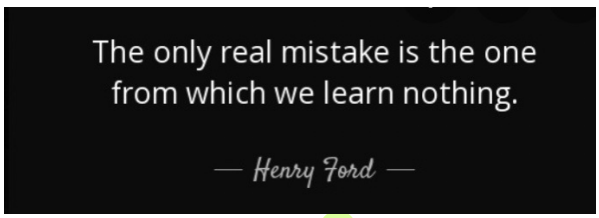
Today Quote



{ Do mistakes → learn from them → grow. }

Today's content.

→ Comparing two Algo's.
  a) using execution time
  b) using iterations & graphs.

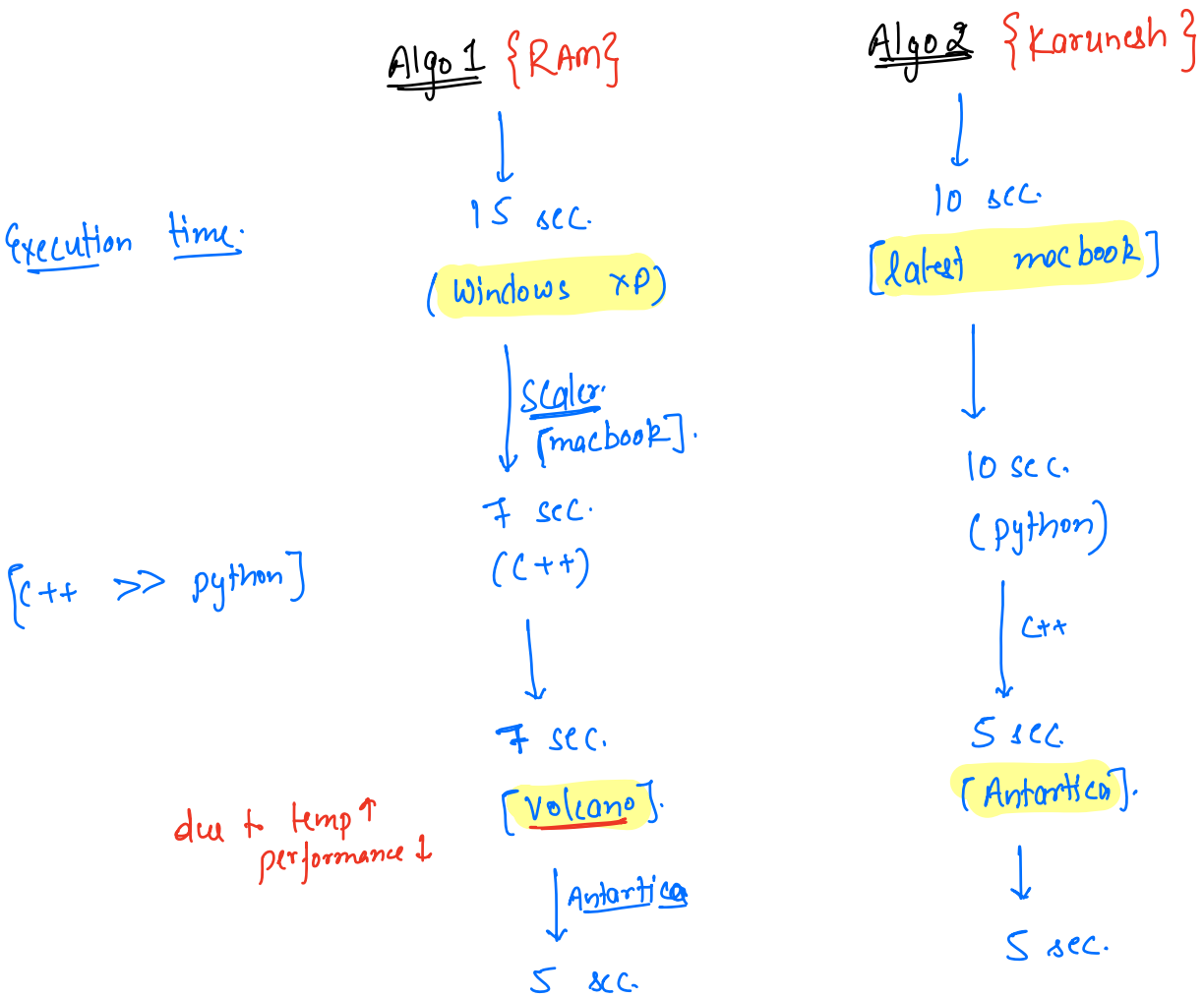
→ Why Big-O needed.
  a) Why lower order terms neglected
  b) Why constant coefficients neglected
  c) Issues in Big-O
  d) Worst case.

