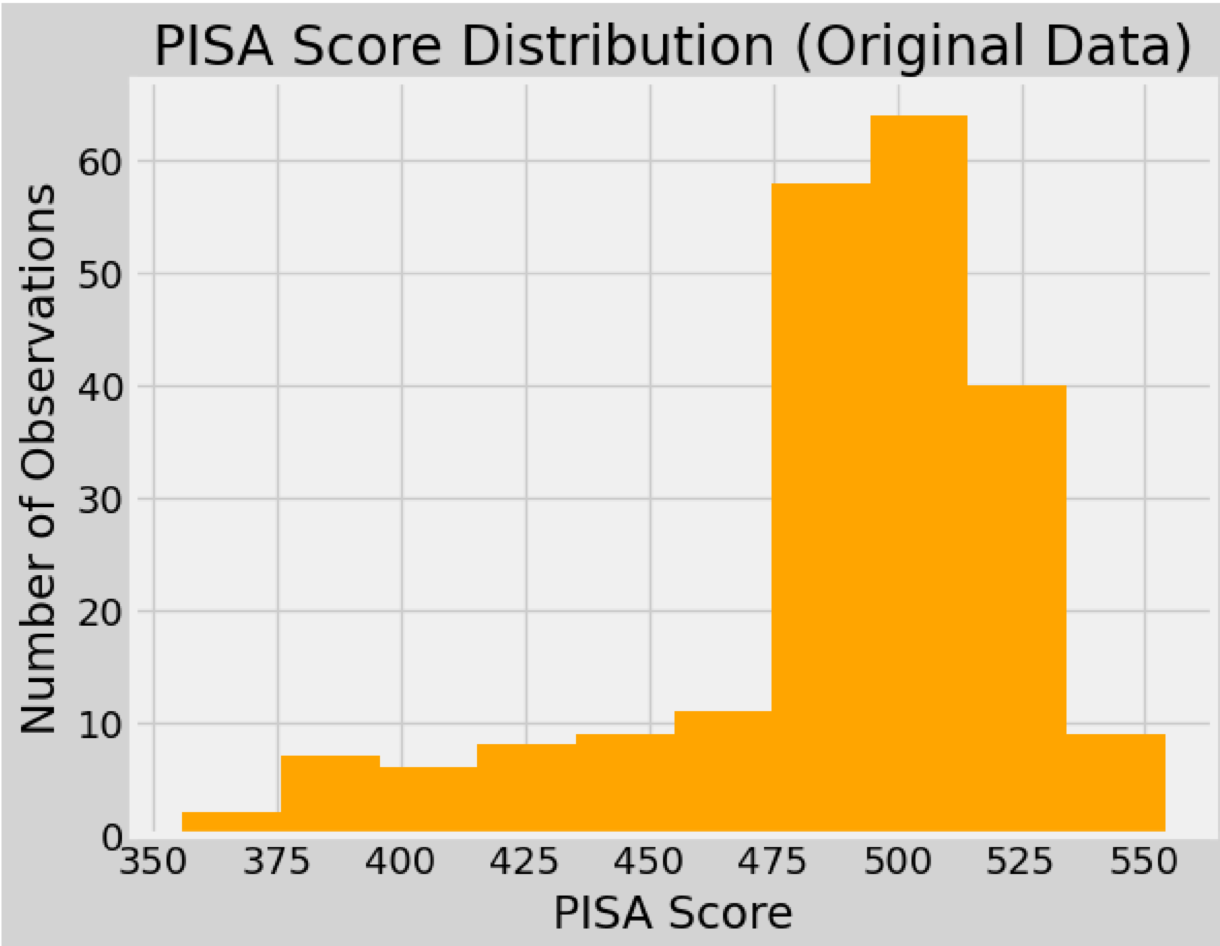
```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 214 entries, 2 to 633
Data columns (total 20 columns):
 #   Column                                 Non-Null Count  Dtype
---  ------                                 --------------  -----
 0   index_code                             214 non-null    object
 1   expenditure_on _education_pct_gdp      202 non-null    float64
 2   mortality_rate_infant                  214 non-null    float64
 3   gini_index                             181 non-null    float64
 4   gdp_per_capita_ppp                     214 non-null    float64
 5   inflation_consumer_prices              214 non-null    float64
 6   intentional_homicides                  187 non-null    float64
 7   unemployment                           214 non-null    float64
 8   gross_fixed_capital_formation          214 non-null    float64
 9   population_density                     214 non-null    float64
 10  suicide_mortality_rate                 214 non-null    float64
 11  tax_revenue                            198 non-null    float64
 12  taxes_on_income_profits_capital        198 non-null    float64
 13  alcohol_consumption_per_capita         37 non-null     float64
 14  government_health_expenditure_pct_gdp  214 non-null    float64
 15  urban_population_pct_total             214 non-null    float64
 16  country                                214 non-null    object
 17  time                                   214 non-null    int64
 18  sex                                    214 non-null    object
 19  rating                                 214 non-null    float64
dtypes: float64(16), int64(1), object(3)
memory usage: 35.1+ KB
```

```
       expenditure_on _education_pct_gdp  mortality_rate_infant
gini_index  \
count                          202.000000             214.000000
181.000000
mean                             5.237384               5.062150
33.411050
std                              1.105318               4.090917
6.816531
min                              3.040150               1.900000
24.400000
25%                              4.460890               3.000000
```

```
28.100000
50%                               5.058085               3.700000
32.700000
75%                               5.796320               5.100000
35.400000
max                               8.448880              25.300000
57.600000

       gdp_per_capita_ppp  inflation_consumer_prices
intentional_homicides  \
count          214.000000                 214.000000
187.000000
mean         36176.526959                   2.300823
3.119124
std          16451.684140                   2.742357
6.122298
min           9587.557951                  -4.478103
0.000000
25%          26081.142138                   0.799862
0.833583
50%          34305.514059                   1.980056
1.095505
75%          42993.833915                   2.991248
1.842371
max         116498.512081                  21.602438
29.581371

       unemployment  gross_fixed_capital_formation  population_density
\
count    214.000000                     214.000000          214.000000

mean       7.643659                      22.507161          133.327098

std        4.021760                       3.942642          133.840583

min        2.246000                      10.770040            2.567036

25%        4.892500                      20.210197           30.425942

50%        6.843000                      22.454439          101.872663

75%        9.333500                      24.296864          192.362949

max       24.981000                      36.800286          528.969011


       suicide_mortality_rate  tax_revenue
taxes_on_income_profits_capital  \
count              214.000000   198.000000
198.000000
```

```
mean                    13.584112    20.119590
27.827355
std                      6.282140     5.748563
13.303791
min                      2.200000     7.903518
7.173176
25%                      9.675000    15.526577
17.681421
50%                     12.900000    20.951774
25.383772
75%                     16.925000    24.172369
34.128627
max                     34.200000    33.921619
65.517310

       alcohol_consumption_per_capita
government_health_expenditure_pct_gdp  \
count                               37.000000
214.000000
mean                                 9.452505
5.939307
std                                  2.721759
1.755618
min                                  1.895130
2.413703
25%                                  7.847910
4.473900
50%                                  9.916870
6.045355
75%                                 11.462780
7.382606
max                                 15.058720
9.276339

       urban_population_pct_total         time      rating
count                  214.000000   214.000000  214.000000
mean                    77.345145  2010.808411  489.242991
std                     11.020002     5.106906   37.238186
min                     51.758000  2003.000000  356.000000
25%                     68.733000  2006.000000  482.000000
50%                     79.628500  2012.000000  495.000000
75%                     85.680750  2015.000000  512.750000
max                     98.001000  2018.000000  554.000000

# plots histogram of PISA score distribution in original data
plt.figure().set_facecolor("lightgrey")
plt.hist(pisa_df_tot.rating, color = "orange")
plt.title("PISA Score Distribution (Original Data)")
plt.ylabel("Number of Observations")
```

```
plt.xlabel("PISA Score")
plt.show()
```



DATA CLEANING and EXPLORATION

```
# counts and prints number of NA values per column
np.sum(pisa_df_tot.isna(),axis=0)
```

```
index_code                              0
expenditure_on _education_pct_gdp      12
mortality_rate_infant                   0
gini_index                             33
gdp_per_capita_ppp                      0
inflation_consumer_prices               0
intentional_homicides                  27
unemployment                            0
gross_fixed_capital_formation           0
```

```
population_density                            0
suicide_mortality_rate                        0
tax_revenue                                  16
taxes_on_income_profits_capital              16
alcohol_consumption_per_capita              177
government_health_expenditure_pct_gdp         0
urban_population_pct_total                    0
country                                       0
time                                          0
sex                                           0
rating                                        0
dtype: int64
```

```
# defining function to impute data with the mean for the column
def fill_missing_with_mean(group):
  '''fill the missing values with the mean of each feature (column)
for each country.'''
  columns_to_fill = ['expenditure_on _education_pct_gdp',
'gini_index', 'intentional_homicides',
                       'tax_revenue',
'taxes_on_income_profits_capital', 'alcohol_consumption_per_capita']
  for column in columns_to_fill:
      group[column] = group[column].fillna(group[column].mean())
  return group

# apply the function to each group (country)
pisa_df_tot = pisa_df_tot.groupby('country',
group_keys=False).apply(fill_missing_with_mean)

# glance at data head after imputing
display(pisa_df_tot.head())

# counts and prints number of NA values per column after imputing
np.sum(pisa_df_tot.isna(),axis=0)
```

```
   index_code  expenditure_on _education_pct_gdp
mortality_rate_infant  \
2    AUS-2003                           5.246357
4.9
5    AUS-2006                           4.738430
4.7
8    AUS-2009                           5.081320
4.2
11   AUS-2012                           4.866670
3.6
14   AUS-2015                           5.315520
3.3

    gini_index  gdp_per_capita_ppp  inflation_consumer_prices  \
2         33.5        30121.818418                   2.732596
```

```
5         33.9        34846.715630                           3.555288
8         33.9        40312.395119                           1.771117
11        33.9        42866.604330                           1.762780
14        33.9        46292.095439                           1.508367

    intentional_homicides  unemployment  gross_fixed_capital_formation
\
2                1.533073         5.933                      26.050295

5                1.372940         4.785                      27.789132

8                1.214170         5.565                      27.601846

11               1.069106         5.225                      27.404168

14               0.990754         6.055                      26.202336


    population_density  suicide_mortality_rate  tax_revenue  \
2             2.567036                    10.5    24.299970
5             2.662089                    10.6    24.511772
8             2.823588                    11.2    22.021006
11            2.959200                    11.7    21.097483
14            3.100113                    13.2    21.866424

    taxes_on_income_profits_capital  alcohol_consumption_per_capita  \
2                         62.726546                         10.3903
5                         65.231562                         10.3903
8                         64.951075                         10.3903
11                        65.517310                         10.3903
14                        64.893433                         10.3903

    government_health_expenditure_pct_gdp  urban_population_pct_total
country  \
2                                5.623778                      84.343
AUS
5                                5.719998                      84.700
AUS
8                                6.244110                      85.063
AUS
11                               6.179476                      85.402
AUS
14                               7.234067                      85.701
AUS

    time  sex  rating
2   2003  TOT   524.0
5   2006  TOT   520.0
8   2009  TOT   514.0
```

```
11  2012  TOT   504.0
14  2015  TOT   494.0
```

```
index_code                                 0
expenditure_on _education_pct_gdp          0
mortality_rate_infant                      0
gini_index                                12
gdp_per_capita_ppp                         0
inflation_consumer_prices                  0
intentional_homicides                      6
unemployment                               0
gross_fixed_capital_formation              0
population_density                         0
suicide_mortality_rate                     0
tax_revenue                                6
taxes_on_income_profits_capital            6
alcohol_consumption_per_capita             2
government_health_expenditure_pct_gdp      0
urban_population_pct_total                 0
country                                    0
time                                       0
sex                                        0
rating                                     0
dtype: int64
```

