PHYS 410/510 Fall 2024 Homework 4

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I started using Julia for this homework assignment as I've been fighting small Python syntax and just feel more comfortable with Julia at this point. Tom said it was okay by him!

Problem 1 (Frequency modulation).

Solution.

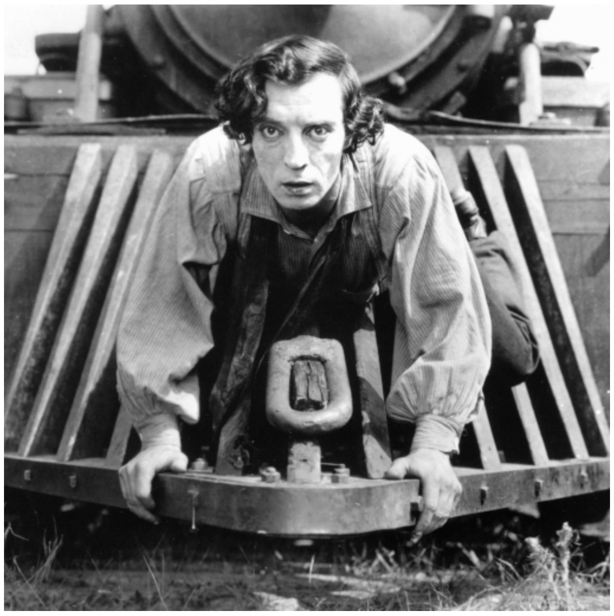


Figure 1: Buster Keaton

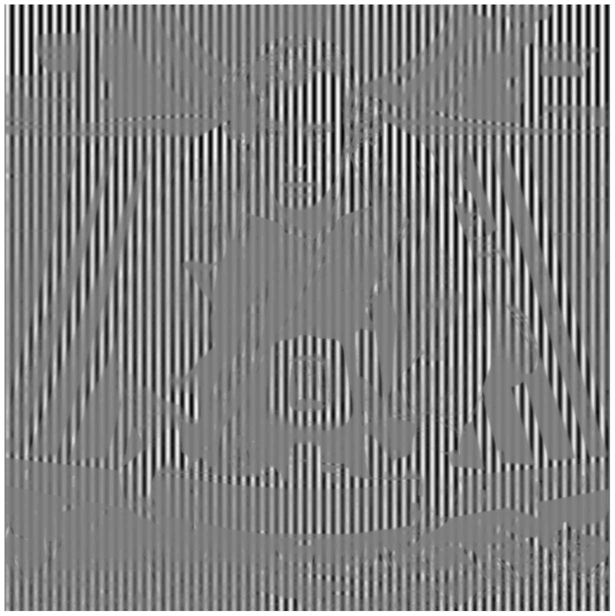


Figure 2: ...after applying a sine wave.

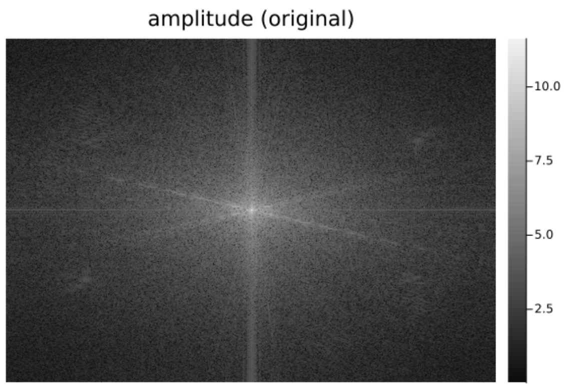


Figure 3: Amplitude array of original image after a Fourier transform.

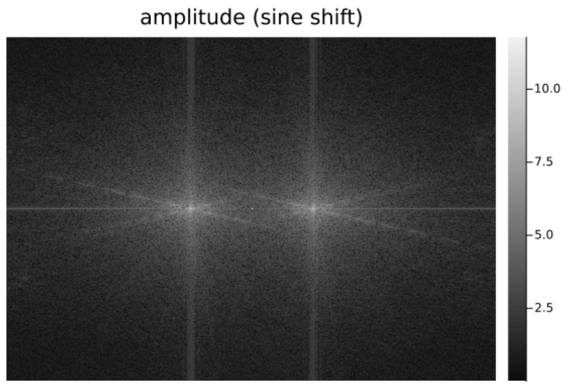


Figure 4: Amplitude array of sine modded image after a Fourier transform.

