Properties of inner products

Quiz, 5 questions

5/5 points (100.00%)

Congratulations! You passed!

Next Item

1 / 1 points

1.
The function

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

- [x] bilinear

Correct
Yes:

- β is symmetric. Therefore, we only need to show linearity in one argument.
- For any $\lambda \in \mathbb{R}$ it holds that $\beta(\mathbf{x} + \lambda\mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda\beta(\mathbf{z}, \mathbf{y})$. This holds because of the rules for vector-matrix multiplication and addition.

- [ ] not an inner product

Un-selected is correct

- [x] positive definite

Correct
Yes, the matrix has only positive eigenvalues and $\beta(\mathbf{x}, \mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{0}$ and $\beta(\mathbf{x}, \mathbf{x}) = 0 \iff \mathbf{x} = \mathbf{0}$

- [x] an inner product

Correct
It's symmetric, bilinear and positive definite. Therefore, it is a valid inner product.

- [ ] not bilinear

Un-selected is correct

- [ ] not positive definite

Un-selected is correct

- [ ] not symmetric

Un-selected is correct

- [x] symmetric

Correct
Yes: $\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$

1 / 1 points

Properties of inner products

5/5 points (100.00%)

2. Quiz, 5 questions

The function

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

- [ ] an inner product

Un-selected is correct

- [ ] bilinear

Correct

Correct:

- β is symmetric. Therefore, we only need to show linearity in one argument.
- $\beta(\mathbf{x} + \lambda\mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda\beta(\mathbf{z}, \mathbf{y})$. This holds because of the rules for vector-matrix multiplication and addition.

- [ ] not symmetric

Un-selected is correct

- [ ] positive definite

Un-selected is correct

- [ ] not an inner product

Correct

Correct: Since β is not positive definite, it cannot be an inner product.

- [ ] symmetric

Correct

Correct: $\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$

- [ ] not positive definite

Correct

With $x = [1, 1]^T$ we get $\beta(\mathbf{x}, \mathbf{x}) = 0$. Therefore β is not positive definite.

- [ ] not bilinear

Un-selected is correct

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1 / 1 points

3.

The function

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

- [ ] symmetric

Un-selected is correct

Properties of inner products

5/5 points (100.00%)

Quiz, 5 questions

- [x] not symmetric

Correct

Correct: If we take $\mathbf{x} = [1, 1]^T$ and $\mathbf{y} = [2, -1]^T$ then $\beta(\mathbf{x}, \mathbf{y}) = 0$ but $\beta(\mathbf{y}, \mathbf{x}) = 6$. Therefore, β is not symmetric.

- [x] bilinear

Correct

Correct.

- [ ] not bilinear

Un-selected is correct

- [ ] an inner product

Un-selected is correct

- [x] not an inner product

Correct

Correct: Symmetry is violated.

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1 / 1 points

4.
The function

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{y}$$

is

- [x] symmetric

Correct

It is the dot product, which we know already. Therefore, it is symmetric.

- [ ] not positive definite

Un-selected is correct

- [x] positive definite

Correct

It is the dot product, which we know already. Therefore, it is positive definite.

- [ ] not symmetric

Un-selected is correct

- [ ] not an inner product

Un-selected is correct

an inner product

Properties of inner products

5/5 points (100.00%)

Correct
It is the dot product, which we know already. Therefore, it is also an inner product.

bilinear

Correct
It is the dot product, which we know already. Therefore, it is positive bilinear.

not bilinear

Un-selected is correct

1 / 1 points

5.
For any two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$ write a short piece of code that defines a valid inner product.

```
import numpy as np

def dot(a, b):
  """Compute dot product between a and b.
  Args:
    a, b: (2,) ndarray as R^2 vectors

  Returns:
    a number which is the dot product between a, b
  """

  dot_product = a @ b

  return dot_product

# Test your code before you submit.
a = np.array([1,0])
b = np.array([0,1])
print(dot(a,b))
```

Run

Reset

Correct Response
Good job!