

# EXERCISES FOR INF3320

## LINEAR, AFFINE, AND PROJECTIVE TRANSFORMS (PART I)

14/09/202

1. A point  $\mathbf{p} = [x, y]$  in  $\mathbb{R}^2$  can be represented in polar coordinates  $[\rho, \phi]$  with

$$x = \rho \cos \phi, \quad y = \rho \sin \phi.$$

- (a) What is the new position  $\mathbf{p}' = [x', y']$  after a rotation of  $\theta$  about the origin?  
(b) Write down a  $2 \times 2$  matrix for the transformation from  $\mathbf{p}$  to  $\mathbf{p}'$ , i.e. find  $\mathbf{M}$  such that

$$\mathbf{p}' = \mathbf{M}\mathbf{p}.$$

- (c) Use the solution of the previous exercise to find a  $4 \times 4$  rotation matrix that represent a rotation of  $\theta$  in  $\mathbb{R}^3$  about the  $y$ -axis.

2. Let  $\mathbf{M}_1, \mathbf{M}_2, \dots, \mathbf{M}_n$  be a sequence of transformations, where each  $\mathbf{M}_i$  is either a rotation around the  $z$ -axis or an arbitrary translation in  $\mathbb{R}^3$ .

Show that

$$\mathbf{M}_1\mathbf{M}_2 \cdots \mathbf{M}_n = \mathbf{T}\mathbf{R}$$

where  $\mathbf{T}$  is a translation and  $\mathbf{R}$  is a rotation around the  $z$ -axis.

3. (a) Write down a matrix  $\mathbf{M}$  that mirrors points about the  $xy$ -plane.

Given an angle  $\theta$  and a normalized axis of rotation  $\mathbf{a}$ , let  $\mathbf{R}_{\mathbf{a},\theta}$  be the corresponding rotation matrix. Given a translation vector  $\mathbf{t}$ , let  $\mathbf{T}_{\mathbf{t}}$  be the corresponding translation matrix. Using  $\mathbf{M}$ ,  $\mathbf{R}_{\mathbf{a},\theta}$  and  $\mathbf{T}_{\mathbf{t}}$ ,

- (b) specify a  $4 \times 4$ -matrix that mirrors points about a plane through the origin, and then  
(c) specify a  $4 \times 4$ -matrix that mirrors points about any plane.

4. Let  $\mathbf{p}$  be a point in  $\mathbb{R}^3$ ,  $\mathbf{n}$  a surface normal, and  $\mathbf{M}$  a  $4 \times 4$  non-singular homogenous transformation matrix.

Using a right hand coordinate system (as OpenGL do):

- (a) How do we apply  $\mathbf{M}$  on  $\mathbf{p}$  to find the transformed point  $\mathbf{p}'$ ?  
(b) How do we apply  $\mathbf{M}$  on  $\mathbf{n}$  to find the transformed surface normal  $\mathbf{n}'$ ? Why?