Since there are some countries that don't have any entry for certain variables we can just drop those countries.
Countries: BEL, JPN, NZL, CRI, LTU.
We also choose to drop countries with the characteristically low PISA scores in comparison to other countries in dataset to normalize the score distribution.
Countries: MEX, BRA, CHL, TUR, COL

```
# dropping countries with lone NaN values
# last 5 countires in list are lowest scoring countries
countries_to_drop = ['BEL', 'JPN',
'NZL','CRI','LTU','MEX','BRA','CHL','TUR','COL']
pisa_df_tot =
pisa_df_tot.drop(pisa_df_tot[pisa_df_tot['country'].isin(countries_to_
drop)].index)

# 'sex' is a redundant column as we already control the dataset on
sex='TOT'
pisa_df_tot = pisa_df_tot.drop(['sex'],axis=1)

# informative 'index_code' column can be used to replace default index
pisa_df_tot.reset_index(drop=True,inplace=True)
pisa_df_tot.set_index('index_code',inplace=True)

# printing remaining number of countries in the dataset
```

```
print("Number of remaining countries in
dataset:",len(pisa_df_tot.country.unique()))
print("\n")

# glance at data head after cleaning
display(pisa_df_tot.head())

# counts and prints number of NA values per column after removing
countries
print(np.sum(pisa_df_tot.isna(),axis=0))
```

```
Number of remaining countries in dataset: 29



            expenditure_on _education_pct_gdp
mortality_rate_infant  \
index_code

AUS-2003                             5.246357                    4.9

AUS-2006                             4.738430                    4.7

AUS-2009                             5.081320                    4.2

AUS-2012                             4.866670                    3.6

AUS-2015                             5.315520                    3.3


            gini_index  gdp_per_capita_ppp  inflation_consumer_prices
\
index_code

AUS-2003          33.5        30121.818418                   2.732596

AUS-2006          33.9        34846.715630                   3.555288

AUS-2009          33.9        40312.395119                   1.771117

AUS-2012          33.9        42866.604330                   1.762780

AUS-2015          33.9        46292.095439                   1.508367


            intentional_homicides  unemployment  \
index_code
AUS-2003                 1.533073         5.933
AUS-2006                 1.372940         4.785
AUS-2009                 1.214170         5.565
AUS-2012                 1.069106         5.225
```

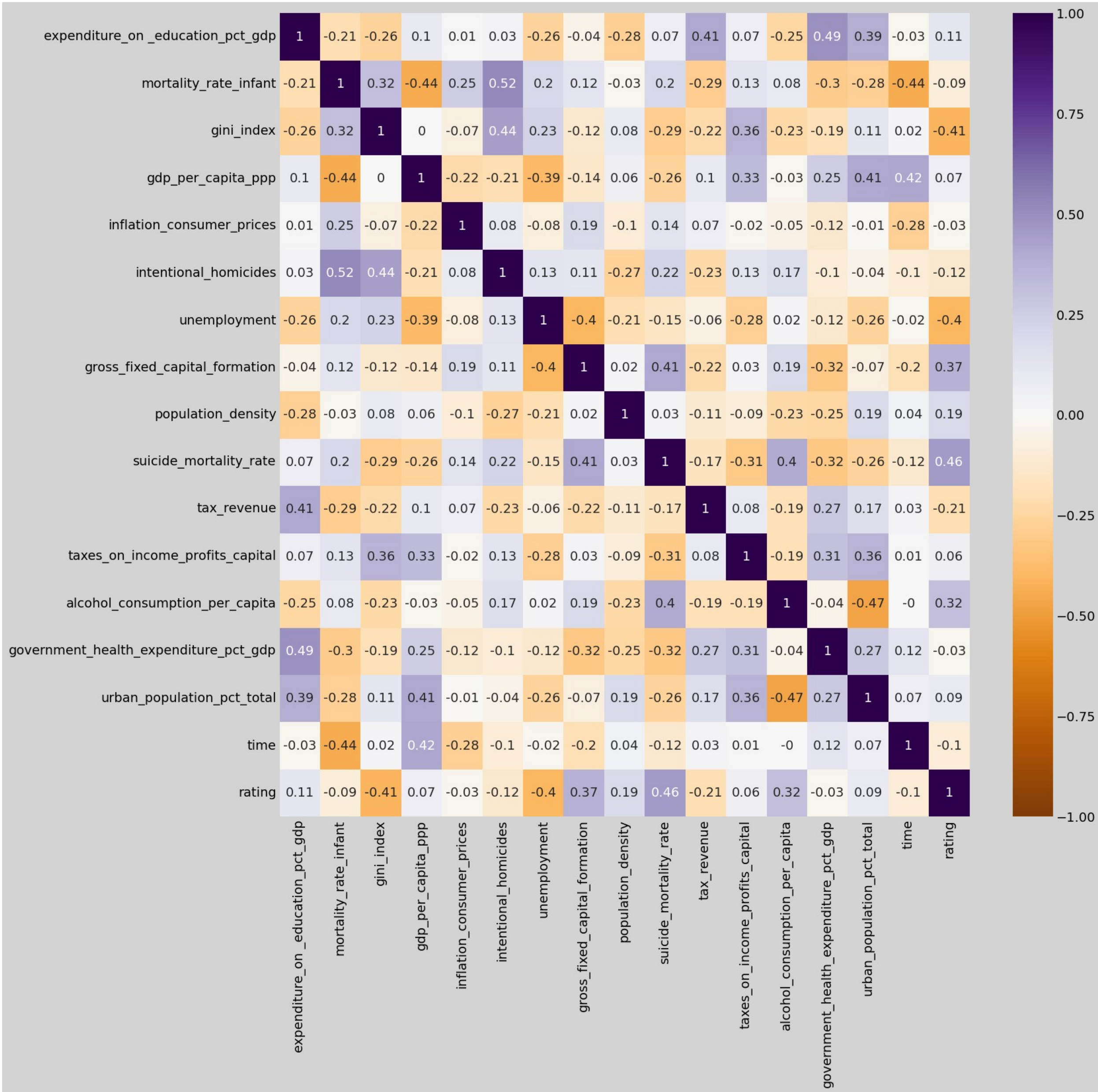
```
AUS-2015                  0.990754         6.055

            gross_fixed_capital_formation  population_density  \
index_code
AUS-2003                        26.050295            2.567036
AUS-2006                        27.789132            2.662089
AUS-2009                        27.601846            2.823588
AUS-2012                        27.404168            2.959200
AUS-2015                        26.202336            3.100113

            suicide_mortality_rate  tax_revenue  \
index_code
AUS-2003                      10.5    24.299970
AUS-2006                      10.6    24.511772
AUS-2009                      11.2    22.021006
AUS-2012                      11.7    21.097483
AUS-2015                      13.2    21.866424

            taxes_on_income_profits_capital
alcohol_consumption_per_capita  \
index_code

AUS-2003                             62.726546
10.3903
AUS-2006                             65.231562
10.3903
AUS-2009                             64.951075
10.3903
AUS-2012                             65.517310
10.3903
AUS-2015                             64.893433
10.3903

            government_health_expenditure_pct_gdp
urban_population_pct_total  \
index_code

AUS-2003                                    5.623778
84.343
AUS-2006                                    5.719998
84.700
AUS-2009                                    6.244110
85.063
AUS-2012                                    6.179476
85.402
AUS-2015                                    7.234067
85.701

           country  time  rating
index_code
```

```
AUS-2003       AUS  2003   524.0
AUS-2006       AUS  2006   520.0
AUS-2009       AUS  2009   514.0
AUS-2012       AUS  2012   504.0
AUS-2015       AUS  2015   494.0
```

```
expenditure_on _education_pct_gdp        0
mortality_rate_infant                    0
gini_index                               0
gdp_per_capita_ppp                       0
inflation_consumer_prices                0
intentional_homicides                    0
unemployment                             0
gross_fixed_capital_formation            0
population_density                       0
suicide_mortality_rate                   0
tax_revenue                              0
taxes_on_income_profits_capital          0
alcohol_consumption_per_capita           0
government_health_expenditure_pct_gdp    0
urban_population_pct_total               0
country                                  0
time                                     0
rating                                   0
dtype: int64
```

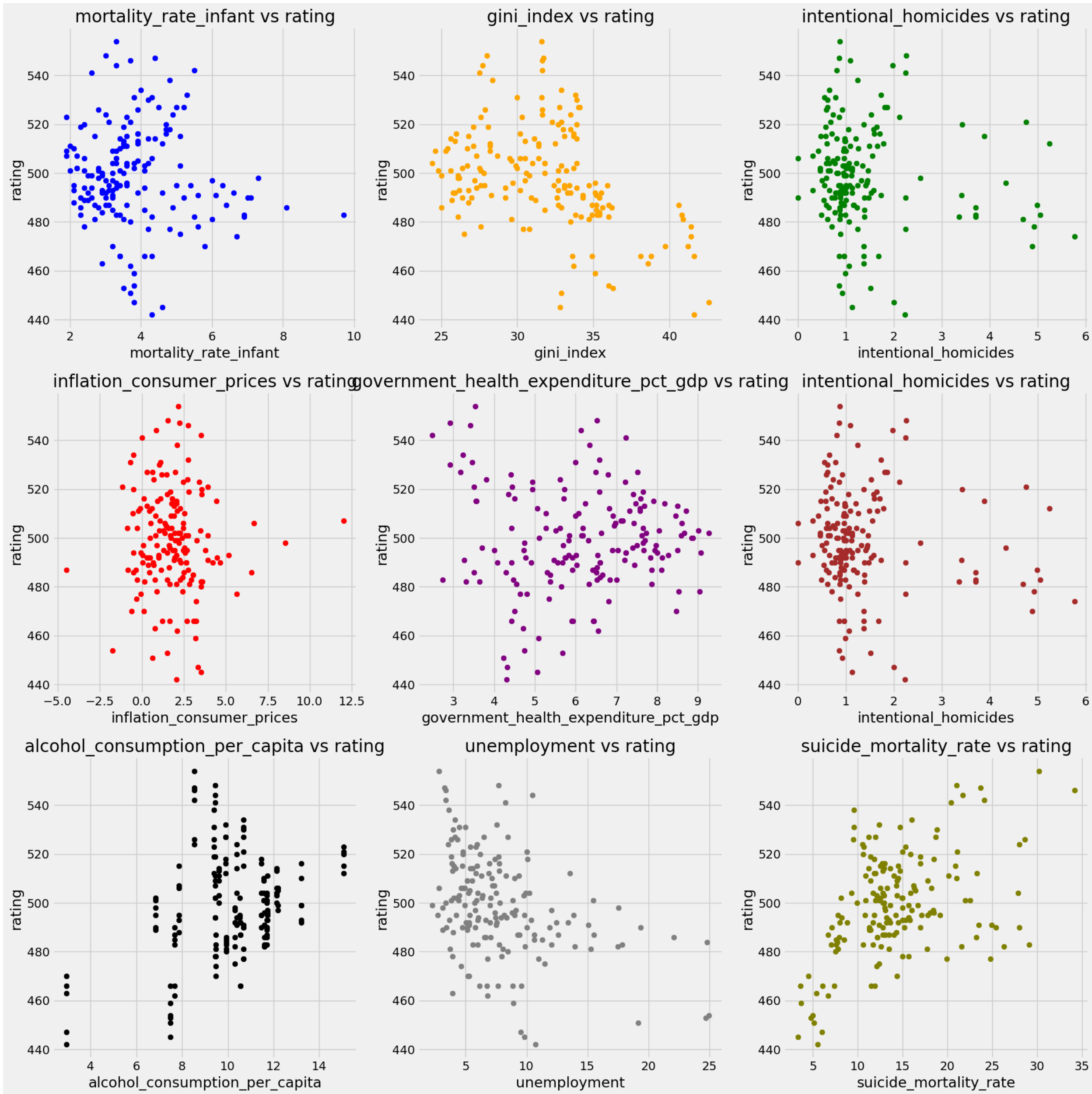
EXPLORATION

```
# plot correlation matrix with all numerical variables
corr_matrix = pisa_df_tot.corr(numeric_only=True).round(2)
plt.figure(figsize=(15,15)).set_facecolor("lightgrey")
sns.heatmap(data=corr_matrix,annot=True,vmin=-1,vmax=1,cmap="PuOr")
plt.show()
```




```python
# exploratory data analysis based on correlation matrix

# list of features that has quite high correlation score
features = ["mortality_rate_infant", "gini_index",
"intentional_homicides",
            "inflation_consumer_prices",
"government_health_expenditure_pct_gdp", "intentional_homicides",
            "alcohol_consumption_per_capita", "unemployment",
"suicide_mortality_rate"
           ]
colors = ['blue', 'orange', 'green', 'red', 'purple', 'brown',
'black', 'gray', 'olive']
```

```
# create subplots
fig, axes = plt.subplots(3, 3, figsize=(18, 18))

# flatten axes for easy iteration
axes = axes.flatten()

# plot scatter plots for each feature
for i, (feature, color) in enumerate(zip(features, colors)):
    ax = axes[i]
    ax.scatter(pisa_df_tot[feature],
pisa_df_tot['rating'],color=color)
    ax.set_xlabel(feature)
    ax.set_ylabel('rating')
    ax.set_title(f'{feature} vs rating')

# adjust layout
plt.tight_layout()
plt.show()
```

