Week 3 – Submit Exercises 15, 17 and 19 in Learning Zone

Exercise 15. *A 'cat lover' has trouble deciding how much food to give to his cats at tea time. They vary in size and he has noticed that they make more noise (miaows) the hungrier they are. A fuzzy expert was called in to help the owner out and after lengthy interviews decided that it is a two input, one output, four rule, problem. The structure is shown in the diagram of Fig. 1.*

1. *Explain how this system could be implemented practically as a type one fuzzy inferencing system (FIS) using the Mamdani model. Your explanation should cover what happens at each stage of the inferencing process and you should use diagrams to illustrate your answer as much as possible.*

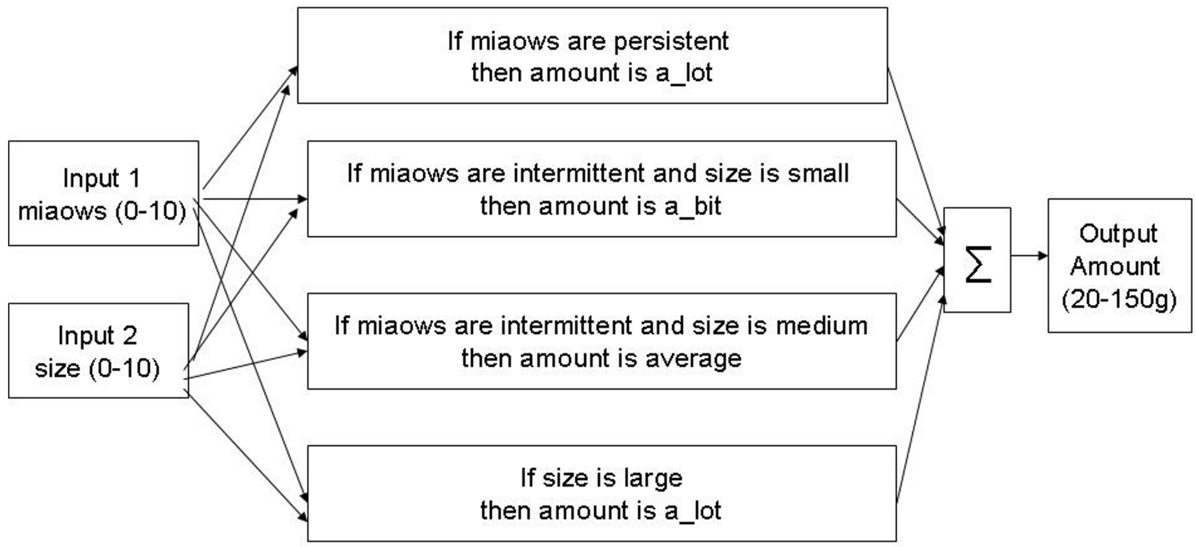

Figure 1: Diagram for 'cat lover' FIS

2. *Discuss some of the issues associated with developing a FIS.*

Exercise 17. *Fuzziness can be introduced at several points in the existing models of decision making. Bellman and Zadeh in 1970 suggested a fuzzy model of decisions that must accommodate certain constraints C and goals G. Suppose we must choose one of four different jobs $a, b, c,$ and d, the salaries of which are given by the function f such that:*

$$f(a) = 30,000, f(b) = 25,000, f(c) = 20,000 \text{ and } f(d) = 15,000.$$

The goal is to choose the job that will give us a high salary given the constraints that the job is interesting and within close driving distance.

- *The first constraint of interest value is represented by the fuzzy set*

$$C_1 = \{(0.4, a), (0.6, b), (0.8, c), (0.6, d)\}.$$

- *The second constraint concerning the driving distance to each job is defined by the fuzzy set*

$$C_2 = \{(0.1, a), (0.9, b), (0.7, c), (1, d)\}.$$

- *The fuzzy goal G of a high salary is defined by the membership function*

$$\mu_G(x) = \begin{cases} 0 & , \text{for} \quad x < 13,000; \\ 1 - 0.00125\left(\frac{x}{1000} - 40\right)^2 & , \text{for} \quad 13,000 \leq x \leq 40,000; \\ 1 & , \text{for} \quad x > 40,000. \end{cases}$$

Which is the best job when applying Bellman and Zadeh's fuzzy decision model?