→ Space Complexity

→ T.L.E
  a) Why TLE occurs & info about online editors.
  b) How to approach any given problem.
  c) Importance of constraints.

Task: Given N elements. Sort them in increasing order.

- arr = { 3 2 7 5 1 11 } ⇒ { 1 2 3 5 7 11 }
- input size [N] → 10^4



Execution time → It depends on so many external factors → hence we can't compare performance of 2 algo's on the basis of execution time.

iterations

```
for ( i = 1 ; i <=N ; i++) {        [iterations : N]
    print(i)
}
```

[Conclusion 1: To compare two algorithms, calculate their no. of iterations, based on input size.]

Algo 1. {Venkat}
$100 \log_2 N$

Algo 2. {Dhiraj}
$N/10$

till $N < 3500$ — Dhiraj's Algo will perform better.

$N > 3500$ — Venkat's Algo will perform better

Ind vs Aus : 1 CR

Google results : millions of results.

Baby shark : 11 Billion.

∴ Pick an algo, that will perform better for very large inputs.

⇒ In real world, data is always increasing.

Can we really draw 1000's of graph ? ⇒ No.

In order to make this comparison simpler

Asymptotic Analysis of Algorithms

↳ Analysis of performance of algorithms for very inputs.

↳ Big-O Notation.

Algo-1. {Venkat}

$100 \log_2 N$

Algo-2. {Dhiraj}

$N/10$

Big-O. $O(\log_2 N)$ is better than $O(N)$

Biggest Advantage of using Big-O

We can directly tell which algorithm is better just by looking at Big-O notation.

[No need to draw graphs].

Steps to calculate Big-O

1. Calculate no. of iterations.
2. Neglect lower order terms?
3. Neglect constant / co-efficients. ?

Neglect lower order terms ?

Q) iterations → $N^2 + 10N$

i/p size	total iterations	% of lower order terms in total iterations
$N = 10$	200	$\frac{100}{200} \times 100\% = 50\%$
$N = 100$	$10^4 + 10^3$	$\frac{10^3}{10^4+10^3} \times 100\% \approx 9\%$
$N = 1000$	$10^6 + 10^4$	$\frac{10^4}{10^6+10^4} \times 100\% \approx 1\%$
$N = 10^4$	$(10^4)^2 + 10 \cdot 10^4$ $\Rightarrow 10^8 + 10^5$	$\frac{10^5}{10^8+10^5} \times 100\% \approx 0.1\%$

[Observation : As input size ↑s, contribution of lower order terms ↓es.]

Neglect constant co-efficients ?

Q:

Algo 1 (Manas)	Algo 2 (Vaibhav)	for very large i/p.
$10 \log_2 N$	N	Manas
$100 \log_2 N$	N	Manas
$1000 \log_2 N$	$N/10$	Manas
$10N$	$N^2/10$	Manas.
$N \log N$	$100N$	Vaibhav.

{Growth rate of $N^2 >>> N$}

[* Growth rate :].

Issues in Big-O

Q)

	Algo 1.	Algo 2.
	$10^3 N$	N^2
Big-O.	$O(N)$	$O(N^2)$

claim 1. : For every input, algo 1 will perform better. : false.

$N = 10$	10^4	10^2	{Algo 2 is better}
$N = 100$	10^5	10^4	{Algo 2 is " }
$N = 1000$	10^6	10^6	{same}
$N = 10^3 + 1$	$10^3(10^3+1)$	$(10^3+1)(10^3+1)$	{Algo. 1 is better}

claim 2. : For every input $> 10^3$, algo. 1 will perform better.

Final claim: When we compare given 2-algorithms using Big-O notations, algo. 1 will perform better for all the values
> 1000.
> certain value.

⇓

Threshold value.

→ After this threshold value, Big-O holds.
→ Don't worry about threshold value.

Issues in Big-O

Q.)

	Algo-1	Algo-2	
	$2N^2 + 4N$	$3N^2$	{Actually, algo 1 will perform better}
Bigo :	$O(N^2)$	$O(N^2)$	{Both are same according to Big-O}

$\boxed{2N^2} + 4N$ $\quad$ $\boxed{2N^2} + N^2$ $\quad$ growth rate of $N^2 >> N$

Q.)

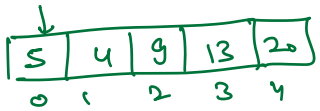
	$5N^3 + 6N^2$	$4N^3 + 10N$	{Actually, algo 2 will perform better}
Big O :	$O(N^3)$	$O(N^3)$	{Both are same according to Big-O}

Observation : If two algo's are having same Big-O notation, then we can't really compare them using Big-O notation.

