Weekly Exercises Fuzzy Logic

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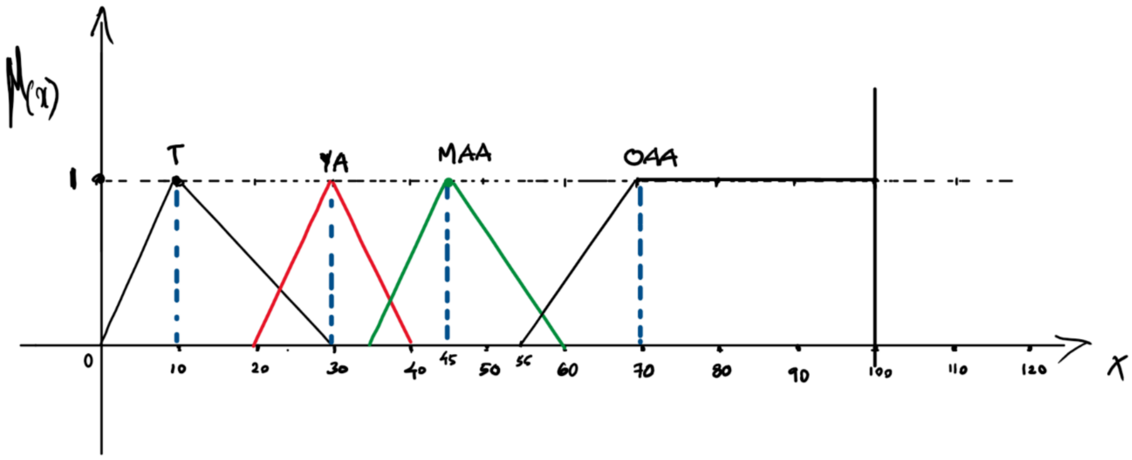
Exercise 3

Linguistic variables in fuzzy logic are used to refer to and define qualitative concepts of natural language. Representation of the linguistic variable “age” can have different ways. When it refers to a person, the following presents a system where age refers to a person.

Consider a complex system that involves customer satisfaction rating. Along with the rating, the age group is also significant. While considering age as a factor(variable), which refers to a person, it can have the following term set.

1. Teen (T): (0 to 30) where 10 represents peak age.
2. Young adults (YA): (20-40) where 30 represents peak age.
3. Middle-aged adults (MAA): (35-60) where 45 represents peak age.
4. Old-aged adults (OAA): (55-100) where 70 represents peak age.

With triangular function, the membership function distribution for this variable “age” can be drawn as given below.

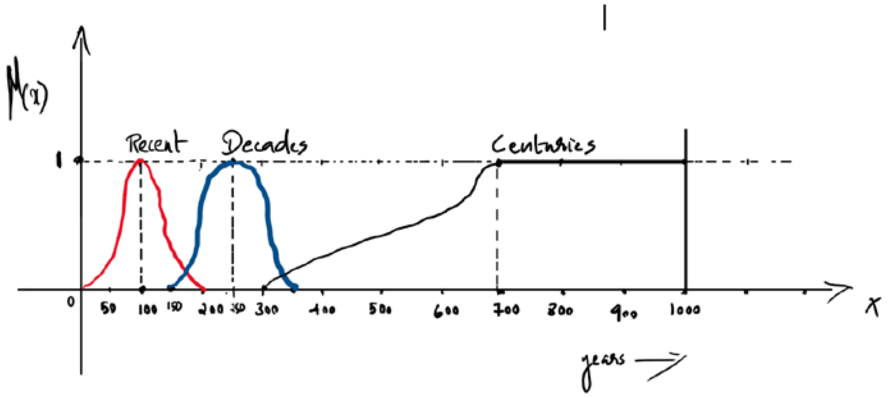


Does your modelling of this variable apply to a building or work of art?

This is not possible to apply the age variable into a system where age is a parameter that shows the age of the building or work of art. Because, in the above system, the age refers to a person.

For a building or an art, the linguistic variables for the factor “age” can take the following term set.

1. Recent (0 years - 200 years; peak is at 100 years)
2. Decades (150 years - 350 years; peak is at 250 years)
3. Centuries (300 years -1000 years; peak is at 700 years)



Exercise 5

PART A

The Membership functions of the given two fuzzy sets A and B are both same, which is depicted below.

