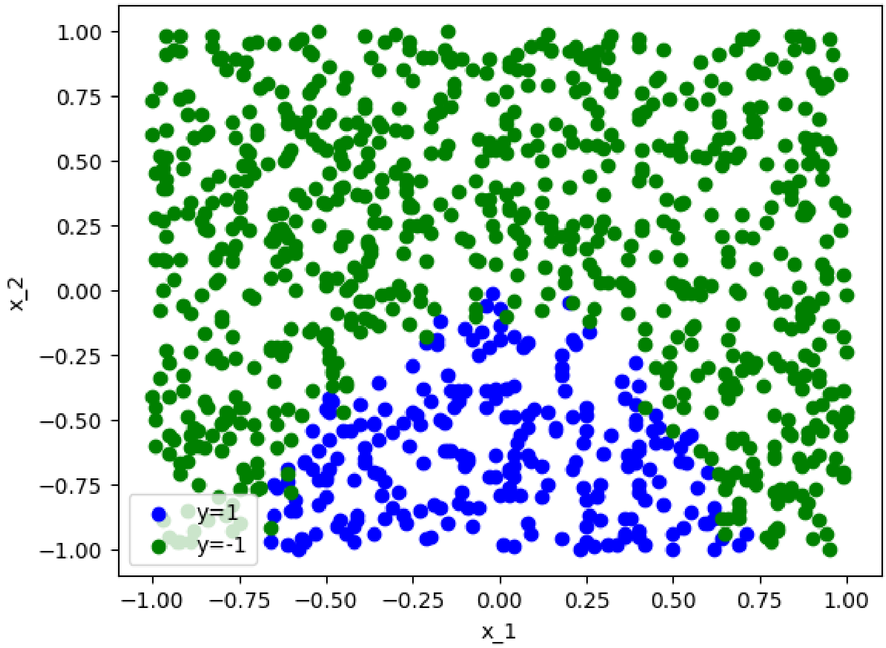
Week2 Assignment Report

Dataset id: 6-6-6

(a)



Visualised data graph

(i) Using numpy to load the data set from the txt file, then visualize it by matplotlib. The markers indicating each class are shown on the bottom left. The graph is also similar to the graph in the instruction document.

(ii) The decision boundary calculated by a logistic model can be written as:

$$\theta_0 + \theta_1 x_1 + \theta_2 x_2 = 0$$

And we need to do the training by using sklearn’s LogisticRegression function, in addition, set solver to 'saga' as the 'saga' can be faster for large dataset. Then we can get the coefficient set and the intercept: [0.08619205 5.06610909] and [1.78829503], indicating:

$$\theta_0 = 1.78829503, \theta_1 = 0.08619205, \theta_2 = 5.06610909$$

Thus, the boundary can be written as:

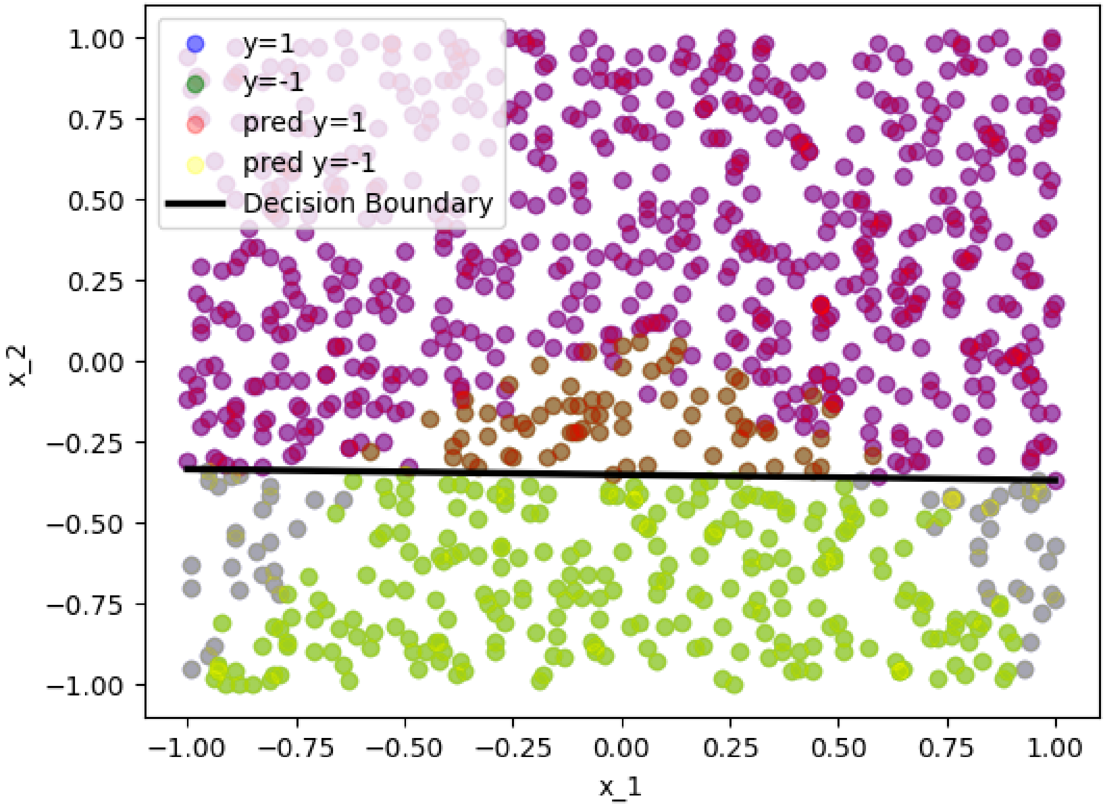
$$0.08619205x_1 + 5.06610909x_2 + 1.788295031 = 0$$

