



1. For each given recurrence relation, find a closed-form solution and check it by substituting it back to the recurrence relation.
 - (a) $a_n = a_{n-1} + 1$ with $a_1 = 3$
 - (b) $a_n = a_{n-1} + n$ with $a_0 = 2$
 - (c) $a_n = 2a_{n-1} + 2$ with $a_0 = 1$
 - (d) $a_n = 4a_{n-1} + 5a_{n-2}$ with $a_0 = 1, a_1 = 17$
 - (e) $a_n = 6a_{n-1} + 9a_{n-2}$ with $a_0 = 2, a_1 = 3$
 - (f) $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ with $a_{0,1,2} = 3, 2, 6$
2. Solve the following recurrences by applying [Master theorem](#). For the cases where Master theorem does not apply, use [Akra-Bazzi method](#). In cases where neither of these two theorems apply, explain why and solve the recurrence relation by examining the recursion tree. Solutions must be in the form $T(n) \in \Theta(\dots)$.
 - (a) $T(n) = 2T(n/2) + n$
 - (b) $T(n) = T(3n/4) + T(n/4) + n$
 - (c) $T(n) = 3T(n/2) + n$
 - (d) $T(n) = 2T(n/2) + n/\log n$
 - (e) $T(n) = 6T(n/3) + n^2 \log n$
 - (f) $T(n) = T(3n/4) + n \log n$
 - (g) $T(n) = T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil) + n$
 - (h) $T(n) = T(n/2) + T(n/4) + 1$
 - (i) $T(n) = T(n/2) + T(n/3) + T(n/6) + n$
 - (j) $T(n) = 2T(n/3) + 2T(2n/3) + n$
 - (k) $T(n) = \sqrt{2n}T(\sqrt{2n}) + \sqrt{n}$
 - (l) $T(n) = \sqrt{2n}T(\sqrt{2n}) + n$
3. Consider a recurrence relation $a_n = 2a_{n-1} + 2a_{n-2}$ with $a_0 = a_1 = 2$. Solve it (*i.e.* find a closed formula) and show how it can be used to estimate the value of $\sqrt{3}$ (hint: observe $\lim_{n \rightarrow \infty} a_n/a_{n-1}$). After that, devise an algorithm for constructing a recurrence relation with integer coefficients that can be used to estimate the square root \sqrt{k} of a given integer k .