Given Membership function:

$$\mu_A(x) = \mu_B(x) = \begin{cases} 1-|x| , & -1 \le x \le 1 \\ 0 , & \text{otherwise} \end{cases}$$

Evaluating for different 'x' values

x	$\mu(x) = (1-\lvert x\rvert)$
-1	0
-0.75	0.25
-0.5	0.5
-0.25	0.75
0	1
0.25	0.75
0.5	0.5
0.75	0.25
1	0

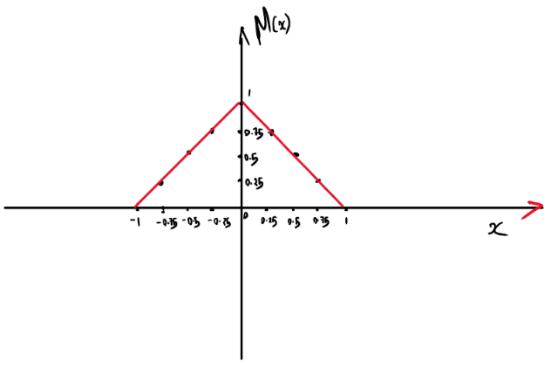


Figure 1

The fuzzy sets A and B can be defined as:

$A = \{(-1, 0), (-0.75, 0.25), (-0.5, 0.5), (-0.25, 0.75), (0, 1),$

$(0.25, 0.75), (0.5, 0.5), (0.75, 0.25), (1, 0)\}$

$B = \{(-1, 0), (-0.75, 0.25), (-0.5, 0.5), (-0.25, 0.75), (0, 1),$

$(0.25, 0.75), (0.5, 0.5), (0.75, 0.25), (1, 0)\}$

Membership functions of A and B:

$\mu_A(x) = (0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0)$

$\mu_B(x) = (0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0)$

Note both A and B are the same since both share the same membership function.

a) Sketch $\mu_{A\cap B}$ for minimum and product t-norms.

Minimum T-norm

$T(A, B) = min(\mu_A(x), \mu_B(x))$

$T(A, B) = (0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0)$

Resulted fuzzy set:

R = $\{(-1, 0), (-0.75, 0.25), (-0.5, 0.5), (-0.25, 0.75), (0, 1),$

$(0.25, 0.75), (0.5, 0.5), (0.75, 0.25), (1, 0)\}$

Since both A and B are same, the minimum would also be same, so the distribution of this membership function would be same as the distribution given in Figure 1.

$\mathbf{Product\ T - norm}$

$T(A, B) = \mu_A(x)\ \mathrm{X}\ \mu_B(x)$

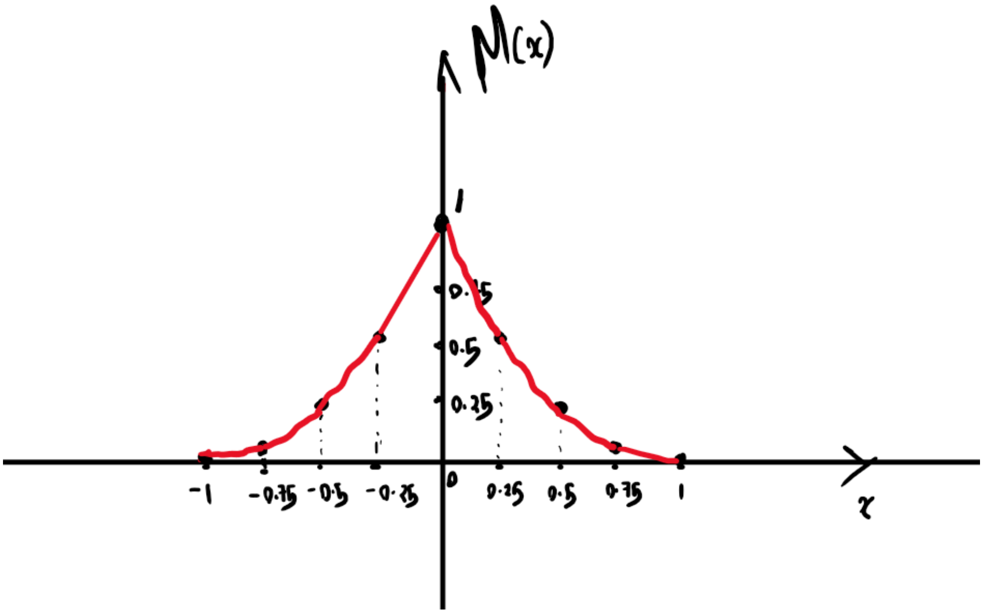
$$T(A, B) = (0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0) * (0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0)$$