{ code → iterations → Big-O }

Code: Searching for an element = k

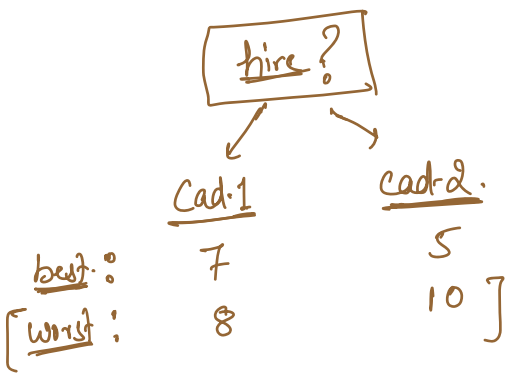
```
boolean search ( int [] arr , int K){
        int n = arr.length ;
        for ( int i = 0 ; i < n ; i++){
            if (arr[i] == K ) return true;
        }
        return false ;
}
```



iterations.

best-case. → 1

worst-case → N



Code. → Time Complexity.

Code. → Space Complexity.

int → 4 B

long → 8 B

Space Complexity

① 4B

```
void fun (int N){
    4B : int x = N;
    4B : int y = x * x;
    8B : long z = x + y;
}
```

total memory :

4B + 4B + 4B + 8B

= 20 B

② 4B

```
void fun (int N){
    4B : int x = N;
    4B : int y = x * x;
    8B : long z = x + y;
    4N B : int[] arr = new int [N];
}
```

total memory :

(20 + 4N) B

S.C → O(N)

③ 4B

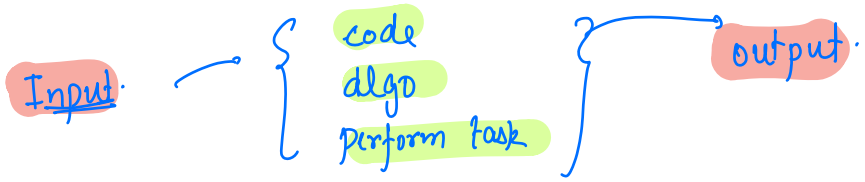
```
void fun (int N){
    4B ✓ : int x = N;
    4B ✓ : int y = x * x;
    8B ✓ : long z = x + y;
    4N ✓ B : int[] arr = new int [N];
    8N² ✓ B : long [][] l = new long [N][N];
}
```

total memory :

4B + 4B + 4B + 8B +

4N B + 8N² B

S.C → O(N²)



Space Complexity Of An Algorithm :

It is amount of extra space that is needed by your algorithm, other than input and output space.

Q) Max in an array ?

```
int maxInArray(int[] arr) {        // i/p.
    int ans = arr[0];
    for(int i = 1; i < arr.length; i++){
        ans = max(ans, arr[i])
    }
    return ans;                    // o/p.
}
```

T.C → O(N)
S.C → O(1) → constant

D.S.A → 600 problems → T.C and S.C

Q:)

```
int task ( int[] arr, int n){
    int[] pf = new int [n]; //4N
      pf [0] = arr[0];
    for( int i= 1 ; i < n ; i++){
          pf[i] = pf[i-1] + arr[i];
    }
    // few more calculations
}
```

i/p i/p

S.C → O(N)

T.C → O(N)

[In most of the cases, first reduce the T.C]

Google → 0.1 sec.

⇓

[5 sec.] ⇒ {great losses.}

Microsoft → 0.01 sec

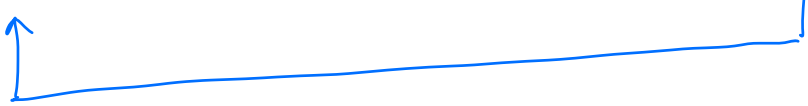
⇓

0.05 sec.

TLE (Time Limit Exceeded)

Shilpa → {Google} → {Hiring challenge} → 2 Qs → duration 1 hr.

Q) idea → code → submit → Time Limit Exceeded.



Remarkable idea:

Without even writing a single piece of code, we can tell whether this approach is going to work or not.

Online Editor

Codes are executed on their code server.

⇓

Processing speed = 1 GHz.

$= 10^9$ instructions/sec.

⇓

⇒ Code should be executed in 1 sec

- → variable declaration
- → add/sub/ mult
- → function calling
- → comparing

observation: Our code can have at max 10^8 instructions.

pseudo-code.

```
int countfactors ( int N){
int c = 0;
for( int i = 1 ; i <= N ; i++){
    if( N % i == 0) { c++ }
}
return c;
}
```

iterations : N

instructions ≈ 6N

Approx. 1.