Then we can rewrite the equation to:

$$x_2 = \frac{1.78829503 + 0.08619205x_1}{-5.06610909}$$

We can also tell from the parameter values we obtained above, the θ1 is a very small number close to zero which means it will make feature x1 have less impact on predictions. So feature x2 has most influence on the prediction.

(iii) Then according to the equation above, we can get the decision boundary. As the following graph, the black line in the center is the decision boundary.



Decision Boundary

(iv) The decision boundary is linear but the dataset is non-linear, therefore, it is reasonable that the predictions failed to match the labels. According to the distribution of the points, the boundary should be quadratic. Thus instead of changing hyperparameters, it might be more efficient changing to a quadratic model.

(b)

(i) First we need to set a list of candidate values of 'C' : 0.001, 0.01, 1, 10, 100, 1000. Then we can use LinearSVC function in sklearn to train to obtain the new parameter values:

$$When\ C = 0.001, \theta_0 = 0.24587642, \theta_1 = 0.023121, \theta_2 = 0.45683574$$

$$When\ C = 0.01, \theta_0 = 0.41712681, \theta_1 = 0.02030266, \theta_2 = 1.17380565$$

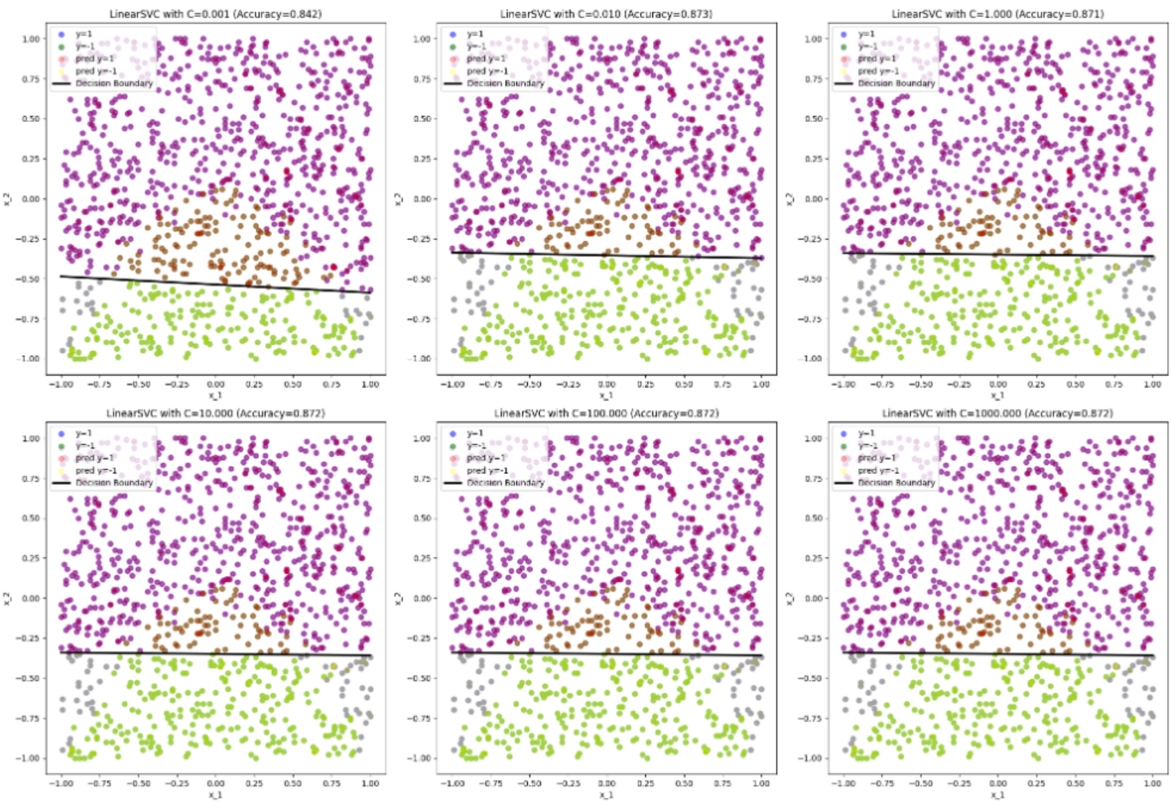
$$When\ C = 1, \theta_0 = 0.64770945, \theta_1 = 0.01637619, \theta_2 = 1.8494905$$