$T(A, B) = (0, 0.0625, 0.25, 0.5625, 1, 0.5625, 0.25, 0.0625, 0)$

Resulted fuzzy set:

R = $\{(-1, 0), (-0.75, 0.0625), (-0.5, 0.25), (-0.25, 0.5625), (0, 1),$

$(0.25, 0.5625), (0.5, 0.25), (0.75, 0.0625), (1, 0)\}$



Observation: The resulted graph of product t-norm has a more narrowed distribution compared to the the normal distribution of A and B.

b) **Sketch μAUB for maximum and probabilistic sum (algebraic sum) t-conorms.**

Maximum T-conorms

$S(A, B) = max(\mu_A(x), \mu_B(x))$

$S(A, B) = (0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0)$

Resulted fuzzy set:

R = $\{(\mathbf{-1}, \mathbf{0}), (\mathbf{-0.75}, \mathbf{0.25}), (\mathbf{-0.5}, \mathbf{0.5}), (\mathbf{-0.25}, \mathbf{0.75}), (\mathbf{0}, \mathbf{1}),$

$(\mathbf{0.25}, \mathbf{0.75}), (\mathbf{0.5}, \mathbf{0.5}), (\mathbf{0.75}, \mathbf{0.25}), (\mathbf{1}, \mathbf{0})\}$

Since both A and B are same, , the minimum would also be same, so the distribution of this membership function would be same as the distribution given in Figure 1.

Probabilistic sum (Algebraic sum)T − conorms

$$S(A,B) = \mu_A(x) + \mu_B(x) - \mu_A(x) \text{ X } \mu_B(x)$$

$$S(A,B) = (0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0)$$
$$+ (0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0)$$
$$- (0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0)$$
$$* (0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0)$$

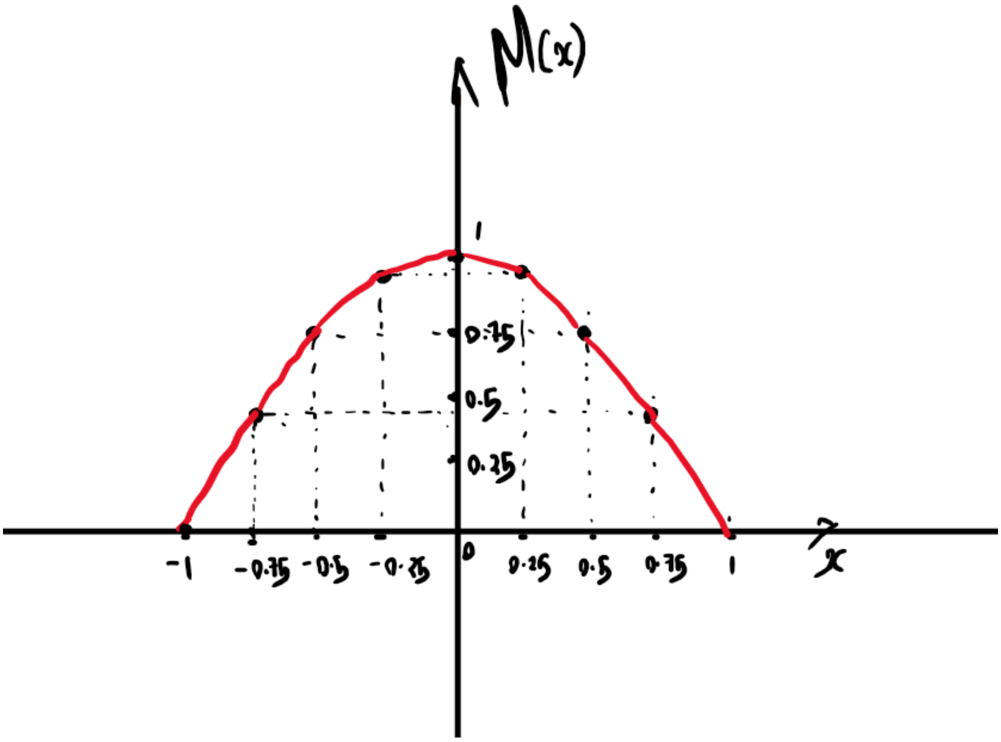
$$S(A,B) = (0, 0.5, 1, 1.5, 2, 1.5, 1, 0.5, 0)$$
$$- (0, 0.0625, 0.25, 0.5625, 1, 0.5625, 0.25, 0.0625, 0)$$