The amplitude array of the original image is shown in Figure 3 whereas the amplitude array of the image after a sine modification is shown in Figure 4. As expected, we can see a frequency array in the original image with a central peak. In the sine modulated image, we see two peaks instead. These two peaks (I believe) correspond to the $\pm$ values of a sine wave. We lose some intensity of the peak at the center because most of the image is washed out by the sine mod, but we can still see faint traces of it.

Problem 2 (Quantized aggregates).

Solution. Alright, so my approach goes like this: I first made sure I could at least extract the emitters with the non-noise image. I wrote a function that iterates through multiples of 33px in the x and y directions and pulled just those pixels, given that the problem states we can take that as a truth. I verified this worked because the resulting array had length 100. I looked at these values and saw that they had some distinct gaps in their 'typical' value. Plotting them in a histogram gave me the results seen in Figure 5. This gave me distinct peaks to look at for classifying monomers, dimers, and trimers. I then wrote a function sorting these and counting them, which gave me the correct result of 69, 20, and 11 monomers, dimers, and trimers respectively. The trouble arose when I tried this with the noisy image, where the histogram can be seen in Figure 6.

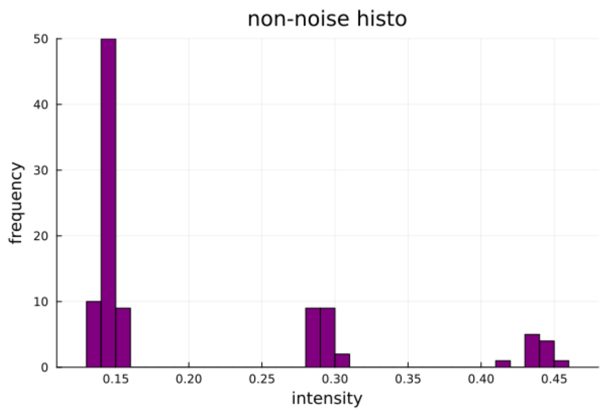


Figure 5: Emitter values in the no-noise image.

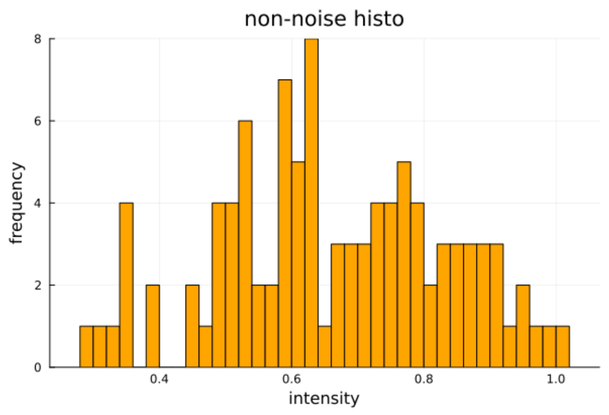


Figure 6: Emitter values in the image that contained a gradient noise overlay.

At this point I needed to find a way to subtract the noise. My process was to blur out the background, emphasizing the emitter pixels, then subtract that blurred background from the original image. From there, I would try to extract the emitters using the functions I wrote before. I then looked at a sorted list of those values and found certain 'jumps' in the values, which is where I set the values for counting and arrived at what I believe is the solution: 25 monomers, 11 dimers, and 64 trimers. The main two functions used are in Figures 9 and 10, and the function used for the Gaussian blur is a part of the `Images` package in Julia.

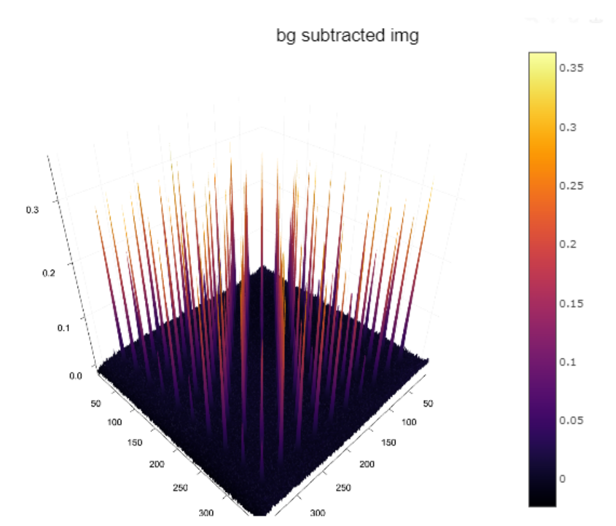


Figure 7: Noisy emitters after subtracting background (iso view).