In our code, 1 iteration contains 10 instructions.

Our code can have at max = 10^9 instructions.

= 10×10^8 instructions.

[Max iterations possible = 10^8 iterations.]

$10^8 \times 10$ instructions.

10^8 iterations.

Approx: [on an avg. code is having 50-60 lines]

In our code, 1 iteration contains 100 instructions.

Our code can have at max = 10^9 instructions.

= $10^7 \times 10^2$ instructions.

[Max iterations possible = 10^7 iterations]

[In general, code iterations $\approx$ [$10^7 \sim 10^8$]]

Assumption → [Execution time for 10^8 iterations = 1 sec]

Structure to solve a question

Read Question
↓
Understand Question
↓
logic
↓
check correctness of logic
↓
code.

constraints [Most useless info]

$1 <= N <= 10^5$

idea → pseudo-code in your think

- 1 loop
- nested-loop
- sorting.

idea → T.C → $O(N^2)$

⇒ 10^{10} iterations.

Not going to work.

⇓

[think of the optimized approach]

Eg: $1 <= N <= 10^3$

idea → pseudo-code → T.C → $O(N^2)$

largest value, $N = 10^3$

iterations = $(10^3)^2 = 10^6$ iterations;

⇓

[Directly go with the approach that you are thinking.]

Eg: $1 \le N \le 10^4$

idea → pseudo code → T.C → $O(N^2)$

$N = 10^4$, no. of iterations = $(10^4)^2$

$= 10^8$ iterations.

{hit / miss}

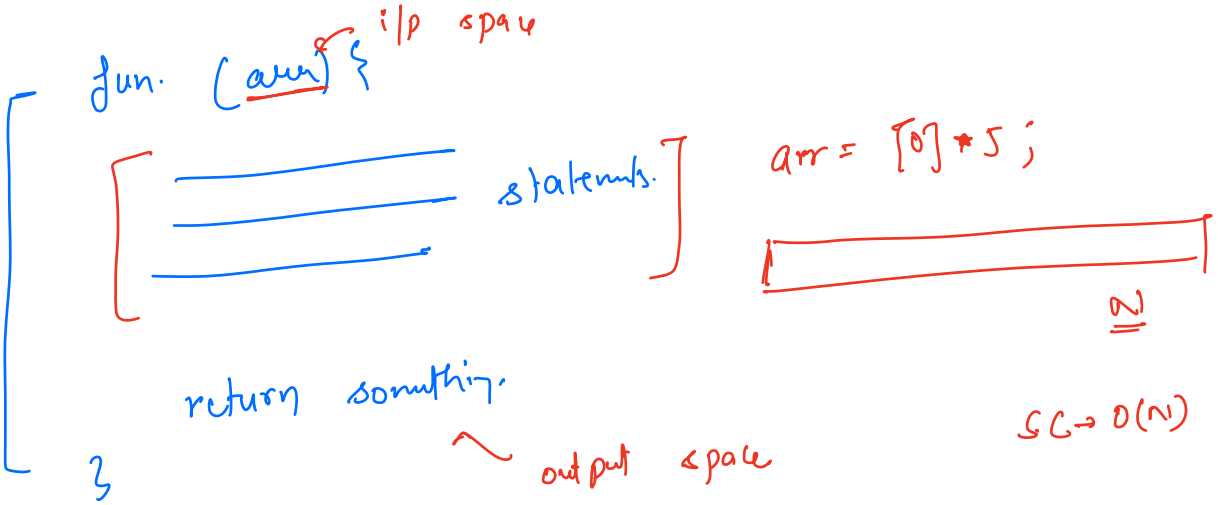
⇓

{think of optimising the code}

→ modular arithmetic.

	Algo1	Algo2
best-case	5	5000
avg-case	10,000	11,000
worst-case	10^7	10^5 iterations.

→ Big-O.