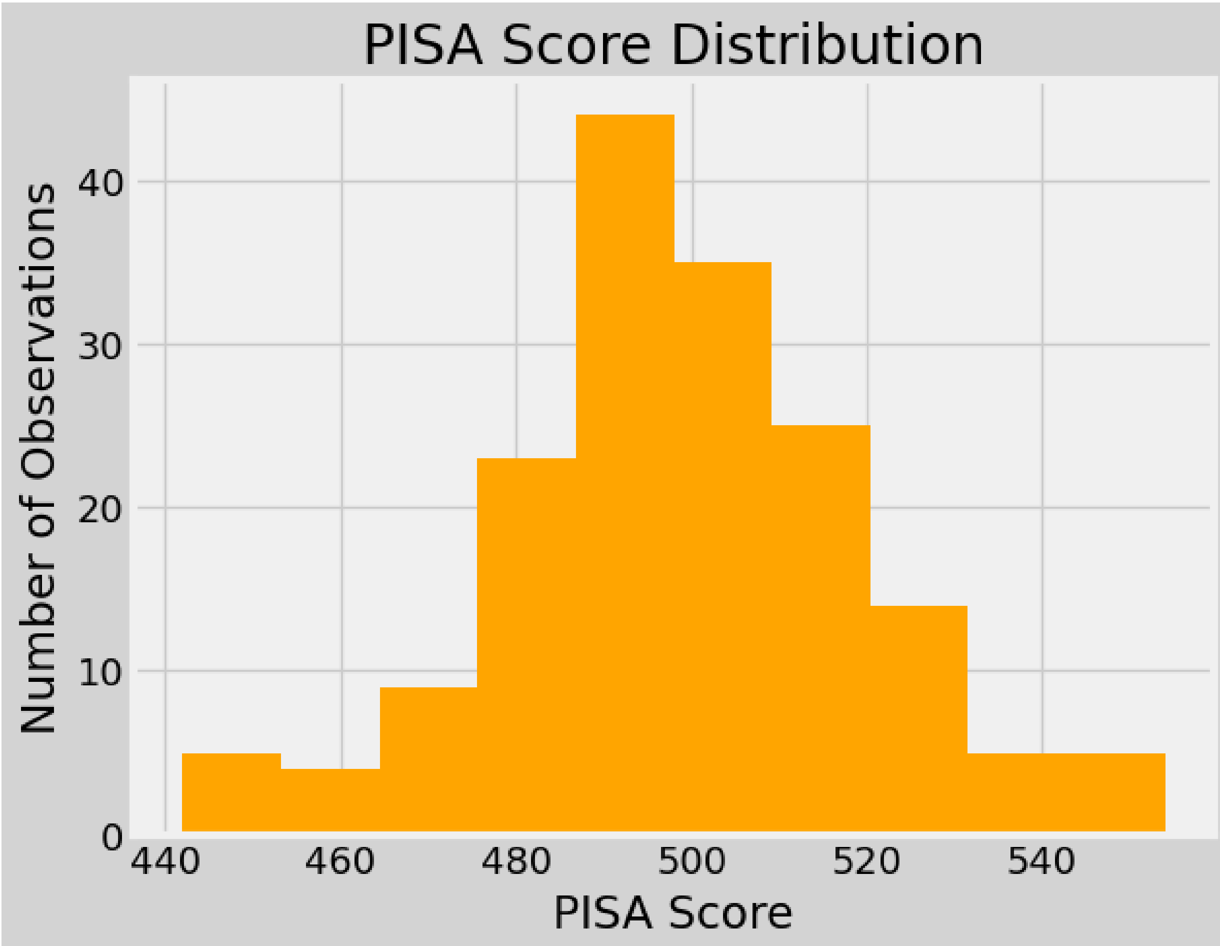



```
# scaling and transforming numerical variables
scaler = StandardScaler()

num_X = pisa_df_tot.select_dtypes(exclude='object')
num_columns = num_X.drop(['rating','time'],axis=1).columns
# (time is a number, but it is indeed a categorical variable!)

# uses standard scaler to transform all numerical variables
pisa_df_tot[num_columns]=
scaler.fit_transform(pisa_df_tot[num_columns])
# one-hot-encodes categorical variables of country and time
country_col = pisa_df_tot.country
```

```
pisa_df_tot =
pd.get_dummies(pisa_df_tot,columns=['country','time'],drop_first=True,
dtype=int)
pisa_df_tot["country"] = country_col
display(pisa_df_tot.head())
```

```
            expenditure_on _education_pct_gdp
mortality_rate_infant  \
index_code

AUS-2003                            -0.029140               0.831882

AUS-2006                            -0.496382               0.679648

AUS-2009                            -0.180958               0.299063

AUS-2012                            -0.378414              -0.157639

AUS-2015                             0.034482              -0.385990


            gini_index  gdp_per_capita_ppp  inflation_consumer_prices
\
index_code

AUS-2003      0.462126           -0.546819                   0.507257

AUS-2006      0.559458           -0.258194                   0.968513

AUS-2009      0.559458            0.075682                  -0.031812

AUS-2012      0.559458            0.231708                  -0.036486

AUS-2015      0.559458            0.440957                  -0.179127


            intentional_homicides  unemployment  \
index_code
AUS-2003                 0.142289     -0.426994
AUS-2006                -0.005489     -0.695237
AUS-2009                -0.152009     -0.512982
AUS-2012                -0.285880     -0.592426
AUS-2015                -0.358186     -0.398488

            gross_fixed_capital_formation  population_density  \
index_code
AUS-2003                         0.872513           -1.001173
AUS-2006                         1.294253           -1.000452
AUS-2009                         1.248829           -0.999228
AUS-2012                         1.200883           -0.998200
AUS-2015                         0.909389           -0.997132
```

```
            suicide_mortality_rate  ...  country_SVK  country_SVN  \
index_code                          ...
AUS-2003                 -0.660127  ...            0            0
AUS-2006                 -0.642575  ...            0            0
AUS-2009                 -0.537262  ...            0            0
AUS-2012                 -0.449501  ...            0            0
AUS-2015                 -0.186219  ...            0            0

            country_SWE  country_USA  time_2006  time_2009  time_2012
\
index_code

AUS-2003              0            0          0          0          0

AUS-2006              0            0          1          0          0

AUS-2009              0            0          0          1          0

AUS-2012              0            0          0          0          1

AUS-2015              0            0          0          0          0


            time_2015  time_2018  country
index_code
AUS-2003            0          0      AUS
AUS-2006            0          0      AUS
AUS-2009            0          0      AUS
AUS-2012            0          0      AUS
AUS-2015            1          0      AUS

[5 rows x 50 columns]

# plots histogram of PISA score distribution after data cleaning
plt.figure().set_facecolor("lightgrey")
plt.hist(pisa_df_tot.rating, color = "orange")
plt.title("PISA Score Distribution")
plt.ylabel("Number of Observations")
plt.xlabel("PISA Score")
plt.show()
```



