Type 2 Fuzzy Logic

Fuzzy Logic

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Introduction & Learning Outcomes

In this lesson we will firstly review the following:

- How do we define our fuzzy sets?
- Success of Type-1 fuzzy logic

At the end of this lesson you should have an understanding of the following type 2 concepts:

- Type-2 fuzzy sets
  - Footprint of uncertainty
  - Union
  - Intersection
- Interval valued fuzzy sets
- Embedded fuzzy sets
- Overview
- Fuzzification

These notes are supported by directed reading which has been made available for you.

Defining fuzzy sets

- We need fuzzy sets to build fuzzy systems
- How do we get them?
  - Trial and error?
  - Talking to experts?

How do we go
about doing this?

Defining fuzzy sets - Exemplification

- builds a membership function from a number of samples.
  - To define a membership function ‘tall’ the expert would be asked to
    - describe a number of heights as tall
    - using terms such as true, more or less true, etc.
  - these would be assigned membership values.
  - For instance ‘more or less true’ might be given a value 0.8.
    - E.g. Amongst male heights, how tall is 5ft 8ins (choose one answer)? *‘A very small bit true’, ‘a bit true’, <u>‘more or less true’, </u>‘a lot true’*

Defining fuzzy sets - Direct rating

- Presents randomly selected members of the fuzzy set, where some measure is available for that member.
- The expert is then asked a question e.g. ‘How tall is Michael Jordan?’
  - (so we know his height but we want to know how would the expert describe someone of that height)
- Expert responds by using a sliding scale to indicate the tallness.

Defining fuzzy sets - Polling

- ‘Do you agree that Michael Jordan is tall?’
  - This question is asked of a **number of experts**
  - the **ratio of yes** responses **to the total** responses provides the membership value for Michael Jordan.
- *E.g. if 7 out of 10 think he is tall then we would say he is tall to degree 0.7*

Defining fuzzy sets - Reverse rating

- Give the experts a set of instances – e.g. a list of all our males and their heights, then ask them to do the following:
  - *‘Identify a man whose height indicates that he has the degree 0.5 of membership in the fuzzy set tall’*
- This can be quite a good way of doing the representation, providing the expert has an understanding of the upper and lower limits,

Success of (Type-1) Fuzzy Logic

- Successful Applications in Control
- Small systems work well
- Not delivered very well in non control applications
- Much work to be done in the modeling of human expertise
- Human Expertise is VERY fuzzy

What's Fuzzy about Type-1 Fuzzy Logic?

- Not very much really….
  - Most systems deal with crisp input.
  - This is converted to a number in [0,1]
  - The inferencing is crisp
  - and we 'defuzzify' !

So....

Lets make things more fuzzy...

- How can we go about this in practise?
- Can do it by blurring the membership function

Type-n Fuzzy Sets

We are only interested
In type 2 for now

- A fuzzy set is of type *n*, *n* = 2, 3, . . . if its membership function ranges over fuzzy sets of type *n*-1. The membership function of a fuzzy set of type-1 ranges over the interval [0,1].

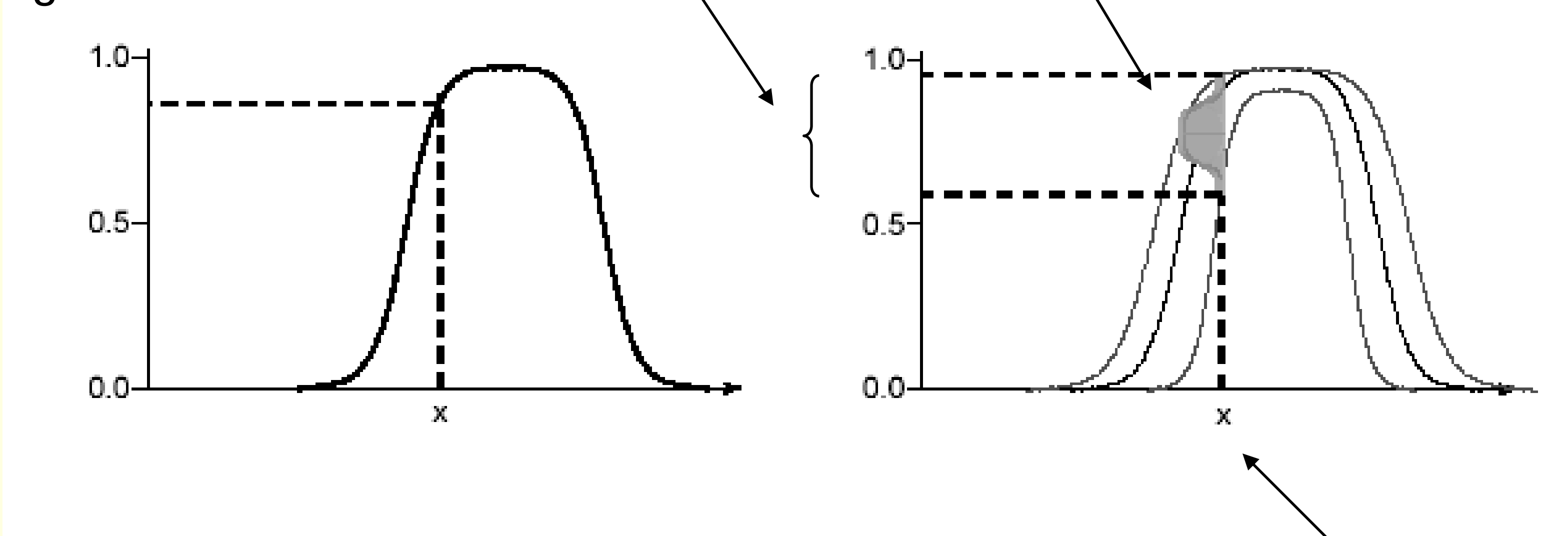
Zadeh, L.A., The Concept of a Linguistic Variable and its Application to Approximate Reasoning - I, *Information Sciences*, 8,199–249, 1975