$5n^3 + 6n^2$ $\quad$ $4n^3 + 10n^2$

$\Downarrow$

$4n^3 + n^3 + 6n^2$ $\quad$ $4n^3 + 10n^2$

[growth rate of n^3 >> n^2]

Doubts :

table.

```
for( i=n ; i>0 ; i/=2){
    if( i%2 == 0){
        for(j=1 ; j<=n+n ; j+=2){
        }
    }
}
```

i	j	no. of iterations
N	$[1, N^2]$	$\frac{N^2+1}{2}$
$N/2$	$[1, N^2]$	$N^2/2$
$N/4$	$[1, N^2]$	$N^2/2$
1		1
1		$N^2/2$

$j=1 ; j<=n ; j+=2$

$\Downarrow$

$\frac{n+1}{2}$

T.C → $\boxed{\frac{N^2}{2} \cdot \log_2 N}$

```
for( i=1 ; j=1 ; j<=n ; i++){
    print(i+j);
    if( i%n == 0){
        j++;
    }
}
```

j	i	iterations
1	$[1, n]$	n
2	$[n+1, 2n]$	n
1		1
1		1
1		1
n		n

no. of iterations = n^2

i : 1 2 3 4 —— N N+1 N+2 N+3 —— (N+N) (2N)

j : 1 1 1 1 -- 1 2 2 2 -- .

1. 2.

Assymptotic notations. ⇒ [Θ → but can /avg. case
Ω →]

```
int j = 0
for( int i = 0; i < n ; i++){
    while ( j <= i ){
        print(—)
        j++;
    }
}
```

```
for( i = 0; i < n ; i++){
    for ( j = 0 ; j <= i ; j++){
        print —
    }
}
```

table.

i	j	iteration.
0	[0,0]	1
1	[1,1]	1 + 1
2	[2,2]	1
!		!
N	[N,N]	1 + 1

⇒ [total no. of iteration = N]

Q3

[j = 1]

$\{1 \le N \le 10^9\}$

$O(N^2) \Rightarrow$ iteration $= 10^{18}$

$\Downarrow$

{T.L.E}.

$\{1 \le N \le 30\}$

T.C $\rightarrow O(2^N)$

$= 2^{30} \approx 10^9$

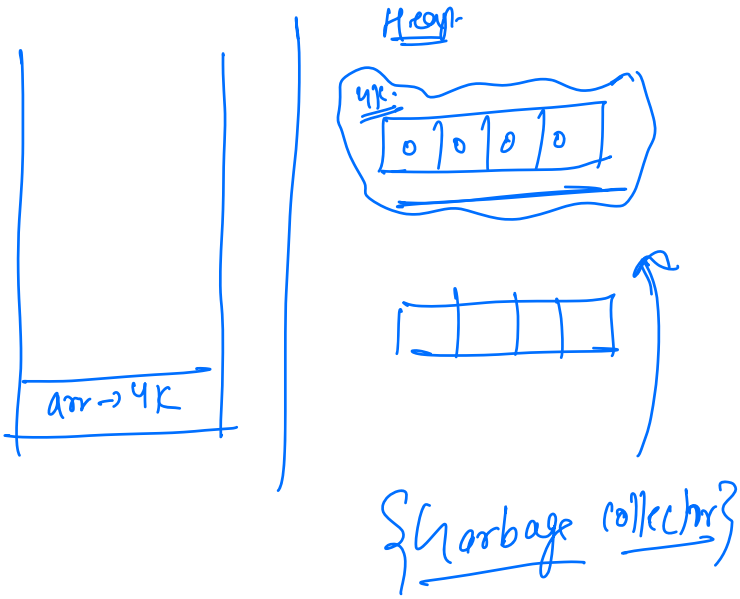
```
for( i = 1 ; i <= 100 ; i++){
    =
}
```

$i \rightarrow [1, 100]$

T.C $\rightarrow O(1)$

```
for( i = 1 ; i <= n ; i++) {
    int[] arr = new int[N];
}
```

N



{Garbage collector}

At max → N space

S.C → O(N)

```
i = 1;
while ( i <= N){
    print ( name)
}
```

intermediate

⇓

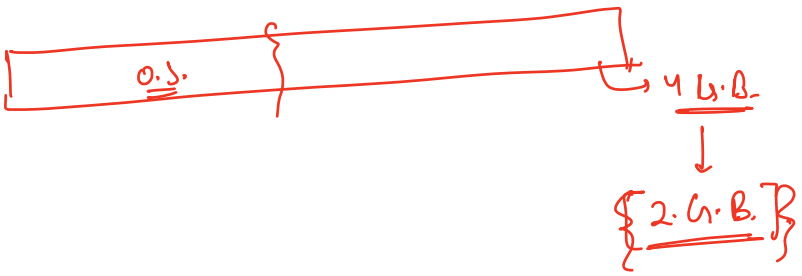
2 months

Advanced

⇓

4.5 | 5 months.

→ {4 months}.



1 int → 4 Bytes.

$(n * 4)$ bytes $= 2 \times 10^9$.

$$n = \frac{2 \times 10^9}{4 \times 2} = 5 \times 10^8.$$