$$S(A,B) = (0, 0.4375, 0.75, 0.9375, 1, 0.9375, 0.75, 0.4375, 0)$$

Resulted fuzzy set:

R = $\{(\mathbf{-1}, \mathbf{0}), (\mathbf{-0.75}, \mathbf{0.4375}), (\mathbf{-0.5}, \mathbf{0.75}), (\mathbf{-0.25}, \mathbf{0.9375}), (\mathbf{0}, \mathbf{1}),$

$(\mathbf{0.25}, \mathbf{0.9375}), (\mathbf{0.5}, \mathbf{0.75}), (\mathbf{0.75}, \mathbf{0.4375}), (\mathbf{1}, \mathbf{0})\}$



Observation: In the above figure, the resultant graph of probabilistic sum is broader than the given distribution graphs for A and B.

PART B

The Membership functions of the given two fuzzy sets A and B are both same, which is depicted below.

The given membership function of fuzzy sets A and B are given below.

$$\mu_A(x) = e^{-\frac{1}{2}(x-3)^2} \quad \& \quad \mu_B(x) = e^{-\frac{1}{2}(x-4)^2}$$

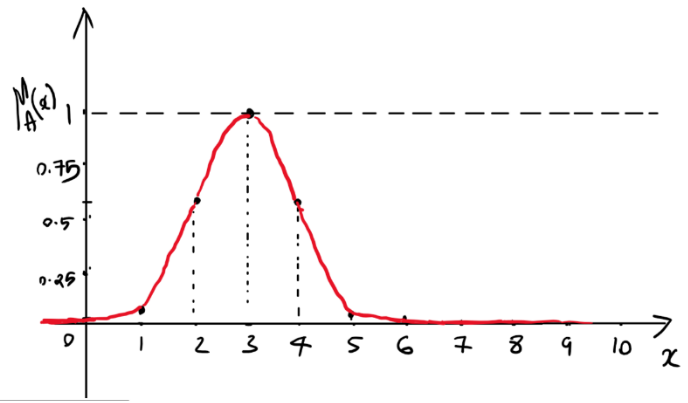
with $\mu_A(x)$,

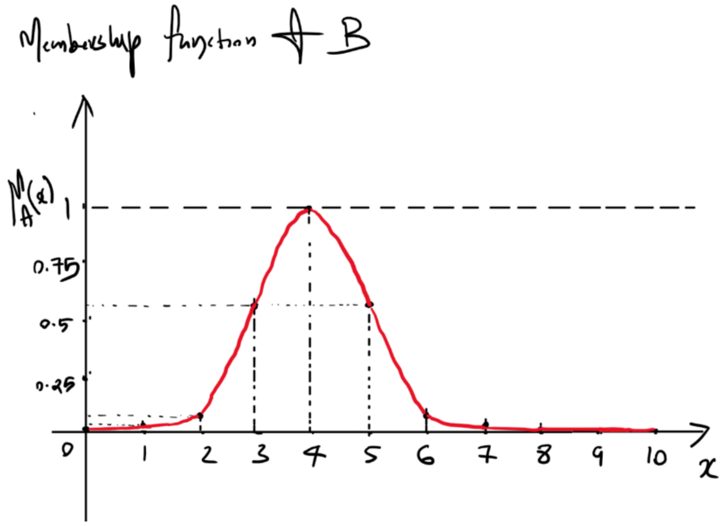
x	$\mu_A(x)$
0	0.0111
1	0.1353
2	0.6065
3	1
4	0.6065
5	0.1353
6	0.0111
7	3.35×10^{-4}
8	3.7×10^{-6}

with $\mu_B(x)$,

x	$\mu_B(x)$
0	3.55×10^{-4}
1	0.0111
2	0.1353
3	0.6065
4	1
5	0.6065
6	0.1353
7	0.0111
8	3.35×10^{-4}

