Maths Crib Sheet 3 –Conditional Probability, Bayes Theorem

Conditional Probability

$p(A, B)$ Is the joint probability of events A and B, this is the probability of events A and B happening at the same time

$p(A|B)$ Is the conditional probability of A given B, this is the probability the event A will occur given that event B has occurred.

Properties:

$$p(A|B) + p(\neg A|B) = 1$$

$$p(A, B) = p(B\,|A) \times p(A)$$

Which is rearranged to give Bayes law:

$$p(B|A) = \frac{p(A,B)}{p(A)} = \frac{p(A|B) * p(B)}{p(A)}$$

Bayes Law for testing a hypothesis:

H is a hypothesis (something we wish to test the truth of), E is the available evidence, then:

$$p(H\,|E) = \frac{p(E\,|H) \times p(H)}{p(E)}$$

p(E | H) is the likelihood of the data, given the hypothesis.
p(H) is the prior probability of the hypothesis.
p(E) is the prior probability of the evidence (used to normalise the probabilities)
p(H | E) is the posterior probability of the hypothesis – the probability that H is true given the evidence E.

Recursive Form of Bayes Law:

Bayes law can be used to fuse evidence gathered over time, let the evidence at time t be E_t then:

$$p(H|E_t) = \frac{p(E_t\,|H) \times p(H\,|E_{t-1})}{p(E_t\,|H) \times p(H\,|E_{t-1}) + p(E_t\,|\neg H) \times p(\neg H|E_{t-1})}$$