LINEAR, RIDGE, and LASSO MODELS: SELECTING BEST MODEL

```
# train and test dataset split
x_df = pisa_df_tot.loc[:,pisa_df_tot.columns != "rating"]
y_df = pisa_df_tot["rating"]

x_train, x_test, y_train, y_test = train_test_split(x_df, y_df,
test_size = 0.2)

# defining 7 different models
model1x = x_train[["gini_index","unemployment"]]
model2x =
x_train[["mortality_rate_infant","intentional_homicides","suicide_mort
ality_rate"]]
model3x =
x_train[["government_health_expenditure_pct_gdp","expenditure_on
_education_pct_gdp"]]
model4x = x_train[["population_density","urban_population_pct_total"]]
model5x =
x_train[["tax_revenue","taxes_on_income_profits_capital","gdp_per_capi
```

```
ta_ppp"]]
model6x =
x_train[["gini_index","mortality_rate_infant","intentional_homicides",
"suicide_mortality_rate", \

"alcohol_consumption_per_capita","government_health_expenditure_pct_gd
p"]]
model7x =
x_train[["alcohol_consumption_per_capita","intentional_homicides","urb
an_population_pct_total"]]

# calcualtes best RIDGE and LASSO alphas for each model using cross-
validation from training dataset
# RIDGE
ridgeCV = RidgeCV(alphas =
np.arange(0.01,100,0.05)).fit(model1x,y_train)
alpha1r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 1:", alpha1r)

ridgeCV = RidgeCV(alphas =
np.arange(0.01,100,0.05)).fit(model2x,y_train)
alpha2r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 2:", alpha2r)

ridgeCV = RidgeCV(alphas =
np.arange(0.01,100,0.05)).fit(model3x,y_train)
alpha3r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 3:", alpha3r)

ridgeCV = RidgeCV(alphas =
np.arange(0.01,100,0.05)).fit(model4x,y_train)
alpha4r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 4:", alpha4r)

ridgeCV = RidgeCV(alphas =
np.arange(0.01,100,0.05)).fit(model5x,y_train)
alpha5r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 5:", alpha5r)


ridgeCV = RidgeCV(alphas =
np.arange(0.01,100,0.05)).fit(model6x,y_train)
alpha6r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 6:", alpha6r)

ridgeCV = RidgeCV(alphas =
np.arange(0.01,100,0.05)).fit(model7x,y_train)
alpha7r = ridgeCV.alpha_
print("Most appropriate Ridge alpha for model 7:", alpha7r, "\n")
```

```python
# LASSO
lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model1x,y_train)
alpha1l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 1:", alpha1l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model2x,y_train)
alpha2l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 2:", alpha2l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model3x,y_train)
alpha3l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 3:", alpha3l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model4x,y_train)
alpha4l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 4:", alpha4l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model5x,y_train)
alpha5l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 5:", alpha5l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model6x,y_train)
alpha6l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 6:", alpha6l)

lassoCV = LassoCV(cv = None, n_alphas = 100).fit(model7x,y_train)
alpha7l = lassoCV.alpha_
print("Most appropriate LASSO alpha for model 7:", alpha7l)
```

```
Most appropriate Ridge alpha for model 1: 5.96
Most appropriate Ridge alpha for model 2: 9.01
Most appropriate Ridge alpha for model 3: 99.96000000000001
Most appropriate Ridge alpha for model 4: 92.71000000000001
Most appropriate Ridge alpha for model 5: 79.26
Most appropriate Ridge alpha for model 6: 25.160000000000004
Most appropriate Ridge alpha for model 7: 1.01

Most appropriate LASSO alpha for model 1: 0.009035105986016721
Most appropriate LASSO alpha for model 2: 0.6709563385389306
Most appropriate LASSO alpha for model 3: 2.473217768450261
Most appropriate LASSO alpha for model 4: 0.033625551095921184
Most appropriate LASSO alpha for model 5: 1.2976570742114053
Most appropriate LASSO alpha for model 6: 0.4414444674193534
Most appropriate LASSO alpha for model 7: 0.007559802997655883
```

```python
# implementing K-fold to test for best model (manual)

# initializes k group IDs, creating modified training dataframes for
cross-validation
kgroup_ids = np.arange(1,6)
```

```
kgroups_array = np.repeat(kgroup_ids,27)
np.random.shuffle(kgroups_array)
x_train["k_group"] = kgroups_array
y_train_cv= pd.DataFrame({'rating':y_train,'k_group':kgroups_array})

y_train1 = y_train_cv.loc[y_train_cv["k_group"] != 1]['rating']

# implementing Kfold to select the best model
ridge_1_mses = []
ridge_2_mses = []
ridge_3_mses = []
ridge_4_mses = []
ridge_5_mses = []
ridge_6_mses = []
ridge_7_mses = []

linear_1_mses = []
linear_2_mses = []
linear_3_mses = []
linear_4_mses = []
linear_5_mses = []
linear_6_mses = []
linear_7_mses = []

lasso_1_mses = []
lasso_2_mses = []
lasso_3_mses = []
lasso_4_mses = []
lasso_5_mses = []
lasso_6_mses = []
lasso_7_mses = []

best_r2s = []

for k in kgroup_ids:
  print('working on fold', k, "...")
  x_train1 = x_train.loc[x_train["k_group"] != k]
  y_train1 = y_train_cv.loc[y_train_cv["k_group"] != k]['rating']


  model1x = x_train1[["gini_index","unemployment"]]
  model2x =
x_train1[["mortality_rate_infant","intentional_homicides","suicide_mor
tality_rate"]]
  model3x =
x_train1[["government_health_expenditure_pct_gdp","expenditure_on
_education_pct_gdp"]]
  model4x =
x_train1[["population_density","urban_population_pct_total"]]
  model5x =
```

```
x_train1[["tax_revenue","taxes_on_income_profits_capital","gdp_per_cap
ita_ppp"]]
  model6x =
x_train1[["gini_index","mortality_rate_infant","intentional_homicides"
,"suicide_mortality_rate", \

"alcohol_consumption_per_capita","government_health_expenditure_pct_gd
p"]]
  model7x =
x_train1[["alcohol_consumption_per_capita","intentional_homicides","ur
ban_population_pct_total"]]


  model1tst = x_test[["gini_index","unemployment"]]
  model2tst =
x_test[["mortality_rate_infant","intentional_homicides","suicide_morta
lity_rate"]]
  model3tst =
x_test[["government_health_expenditure_pct_gdp","expenditure_on
_education_pct_gdp"]]
  model4tst =
x_test[["population_density","urban_population_pct_total"]]
  model5tst =
x_test[["tax_revenue","taxes_on_income_profits_capital","gdp_per_capit
a_ppp"]]
  model6tst =
x_test[["gini_index","mortality_rate_infant","intentional_homicides","
suicide_mortality_rate", \

"alcohol_consumption_per_capita","government_health_expenditure_pct_gd
p"]]
  model7tst =
x_test[["alcohol_consumption_per_capita","intentional_homicides","urba
n_population_pct_total"]]

  ridgereg = Ridge(alpha = alpha1r).fit(model1x,y_train1)
  predicted_y1 = ridgereg.predict(model1tst)
  print("-----------")
  # print("MSE for Model 1 Ridge
regression:",mean_squared_error(y_test,predicted_y1))
  ridge_1_mses.append(mean_squared_error(y_test,predicted_y1))

  ridgereg = Ridge(alpha = alpha2r).fit(model2x,y_train1)
  predicted_y2 = ridgereg.predict(model2tst)
  # print("MSE for Model 2 Ridge
regression:",mean_squared_error(y_test,predicted_y2))
  ridge_2_mses.append(mean_squared_error(y_test,predicted_y2))

  ridgereg = Ridge(alpha = alpha3r).fit(model3x,y_train1)
  predicted_y3 = ridgereg.predict(model3tst)
```

```
  # print("MSE for Model 3 Ridge
regression:",mean_squared_error(y_test,predicted_y3))
  ridge_3_mses.append(mean_squared_error(y_test,predicted_y3))

  ridgereg = Ridge(alpha = alpha4r).fit(model4x,y_train1)
  predicted_y4 = ridgereg.predict(model4tst)
  # print("MSE for Model 4 Ridge
regression:",mean_squared_error(y_test,predicted_y4))
  ridge_4_mses.append(mean_squared_error(y_test,predicted_y4))

  ridgereg = Ridge(alpha = alpha5r).fit(model5x,y_train1)
  predicted_y5 = ridgereg.predict(model5tst)
  # print("MSE for Model 5 Ridge
regression:",mean_squared_error(y_test,predicted_y5))
  ridge_5_mses.append(mean_squared_error(y_test,predicted_y5))

  ridgereg = Ridge(alpha = alpha6r).fit(model6x,y_train1)
  predicted_y6 = ridgereg.predict(model6tst)
  # print("MSE for Model 6 Ridge
regression:",mean_squared_error(y_test,predicted_y6))
  ridge_6_mses.append(mean_squared_error(y_test,predicted_y6))
  best_r2s.append(r2_score(y_test,predicted_y6))

  ridgereg = Ridge(alpha = alpha7r).fit(model7x,y_train1)
  predicted_y7 = ridgereg.predict(model7tst)
  # print("MSE for Model 7 Ridge
regression:",mean_squared_error(y_test,predicted_y7))
  ridge_7_mses.append(mean_squared_error(y_test,predicted_y7))

  linearReg = LinearRegression().fit(model1x,y_train1)
  predicted_y1 = linearReg.predict(model1tst)
  # print("MSE for Model 1 Linear
regression:",mean_squared_error(y_test,predicted_y1))
  linear_1_mses.append(mean_squared_error(y_test,predicted_y1))

  linearReg = LinearRegression().fit(model2x,y_train1)
  predicted_y2 = linearReg.predict(model2tst)
  # print("MSE for Model 2 Linear
regression:",mean_squared_error(y_test,predicted_y2))
  linear_2_mses.append(mean_squared_error(y_test,predicted_y2))

  linearReg = LinearRegression().fit(model3x,y_train1)
  predicted_y3 = linearReg.predict(model3tst)
  # print("MSE for Model 3 Linear
regression:",mean_squared_error(y_test,predicted_y3))
  linear_3_mses.append(mean_squared_error(y_test,predicted_y3))

  linearReg = LinearRegression().fit(model4x,y_train1)
  predicted_y4 = linearReg.predict(model4tst)
  # print("MSE for Model 4 Linear
```

```
regression:",mean_squared_error(y_test,predicted_y4))
  linear_4_mses.append(mean_squared_error(y_test,predicted_y4))

  linearReg = LinearRegression().fit(model5x,y_train1)
  predicted_y5 = linearReg.predict(model5tst)
  # print("MSE for Model 5 Linear
regression:",mean_squared_error(y_test,predicted_y5))
  linear_5_mses.append(mean_squared_error(y_test,predicted_y5))

  linearReg = LinearRegression().fit(model6x,y_train1)
  predicted_y6 = linearReg.predict(model6tst)
  # print("MSE for Model 6 Linear
regression:",mean_squared_error(y_test,predicted_y6))
  linear_6_mses.append(mean_squared_error(y_test,predicted_y6))

  linearReg = LinearRegression().fit(model7x,y_train1)
  predicted_y7 = linearReg.predict(model7tst)
  # print("MSE for Model 7 Linear
regression:",mean_squared_error(y_test,predicted_y7))
  linear_7_mses.append(mean_squared_error(y_test,predicted_y7))

  lassoreg = Lasso(alpha = alpha1l).fit(model1x,y_train1)
  predicted_y1 = lassoreg.predict(model1tst)
  # print("MSE for Model 1 LASSO
regression:",mean_squared_error(y_test,predicted_y1))
  lasso_1_mses.append(mean_squared_error(y_test,predicted_y1))

  lassoreg = Lasso(alpha = alpha2l).fit(model2x,y_train1)
  predicted_y2 = lassoreg.predict(model2tst)
  # print("MSE for Model 2 LASSO
regression:",mean_squared_error(y_test,predicted_y2))
  lasso_2_mses.append(mean_squared_error(y_test,predicted_y2))

  lassoreg = Lasso(alpha = alpha3l).fit(model3x,y_train1)
  predicted_y3 = lassoreg.predict(model3tst)
  # print("MSE for Model 3 LASSO
regression:",mean_squared_error(y_test,predicted_y3))
  lasso_3_mses.append(mean_squared_error(y_test,predicted_y3))

  lassoreg = Lasso(alpha = alpha4l).fit(model4x,y_train1)
  predicted_y4 = lassoreg.predict(model4tst)
  # print("MSE for Model 4 LASSO
regression:",mean_squared_error(y_test,predicted_y4))
  lasso_4_mses.append(mean_squared_error(y_test,predicted_y4))

  lassoreg = Lasso(alpha = alpha5l).fit(model5x,y_train1)
  predicted_y5 = lassoreg.predict(model5tst)
  # print("MSE for Model 5 LASSO
regression:",mean_squared_error(y_test,predicted_y5))
  lasso_5_mses.append(mean_squared_error(y_test,predicted_y5))
```

```
  lassoreg = Lasso(alpha = alpha6l).fit(model6x,y_train1)
  predicted_y6 = lassoreg.predict(model6tst)
  # print("MSE for Model 6 LASSO
regression:",mean_squared_error(y_test,predicted_y6))
  lasso_6_mses.append(mean_squared_error(y_test,predicted_y6))

  lassoreg = Lasso(alpha = alpha7l).fit(model7x,y_train1)
  predicted_y7 = lassoreg.predict(model7tst)
  # print("MSE for Model 7 LASSO
regression:",mean_squared_error(y_test,predicted_y7))
  lasso_7_mses.append(mean_squared_error(y_test,predicted_y7))
  print(' ')

print("Ridge Model 1 Mean MSE:",np.mean(ridge_1_mses))
print("Ridge Model 2 Mean MSE:",np.mean(ridge_2_mses))
print("Ridge Model 3 Mean MSE:",np.mean(ridge_3_mses))
print("Ridge Model 4 Mean MSE:",np.mean(ridge_4_mses))
print("Ridge Model 5 Mean MSE:",np.mean(ridge_5_mses))
print("Ridge Model 6 Mean MSE:",np.mean(ridge_6_mses))
print("Ridge Model 7 Mean MSE:",np.mean(ridge_7_mses))
print(" ")
print("Linear Model 1 Mean MSE:",np.mean(linear_1_mses))
print("Linear Model 2 Mean MSE:",np.mean(linear_2_mses))
print("Linear Model 3 Mean MSE:",np.mean(linear_3_mses))
print("Linear Model 4 Mean MSE:",np.mean(linear_4_mses))
print("Linear Model 5 Mean MSE:",np.mean(linear_5_mses))
print("Linear Model 6 Mean MSE:",np.mean(linear_6_mses))
print("Linear Model 7 Mean MSE:",np.mean(linear_7_mses))
print(" ")
print("LASSO Model 1 Mean MSE:",np.mean(lasso_1_mses))
print("LASSO Model 2 Mean MSE:",np.mean(lasso_2_mses))
print("LASSO Model 3 Mean MSE:",np.mean(lasso_3_mses))
print("LASSO Model 4 Mean MSE:",np.mean(lasso_4_mses))
print("LASSO Model 5 Mean MSE:",np.mean(lasso_5_mses))
print("LASSO Model 6 Mean MSE:",np.mean(lasso_6_mses))
print("LASSO Model 7 Mean MSE:",np.mean(lasso_7_mses))
print(" ")

print("Best Model (Ridge 6) Mean R^2:",np.mean(best_r2s))
```

```
working on fold 1 ...
-----------

working on fold 2 ...
-----------

working on fold 3 ...
-----------
```

```
working on fold 4 ...
-----------

working on fold 5 ...
-----------

Ridge Model 1 Mean MSE: 348.030073368065
Ridge Model 2 Mean MSE: 345.24193578765033
Ridge Model 3 Mean MSE: 431.8989196823908
Ridge Model 4 Mean MSE: 429.2027447216643
Ridge Model 5 Mean MSE: 413.136386745325
Ridge Model 6 Mean MSE: 336.05341183074245
Ridge Model 7 Mean MSE: 409.1743014925128

Linear Model 1 Mean MSE: 349.6230579375316
Linear Model 2 Mean MSE: 345.6865492881978
Linear Model 3 Mean MSE: 426.97797907901924
Linear Model 4 Mean MSE: 430.2551745905697
Linear Model 5 Mean MSE: 419.63504795191665
Linear Model 6 Mean MSE: 337.429379618244
Linear Model 7 Mean MSE: 410.68525388631633

LASSO Model 1 Mean MSE: 349.5833427854218
LASSO Model 2 Mean MSE: 348.9848770294435
LASSO Model 3 Mean MSE: 435.8617145144079
LASSO Model 4 Mean MSE: 430.2436627504747
LASSO Model 5 Mean MSE: 407.10899545782615
LASSO Model 6 Mean MSE: 336.32302023314764
LASSO Model 7 Mean MSE: 410.55284349573355

Best Model (Ridge 6) Mean R^2: 0.22449780296456773
```

```
# recursively implementing K-fold to test for best model a number of
times
# will contextualize how the ORIGINAL test-train split affects best
model

# a list of the 7 chosen possible models, in order model 1-7
models = [["gini_index","unemployment"],

["mortality_rate_infant","intentional_homicides","suicide_mortality_ra
te"],
          ["government_health_expenditure_pct_gdp","expenditure_on
_education_pct_gdp"],
          ["population_density","urban_population_pct_total"],

["tax_revenue","taxes_on_income_profits_capital","gdp_per_capita_ppp"]
,

["gini_index","mortality_rate_infant","intentional_homicides","suicide
```

```
_mortality_rate", \

"alcohol_consumption_per_capita","government_health_expenditure_pct_gd
p"],

["alcohol_consumption_per_capita","intentional_homicides","urban_popul
ation_pct_total"]]

# for each K-fold CV, records best model (out of 21), and their MSEs
and R2s
best_models = np.array([])
best_mses = np.array([])
best_r2s = np.array([])

# THIS PART WILL TAKE LONG TO RUN: CAN LIMIT K-FOLDS TO ONE:
range(100) -> range(1)
for split in notebook.tqdm(range(100)):
  # creates a shuffled K-fold indicies of given folds (default=5)
  k = 5
  kfold = KFold(n_splits=k,shuffle=True)

  model_mses = np.zeros((3,7))
  model_r2s = np.zeros((3,7))

  # does a randomized initial test-train split before running K-Fold
CV
  x_traink, x_testk, y_traink, y_testk = train_test_split(x_df, y_df,
test_size = 0.2)

  # iterates over each of the models
  for m,model in enumerate(models):
    # calculates best fitting RIDGE and LASSO alphas using new initial
training dataframe and model
    ridgeCV = RidgeCV(alphas =
np.arange(0.01,100,0.05)).fit(x_traink[model],y_traink)
    ralpha = ridgeCV.alpha_
    lassoCV =
LassoCV(cv=None,n_alphas=500).fit(x_traink[model],y_traink)
    lalpha = lassoCV.alpha_

    # for each K-fold, record that fold's MSE and R2 values
    mses = np.zeros((3,k))
    r2s = np.zeros((3,k))
    # fold number (0 to k-1)
    ki = 0

    # performs K-fold CV
    for train_index,test_index in kfold.split(x_traink):
      # test and train split for specific fold, from initial training
dataframe and model
```