Membership function of A :





The fuzzy sets A and B can be defined as:

$A = \{(0, 0.0111), (1, 0.1353), (2, 0.6065), (3, 1), (4, 0.6065),$

$(5, 0.1353), (6, 0.0111), (7, 0.000335), (8,0.0000037)\}$

$A = \{(0, 0.000355), (1, 0.0111), (2, 0.1353), (3, 0.6065), (4, 1),$

$(5, 0.6065), (6, 0.1353), (7, 0.0111), (8,0.000335)\}$

Membership functions of A and B:

$\mu_A(x) = (0.0111, 0.1353, 0.6065,1,0.6065, 0.1353, 0.0111,0.000335,0.0000037)$

$\mu_B(x) \ = (0.000355, 0.0111, 0.1353, 0.6065,1,0.6065,0.1353,0.0111,0.000335)$

a) Sketch $\mu_{A\cap B}$ for minimum and product t-norms.

Minimum T-norm

$T(A, B) = min(\mu_A(x), \mu_B(x))$

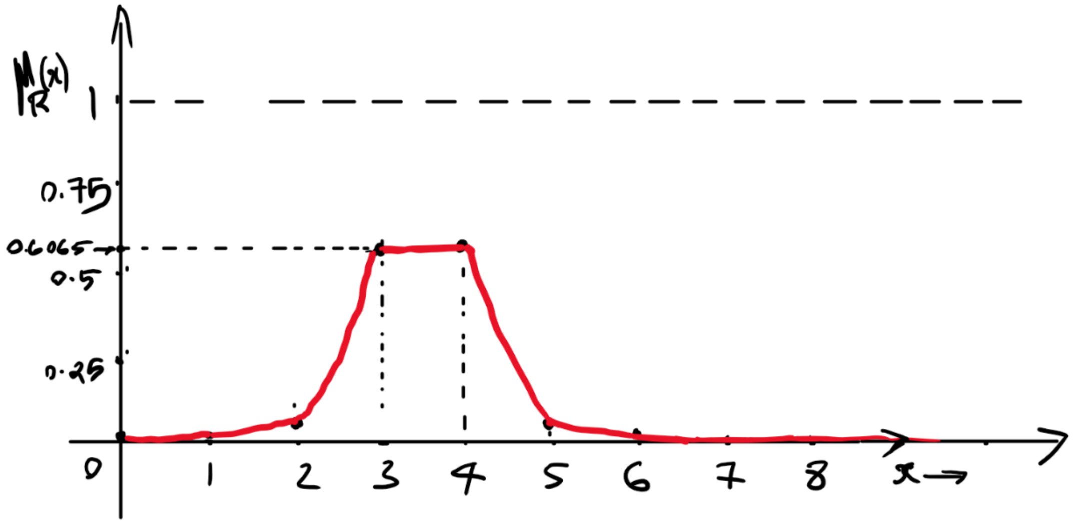
$T(A, B) = (0.000355,0.0111,0.1353,0.6065,0.6065,0.1353,0.0111,$

$0.000335,0.0000037)$

Resulted fuzzy set:

R = $\{(0, 0.000355), (1, 0.0111), (2, 0.1353), (3, 0.6065), (4, 0.6065),$

$(5, 0.1353), (6, 0.0111), (7, 0.000335), (8, 0.0000037)\}$



Observation: The resulted distribution shows that every element has at least some degree of membership, but nothing has the complete membership. The highest value of membership is 0.6065 for x=3 and x=4

