Exercises for INF3320

Parameterization and Barycentric Coordinates

17/10/2010

1. Suppose you are given a point $\mathbf{p}$ and a triangle $[\mathbf{a}, \mathbf{b}, \mathbf{c}]$ in $\mathbb{R}^3$.

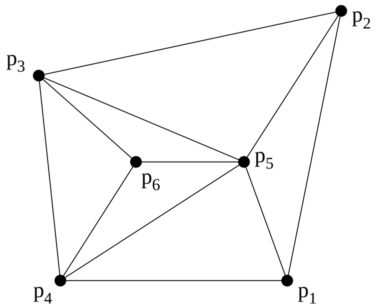
   Create a procedure to determine whether the projection of this point onto a plane of the triangle lays in the triangle, and if so, find its barycentric coordinates in relation to that triangle.

   Hint 1: Determining if a point is inside a triangle is a $2D$-problem, while this problem is in $3D$. By projecting all points onto an appropriate plane we can simplify the problem significantly. Since barycentric coordinated do not change under such projection, the solution of a $2D$-problem is a solution of a $3D$-problem.
   Hint 2: Plane of the triangle is not necessarily the best option to solve the problem in. It's much easier to use the XY, YZ or ZX plane, because then we can perform the projection by just throwing one of the coordinates of the points away. It is best to use the plane that is most similarly oriented as the plane of a triangle. We can find it by looking at the components of normal vector of the triangle. If the x-coordinate is the greatest, then we project points down to the YZ-plane by throwing away the x-coordinate. Otherwise, if the y-coordinate is greatest, we can throw it away. Etc.
   Hint 3: The signed area of the triangle composed by a point and one of the edges of original triangle tells you on which side of the edge point lies on.

2. Create a procedure that given a sequence of points finds a uniform parameterization of these points over the interval $[0, 1]$.

3. Create a procedure that given a sequence of points finds a chord length parameterization of these points over the interval $[0, 1]$.

4. Suppose that M is a triangular mesh in the chart below with nodes $\mathbf{p}_1, \ldots, \mathbf{p}_6 \in \mathbb{R}^3$.



   Let $\psi : M \to \mathbb{R}^2$ be a piecewise linear mapping. Where $\mathbf{v}_i = \psi(\mathbf{p}_i)$ positions the edge by setting

$$\mathbf{v}_1 = \begin{bmatrix}1\\0\end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix}1\\1\end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix}0\\1\end{bmatrix}, \qquad \mathbf{v}_4 = \begin{bmatrix}0\\0\end{bmatrix}.$$

   Positions of the inner nodes are given by requiring each of the two points $\mathbf{v}_5$ and $\mathbf{v}_6$ shall be in barycenter (average) of neighboring points. Set up the equation system, solve it and mark the point in the final mesh.

5. In this exercise you are going to derive the formulas for perspective correct interpolation that were presented on the lecture. We will do this in steps.

- Step 1: Consider a camera at the origin with the screen distance d away. Suppose you are given a vertex $\mathbf{v}$ in $\mathbb{R}^3$, where $\mathbf{v} = (x, y, z)$ are its coordinates. Write down a relationship between the vertex $\mathbf{v}$ and its projection $\mathbf{p} = (x', y', z')$ to a plane at a distance d from the camera. (Hint: This was on the first lecture).
- Step 2: We will be using the notation from step 1. You are given two vertices describing a line segment $[\mathbf{v}_1, \mathbf{v}_2]$ in a view space and a point $\mathbf{p}$ laying on the projection of that line segment to the screen:

  $$\mathbf{p} = \alpha_1 \mathbf{p}_1 + \alpha_2 \mathbf{p}_2 \text{ where } \alpha_1 + \alpha_2 = 1$$

  Given α_1 and α_2 find a point $\mathbf{v}$ on line segment $[\mathbf{v}_1, \mathbf{v}_2]$ such that it projects to $\mathbf{p}$. Represent this point as a convex combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.
- Step 3: Again we will be using the notation from step 1. You are given three vertices describing a triangle $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]$ in view space and a point $\mathbf{p}$ laying in the projection of that triangle to the screen. We describe this point using barycentric coordinates $(\alpha_1, \alpha_2, \alpha_3)$ with respect to projected triangle $[\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3]$:

  $$\mathbf{p} = \alpha_1 \mathbf{p}_1 + \alpha_2 \mathbf{p}_2 + \alpha_3 \mathbf{p}_3 \text{ where } \alpha_1 + \alpha_2 + \alpha_3 = 1$$

  Given α_1, α_2 and α_3 find a point $\mathbf{v}$ in the triangle $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]$ such that it projects to $\mathbf{p}$. Represent that point in barycentric coordinates with respect to triangle $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]$.

6. The formulas given on the lectures and derived in the previous problem are used to perform a perspective correct interpolation of attributes assigned to vertices (like color, normals, texture coordinates) over the whole primitive. Find an example of line segment, texture coordinate assignment and a texture that will give different results if interpolated linearly in screen space and different results if interpolated in a perspective correct manner in object space. Verify, using the formulas, that the results are indeed different. (Hint: Find a point that will have different texture coordinates when different interpolation methods are used).