Type 2 fuzzy sets

- Imagine blurring a type 1 membership function.
- There is no longer a single value for the membership function for any x value, there are a few

Quite tall

e.g. Tallness



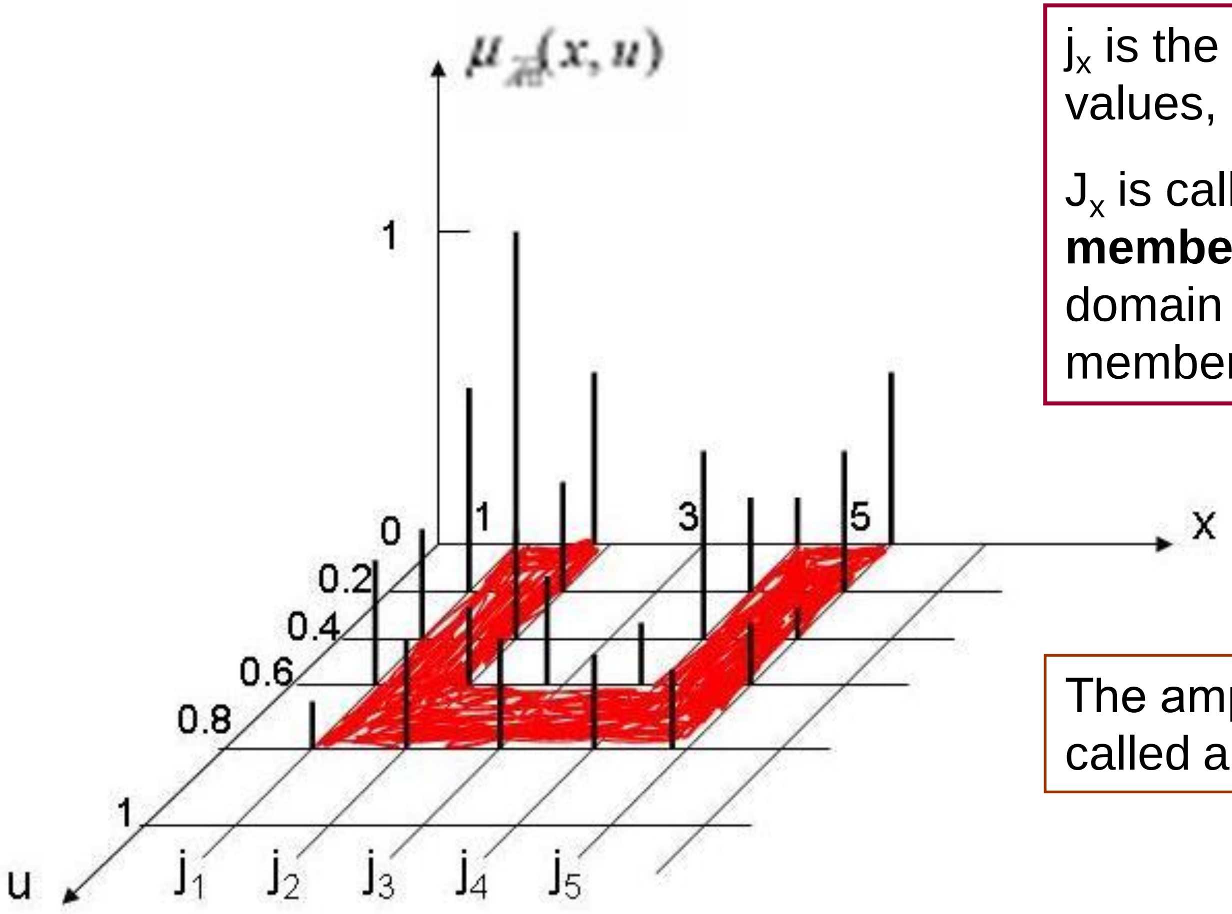
e.g. Joe Bloggs

Type 2 fuzzy sets

- These values need not all be the same
- We can therefore assign an amplitude distribution to all of the points
- Doing this creates a 3-D membership function i.e. a type 2 membership function
- This characterises a fuzzy set

(see diagram on next slide)

Example of a type 2 membership function. The shaded area is called the 'Footprint of Uncertainty' (FOU)



j_x is the set of possible u values, i.e. $j_3 = [0.6, 0.8]$

J_x is called the **primary membership** of x and is the domain of the secondary membership function.