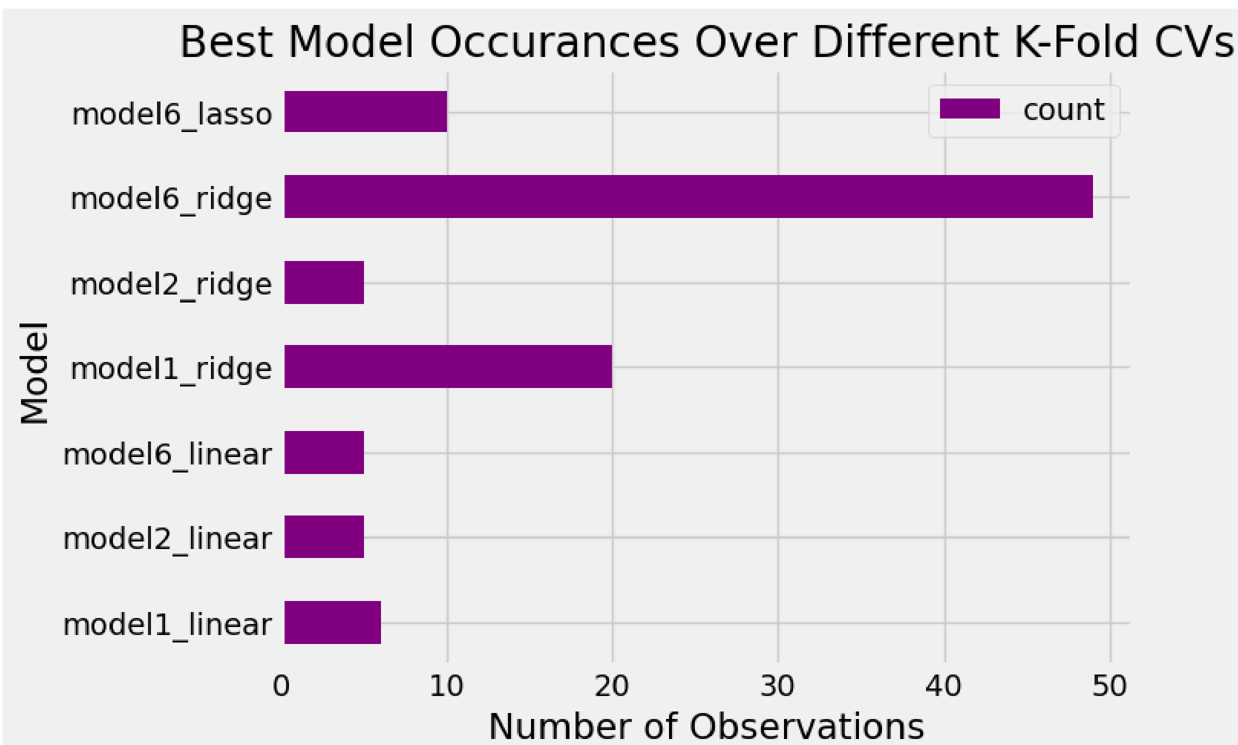
```
      X_tr, X_te, y_tr, y_te = x_traink.iloc[train_index],
x_traink.iloc[test_index], y_traink.iloc[train_index],
y_traink.iloc[test_index]

      # performs linear regression on training fold, predicts y using
test fold
      linearReg = LinearRegression().fit(X_tr[model],y_tr)
      predicted_y = linearReg.predict(X_te[model])
      mses[0][ki] = mean_squared_error(y_te,predicted_y)
      r2s[0][ki] = r2_score(y_te,predicted_y)

      # performs RIDGE regression on training fold, predicts y using
test fold
      ridgeReg = Ridge(alpha = ralpha).fit(X_tr[model],y_tr)
      predicted_y = ridgeReg.predict(X_te[model])
      mses[1][ki] = mean_squared_error(y_te,predicted_y)
      r2s[1][ki] = r2_score(y_te,predicted_y)

      # performs LASSO regression on training fold, predicts y using
test fold
      lassoReg = Lasso(alpha = lalpha).fit(X_tr[model],y_tr)
      predicted_y = lassoReg.predict(X_te[model])
      mses[2][ki] = mean_squared_error(y_te,predicted_y)
      r2s[2][ki] = r2_score(y_te,predicted_y)

      # increases fold number
      ki += 1

    # across all folds, calculates the mean MSE
    model_mses[0][m] = np.mean(mses[0])
    model_mses[1][m] = np.mean(mses[1])
    model_mses[2][m] = np.mean(mses[2])

    # across all folds, calculates the mean R2 score
    model_r2s[0][m] = np.mean(r2s[0])
    model_r2s[1][m] = np.mean(r2s[1])
    model_r2s[2][m] = np.mean(r2s[2])

  # out of a given K-Fold CV, appends best model, and its MSE and R2
  best_models = np.append(best_models,np.argmin(model_mses))
  best_mses = np.append(best_mses,np.min(model_mses))
  best_r2s = np.append(best_r2s,model_r2s[np.argmin(model_mses)//7]
[np.argmin(model_mses)%7])

print("Best Models (#):",best_models)
print("Best Model MSE:",best_mses)
print("Best Model R^2:",best_r2s)
```

{"model_id":"f67600c6e2a24411b44f762096f7a4b4","version_major":2,"vers
ion_minor":0}

```
Best Models (#): [ 7.  8. 12.  5.  1.  7. 12. 12. 12. 12. 12.  0.  7.
12.  5. 19. 12. 19.
  0. 19.  7.  7. 12. 12.  1. 19.  7.  7. 12. 12.  8. 12. 12. 12. 12.
12.
 12. 19.  7. 12.  7. 12.  0. 12. 12. 12. 19. 12.  7. 12.  0.  5.  5.
7.
 12. 12. 19. 12. 12. 12. 12.  8.  7.  7.  7. 12. 12.  1. 12. 19. 12.
7.
  7.  5. 12. 12. 19. 12. 19.  1. 12. 12. 12. 12.  7. 12. 12.  8. 12.
12.
 12. 12.  8.  1.  7. 12.  7.  7.  0.  0.]
Best Model MSE: [324.26169157 301.59536948 334.54295436 333.86602521
310.92577076
 306.30604768 316.23188624 293.21461502 340.50531055 332.49428351
 335.95854873 318.88896443 308.93343671 328.70345973 323.51321629
 304.90838875 309.40546515 327.60486263 329.47162582 312.27507599
 350.08862812 312.5815229  344.80496493 335.17902564 318.08228269
 313.45059923 327.31112444 327.4336232  359.59052334 332.26992543
 313.75529574 290.27651095 303.40757796 270.57183917 337.1653541
 330.39562629 347.3398795  285.00974694 311.95726817 338.31546224
 321.16356575 331.27535585 296.45381751 329.64092784 289.74159367
 339.69579174 334.82136214 291.66169041 293.58286429 302.70480685
 320.06964839 313.40873098 294.32080421 345.54541267 324.69376476
 299.48611478 337.68235155 297.06806331 337.19769647 316.38276524
 281.8228077  340.41877254 320.13909966 287.36115712 320.8692162
 319.99187508 320.35495003 334.2169424  330.30596196 300.02718358
 327.53752575 317.60599811 309.37904528 306.4347881  324.34741628
 322.79389878 322.39098658 314.11439445 323.3793774  319.53797618
 329.92853867 336.02640699 320.51645237 325.09582641 307.49531227
 308.03975115 337.36720691 327.92953415 314.89601368 312.82864106
 324.49649328 314.07103748 307.6356152  335.70545744 334.17883775
 320.62120772 327.8438504  304.80396453 296.85030683 303.2063172 ]
Best Model R^2: [0.23524283 0.2329234  0.26902664 0.12564711
0.25838466 0.2501753
 0.25145202 0.29762884 0.21266328 0.18612768 0.28442602 0.21976035
 0.21567122 0.26907972 0.15098136 0.25088636 0.14663318 0.22595114
 0.25708259 0.18087079 0.25731593 0.17525782 0.14865546 0.25698496
 0.21448967 0.28392619 0.14821688 0.17711886 0.23484425 0.26908705
 0.24128525 0.29398777 0.32641148 0.30762301 0.22297186 0.17498078
 0.18712769 0.27439243 0.19650276 0.26639014 0.15349274 0.28822626
 0.23080729 0.19418514 0.23182475 0.21905316 0.28523682 0.26388005
 0.22457687 0.23679096 0.17680572 0.18704896 0.29740885 0.1210857
 0.21281665 0.25656137 0.27175736 0.16636146 0.11817928 0.16521081
 0.29335891 0.19197067 0.19289006 0.19505283 0.18714827 0.25567163
 0.24258664 0.15529273 0.21021812 0.35429824 0.26605735 0.21793912
 0.17351692 0.21634219 0.18678703 0.19649822 0.24395725 0.23622035
 0.12282165 0.22658188 0.22386951 0.26508616 0.28652673 0.16079967
 0.1543984  0.20087495 0.184429   0.165907   0.11644778 0.27728154
```

```
 0.23937888 0.27133759 0.18687607 0.21579371 0.19338743 0.30977655
 0.23577436 0.22451223 0.30783951 0.22631346]
```

```python
# results from recursive K-Fold tries

# matches model number to an informative name
models_s = []
for mo in range(21):
  if mo//7==0:
    models_s.append("model"+str(mo%7+1)+"_linear")
  elif mo//7==1:
    models_s.append("model"+str(mo%7+1)+"_ridge")
  else:
    models_s.append("model"+str(mo%7+1)+"_lasso")

# gets best models, and their occurances over the number of K-Folds
x,height = np.unique(best_models,return_counts=True)
best_models_df = pd.DataFrame({"model":x,"count":height})
best_models_df["model"] =
best_models_df["model"].replace(np.arange(21),models_s)
# plots histogram of best model over multiple K-Folds
best_models_df.plot.barh(x="model",y="count",color="purple")
plt.title("Best Model Occurances Over Different K-Fold CVs")
plt.ylabel("Model")
plt.xlabel("Number of Observations")
plt.show()
```