Mathematical Definition of a Type-2 Fuzzy Set. Let X be a universe of discourse. A fuzzy set A on X is characterised by a membership function $\mu_A : X \to [0,1]$, and it is expressed as follows:

$$A = \{(x, \mu_A(x));\ \mu_A(x) \in [0,1]\, \forall x \in X\}. \tag{6}$$

Alternative representation of a fuzzy set A with continuous and discrete universe of discourses, respectively, are:

$$A = \int_{x \in X} \mu_A(x)/x; \qquad \text{and} \qquad A = \sum_{x \in X} \mu_A(x)/x. \tag{7}$$

Note that the membership grades of A are crisp numbers. This type of fuzzy set is also referred to as a type-1 fuzzy set.

In the following we will use the notation $U = [0,1]$. Let $\tilde{P}(U)$ be the set of fuzzy sets in U. A type-2 fuzzy set $\tilde{A}$ in X is a fuzzy set whose membership grades are themselves fuzzy.

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x));\ \mu_{\tilde{A}}(x) \in \tilde{P}(U)\, \forall x \in X\}. \tag{8}$$

Since $\mu_{\tilde{A}} : X \to \tilde{P}(U)$, it follows that $\forall x \in X\, \exists J_x \subseteq U$ such that $\mu_{\tilde{A}}(x) : J_x \to U$. Applying (6), we have:

$$\mu_{\tilde{A}}(x) = \{(u, \mu_{\tilde{A}}(x)(u));\ \mu_{\tilde{A}}(x)(u) \in U\, \forall u \in J_x \subseteq U\}. \tag{9}$$

J_x is called the primary membership of x and $\mu_{\tilde{A}}(x)$ is called the secondary membership of x. In most practical applications X and U coincide. Putting (8) and (9) together we have

$$\tilde{A} = \{(x, (u, \mu_{\tilde{A}}(x)(u)));\ \mu_{\tilde{A}}(x)(u) \in U,\, \forall x \in X \wedge \forall u \in J_x \subseteq U\}. \tag{10}$$

This is the 'vertical representation' of a type-2 fuzzy set, and it is used to define the concept of an *embedded set* of a type-2 fuzzy set, which is fundamental to the definition of the *centroid* of a type-2 fuzzy set. It is usual to denote the secondary membership of x as follows: $\mu_{\tilde{A}}(x) \equiv \mu_{\tilde{A}(x)}$.

Exercise 19. *Let $\tilde{A}$ and $\tilde{B}$ be type-2 fuzzy sets defined on the same primary domain X. Suppose the secondary membership for a particular element x in these two type-2 fuzzy sets are the following type-1 fuzzy sets:*

$$\mu_{\tilde{A}(x)} = \{(0.5, 0.8), (0.3, 0.7), (0.8, 0.2)\}, \qquad \Big[\textit{elements} \quad \Big(v, \mu_{\tilde{A}(x)}(v)\Big)\Big]$$
$$\mu_{\tilde{B}(x)} = \{(0.2, 0.6), (0.3, 0.4), (0.6, 0.2)\}. \qquad \Big[\textit{elements} \quad \Big(w, \mu_{\tilde{B}(x)}(w)\Big)\Big]$$

An alternative representation of these are

$$\mu_{\tilde{A}(x)} = 0.8/0.5 + 0.7/0.3 + 0.2/0.8,$$
$$\mu_{\tilde{B}(x)} = 0.6/0.2 + 0.4/0.3 + 0.2/0.6.$$

Calculate the secondary membership for x in the type-2 fuzzy sets $\tilde{A} \sqcup \tilde{B}$ and $\tilde{A} \sqcap \tilde{B}$ obtained with the join and meet operations for the union and intersection of $\tilde{A}$ and $\tilde{B}$, i.e. obtain the type-1 fuzzy sets: $\mu_{\tilde{A}\cup\tilde{B}(x)} \equiv \mu_{\tilde{A}(x)} \sqcup \mu_{\tilde{B}(x)}$ and $\mu_{\tilde{A}\cap\tilde{B}(x)} \equiv \mu_{\tilde{A}(x)} \sqcap \mu_{\tilde{B}(x)}$.

$$\textit{Join:} \qquad \mu_{\tilde{A}(x)} \sqcup \mu_{\tilde{B}(x)}(u) = \sup_{\substack{u=\max\{v,w\} \\ v,w\in X}} \min\{\mu_{\tilde{A}(x)}(v), \mu_{\tilde{B}(x)}(w)\}.$$

$$\textit{Meet:} \qquad \mu_{\tilde{A}(x)} \sqcap \mu_{\tilde{B}(x)}(u) = \sup_{\substack{u=\min\{v,w\} \\ v,w\in X}} \min\{\mu_{\tilde{A}(x)}(v), \mu_{\tilde{B}(x)}(w)\}.$$