Product T-norm

$T(A,B) = \mu_A(x) \text{ X } \mu_B(x)$

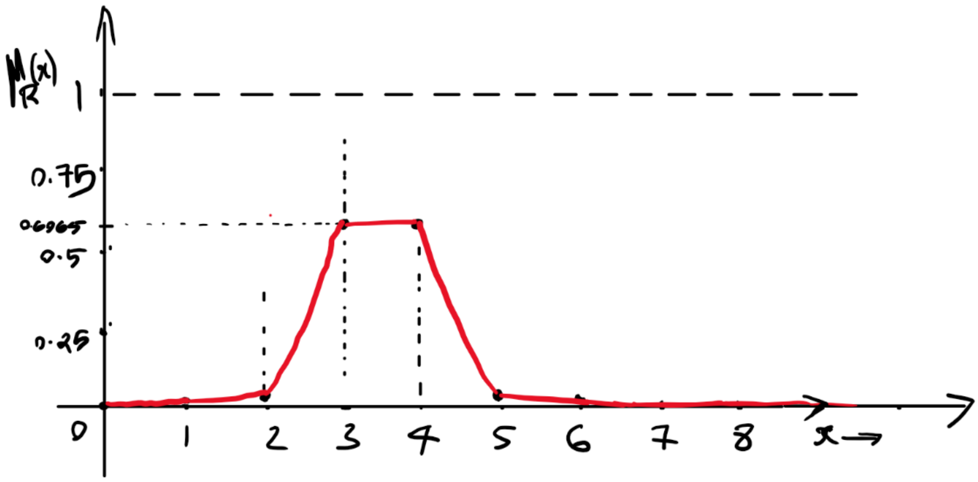
$T(A,B) =$
$(0.0111, 0.1353, 0.6065, 1, 0.6065, 0.1353, 0.0111, 0.000335, 0.0000037)$
*($0.000355, 0.0111, 0.1353, 0.6065, 1, 0.6065, 0.1353, 0.0111, 0.000335)$

$T(A,B) = (0, 0.0015, 0.0821, 0.6065, 0.6065, 0.0821, 0.0015, 0, 0)$

Resulted fuzzy set:

R = $\{(0,0), (1, 0.0015), (2, 0.0821), (3, 0.6065), (4, 0.6065),$

$(5, 0.0821), (6, 0.0015), (7, 0), (8, 0)\}$



Observation: The resulted distribution shows that every element has at least some degree of membership, but nothing has the complete membership. The highest value of membership is 0.6065 for x=3 and x=4

b) **Sketch $\mu_{A \cup B}$ for maximum and probabilistic sum (algebraic sum) t-conorms.**

Maximum T-conorms

$S(A, B) = max(\mu_A(x), \mu_B(x))$

$S(A, B) = (0.0111, 0.1353, 0.6065, 1, 1, 0.6565, 0.1353, 0.0111, 0.000335)$

Resulted fuzzy set:

R = $\{(0, 0.0111), (1, 0.1353), (2, 0.6065), (3, 1), (4, 1),$

$(5, 0.6565), (6, 0.1353), (7, 0.0111), (8, 0.000335)\}$

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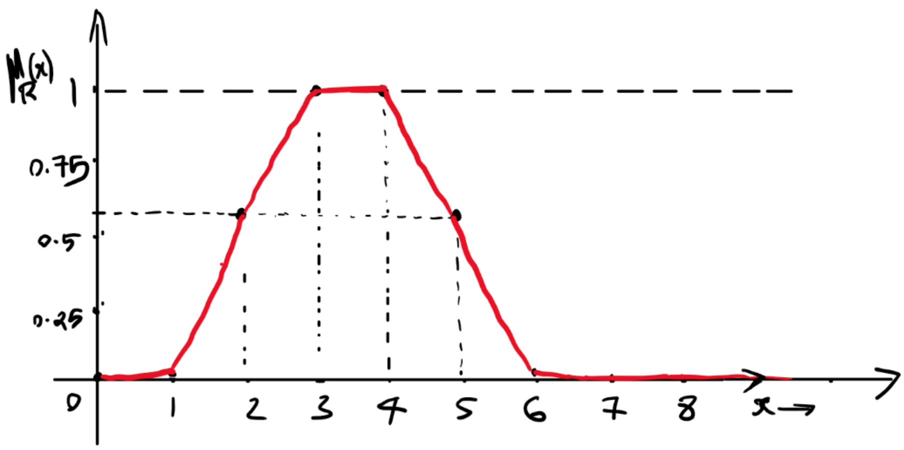


