Today's Quote →

> **The man who asks a question is a fool for a minute, the man who does not ask is a fool for life.**

{ Pen and Paper }

Content

no. of iterations

- → Time and Space Complexity
- → Assymptotic Analysis
- → Big-O Notation
- → T.L.E → Time Limited Exceeded.

TC-2.

① Sum of first N natural numbers $= \frac{N(N+1)}{2}$.

② How many no's are there in this range $[3, 10]$.

↳ 8.

$[a, b] = b - a + 1$

$[3, 10] \Rightarrow 10 - 3 + 1 = 8$

$[a, b) = b - a$

$(a, b) = b - a - 1$

{ [→ inclusive
(→ exclusive }

(3) How many times do we need to divide N by 2 to reduce it to 1?

$N = 64 \longrightarrow 32 \longrightarrow 16 \longrightarrow 8 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1$

$N = 64 = 2^6$ No. of steps $\to$ 6

$N = 1024 = 2^{10}$ No. of steps $\to$ 10

[$\log_2 N$ times]

(4) $$\boxed{\log_a a^x = x}$$

$\log_2 2^{10} = 10$

$\log_3 3^5 = 5.$

Arithmetic Progression

$\Rightarrow$ difference b/w any two consecutive terms is fixed/constant

$$4, \underbrace{7}_{3}, \underbrace{10}_{3}, \underbrace{13}_{3}, \underbrace{16}_{3}, \underbrace{19}_{3}, \ldots$$

$d = $ 3

$a \quad a+d \quad a+2d \quad a+3d \quad a+4d \quad \ldots \quad a+(n-1)d$

first term $= a$

common diff $= d$

[Sum of N terms of an A.P $= \frac{n}{2}\left[2a + (n-1)d\right]$]

Geometric Progression (G.P)

ratio of 2 consecutive terms → constant.

3, 6, 12, 24, 48, 96, ---

Common ratio = 2 2 2 2 2
(r)

$a, ar, ar^2, ar^3, ar^4, \text{------} ar^{n-1}$

first term = a
Common ratio = r

$$\text{Sum of N terms of G.P} = \frac{a[r^n - 1]}{[r-1]} \quad r > 1$$

$$= \frac{a[1 - r^n]}{[1-r]} \quad r < 1$$

→ {class-10 NCERT}

Q1.) void fun(int N){

```
    s = 0;
    for( i = 1 ; i <= N ; i++){
        s = s+i;
    }
    return s;
}
```

i : [1, N] $\Rightarrow N - \cancel{1} + \cancel{1} = N$ iterations.

Q2.) int fun(int N){

```
    s = 0;
    for (i = 1 ; i <= N ; i = i + 2){
        s = s + i ;
    }
}
```

iterations.

N = 12   i → 1, 3, 5, 7, 9, 11 $\frac{12}{2} \Rightarrow [6]$

N = 13   i → 1, 3, 5, 7, 9, 11, 13 $7 = \frac{(13+1)}{2} = \frac{14}{2}$

$\frac{(n+1)}{2}$

no. of iterations $= \frac{(n+1)}{2}$.

Q3.)
```
void func (int N, int M){
    for( i=1 ; i<=N ; i++){
        if (i%2 ==0){
            print(i)
        }
    }
    for ( i=1 ; i<=M; i++) {
        if ( i%2 ==0){
            print(i)
        }
    }
}
```
: $i \rightarrow [1, N]$
$= N$

: $i \rightarrow [1, M]$
$= M$

total no. of iterations = N+M.

Q4.)
```
int fun (int N){
    s=0;
    for (i=0 ; i<=100 ; i++){
        s=s+i+i^2;
    }
    return s;
}
```
: $i \rightarrow [0, 100]$

$100 - 0 + 1 = 101$

no. of iterations = 101

Q5) void fun(N){

```
void fun(N){
    for( i=1 ; i*i <= N ; i++){
        s = 1+i2 ;
    }
    return s ;
}
```

$i * i <= N$

$i^2 <= N$

$i <= \sqrt{N}$

$i : [1, \sqrt{N}]$

no. of iterations = $\sqrt{n}$.