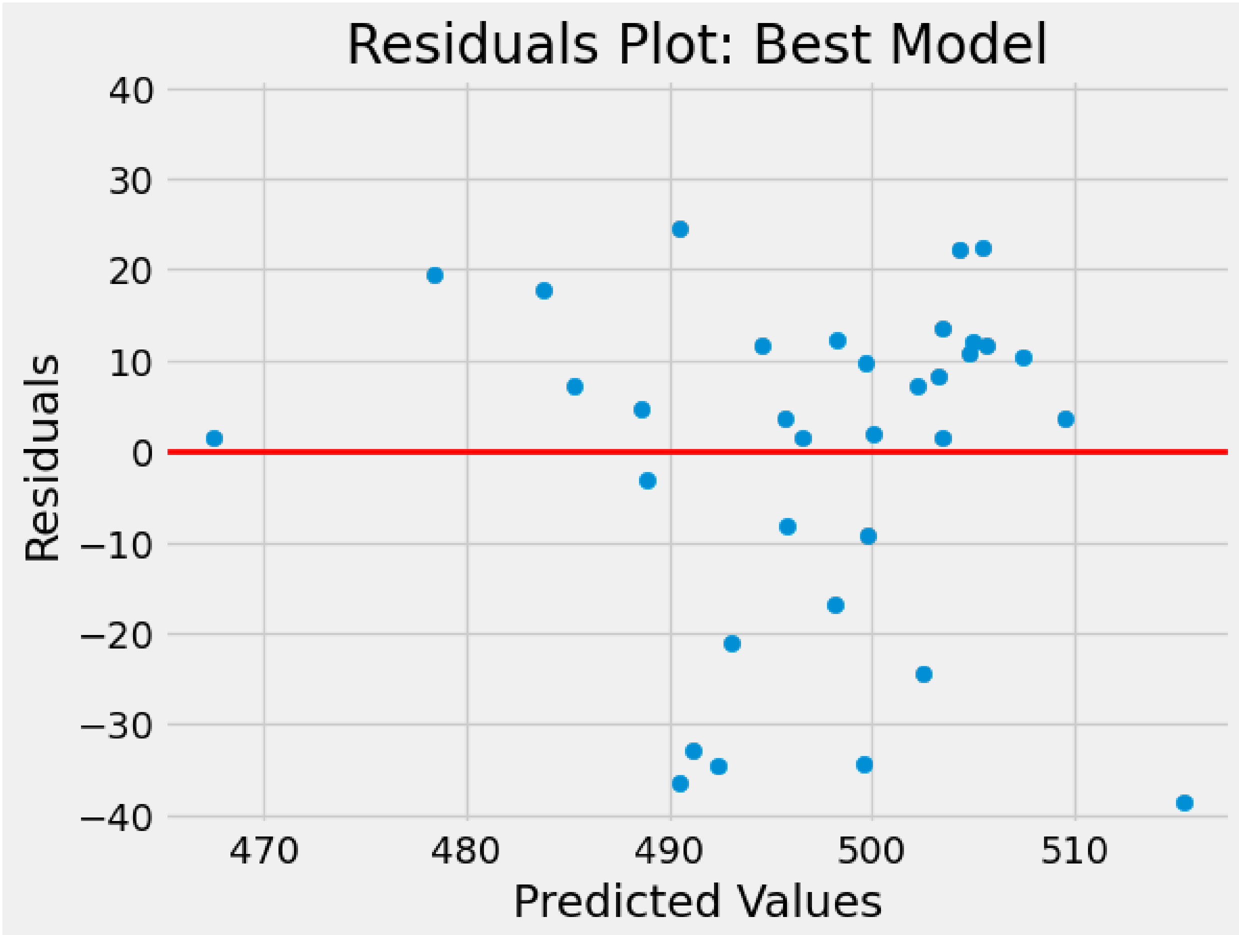
```
# choosing best model based on histogram; running a normal CV using
original test-train split

# best model: model 6, RIDGE
model6_train =
x_train[["gini_index","mortality_rate_infant","intentional_homicides",
"suicide_mortality_rate", \

"alcohol_consumption_per_capita","government_health_expenditure_pct_gd
p"]]
model6_test =
x_test[["gini_index","mortality_rate_infant","intentional_homicides","
suicide_mortality_rate", \

"alcohol_consumption_per_capita","government_health_expenditure_pct_gd
p"]]

# recalculates and saves RIDGE alpha for clarity
best_ridgeCV = RidgeCV(alphas =
np.arange(0.01,100,0.05)).fit(model6_train,y_train)
best_alpha = best_ridgeCV.alpha_

# performs regression, predicts using test dataframe
best_ridgereg = Ridge(alpha=best_alpha).fit(model6_train,y_train)
best_predicted_y = best_ridgereg.predict(model6_test)

# calculates MSE and r2
best_mse = mean_squared_error(y_test,predicted_y6)
best_r2 = r2_score(y_test,predicted_y6)
```

BEST MODEL ANALYSIS

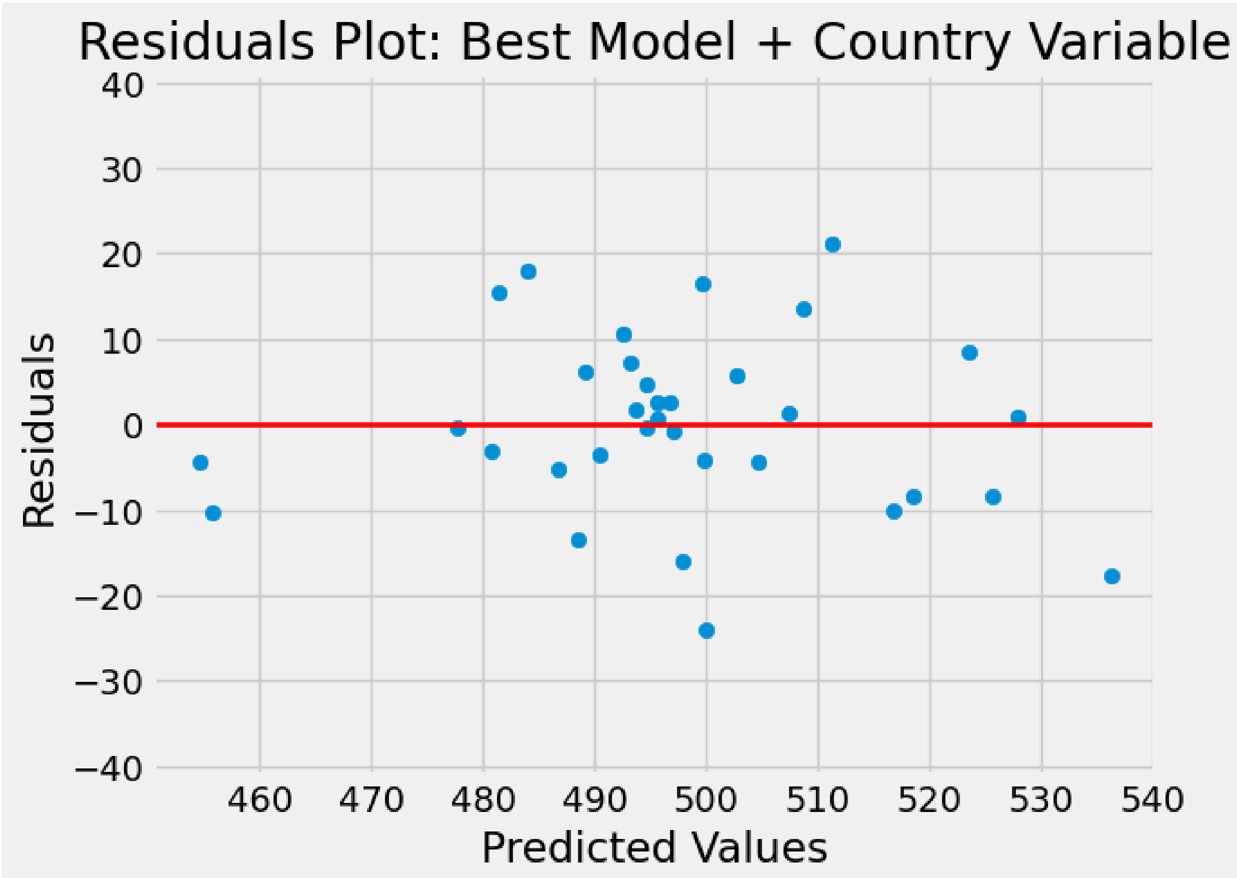
```
# looking at the best model's residuals & comparing when we also
control for country
print("MSE for Model 6 Ridge regression:",best_mse)

# plotting the residuals for the best model
plt.scatter(best_predicted_y,best_predicted_y - y_test)
plt.ylim(-41,41)
plt.axhline(0,color='red',linewidth = 2)
plt.title('Residuals Plot: Best Model')
plt.xlabel("Predicted Values")
plt.ylabel("Residuals")
plt.show()


# getting the MSE for the best model + controlling for country
country_control =
x_train[["gini_index","mortality_rate_infant","intentional_homicides",
"suicide_mortality_rate", \
```

```
"alcohol_consumption_per_capita","government_health_expenditure_pct_gd
p",'country_AUT', 'country_CAN', \
                           'country_CHE', 'country_CZE','country_DEU',
'country_DNK', 'country_ESP', 'country_EST','country_FIN', \
                           'country_FRA', 'country_GBR',
'country_GRC','country_HUN', 'country_IRL', 'country_ISL',
'country_ISR','country_ITA', \
                           'country_KOR', 'country_LUX',
'country_LVA','country_NLD', 'country_NOR', 'country_POL',
'country_PRT','country_SVK', \
                           'country_SVN', 'country_SWE',
'country_USA']]

ridgeCV =
RidgeCV(alphas=np.arange(0.01,100,0.05)).fit(country_control,y_train)
print("Most appropriate Ridge alpha for model 6 (with country
control):", ridgeCV.alpha_)

country_control_tst =
x_test[["gini_index","mortality_rate_infant","intentional_homicides","
suicide_mortality_rate", \

"alcohol_consumption_per_capita","government_health_expenditure_pct_gd
p",'country_AUT', 'country_CAN', \
                           'country_CHE', 'country_CZE','country_DEU',
'country_DNK', 'country_ESP', 'country_EST','country_FIN', \
                           'country_FRA', 'country_GBR',
'country_GRC','country_HUN', 'country_IRL', 'country_ISL',
'country_ISR','country_ITA', \
                           'country_KOR', 'country_LUX',
'country_LVA','country_NLD', 'country_NOR', 'country_POL',
'country_PRT','country_SVK', \
                           'country_SVN', 'country_SWE',
'country_USA']]

ridgereg = Ridge(alpha=ridgeCV.alpha_).fit(country_control,y_train)
predicted_y_cc = ridgereg.predict(country_control_tst)
print("MSE for Model 6 + Country Variable Ridge
regression:",mean_squared_error(y_test,predicted_y_cc))

plt.scatter(predicted_y_cc,predicted_y_cc-y_test)
plt.axhline(0,color='red',linewidth = 2)
plt.ylim(-41,41)
plt.title('Residuals Plot: Best Model + Country Variable')
plt.xlabel("Predicted Values")
plt.ylabel("Residuals")
plt.show()
```

```
MSE for Model 6 Ridge regression: 337.1024391863911
```