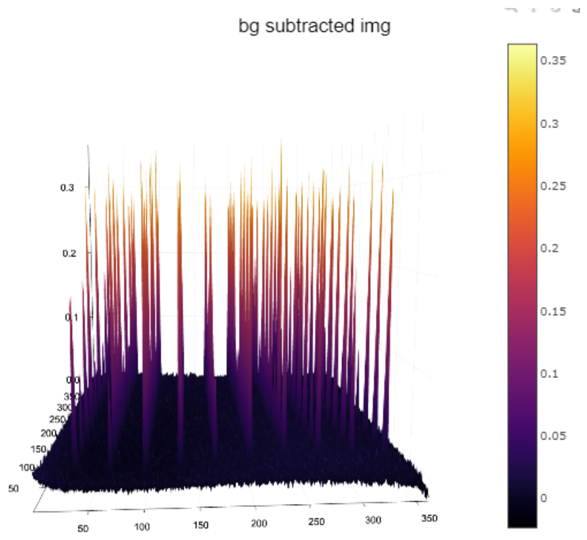
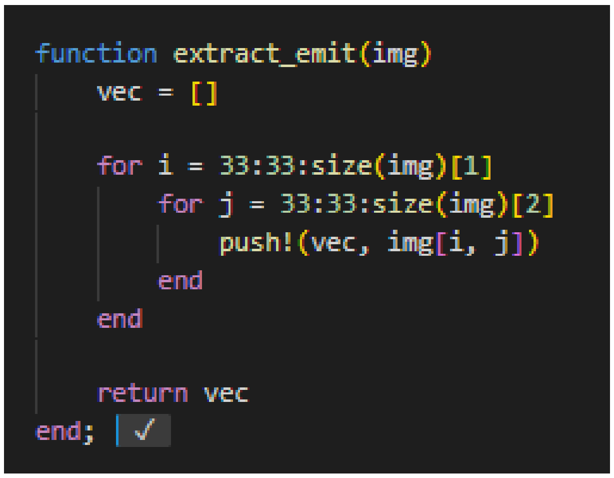
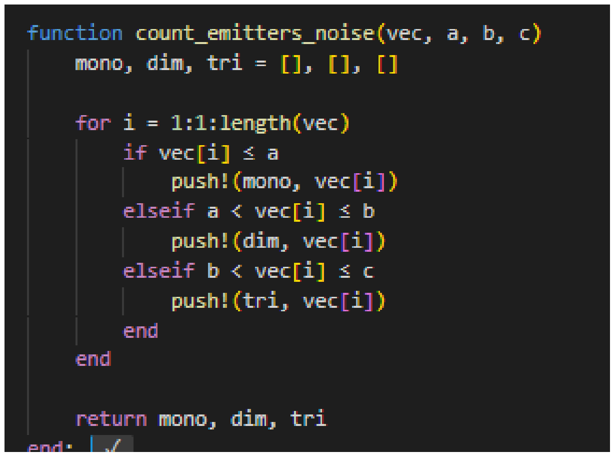


Figure 8: Noisy emitters after subtracting background (side view).



```
function extract_emit(img)
    vec = []

    for i = 33:33:size(img)[1]
        for j = 33:33:size(img)[2]
            push!(vec, img[i, j])
        end
    end

    return vec
end; ✓
```

Figure 9: Function for extracting only the emitter pixels.