The amplitude of the 'sticks' is called a **secondary grade**

Type-2 Fuzzy Sets

A type-2 fuzzy set, $\tilde{A}$, is characterised by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$

For all x in X

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\}$$

One of the x,u intersections

Its ‘stick’ height

For all u between 0 and 1

Mendel, J.M. and John, R.I., Type-2 Fuzzy Sets Made Simple, IEEE Transactions on Fuzzy Systems,10(2),117–127

Footprint of Uncertainty - FOU

- The FOU is the union of all primary memberships
- It is the region bounded by all of the ‘j’ values i.e. the red shaded region on the earlier slide.
- FOU is useful because:
  - Focuses our attention on uncertainties (blurriness!)
  - Allows us to <u>depict</u> a type 2 fuzzy set graphically in 2 dimensions instead of 3.
  - The shaded FOUs <u>imply</u> the 3rd dimension on top of it.

Appealing

"Type-2 fuzzy sets are fuzzy sets whose grades of membership are themselves fuzzy. They are intuitively appealing because grades of membership can never be obtained precisely in practical situations."

Dubois, D. and Prade, H.,Fuzzy Sets and Systems: Theory and Applications, Academic Press,1980

Comparison with type-1

- Type-1 fuzzy sets are two dimensional
- Type-2 fuzzy sets are three dimensional

- Type one membership grades are in [0,1]
- We can have linguistic grades with type-2

- Type-1 fuzzy systems are computationally cheap
- Type-2 fuzzy systems are computationally expensive (but..)

Union (known as Join for type-2)

The union ($\cup$) of two type-2 fuzzy sets ($\tilde{A}$, $\tilde{B}$) corresponding to $\tilde{A}$ OR $\tilde{B}$ is given by:

$$
\begin{aligned}
A \cup B \Leftrightarrow \mu_{\tilde{A}\cup\tilde{B}}(x) &= \mu_{\tilde{A}}(x) \sqcup \mu_{\tilde{B}}(x) \\
&= \sum_{i,j} (f(u_i) \wedge g(w_j))/(u_i \vee w_j)
\end{aligned}
$$

Their membership grades

The elements – or values of u along a particular primary membership (j_x)

The join between 2 two secondary membership functions must be performed between every possible pair of primary memberships.

See next slide for example

Example – join of type-2 fuzzy sets

- Given two type-2 fuzzy sets $\tilde{A}$ and $\tilde{B}$, and given the following membership functions for the element *x* calculate the union (or join) using the minimum t-norm and maximum t-conorm.

- $\mu_{\tilde{A}}(x) = 0.5/0 + 0.7/0.1$

- $\mu_{\tilde{B}}(x) = 0.3/0.4 + 0.9/0.8$

The elements

Their membership grades

Example – join of type-2 fuzzy sets

So we perform the join operation on all combinations:

$$\mu_{\tilde{A}}(x) = 0.5/0 + 0.7/0.1$$

$$\mu_{\tilde{B}}(x) = 0.3/0.4 + 0.9/0.8$$

Which gives us:

0.3/0.4 + 0.5/0.8 + 0.3/0.4 + 0.7/0.8

=0.3/0.4 + 0.7/0.8

Note – we take the max where
There are 2 elements that are the same
But with different membership
grades

Intersection (known as Meet for type-2)

The intersection ($\cap$) of two type-2 fuzzy sets ($\tilde{A}$, $\tilde{B}$) corresponding to $\tilde{A}$ AND $\tilde{B}$ is given by:

$$\begin{aligned}\tilde{A} \cap \tilde{B} \Leftrightarrow \mu_{\tilde{A}\cap\tilde{B}}(x) &= \mu_{\tilde{A}}(x) \sqcap \mu_{\tilde{B}}(x) \\ &= \sum_{i,j} (f(u_i) \wedge g(w_j))/(u_i \wedge w_j)\end{aligned}$$

The meet between 2 two secondary membership functions must be performed between every possible pair of primary memberships.