Q6)

```
void fun( int N){
    i = N ;
    while ( i > 1){
        i = i / 2;
    }
}
```

iterations	value of i
1	$N/2^1$
2	$N/2^2$
3	$N/2^3$
4	$N/2^4$
5	$N/2^5$
⋮	
	1 .

$$\frac{N}{2^0} \rightarrow \frac{N}{2^1} \rightarrow N/2^2 \rightarrow N/2^3 \rightarrow N/2^4 \rightarrow N/2^5 \rightarrow ----1$$

if $i == 1$, loop will break.

After K iteration, loop will break .

$$\Rightarrow \quad i = \boxed{1 = \frac{N}{2^K}}$$

$$2^k = N$$

$$\log_2 2^k = \log_2 N$$

$$\Rightarrow \boxed{k = \log_2 N}$$

\# no. of iterations $= \log_2 N$

Q)

```
void fun(int N){
    i = N;
    while (i > 1){
        i = i/3;
    }
}
```

$$N \to N/3 \to \frac{N}{9} \to N/27 \to \frac{N}{81} \to - - -- 1.$$

After k iterations, loop will break.

$$1 = \frac{N}{3^k}$$

$$3^k = N$$

$$\log_3 3^k = \log_3 N \Rightarrow \boxed{k = \log_3 N}$$

Amazon MCQ → Multiple choice question

Q7)
```
void fun(int N){
    s=0;
    for(i=0; i<N; i=i*2){
        s=s+i;
    }
}
```

$0 \to 0 \to 0 \to 0 \to 0$

no. of iterations → infinite.

Q8)
```
void fun(N){
    s=0;
    for(i=1; i<N; i=i*2){
        s=s+i;
    }
}
```

iterations	value of i
1	2
2	2^2
3	2^3
4	2^4
5	2^5
⋮	⋮

$1 \to 2 \to 4 \to 8 \to 16 \to$

if i == N, loop will break.

After k iterations, loop will break.

$$N = 2^k$$

$$\log_2 N = \log_2 2^k \Rightarrow \boxed{\log_2 N = k}$$

no. of iterations = $\log_2 n$.

Nested loops

Q9.)

```
void fun (int N){
    for( i = 1 ; i <= 10 ; i++){
        for( j = 1 ; j <= N ; j++){
            print( -- )
        }
    }
}
```

table:

value of i	j	iterations
1	[1, N]	N +
2	[1, N]	N +
3	[1, N]	N +
⋮		
10	[1, N]	N

total no. of iteration = 10N

Q10.)

```
void fun ( int N){
    for( i = 1 ; i <= N ; i++){
        for( j = 1 ; j <= N ; j++){
            print( i * j );
        }
    }
}
```

table

i	j	iterations
1	[1, N]	N +
2	[1, N]	N +
3	[1, N]	N +
⋮		⋮ +
N	[1, N]	N

no. of iterations = $N * N = N^2$.

Q11)

```
void fun( int N){
    for( i=1 ; i <= N ; i++){
        for( j=1 ; j <=i ; j++){
            print(i*j);
        }
    }
}
```

i	j	iterations
1	[1, 1]	1 +
2	[1, 2]	2 +
3	[1, 3]	3 +
⋮		⋮ +
N	[1, N]	N

total no. of iterations $= \frac{N(N+1)}{2}$.

Q12.)

```
void fun( int N){
    for( i=1 ; i <=N ; i++){
        for( j=1 ; j<N ; j=j*2){
            print( i*j);
        }
    }
}
```

i	j	iterations
1	[1, N)	$\log_2 N$ +
2	[1, N)	$\log_2 N$ +
3	[1, N)	$\log_2 N$ +
⋮	⋮	⋮ +
N	[1, n)	$\log_2 N$

total no. of iterations $= \log_2 N * N$

$= N \log_2 N$.

Q14)

```
void fun( int N) {
    for (i=1 ; i <= N ; i++) {
        for( j=1 ; j <= 2^i ; j++) {
            print(i*j);
        }
    }
}
```

i	j	iterations
1	$[1, 2]$	2 +
2	$[1, 2^2]$	$4 = 2^2$ +
3	$[1, 2^3]$	$8 = 2^3$ +
⋮		⋮ +
N	$[1, 2^N]$	2^N

total no. of iterations = $2^1 + 2^2 + 2^3 + \text{———} 2^N$

$$= \frac{2[2^N - 1]}{[2-1]} = \underline{\underline{2[2^N - 1]}}$$

$$\frac{a \cdot [r^n - 1]}{[r-1]}$$

Q:

```
for( i= N ; i > 0 ; i = i/2){
    for( j=1 ; j <= i ; j++){
        print(i*j)
    }
}
```

table

i	j	iterations
N	[1, N]	$N/2^0$ +
N/2	[1, N/2]	$N/2^1$ +
N/4	[1, N/4]	$N/2^2$ +
⋮		$N/2^3$
		⋮
		$\frac{N}{2^{\log_2 N}}$