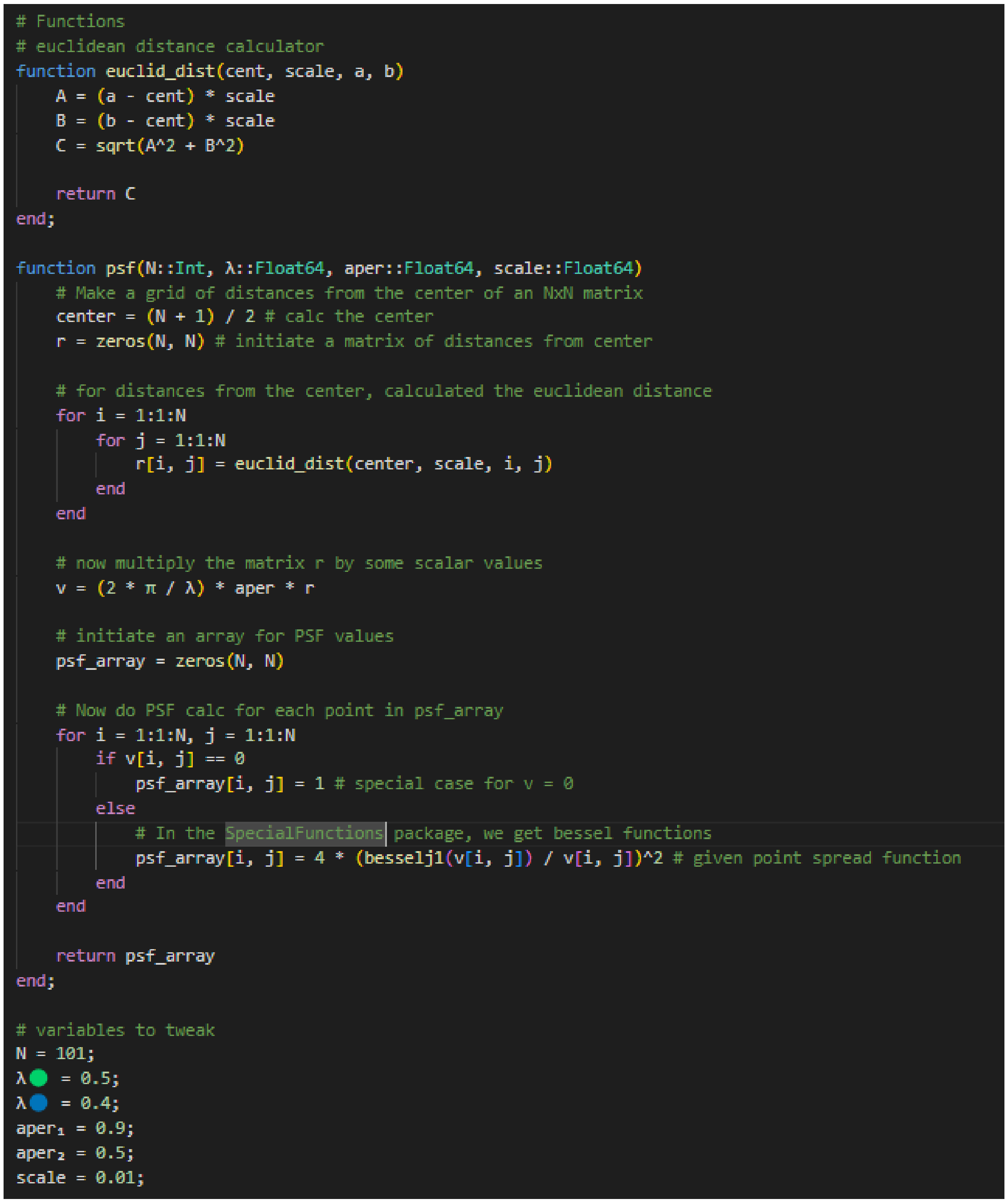


```
function count_emitters_noise(vec, a, b, c)
    mono, dim, tri = [], [], []

    for i = 1:1:length(vec)
        if vec[i] ≤ a
            push!(mono, vec[i])
        elseif a < vec[i] ≤ b
            push!(dim, vec[i])
        elseif b < vec[i] ≤ c
            push!(tri, vec[i])
        end
    end

    return mono, dim, tri
end; ✓
```

Figure 10: Function for counting the emitters within certain values specified by `a`, `b`, and `c`.

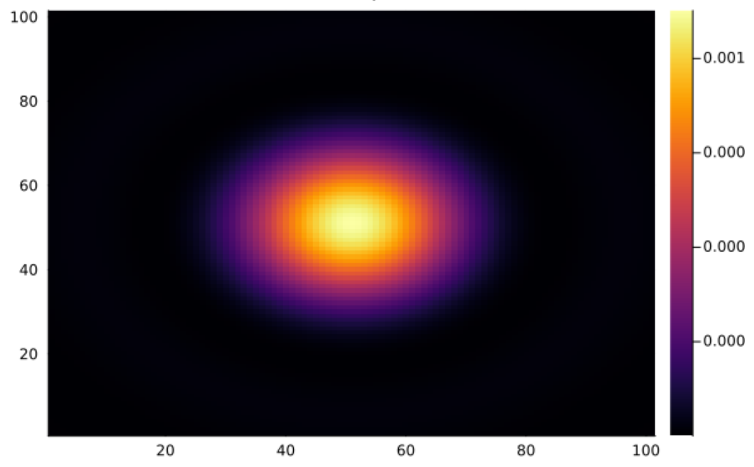
Problem 3 (A high-resolution PSF).