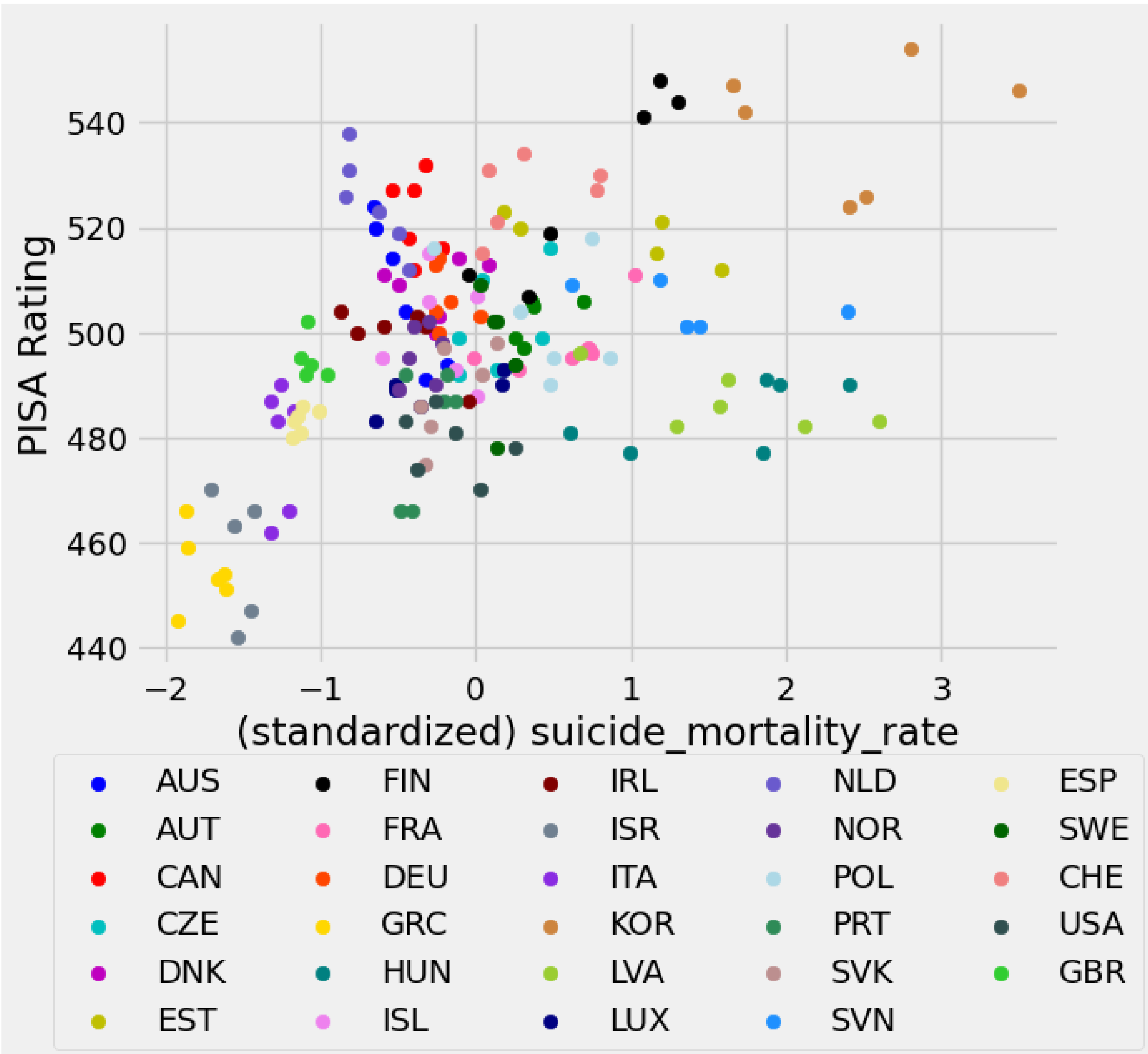


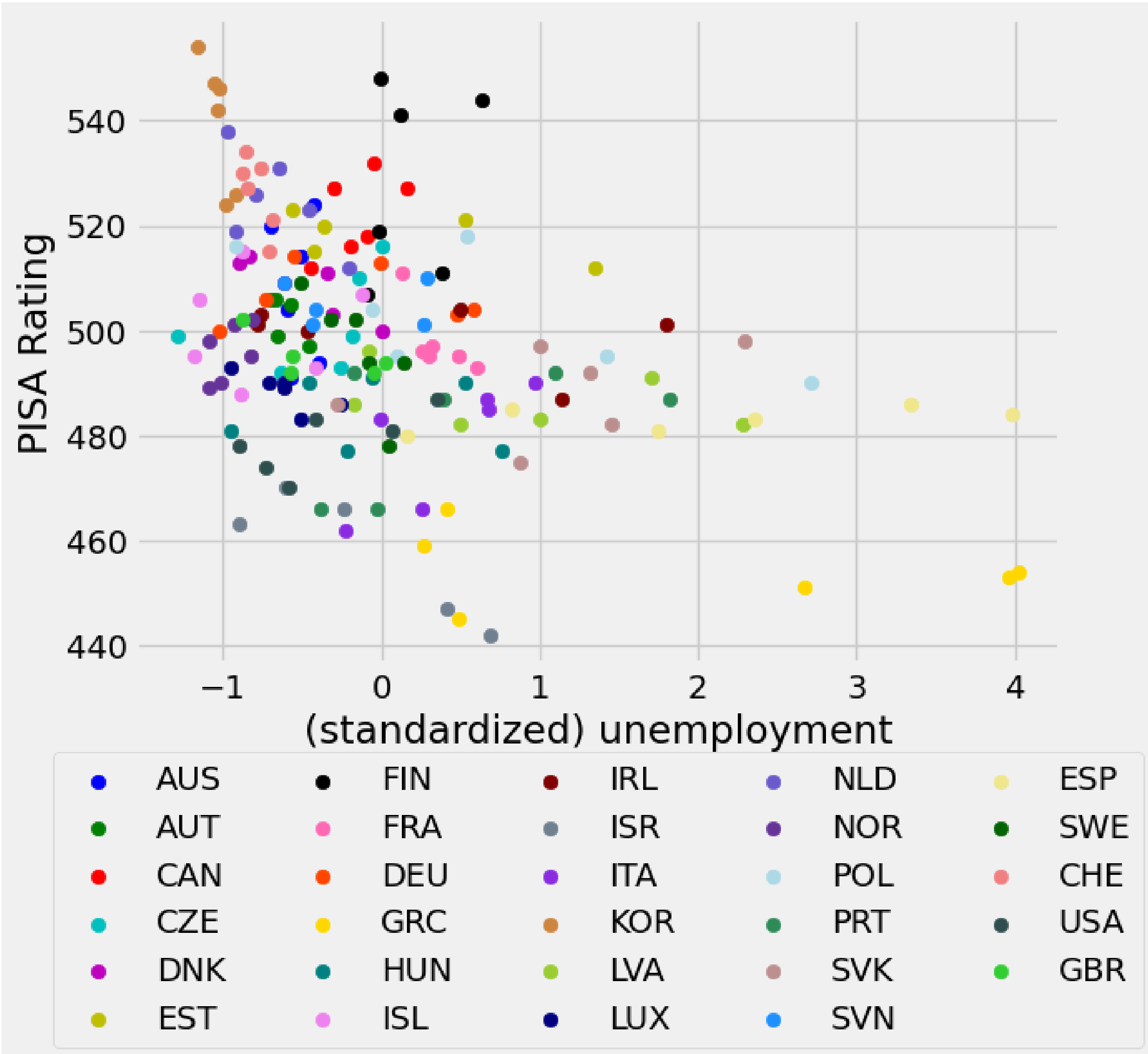

```
Most appropriate Ridge alpha for model 6 (with country control): 0.26
MSE for Model 6 + Country Variable Ridge regression:
106.16160153621229
```




```
# analyze how countries cluster over influential variables and the
PISA score
# example: suicide_mortality_rate

colors =
["b","g","r","c","m","y","k","hotpink","orangered","gold","teal","viol
et","maroon","slategrey","blueviolet","peru","yellowgreen" \
          ,"navy","slateblue","rebeccapurple","lightblue","seagreen","
rosybrown","dodgerblue","khaki","darkgreen","lightcoral","darkslategre
y","limegreen"]

for i,country in enumerate(pisa_df_tot.country.unique()):
  plt.scatter(pisa_df_tot.loc[pisa_df_tot["country"] ==
country].suicide_mortality_rate,pisa_df_tot.loc[pisa_df_tot["country"]
== country].rating, \
              label = country,color = colors[i])
  plt.legend(ncols=5,loc='upper center',bbox_to_anchor=(0.5, -0.11))
  plt.xlabel("(standardized) suicide_mortality_rate")
  plt.ylabel("PISA Rating")
plt.show()

print("\n")
for i,country in enumerate(pisa_df_tot.country.unique()):
```

```
  plt.scatter(pisa_df_tot.loc[pisa_df_tot["country"] ==
country].unemployment,pisa_df_tot.loc[pisa_df_tot["country"] ==
country].rating, \
                label = country,color = colors[i])
  plt.legend(ncols=5,loc='upper center',bbox_to_anchor=(0.5, -0.11))
  plt.xlabel("(standardized) unemployment")
  plt.ylabel("PISA Rating")
plt.show()
```





CLUSTERING ANALYSIS

```
# define dataframe for clustering, with best model including the PISA
score
bestmodel_df =
pisa_df_tot[["gini_index","mortality_rate_infant","intentional_homicid
es", \

"suicide_mortality_rate","alcohol_consumption_per_capita","government_
health_expenditure_pct_gdp","rating"]]

# standardize PISA score
scaler = StandardScaler()
bestmodel_df[["rating"]] =
scaler.fit_transform(bestmodel_df[["rating"]])
```

```python
# display the dataframe
display(bestmodel_df.head())
```

```
C:\Users\daniel\AppData\Local\Temp\ipykernel_22508\2792784023.py:7:
SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation:
https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#
returning-a-view-versus-a-copy
  bestmodel_df[["rating"]] =
scaler.fit_transform(bestmodel_df[["rating"]])
```

```
            gini_index  mortality_rate_infant
intentional_homicides  \
index_code

AUS-2003      0.462126               0.831882               0.142289

AUS-2006      0.559458               0.679648              -0.005489

AUS-2009      0.559458               0.299063              -0.152009

AUS-2012      0.559458              -0.157639              -0.285880

AUS-2015      0.559458              -0.385990              -0.358186


            suicide_mortality_rate  alcohol_consumption_per_capita  \
index_code
AUS-2003                 -0.660127                        0.121143
AUS-2006                 -0.642575                        0.121143
AUS-2009                 -0.537262                        0.121143
AUS-2012                 -0.449501                        0.121143
AUS-2015                 -0.186219                        0.121143

            government_health_expenditure_pct_gdp    rating
index_code
AUS-2003                                -0.330309  1.206663
AUS-2006                                -0.269865  1.015228
AUS-2009                                 0.059373  0.728076
AUS-2012                                 0.018771  0.249488
AUS-2015                                 0.681248 -0.229099
```

```python
# required definitions to perform K-Means Clustering

def distance(pt1, pt2):
    """Return the distance between two points, represented as
arrays"""
```

```python
    return np.sqrt(sum((pt1 - pt2)**2))

def initialize_centroids(df,K):
    random_ids = np.random.permutation(df.shape[0])
    centroids = df.iloc[random_ids[:K]]
    return centroids

def compute_distance(df, centroids):
    K=centroids.shape[0]
    distances_ar = np.zeros((df.shape[0], K))
    for k in range(K):
        point=centroids.iloc[k]
        def distance_from_point(row):
            return distance(point, np.array(row))
        distances_ar[:,k] =
df.apply(distance_from_point,axis=1).values
    return distances_ar

def compute_sse(df, labels, centroids,K):
    distances_ar = np.zeros(df.shape[0])
    for k in range(K):
        point=centroids.iloc[k]
        def distance_from_point(row):
            return distance(point, np.array(row))
        distances_ar[labels == k] = df[labels ==
k].apply(distance_from_point,axis=1).values
    return np.sum(distances_ar)

def compute_centroids(df, labels, K):
    centroids = np.zeros((K, df.shape[1]))
    for k in range(K):
        centroids[k, :] = df[labels == k].mean()
    return centroids

def Kmeans(df,K):
    max_iter=20

    centroids=initialize_centroids(df,K)

    for i in range(max_iter):
            old_centroids = centroids
            dist_matrix = compute_distance(df, old_centroids)
            clust=np.argmin(dist_matrix,axis = 1)
            centroids = pd.DataFrame(compute_centroids(df,clust,K))

    return centroids,clust

def Kmeans_sse(df,K):
    '''performs Kmeans returns centroids and prints sse of each new
centroids'''
```

```
    #define the maximum number of iterations
    max_iter=20

    #initialize centroids
    centroids=initialize_centroids(df,K)

    for i in range(max_iter):
            old_centroids = centroids
            dist_matrix = compute_distance(df, old_centroids)
            clust=np.argmin(dist_matrix, axis=1)
            centroids = pd.DataFrame(compute_centroids(df,clust,K))

    # return the centroids
    return compute_sse(df,clust,old_centroids,K)

# create k and SSE arrays to find best number of clusters

k = np.array([])
sse = np.array([])
for i in np.arange(1,10):
    k = np.append(k,i)
    sse = np.append(sse,Kmeans_sse(bestmodel_df,i))

# elbow plot is used to select number of clusters to use

plt.plot(k,sse)
plt.title("Elbow Plot")
plt.xlabel("K (number of clusters)")
plt.ylabel("Minimum SSE")
plt.show()
```


```
# return centroids and clusters used chosen K=3
centroids,clust = Kmeans(bestmodel_df,3)

# show the clusters created by K-Means Clustering!
pd.set_option('display.max_rows', None)

country_names = bestmodel_df.index
cluster_countries =
pd.DataFrame({"country":country_names,"cluster_label":clust})
cluster_countries

cluster0_countries =
cluster_countries[cluster_countries["cluster_label"] == 0]
display(cluster0_countries)

cluster1_countries =
cluster_countries[cluster_countries["cluster_label"] == 1]
display(cluster1_countries)

cluster2_countries =
```

```
cluster_countries[cluster_countries["cluster_label"] == 2]
display(cluster2_countries)
```

