Fuzzy Sets and Logic
Introduction

MSc IS/ISR

Classical Sets and Logic v Fuzzy Sets and Logic

- Many propositions about the real world are not either true or false, rendering classical logic inadequate for reasoning with such propositions.
- Furthermore, most concepts used in human communication do not have crisp boundaries, rendering classical sets inadequate to represent such concept.
- The main aim of fuzzy logic and fuzzy sets is to overcome the disadvantages of classical logic and classical sets.

Natural Languages and Formal Models

Formal Models

- Classical logic and mathematics assume that we can assign one of the two values, true or false , to each logical proposition or statement.
- If a suitable formal model for a certain problem or task can be specified, conventional mathematics provides powerful tools which help us to solve the problem.
- When we describe such a formal model, we use a terminology which has much more stringent rules than natural language.
- This specification often requires more work and effort, but by using it we can avoid misinterpretations.
- Furthermore, based on such models we can prove or reject hypotheses or derive unknown correlations.

Natural Languages

- However, in our everyday life formal models do not concern the interhuman communication.
- Human beings are able to assimilate easily linguistic information without thinking in any type of formalization of the specific situation.
  - For example, a person will have no problems to accelerate slowly while starting a car, if he is asked to do so.
- If we want to automate this action, it will not be clear at all, how to translate this advice into a well-defined control action.
  - It is necessary to determine a concrete statement based on an unambiguous value, that is, *step on the gas at the velocity of half a centimeter per second*.
  - On the other hand, this kind of information will not be adequate or very helpful for a person.

Natural Languages and Formal Models

- Automated control is usually not based on a linguistic description of heuristic knowledge or knowledge from one's own experience, but it is based on a formal model of the technical or physical system.
  - This method is definitely a suitable approach, especially if there is a good model to be determined.
- However, a completely different technique is to use knowledge formulated in natural language directly for the design of the control strategy.
- In this case, a main problem will be the translation of the verbal description into concrete values:
  - i.e. assigning "step on the gas slowly" into "step on the gas at the velocity of a centimeter per second" as in the above mentioned example

Vagueness

- When describing an object or an action, we usually use uncertain or vague concepts.
- In natural language we hardly ever find exactly defined concepts like supersonic speed for the velocity of a passing airplane.
  - Supersonic speed characterizes an unambiguous set of velocities, because the speed of sound is a fixed entity and therefore it is unambiguously clear whether an airplane flies faster than sound or not.
- Frequently used *vague concepts*, like fast, very big, small and so on, make it impossible to decide unambiguously whether a given value satisfies such a vague concept or not.

Vagueness

- One of the reasons for this is that vague concepts are usually context dependent.
  - fast has a different meaning for airplanes than for cars.
  - Even for cars it is not easy to distinguish clearly between fast and non-fast cars.
  - The difficulty here is not to find a value telling us whether a car (or its top speed) is fast or not, but we had to presuppose that such a value does exist.
  - It is more likely that we will be reluctant to fix such a value because there are velocities we can classify as fast for a car and there are some we can classify as not fast, and in between there is a wide range of velocities which are considered as more or less fast.

Fuzziness versus Uncertainty

- Any notation is said to be fuzzy when its meaning is not fixed by sharp boundaries.
  - This means that the statement can be applied fully, to a certain degree, or not at all.
- The gradual degrees of this membership is also called fuzziness.
- A proposition is fuzzy if it contains gradual predicates.
  - Such propositions may be neither true nor false, but anything in-between, for example, to a certain degree or partially true .
  - Forms of such degrees can be found in natural languages, like very, rather, or almost.

Fuzziness versus Uncertainty

- Why is there a need for vagueness in natural languages?
  - Any language is discrete, but the real world is continuous.
  - To close this gap, we use fuzzy statements.
- Consider the word young for human beings.
  - The more fine-grained the scale of age, for example, going from years to months, weeks, days, etc., the more difficult is to fix the *threshold* below which young fully applies and above which young does not at all.
- So there is a conflict between the linguistic finite term set {young, mature, old} and the numerical representation by a real-valued interval, e.g. [0, 120] years for human beings.
- Fuzzy set theory does not assume any threshold.

Fuzziness versus Uncertainty

- In contrast, uncertainty describes the probability of a well defined proposition.
  - For example, rolling a die will either lead to exactly 6 or not, but not to something around 6.
- Uncertainty also comes from conflicting but precisely observed pieces of information and is usually found in statistics.

Fuzziness versus Uncertainty

- Here are some examples for a better distinction of fuzziness and uncertainty:
  - “This car is about 10 and 15 years old.”
    - This proposition is fuzzy, because we have a lack of information or a lack of ability to measure or evaluate the numerical feature exactly.
  - “This car was probably built in Germany.”
    - This sentence expresses uncertainty about the well-defined proposition made in Germany . Perhaps this uncertainty is coming from statistics like a random experiment.
  - “The car I chose randomly is perhaps very big.”
    - In this example there are both kind of concepts, that is, the uncertain statement perhaps and the fuzzy description very big .

Fuzziness versus Uncertainty

- A good example that demonstrates the conceptual difference between fuzziness and uncertainty was given by James Bezdek.
- In the example, a person who is dying of thirst in the desert is given two bottles of fluid.
  - One bottle's label says that it has a 0.9 membership in the class of fluids known as non-poisonous drinking water.
  - The other bottle's label states that it has a 90% probability of being pure drinking water and a 10% probability of being poison.
  - Which bottle would you choose?
- The "probability bottle" contains poison.
  - This is quite plausible since there was a 1 in 10 chance of it being poisonous
- The "fuzzy bottle" contains swamp water.
  - This also makes sense since swamp water would have a 0.9 membership in the class of non-poisonous fluids

Fuzziness versus Uncertainty

- The point is that
  - probability involves crisp set theory and does not allow for an element to be a partial member in a class.
  - Probability is an indicator of the frequency or likelihood that an element is in a class.
- Fuzzy set theory deals with the similarity of an element to a class.

Fuzzy Set Theory

- The idea of fuzzy sets is to solve this problem by avoiding the sharp separation of conventional sets into two values – complete membership or complete non-membership.
- Instead, fuzzy sets can handle partial membership.
- So, in fuzzy sets we have to determine to what degree or extend an element is a member of this fuzzy set.
- Fuzzy Set Theory Aim – **to provide a systematic framework for dealing with the vagueness and imprecision inherent in human thought processes**

“Close to 1”

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