Solution. This problem was fairly straightforward as everything is provided for us so I'm going to start by posting my function so the comments can be seen to show what's going on. I also love that Julia lets me use unicode for variables (hence the green and blue by λ). Below this you'll see the results of the function I wrote.

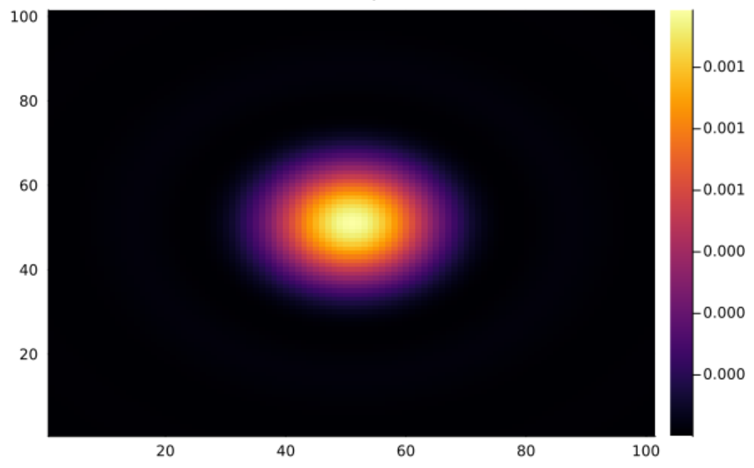


```julia
# Functions
# euclidean distance calculator
function euclid_dist(cent, scale, a, b)
    A = (a - cent) * scale
    B = (b - cent) * scale
    C = sqrt(A^2 + B^2)

    return C
end;

function psf(N::Int, λ::Float64, aper::Float64, scale::Float64)
    # Make a grid of distances from the center of an NxN matrix
    center = (N + 1) / 2 # calc the center
    r = zeros(N, N) # initiate a matrix of distances from center

    # for distances from the center, calculated the euclidean distance
    for i = 1:1:N
        for j = 1:1:N
            r[i, j] = euclid_dist(center, scale, i, j)
        end
    end

    # now multiply the matrix r by some scalar values
    v = (2 * π / λ) * aper * r

    # initiate an array for PSF values
    psf_array = zeros(N, N)

    # Now do PSF calc for each point in psf_array
    for i = 1:1:N, j = 1:1:N
        if v[i, j] == 0
            psf_array[i, j] = 1 # special case for v = 0
        else
            # In the SpecialFunctions package, we get bessel functions
            psf_array[i, j] = 4 * (besselj1(v[i, j]) / v[i, j])^2 # given point spread function
        end
    end

    return psf_array
end;

# variables to tweak
N = 101;
λ🟢 = 0.5;
λ🔵 = 0.4;
aper₁ = 0.9;
aper₂ = 0.5;
scale = 0.01;
```

