Weekly Exercises Fuzzy Logic

IMAT 1223

Week 1 – Submit Exercises 3, 5 and 7 in Learning Zone

Exercise 1. *For classical sets (also referred to as crisp sets in Fuzzy Set Theory) A and B, prove:*

(a) Commutative law: $A \cup B = B \cup A$; $A \cap B = B \cap A$.

(b) Associative law: $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$.

(c) Distributive law: $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$; $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

(d) De Morgan's law: $\overline{A \cup B} = \overline{A} \cap \overline{B}$; $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Exercise 2. *Establish membership functions for*

(a) Real numbers approximately equal to 6.

(b) Integers far from 10.

(c) Integers very far from 10.

Exercise 3. *A variable is something that takes more than one value. If only one value is possible, then the variable is referred to as a constant variable (or singleton in Fuzzy Set Theory). A linguistic variable is a variable that takes linguistic values or terms, i.e, values the variable is described with are words or combinations of words, like fast, slow, very fast, etc. Describe the linguistic variable "age" referred to a person. Does your modelling of this variable apply to a building or work of art?*

Exercise 4. *Describe the variable "weight" and "height" using five descriptors or linguistic terms. Does your modelling apply to both male and female human beings? Words may mean different things not just when applied to different things but they may mean different things for different people.*

Exercise 5. *For fuzzy sets A and B with MFs:* $\mu_A(x) = \mu_B(x) = \begin{cases} 1 - |x|, & -1 < x < 1; \\ 0; & \textit{otherwise}. \end{cases}$.

(a) Sketch $\mu_{A \cap B}$ for minimum and product t-norms.

(b) Sketch $\mu_{A \cup B}$ for maximum and probabilistic sum (algebraic sum) t-conorms.

Do the same when the Mrs are: $\mu_A(x) = e^{-\frac{1}{2}(x-3)^2}$ *and* $\mu_B(x) = e^{-\frac{1}{2}(x-4)^2}$.

Exercise 6. *Prove that the minimum operator is the strongest t-norms while the maximum operator is the weakest t-conorm.*

Exercise 7. *Prove that $T_L = \max\{x + y - 1, 0\}$ $(\forall x, y \in [0, 1])$ is a t-norm, which is known as the Łukasiewicz t-norm. Determine the Łukasiewicz t-conorm, $S_L = \min\{x + y, 1\}$ using the duality property*

$$S_L(x, y) = 1 - T_L(1 - x, 1 - y).$$