$$When\ C = 10, \theta_0 = 0.65346945, \theta_1 = 0.01657394, \theta_2 = 1.86614893$$

$$When\ C = 100, \theta_0 = 0.65407461, \theta_1 = 0.01661755, \theta_2 = 1.86789476$$

$$When\ C = 1000, \theta_0 = 0.65413465, \theta_1 = 0.0166215, \theta_2 = 1.86806845$$

(ii) For better comparison we also need to control the hyperparameters, therefore, all hyperparameters except 'C' are remaining the same during the experiment. And then we generate a plot with several subplots to visualize the results, besides, different 'C' will be evaluated by the 'score' function which calculated the mean accuracy measured with the given dataset to get the accuracies.



Different 'C' values with their accuracies

(iii) From the questions(i)&(ii) we can know that the value of C will affect the decision boundary. According to the graph we got from question (ii) above shows the accuracy trend regarding the change of 'C'. 'C' stands for the regularization strength. A small C would allow the model to accept a smaller margin, if the margin can separate most of the points. On the other hand, a larger C would force the model to maximize the margin and may harm the classification performance. The SVM model can also reach the best performance with a medium value of C, which also helps the model to avoid overfitting or underfitting.

(iv) The shape of the boundaries are similar due to the linearity. The value of C brings big differences to the model's accuracy on the same dataset. However, this hyperparameter has little effect on the logistic regression model.

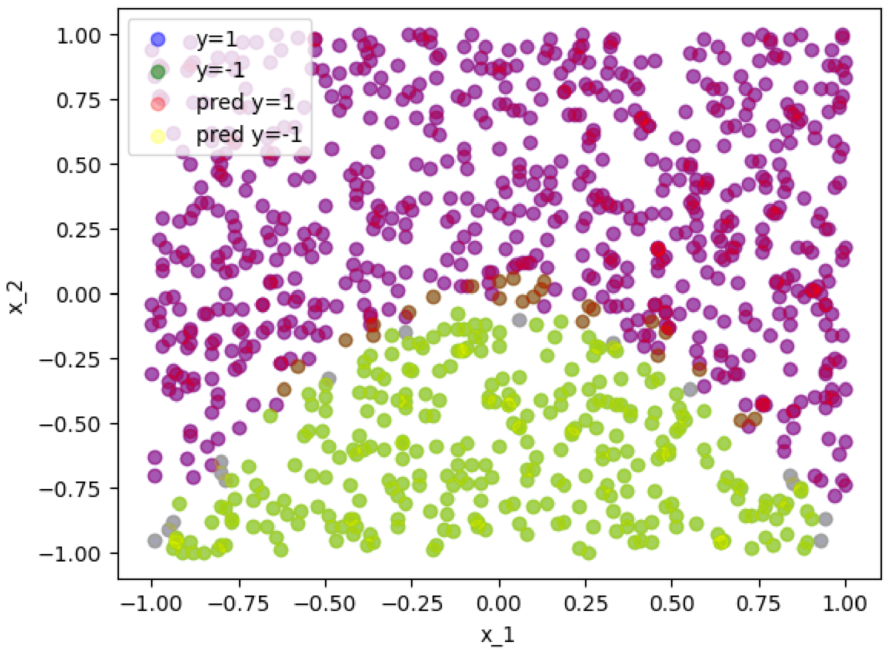
(c)

(i) After adding the additional features, the new model will be:

$$\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2 = 0$$

Then we can use the the same hyperparameters from the questions before to obtain the new parameter values:

$$\theta_0 = 0.38311111, \theta_1 = 0.03970063, \theta_2 = 6.65826619, \theta_3 = 6.65556341, \theta_4 = -1.09207904$$



Visualised data graph with new features

(ii)&(iii) Then we can get the new graph as shown above. The results predicted by the new model are different from the former models. It is apparent that it is difficult to calculate the boundary of a quadratic model. In addition, we can get the accuracy by 'score' function and the accuracy of this model is 0.96096, much higher than the linear model trained with the same hyperparameters.