	country	cluster_label
6	AUT-2003	0
17	CZE-2003	0
18	CZE-2006	0
19	CZE-2009	0
20	CZE-2012	0
21	CZE-2015	0
22	CZE-2018	0
29	EST-2006	0
30	EST-2009	0
31	EST-2012	0
32	EST-2015	0
33	EST-2018	0
34	FIN-2003	0
35	FIN-2006	0
36	FIN-2009	0
58	HUN-2003	0
59	HUN-2006	0
60	HUN-2009	0
61	HUN-2012	0
62	HUN-2015	0
63	HUN-2018	0
87	KOR-2003	0
88	KOR-2006	0
89	KOR-2009	0
90	KOR-2012	0
91	KOR-2015	0
92	KOR-2018	0
93	LVA-2003	0
94	LVA-2006	0
95	LVA-2009	0
96	LVA-2012	0
97	LVA-2015	0
98	LVA-2018	0
99	LUX-2003	0
100	LUX-2006	0
117	POL-2003	0
118	POL-2006	0
119	POL-2009	0
120	POL-2012	0
121	POL-2015	0
122	POL-2018	0
129	SVK-2003	0
130	SVK-2006	0
131	SVK-2009	0
132	SVK-2012	0
134	SVK-2018	0

135	SVN-2006	0
136	SVN-2009	0
137	SVN-2012	0
138	SVN-2015	0
139	SVN-2018	0
152	CHE-2003	0
153	CHE-2006	0
154	CHE-2009	0
155	CHE-2012	0
156	CHE-2015	0
157	CHE-2018	0

	country	cluster_label
0	AUS-2003	1
1	AUS-2006	1
2	AUS-2009	1
3	AUS-2012	1
4	AUS-2015	1
5	AUS-2018	1
7	AUT-2006	1
8	AUT-2012	1
9	AUT-2015	1
10	AUT-2018	1
11	CAN-2003	1
12	CAN-2006	1
13	CAN-2009	1
14	CAN-2012	1
15	CAN-2015	1
16	CAN-2018	1
40	FRA-2003	1
41	FRA-2006	1
42	FRA-2009	1
43	FRA-2012	1
44	FRA-2015	1
45	FRA-2018	1
46	DEU-2003	1
47	DEU-2006	1
48	DEU-2009	1
49	DEU-2012	1
50	DEU-2015	1
51	DEU-2018	1
70	IRL-2003	1
71	IRL-2006	1
72	IRL-2009	1
73	IRL-2012	1
105	NLD-2003	1
123	PRT-2003	1
124	PRT-2006	1
125	PRT-2009	1
126	PRT-2012	1

127	PRT-2015	1
158	USA-2003	1
159	USA-2006	1
160	USA-2009	1
161	USA-2012	1
162	USA-2015	1
163	USA-2018	1
164	GBR-2006	1
165	GBR-2009	1
166	GBR-2012	1
167	GBR-2015	1
168	GBR-2018	1

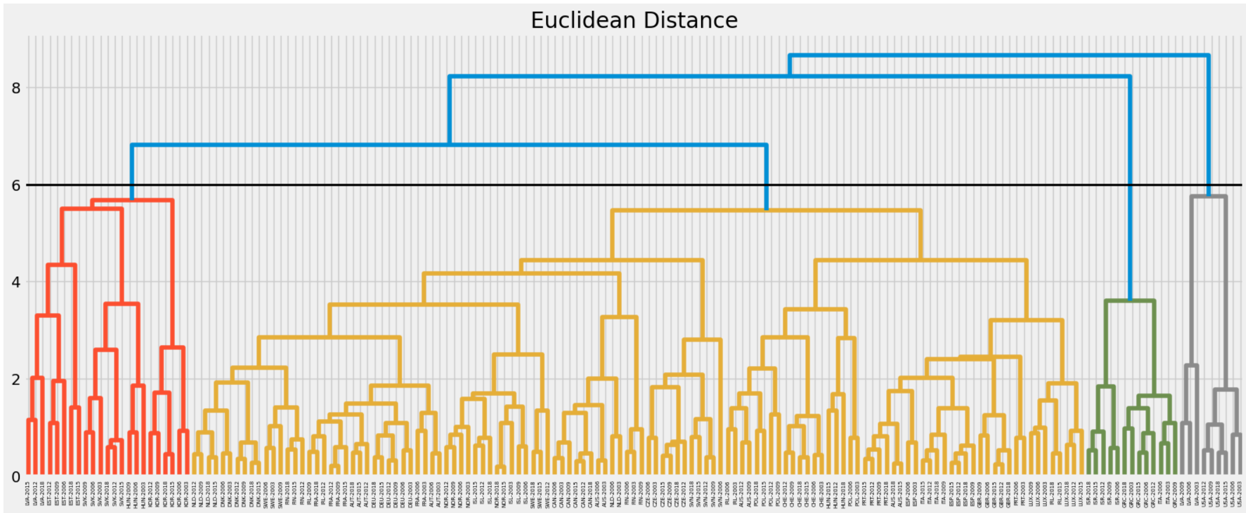
	country	cluster_label
23	DNK-2003	2
24	DNK-2006	2
25	DNK-2009	2
26	DNK-2012	2
27	DNK-2015	2
28	DNK-2018	2
37	FIN-2012	2
38	FIN-2015	2
39	FIN-2018	2
52	GRC-2003	2
53	GRC-2006	2
54	GRC-2009	2
55	GRC-2012	2
56	GRC-2015	2
57	GRC-2018	2
64	ISL-2003	2
65	ISL-2006	2
66	ISL-2009	2
67	ISL-2012	2
68	ISL-2015	2
69	ISL-2018	2
74	IRL-2015	2
75	IRL-2018	2
76	ISR-2006	2
77	ISR-2009	2
78	ISR-2012	2
79	ISR-2015	2
80	ISR-2018	2
81	ITA-2003	2
82	ITA-2006	2
83	ITA-2009	2
84	ITA-2012	2
85	ITA-2015	2
86	ITA-2018	2
101	LUX-2009	2
102	LUX-2012	2

```
103  LUX-2015              2
104  LUX-2018              2
106  NLD-2006              2
107  NLD-2009              2
108  NLD-2012              2
109  NLD-2015              2
110  NLD-2018              2
111  NOR-2003              2
112  NOR-2006              2
113  NOR-2009              2
114  NOR-2012              2
115  NOR-2015              2
116  NOR-2018              2
128  PRT-2018              2
133  SVK-2015              2
140  ESP-2003              2
141  ESP-2006              2
142  ESP-2009              2
143  ESP-2012              2
144  ESP-2015              2
145  ESP-2018              2
146  SWE-2003              2
147  SWE-2006              2
148  SWE-2009              2
149  SWE-2012              2
150  SWE-2015              2
151  SWE-2018              2
```

```
# perform hierarchical clustering
lbls = np.array(bestmodel_df.index)

cluster = linkage(bestmodel_df, method ='complete',metric =
"euclidean")
plt.figure(figsize=(18, 7))
dendrogram(cluster,
            orientation='top',
            labels=lbls,
            distance_sort='descending')
plt.axhline(6,c="black",linewidth = 2)
plt.title("Euclidean Distance")
plt.show()
print()

# print out the clusters, using K=4
labels = fcluster(cluster, t=6.0, criterion='distance')
for k in np.arange(1,1+len(np.unique(labels))):
  print("group",k)
  print(bestmodel_df[labels==k].index.values)
  print('\n')
```




```
group 1
['LVA-2003' 'LVA-2006' 'LVA-2009' 'USA-2003' 'USA-2006' 'USA-2009'
 'USA-2012' 'USA-2015' 'USA-2018']


group 2
['GRC-2003' 'GRC-2006' 'GRC-2009' 'GRC-2012' 'GRC-2015' 'GRC-2018'
 'ISR-2006' 'ISR-2009' 'ISR-2012' 'ISR-2015' 'ISR-2018' 'ITA-2003'
 'ITA-2006']


group 3
['AUS-2003' 'AUS-2006' 'AUS-2009' 'AUS-2012' 'AUS-2015' 'AUS-2018'
 'AUT-2003' 'AUT-2006' 'AUT-2012' 'AUT-2015' 'AUT-2018' 'CAN-2003'
 'CAN-2006' 'CAN-2009' 'CAN-2012' 'CAN-2015' 'CAN-2018' 'CZE-2003'
 'CZE-2006' 'CZE-2009' 'CZE-2012' 'CZE-2015' 'CZE-2018' 'DNK-2003'
 'DNK-2006' 'DNK-2009' 'DNK-2012' 'DNK-2015' 'DNK-2018' 'FIN-2003'
 'FIN-2006' 'FIN-2009' 'FIN-2012' 'FIN-2015' 'FIN-2018' 'FRA-2003'
 'FRA-2006' 'FRA-2009' 'FRA-2012' 'FRA-2015' 'FRA-2018' 'DEU-2003'
 'DEU-2006' 'DEU-2009' 'DEU-2012' 'DEU-2015' 'DEU-2018' 'HUN-2012'
 'HUN-2015' 'HUN-2018' 'ISL-2003' 'ISL-2006' 'ISL-2009' 'ISL-2012'
 'ISL-2015' 'ISL-2018' 'IRL-2003' 'IRL-2006' 'IRL-2009' 'IRL-2012'
 'IRL-2015' 'IRL-2018' 'ITA-2009' 'ITA-2012' 'ITA-2015' 'ITA-2018'
 'LUX-2003' 'LUX-2006' 'LUX-2009' 'LUX-2012' 'LUX-2015' 'LUX-2018'
 'NLD-2003' 'NLD-2006' 'NLD-2009' 'NLD-2012' 'NLD-2015' 'NLD-2018'
 'NOR-2003' 'NOR-2006' 'NOR-2009' 'NOR-2012' 'NOR-2015' 'NOR-2018'
 'POL-2003' 'POL-2006' 'POL-2009' 'POL-2012' 'POL-2015' 'POL-2018'
 'PRT-2003' 'PRT-2006' 'PRT-2009' 'PRT-2012' 'PRT-2015' 'PRT-2018'
 'SVN-2006' 'SVN-2009' 'SVN-2012' 'SVN-2015' 'SVN-2018' 'ESP-2003'
 'ESP-2006' 'ESP-2009' 'ESP-2012' 'ESP-2015' 'ESP-2018' 'SWE-2003'
 'SWE-2006' 'SWE-2009' 'SWE-2012' 'SWE-2015' 'SWE-2018' 'CHE-2003'
 'CHE-2006' 'CHE-2009' 'CHE-2012' 'CHE-2015' 'CHE-2018' 'GBR-2006'
 'GBR-2009' 'GBR-2012' 'GBR-2015' 'GBR-2018']
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group 4
['EST-2006' 'EST-2009' 'EST-2012' 'EST-2015' 'EST-2018' 'HUN-2003'
 'HUN-2006' 'HUN-2009' 'KOR-2003' 'KOR-2006' 'KOR-2009' 'KOR-2012'
 'KOR-2015' 'KOR-2018' 'LVA-2012' 'LVA-2015' 'LVA-2018' 'SVK-2003'
 'SVK-2006' 'SVK-2009' 'SVK-2012' 'SVK-2015' 'SVK-2018']
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