Probabilistic sum (Algebraic sum)T − conorms

$S(A,B) = \mu_A(x) + \mu_B(x) - \mu_A(x) \times \mu_B(x)$

$S(A,B) = (0.0111, 0.1353, 0.6065, 1, 0.6065, 0.1353, 0.0111, 0.000335, 0.0000037)$
$+ (0.000355, 0.0111, 0.1353, 0.6065, 1, 0.6065, 0.1353, 0.0111, 0.000335) -$
$(0.0111, 0.1353, 0.6065, 1, 0.6065, 0.1353, 0.0111, 0.000335, 0.0000037)$
$*(0.000355, 0.0111, 0.1353, 0.6065, 1, 0.6065, 0.1353, 0.0111, 0.000335)$

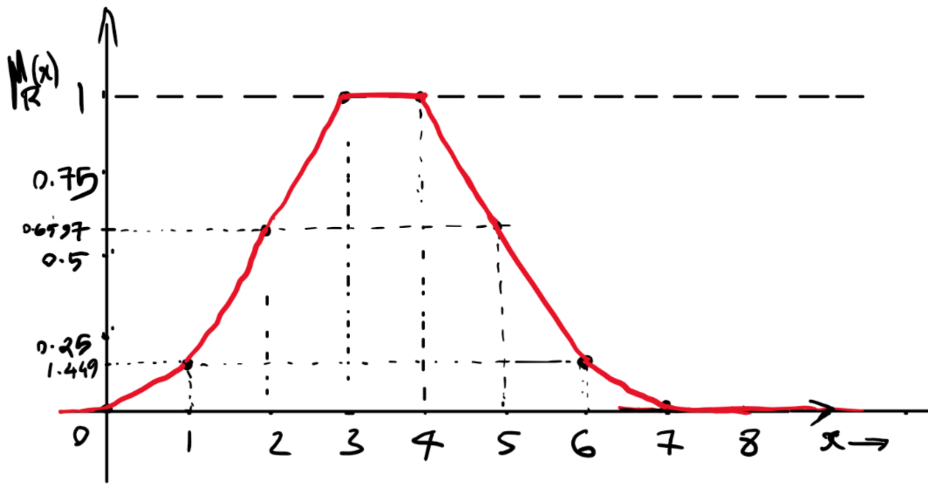
$S(A,B)$
$= (0.0115, 0.1464, 0.7418, 1.6065, 1.6065, 0.7418, 0.1464, 0.0114, 0.0003)$
$- (0.0000, 0.0015, 0.0821, 0.6065, 0.6065, 0.0821, 0.0015, 0, 0)$

$S(A,B) = (0.0115, 0.1449, 0.6597, 1, 1, 0.6597, 0.1449, 0.0114, 0)$

Resulted fuzzy set:

R = $\{(0, 0.0115), (1, 0.1449), (2, 0.6597), (3, 1), (4, 1),$

$(5, 0.6597), (6, 0.1449), (7, 0.0114), (8, 0)\}$



EXERCISE 7

b) Determine the Lukasiewicz t-conorm $S_L = \min\{x+y, 1\}$ using the duality property.

Solution:

Tnorm and Tconorm are dual to each other.

$$\text{so} \quad S(x,y) = 1 - T_L(1-x, 1-y)$$
$$= 1 - \max\{(1-x)+(1-y)-1, 0\}$$
$$= 1 - \max\{1-x-y, 0\}$$

Cases:

(1) $1-x-y \leq 0 \Rightarrow S(x,y) = 1 - \max(0,0)$
$$= 1$$

(2) $1-x-y > 0 \Rightarrow S(x,y) = 1-(1-x-y)$
$$= 1 - x + y - 1$$
$$= x+y$$

from these (1) & (2), the bounds of $S(x,y)$ should be in between '1' and '$x+y$'.

$$1-x-y > 0 \Rightarrow 1 > x+y$$

from this,

$$S(x,y) = 1 - \max(x,y)$$

applying duality property

$$= \min\{1-x, 1-y\}$$

putting the values of bounds

$$= \min\{1-(1-x-y), 1-0\}$$
$$= \min\{x+y, 1\}$$

Hence the proof

a) $T_L(x,y) = \max\{x+y-1, 0\}$

To proof this is tnorm, prove the four laws of Tnorm. with T_L.