See next slide for example

Example – meet of type 2 fuzzy sets

- Given two type-2 fuzzy sets Ã and $\tilde{B}$, and given the following membership functions for the element *x* calculate the intersection (or meet) using the minimum t-norm and maximum t-conorm.

$$\mu_{\tilde{A}}(x) = 0.5/0 + 0.7/0.1$$

$$\mu_{\tilde{B}}(x) = 0.3/0.4 + 0.9/0.8$$

Example – meet of type-2 fuzzy sets

So in the same way we perform the meet operation on all combinations:

$\mu_{\tilde{A}}(x) = 0.5/0 + 0.7/0.1$

$\mu_{\tilde{B}}(x) = 0.3/0.4 + 0.9/0.8$

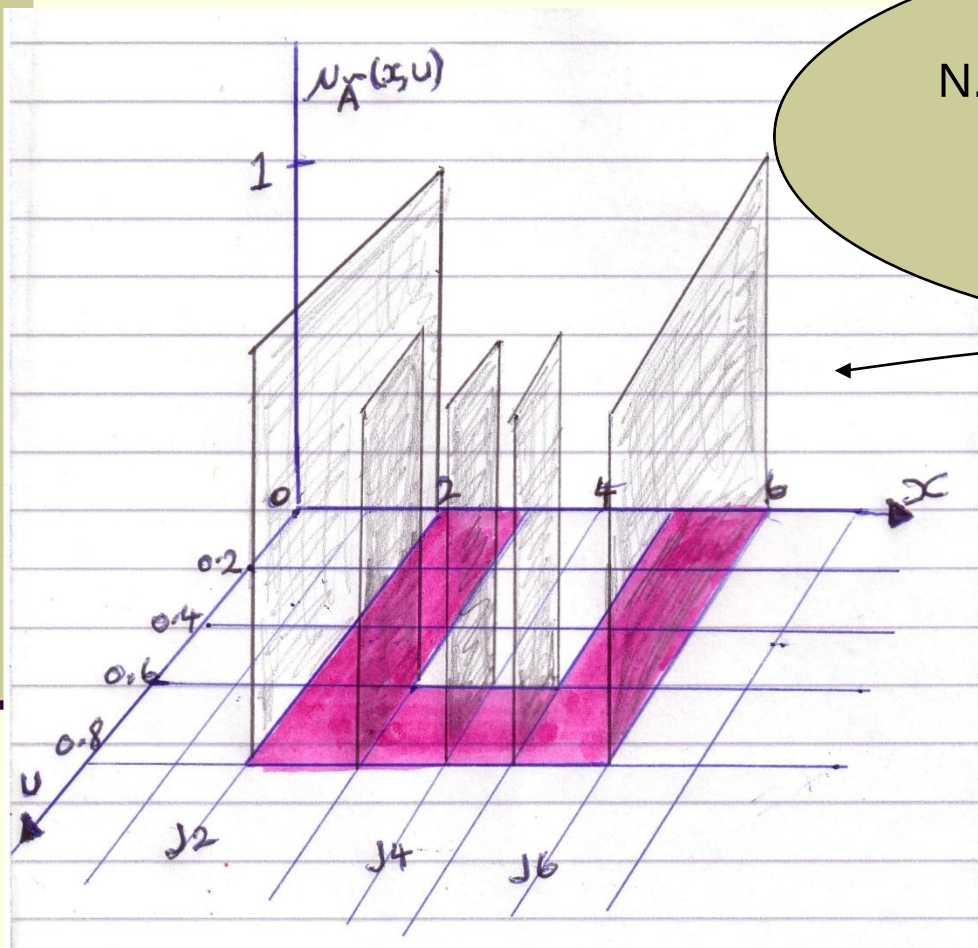
Which gives us:

0.3/0 + 0.5/0 + 0.3/0.1 + 0.7/0.1

=0.5/0 + 0.7/0.1

Note – we take the max where
There are 2 elements that are the same
But with different membership
grades

Interval Valued type-2 fuzzy sets



N.B. these are **intervals** & are not by definition discretised, hence the name.

When the amplitudes of of the secondary membership function all equal 1, we have an interval valued fuzzy set.