Appendix

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import LogisticRegression
from sklearn import svm
# id: 6-6-6

# load the dataset
file = open('week2.txt', 'r')
data = np.array([line.strip().split(',') for line in
file.readlines()]).astype(float)
# Separate the Xs and ys, and create two index lists
x = data[:, :2]
y = data[:, 2]
index1 = [i for i in range(len(y)) if y[i] == 1]
index2 = [i for i in range(len(y)) if y[i] == -1]
# Visualize the data
plt.figure()
plt.scatter(x[index1, 0], x[index1, 1], color='blue', label='y=1')
plt.scatter(x[index2, 0], x[index2, 1], color='green', label='y=-1')
plt.legend()
plt.xlabel('x_1')
plt.ylabel('x_2')
plt.show()

# Using sklearn's LogisticRegression function to train
clf = LogisticRegression(solver='saga').fit(x, y)
pred = clf.predict(x)
pred_index1 = [i for i in range(len(y)) if pred[i] == 1]
pred_index2 = [i for i in range(len(y)) if pred[i] == -1]

# Get the coefficient and intercept
coef = clf.coef_[0]
intercept = clf.intercept_

print(coef)
print(intercept)

# Obtain the decision boundary
x1 = np.linspace(-1, 1, 10)
x2 = (-intercept-x1*coef[0])/coef[1]

# Visualize the result
plt.figure()
plt.scatter(x[index1, 0], x[index1, 1], color='blue', label='y=1',
alpha=0.5)
plt.scatter(x[index2, 0], x[index2, 1], color='green', label='y=-1',
alpha=0.5)
```

```
plt.scatter(x[pred_index1, 0], x[pred_index1, 1], color='red',
label='pred y=1', alpha=0.3)
plt.scatter(x[pred_index2, 0], x[pred_index2, 1], color='yellow',
label='pred y=-1', alpha=0.3)

plt.plot(x1, x2, label='Decision Boundary', color='black', linewidth=2.5)
plt.legend()
plt.xlabel('x_1')
plt.ylabel('x_2')
plt.show()

penalties = [0.001, 0.01, 1, 10, 100, 1000]
accuracies = []
plt.figure(figsize=[20, 20])
for index in np.arange(len(penalties))+1:
    plt.subplot(3, 3, index)

    # Model construction
    clf = svm.LinearSVC(C=penalties[index-1],dual=False).fit(x, y)

    # Predict the values and calculate the accuracy
    acc = clf.score(x, y)
    accuracies.append(acc)
    pred = clf.predict(x)
    pred_index1 = [i for i in range(len(y)) if pred[i] == 1]
    pred_index2 = [i for i in range(len(y)) if pred[i] == -1]

    # Get the coefficient and intercept to obtain the decision boundary
    coef = clf.coef_[0]
    intercept = clf.intercept_
    print(coef)
    print(intercept)

    # Obtain the decision boundary
    x1 = np.linspace(-1, 1, 10)
    x2 = (-intercept-x1*coef[0])/coef[1]

    plt.scatter(x[index1, 0], x[index1, 1], color='blue', label='y=1',
alpha=0.5)
    plt.scatter(x[index2, 0], x[index2, 1], color='green', label='y=-1',
alpha=0.5)

    plt.scatter(x[pred_index1, 0], x[pred_index1, 1], color='red',
label='pred y=1', alpha=0.3)
    plt.scatter(x[pred_index2, 0], x[pred_index2, 1], color='yellow',
label='pred y=-1', alpha=0.3)

    plt.plot(x1, x2, label='Decision Boundary', color='black',
linewidth=2.5)
```

```
    plt.title('LinearSVC with C=%.3f (Accuracy=%.3f)'%(penalties[index-1], acc))
    plt.legend()
    plt.xlabel('x_1')
    plt.ylabel('x_2')

plt.tight_layout()
plt.show()

# Add additional features
x_quad = np.concatenate(([x[:, 0]], [x[:, 1]], [x[:, 0]**2], [x[:, 1]**2])).T
# Fit the model
clf = LogisticRegression(solver='saga').fit(x_quad, y)
score = clf.score(x_quad, y)
# Get the new coefficient and intercept
coef = clf.coef_[0]
intercept = clf.intercept_

print(intercept)
print(coef)
print(score)

pred = clf.predict(x_quad)
pred_index1 = [i for i in range(len(y)) if pred[i] == 1]
pred_index2 = [i for i in range(len(y)) if pred[i] == -1]

# Visualize the result
plt.figure()
plt.scatter(x[index1, 0], x[index1, 1], color='blue', label='y=1', alpha=0.5)
plt.scatter(x[index2, 0], x[index2, 1], color='green', label='y=-1', alpha=0.5)

plt.scatter(x[pred_index1, 0], x[pred_index1, 1], color='red', label='pred y=1', alpha=0.3)
plt.scatter(x[pred_index2, 0], x[pred_index2, 1], color='yellow', label='pred y=-1', alpha=0.3)

plt.legend()
plt.xlabel('x_1')
plt.ylabel('x_2')
plt.show()
```