$1 + \log_2 N \Leftarrow$

total no. of iterations $= N + \frac{N}{2^1} + \frac{N}{2^2} + \frac{N}{2^3} + \text{---} \frac{N}{2^{\log_2 N}}$

$$= N + N \underbrace{\left[\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \text{---} \frac{1}{2^{\log_2 N}} \right]}_{S = \frac{N-1}{N}}$$

$$= N + \cancel{N}\left[\frac{N-1}{\cancel{N}}\right] = \boxed{2N-1}$$

$$S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \text{---} \frac{1}{2^{\log_2 N}}$$

$$S = \frac{\cancel{\frac{1}{2}}\left[1 - \left(\frac{1}{2}\right)^{\log_2 N}\right]}{\cancel{\left(1 - \frac{1}{2}\right)}}$$

$r < 1$

$$\frac{a[1-r^n]}{[1-r]}$$

[$a = 1/2$]

[$r = 1/2$]

[term = $\log_2 N$]

$$S = 1 - \frac{1}{2^{\log_2 N}} = 1 - \frac{1}{N} = \frac{N-1}{N}.$$

$$\Rightarrow \boxed{a^{\log_a x} = x} \Rightarrow \text{Property of log}$$

$$2^{\log_2 N} \Rightarrow N^{\log_2 2} = N^1 = N$$

Comparison of terms- { large value of N }

$$N\,[\log_2 N] < [\sqrt{n}] * N$$

$$N\,[\sqrt{n}] < [n] * N$$

$$\{ \log_2 N < \sqrt{n} < n < N \log N < N\sqrt{N} < N^2 < 2^N \}$$

$N \to 2^{10}$

$$\log_2 2^{10} \Downarrow 10 \qquad \sqrt{2^{10}} \Downarrow 2^5 = 32.$$

$$\log N < \sqrt{N}.$$

$$N^2 \quad (10)^2 \Downarrow 100 \qquad 2^N \quad (2^{10}) \quad 1024.$$

How to calculate Big-O?

what?
why?
→ Next class.

1. Calculate no. of iterations
2. Ignore lower order terms.
3. Ignore constant / co-efficients.

$$f(N) = 10N^2 + \underbrace{100N^1 + 10^3 N^0}_{\times}$$

$$= 10N^2$$

$$\Rightarrow O(N^2)$$

Q.) $f(N) = 4N^2 + \underbrace{3N^1 + 10^5 N^0}_{\times} \Rightarrow O(N^2)$

Q.) $f(N) = \underbrace{4N}_{\times} + 3N\log N + \underbrace{10^6}_{\times} \Rightarrow O(N\log N)$

Q.) $f(N) = \underbrace{4N\log N}_{\times} + \underbrace{3}_{\times} N\sqrt{N} + \underbrace{10^6}_{\times} \Rightarrow O(N\sqrt{N})$

{21 Question.}

Assigment / H.W question.

→ table is the ultimate truth.

$\boxed{i = 6}$

```
while (i > 1) {

    i = i / 7

}
```

$\log_7 6$ ~~↗~~

```
for ( i = 1 ; i <= n ; i *= 2) {
    for ( j = 1 ; j <= n ; j++) {
        // —
    }
}
```

→ {send me the code.}

i	j	iterns
1	[1, N]	N ✓
2	[1, N]	+ N ✓
4	[1, N]	+ N ✓
8	[1, N]	+ N ✓
⋮		+ ⋮

$\boxed{N} \mid [1, N] \mid N$

$N * (\log_2 N + 1)$

$\Rightarrow O(N \log N)$

$\underbrace{1, 2, 4, 8, 16, \text{—} \text{—}}_{K\text{-terms.}}$

$$2^{K-1} = N$$

$$\log_2 2^{K-1} = \log_2 N$$

$$\{ K = \log_2 N + 1 \}$$

$\Rightarrow$ Quiz. $\rightarrow$ { class performance. }

$\hookrightarrow$ { interactive }

$\rightarrow$ All classes.