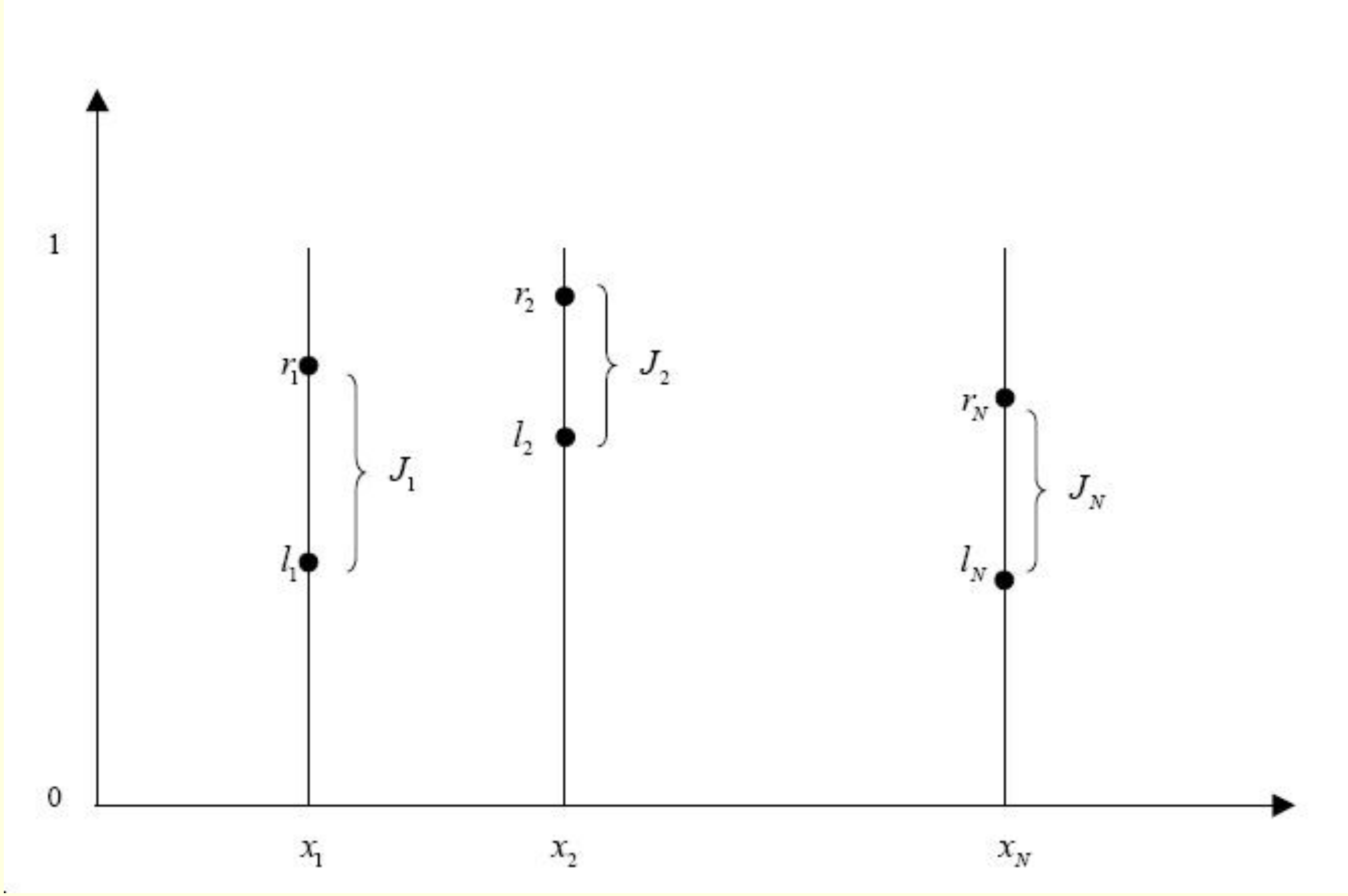
Interval valued type-2 fuzzy sets

An interval valued type-2 fuzzy set (IVFS), $\tilde{A}^{iv}$, is characterised by a type-2 membership function $\mu_{\tilde{A}^{iv}}(x, u)$, where $x \in X$ and $u \in J_x \subseteq [0, 1]$

$$\tilde{A}^{iv} = \{((x, u), 1) | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\}$$

i.e.
$\mu_{\tilde{A}}(x, u) = 1$

Interval valued type-2 fuzzy sets



IVFS in 2-dimensions

Meet and join for IV fuzzy sets

MEET of IVFS

For a given x in $\tilde{F}$ and $\tilde{G}$ the secondary membership function is an interval set over (l_F, r_F) and (l_G, r_G). The secondary membership function of x in $F \sqcap G$ is an interval set over $(l_F \star l_G), (r_F \star r_G)$. where $\star$ is minimum or product.

JOIN of IVFS

For a given x in $\tilde{F}$ and $\tilde{G}$ the secondary membership function is an interval set over (l_F, r_F) and (l_G, r_G). The secondary membership function of x in $F \sqcup G$ is an interval set over $(l_F \vee l_G), (r_F \vee r_G)$. where $\vee$ is maximum.