① Neutral : $T(x,1) = x$

$$T(x,y) = \max\{x+y-1, 0\}$$

$$T(x,1) = \max\{x+1-1, 0\} = \max\{x, 0\} \longrightarrow ①$$

consider two cases based on the value of x (note that bound of x is from $0 \to 1$)

1) $x=0 \Rightarrow$ equation ① becomes $T(x,1) = \max\{0,0\} = 0$

2) $x=1 \Rightarrow$ equation ① becomes $T(1,1) = \max\{1,0\} = 1$

from the result of two above cases

$$T(x,1) = x$$

hence the proof.

② Commutative Law : $T(x,y) = T(y,x)$

Start from LHS,

$$T(x,y) = \max\{x+y-1, 0\}$$

The term which influence $T(x,y)$ is '$x+y-1$'

$$\left.\begin{aligned} x+y-1 < 0 &\Rightarrow T(x,y) = 0 \\ x+y-1 \geq 0 &\Rightarrow T(x,y) = x+y-1 \end{aligned}\right\} \longrightarrow ①$$

Consider RHS,

$$T(y,x) = \max\{y+x-1, 0\}$$

$y+x-1 \Rightarrow$

case 1: $y+x-1 \leq 0 \Rightarrow T(y,x) = 0$

case 2: $y+x-1 > 0 \Rightarrow T(y,x) = y+x-1$ $\Big\} \rightarrow (2)$

from analysis result (1) and (2), it can be seen that results of both are same with respective conditions

$$\therefore \quad T(x,y) = T(y,x)$$

(3) Associative law : $T(x, T(y,z)) = T(T(x,y), z)$

$$T(T(x,y), z) = \max\{T(x,y) + z - 1, 0\}$$

$$= \max\{\max\{x+y-1, 0\} + z - 1, 0\}$$

Cases :

$x+y-1 > 0 \Rightarrow T(T(x,y),z) = \max\{x+y-1+z-1, 0\}$

$= \max\{x+y+z-2, 0\} \rightarrow (1)$

$x+y-1 \leq 0 \Rightarrow T(T(x,y),z) = \max\{z-1, 0\} = 0.$

since $z \in [0,1] \Rightarrow z-1 \leq 0 \Rightarrow z \leq 1$

$\therefore$ (1) would be $x+y+z-2 \geq z-1$

$x+y \geq 1$ $\Big\} \rightarrow (3)$

$$T(x, T(y,z)) = \max\{x + T(y,z) - 1, 0\}$$
$$= \max\{x + \max\{y+z-1, 0\} - 1, 0\}$$

cases:

$y+z-1 \geq 0 \Rightarrow T(x, T(y,z)) = \max\{x+y+z-1-1, 0\}$
$= \max\{x+y+z-2, 0\} \longrightarrow$ (2)

$y+z-1 < 0 \Rightarrow T(x, T(y,z)) = \max\{x-1, 0\}$

since $x \in [0,1] \Rightarrow x-1 \leq 0$
$x \leq 1$

$\therefore$ (2) will be $\left.\begin{array}{l} x+y+z-2 \geq x-1 \\ \underline{\underline{y+z \geq 1}} \end{array}\right\}$ 4

both (3) & (4) follow the same pattern

LHS = RHS

$$\underline{\underline{T(x, T(y,z)) = T(T(x,y), z)}}$$

(4) <u>Monotonicity</u>: $T(a,b) \leq T(c,d)$ if $a \leq c$ and $b \leq d$

$T(a,b) = \max\{a+b-1, 0\}$

cases:

$a+b-1 < 0 \Rightarrow T(a,b) = \max\{(-ve), 0\} = \underline{\underline{0}}$

$a+b-1 \geq 0 \Rightarrow T(a,b) = a+b-1 \longrightarrow$ (1)

$\underline{\underline{a+b \geq 1}}$

$T(c,d) = \max\{c+d-1, 0\}$

cases:

$c+d-1 < 0 \Rightarrow T(c,d) = \underline{0}$

$c+d-1 \geq 0 \Rightarrow T(c,d) = c+d-1 \longrightarrow$ (2)

apply $a \geq c$ & $b \geq d$ in ① and ②

$a+b-1 \geq c+d-1$, because $a+b \geq c+d$

ie $T(a,b) \geq T(c,d)$

$\therefore T(a,b) \geq T(c,d)$ for $a \geq c$ && $b \geq d$

hence the proof