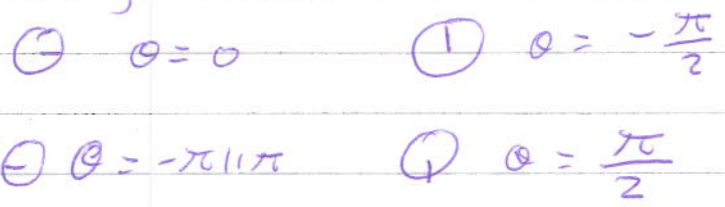
Derivation of Dead Reckoning Equations

Preliminaries

co-ord system

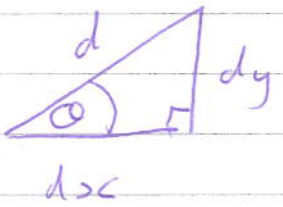
heading



We know

$d = 2\pi r$

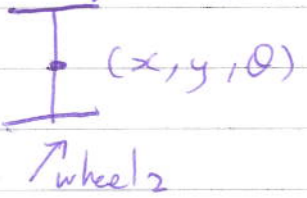
circ = 2π radius



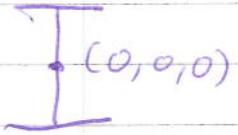
$dx = \cos(\theta)\, d$

$dy = \sin(\theta)\, d$

The robot



Start pos



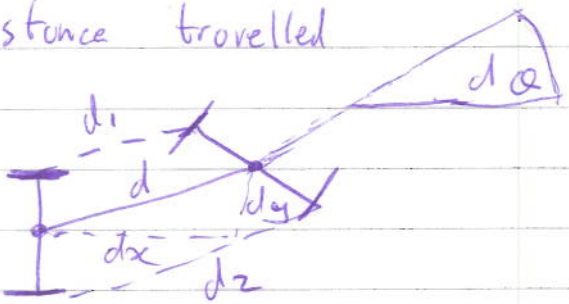
r_w = wheel radius

t_1 = encoder count on wheel 1

t_2 = encoder count on wheel 2

t_R = encoder count for one full rotation of a wheel

Distance travelled



$$d = \frac{d_1 + d_2}{2}$$

time = 0 time = 1

$$d_1 = 2\pi r_w \frac{t_1}{t_r} \qquad d_2 = 2\pi r_w \frac{t_2}{t_r}$$

$$d = \frac{d_1 + d_2}{2} = \frac{2\pi r_w \frac{t_1}{t_r} + 2\pi r_w \frac{t_2}{t_r}}{2}$$

$$d = \frac{2\pi r_w (t_1 + t_2)}{2\, t_r}$$

$$d = \frac{\pi r_w (t_1 + t_2)}{t_r}$$

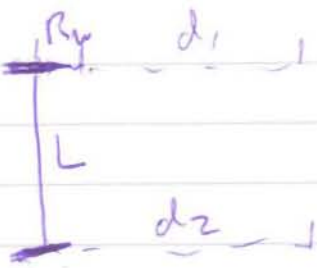
~~$d_{xc} = \cos$~~ $d_{xc} = \cos(\theta)\, d$

$$d_{xc} = \frac{\cos(\theta)\, \pi r_w (t_1 + t_2)}{t_r}$$

$$d_{xc} = \cos(\theta)\,(t_1 + t_2)\,\frac{\pi r_w}{t_r}$$

$$d_y = \sin(\theta)\, d$$

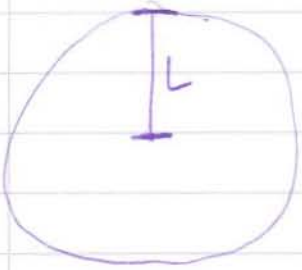
$$d_y = \sin(\theta)\,(t_1 + t_2)\,\frac{\pi r_w}{t_r}$$



$$d_1 = 2\pi r_w \frac{t_1}{t_r}$$

$$d_2 = 2\pi r_w \frac{t_2}{t_r}$$

$$\Delta d = d_2 - d_1 = 2\pi r_w \frac{(t_1 - t_2)}{t_r}$$



For a complete rotation

$$\Delta d = 2\pi L$$

$$\text{As a decimal } [0,1] = \frac{\Delta d}{2\pi L} = \frac{2\pi r_w \frac{(t_1 - t_2)}{t_r}}{2\pi L}$$

Cancel 2π's

$$= \frac{r_w (t_1 - t_2)}{t_r L}$$

extract the constants

$$= (t_1 - t_2) \times \frac{r_w}{t_r L}$$

$$\text{In radians} = 2\pi (t_1 - t_2) \times \frac{r_w}{t_r L}$$

$$\Delta\theta = 2\pi (t_1 - t_2) \times \frac{r_w}{t_r L}$$