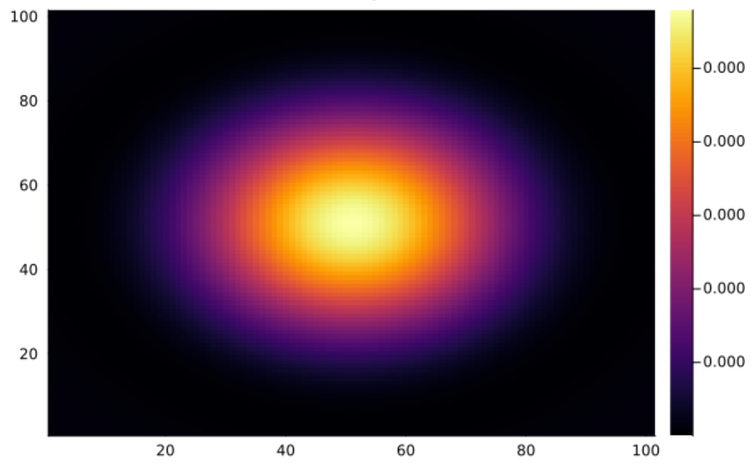
PSF with λ=0.5μm, NA=0.9



PSF with λ=0.4μm, NA=0.9

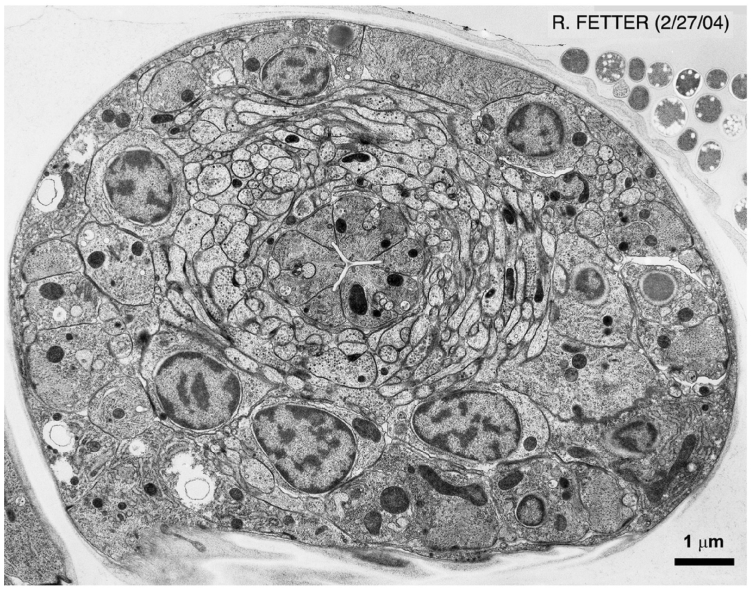


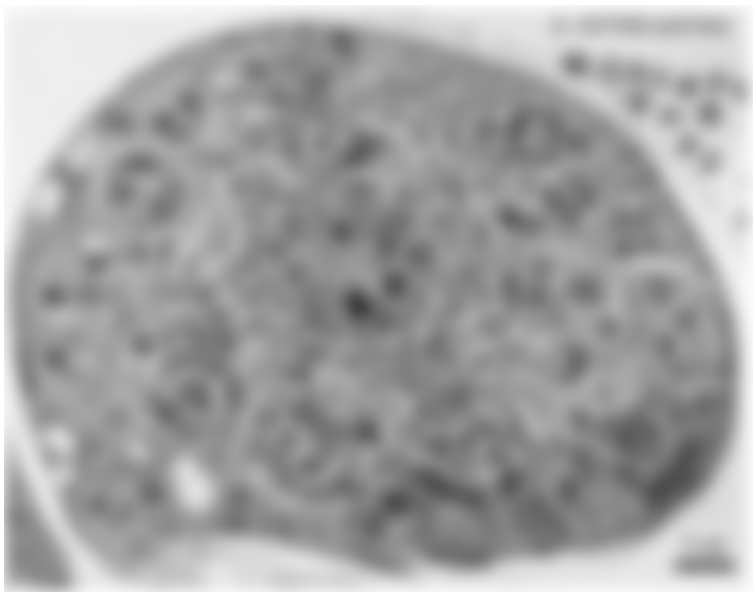
PSF with λ=0.4μm, NA=0.5



Problem 4 (A worse worm image).

Solution. I'm a little confused on this one - would the image just be blurred? My only thought, since it hints at using the PSF from before, would be to apply that as a filter to the image. This makes sense to me conceptually since the PSF is determined by apertures and wavelength so if we used the indicated values and apply that as a filter to the image, it would simulate it as if we're looking at visible light using a specific aperture. The figures below show the original image and what it would look like when using the PSF as a filter. I chose the scale to be 1/100 as that's what we chose for similar scales in the previous problem.





Problem 5 (SNR and Poisson noise).

Solution. I read that one definition of the signal to noise ratio is the mean divided by the standard deviation. For a Poisson distribution, the mean would be equal to N_{photon} or the number of photons expected. The standard deviation would then be $\sqrt{N_{photon}}$. Therefore

$$SNR = \frac{\mu}{\sigma} = \frac{N_{photon}}{\sqrt{N_{photon}}} = \sqrt{N_{photon}}. \tag{1}$$

Problem 6 (Simulated point-sources, part 1).

Solution. Here are my immediate ideas on how to approach this problem:

- First, we obviously need to change our scale for the PSF function from before. Previously I was using $0.01\mu m$ as it was a given, but now we should consider scales of $0.1\mu m$. I think I'm going to have to use some sort of 'nested' grid where the camera grid is this larger size but comprised of smaller grids like we used before. This could be similar to a convolution where we take the average of a grid of say, 3x3 pixels, and set all there values to one - somewhat like a downscaling. I *think* this is the right approach but I'm not sure.
- Once this is done, we're essentially pixelating our image.
- Now I need to play with some PSF stuff related to scaling? I haven't gotten this far yet. It sounds almost like we'll have 'nested' psf's or something but that doesn't quite make sense conceptually.