Exercise

Suppose the interval type-2 membership functions for a particular element x are given by the following:

$$\mu_{\tilde{A}}(x) = 1/0.1 + 1/0.2$$

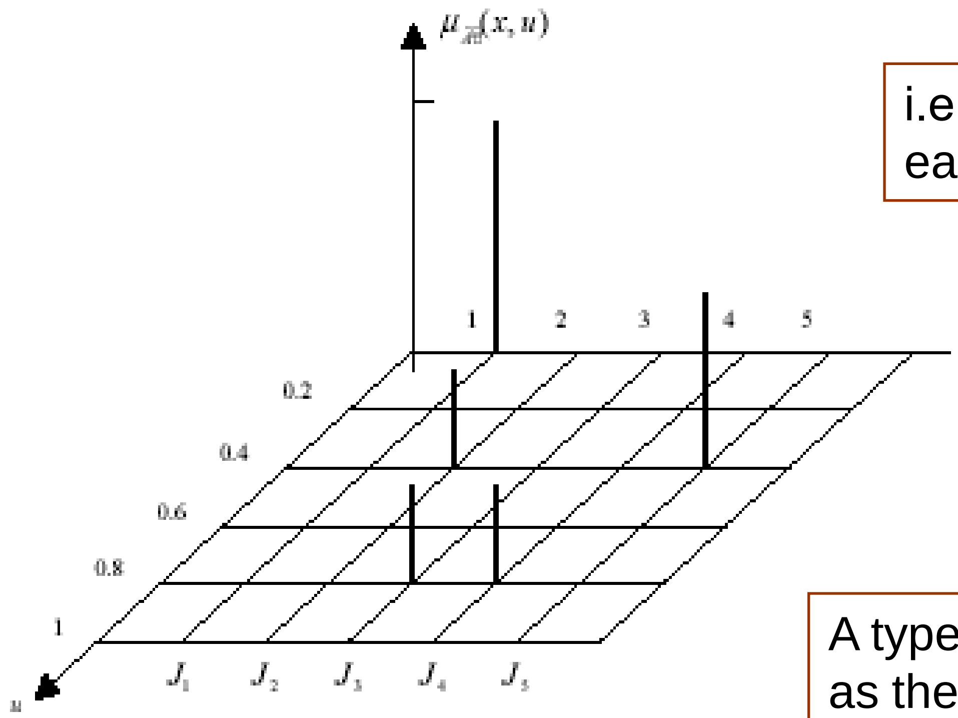
$$\mu_{\tilde{B}}(x) = 1/0.4 + 1/0.8$$

Again – we can't really discretise here but we can take these values as being the upper and lower limits of each interval set

Using min for the t-norm and max for the t-conorm, calculate $\tilde{A} \cap \tilde{B}$ and $\tilde{A} \cup \tilde{B}$ using:

1. the method for calculating the meet and join of any type 2 fuzzy set, and then
2. using the method specifically for interval type-2 sets *(doing this is rather contrived but it does help to illustrate).*

Embedded type-2 fuzzy sets



i.e. we have one ‘stick’ from each primary membership (j line)

A type-2 fuzzy set can be represented as the union of all of its type-2 embedded fuzzy sets.

Embedded type-2 fuzzy sets – more formally

For discrete universes of discourse X and U, an embedded type-2 fuzzy set $\tilde{A}_e$ has N elements, where $\tilde{A}_e$ contains exactly one element from $J_{x_1}, J_{x_2}, \ldots, J_{x_N}$, namely $u_1, u_2, \ldots, u_N$, each with its associated secondary grade $f_{x_i}(u_i)$ $(i = 1, \ldots, N)$, i.e.

$$\tilde{A}_e = \sum_{i=1}^{N} [f_{x_i}(u_i)/u_i]/x_i \qquad u_i \in J_{x_i} \subseteq U = [0, 1]$$

Embedded type-2 fuzzy sets

- So we can say that a type-2 set is made up of the collection (or union) of all of the embedded sets.
- How many of these will there be?
- This idea has been given the name of the Fundamental Decomposition Theorem.

Fundamental Decomposition Theorem

Let $\tilde{A}_e^j$ denote the jth type-2 embedded fuzzy set for type-2 fuzzy set $\tilde{A}$.

$$\tilde{A}_e^j \equiv \{(u_i^j, f_{x_i}(u_i^j)), i = 1, \ldots, N\}$$

where

$$u_i^j \in \{u_{ik}, k = 1, \ldots, M_i\}$$

Here, each u_{ik} is a member of the secondary membership grade for $x_{i.}$ $\tilde{A}$ can be represented as the union of all its type-2 embedded fuzzy sets. That is:

$$\tilde{A} = \sum_{j=1}^{n} \tilde{A}_e^j$$

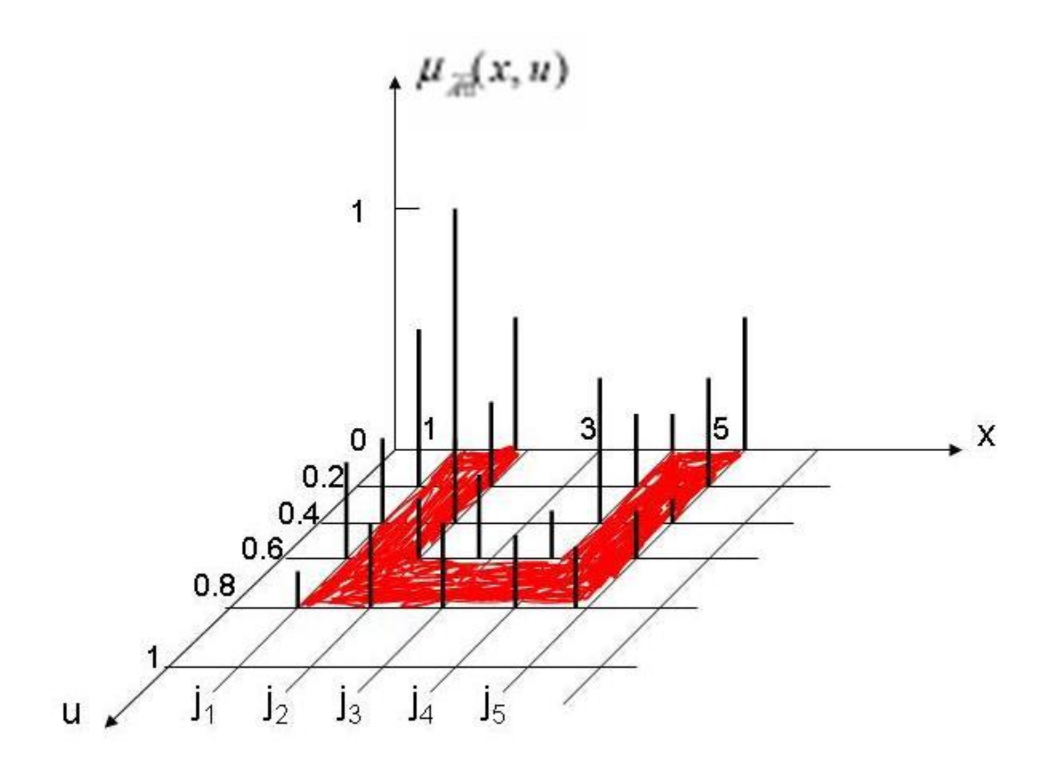
i.e. the union of all the embedded sets

where

$$n \equiv \prod_{i=1}^{N} M_i$$

Indicates how to calculate the number of embedded sets

Calculating the number of embedded sets

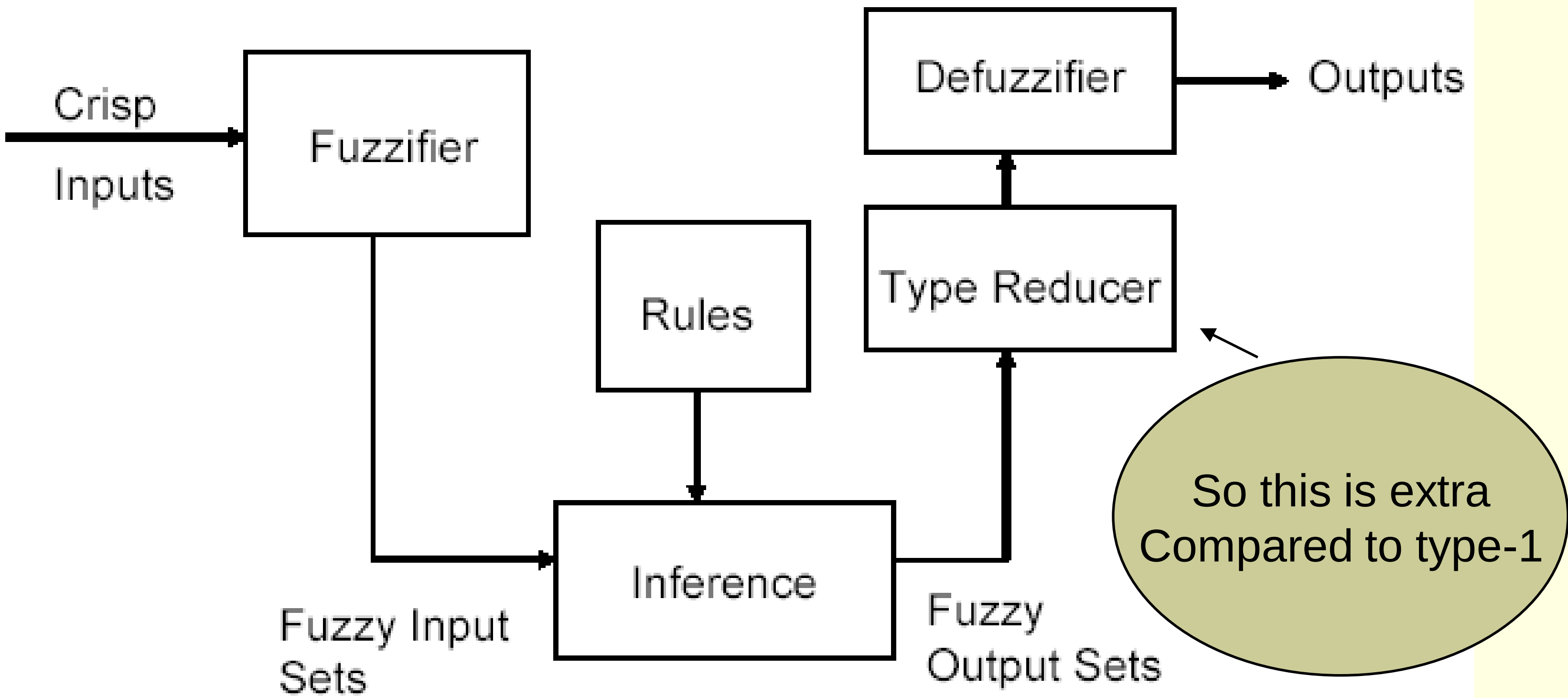


In the above example the number of possible embedded sets ('n' on the previous slide) would be:

5 x 5 x 2 x 5 x 5 = 1250

Type-2 overview

From: Mendel: Uncertain Rule Based Fuzzy Logic Systems



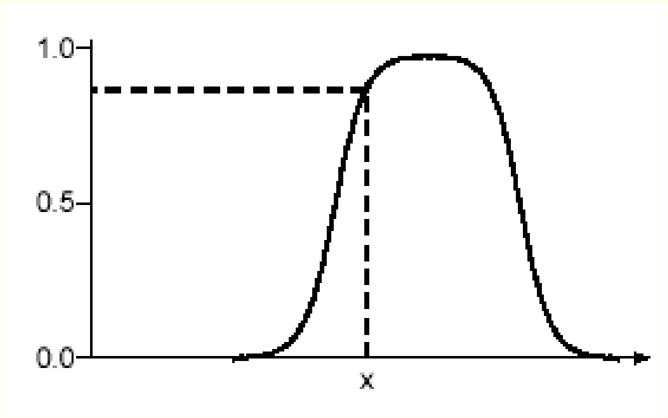
Rules

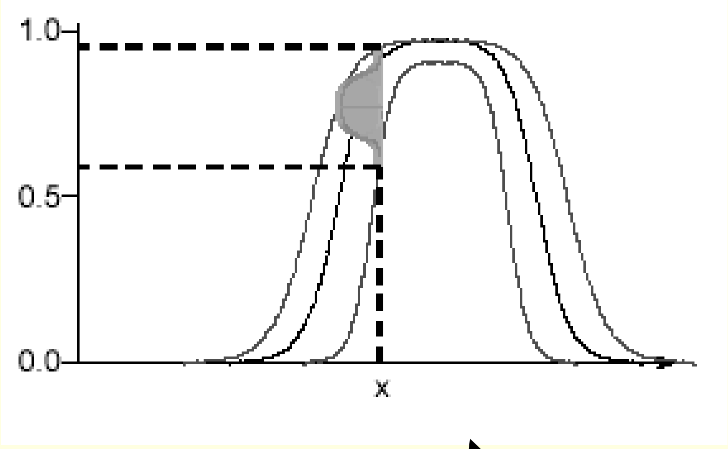
Of the form

$$\textit{If } x_1 \textit{ is } \tilde{\tilde{F}}_1 \textit{ and } \ldots \textit{ and } x_p \textit{ is } \tilde{\tilde{F}}_p \textit{ Then yis } \tilde{\tilde{G}}$$

where $\tilde{\tilde{F}}_i$ are the antecedent type-2 fuzzy sets and $\tilde{\tilde{G}}$ the output fuzzy sets.

Fuzzification





Fuzzifying in type-1
(fairly easy)

Fuzzifying in type-2

We have now covered the following:

- Defining our fuzzy sets
- Success of Type-1
- Type-2 fuzzy sets
  - Footprint of uncertainty
  - Union
  - Intersection
- Interval valued fuzzy sets
- Embedded fuzzy sets
- Overview
- Fuzzification

Review Question:

Suppose the secondary grades (or type-2 membership functions) for a particular element x are given by the following:

$$\mu_{\tilde{A}}(x) = 0.9/0.4 + 0.6/0.2 + 0.2/0.7$$

$$\mu_{\tilde{B}}(x) = 0.5/0.2 + 0.4/0.4 + 0.1/0.5$$

Using min for the t-norm and max for the t-conorm, calculate $\tilde{A} \cap \tilde{B}$ and $\tilde{A} \cup \tilde{B}$.

Discussion board question

- Post your solution to the following on the discussion board:

Write a paragraph on each of the following:

1. why there is a perceived need for the development of type-2 fuzzy sets in fuzzy logic;
2. the areas of application where type-2 fuzzy sets might provide a solution;
3. issues associated with practical implementation of type-2 fuzzy s sets.

N.B. To do this well you will need to have read the support documents on type-2 fuzzy logic, and the paper/s on applications available in Lesson 9.