Weekly Exercises Fuzzy Logic Week 5

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EXERCISE 8

1.
   a) If α ≥ β, then Aα ⊆ Aβ
   b) (A ∪ B)α = Aα ∪ Bα
   c) (A ∩ B)α = Aα ∩ Bα

a) if $\alpha \geqslant \beta$ then $A_\alpha \subseteq A_\beta$

$A \to \mu_A$, $A_\alpha = \{x \mid \mu_A(x) \geqslant \alpha\}$

to prove that $A_\alpha \subseteq A_\beta$, we need to prove each element of A_α belongs to A_β

if $x \in A_\alpha \Rightarrow x \in A_\beta$

$x \in A_\beta$ iff $\mu_A(x) \geqslant \beta$

since $x \in A_\alpha$ is an assumption

$\mu_A(x) \geqslant \alpha \geqslant \beta$

hence the proof

b) $(A \cup B)_\alpha = A_\alpha \cup B_\alpha$

$A_\alpha = \{x \mid \mu_A(x) \geqslant \alpha\} \longrightarrow ①$

based on ①,

$(A \cup B)_\alpha = \{x \mid \mu_{A\cup B}(x) \geqslant \alpha\}$

union of two fuzzy set is evaluated with maximum

$\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$

which is equalent to

$\mu_A(x) \geqslant \alpha$ OR $\mu_B(x) \geqslant \alpha$

ie. Membership function value of at least one of the set has to be greater than α

$x \in A_\alpha$ OR $x \in A_\beta \Leftrightarrow x \in A_\alpha \cup B_\alpha$

hence the proof

c) $(A \cap B)_\alpha = A_\alpha \cap B_\alpha$

we know $A_\alpha = \{x \mid \mu_A(x) \geqslant \alpha\} \longrightarrow ①$

substituting in ①,

$(A \cap B)_\alpha = \{x \mid \mu_{A\cap B}(x) \geqslant \alpha\}$

intersection of two fuzzy set is evaluated with minimum

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$\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$

which is equivalent to,

$\mu_A(x) \geq \alpha$ and $\mu_B(x) \geq \alpha$

Membership function value of both of the set has to be greater than α

$x \in A_\alpha$ and $x \in B_\alpha \iff x \in A_\alpha \cap B_\alpha$

ie $(A \cap B)_\alpha = A_\alpha \cap B_\alpha$

hence, the proof

2.

Given the discrete fuzzy set A = {(0.2, x1), (0.4, x2), (0.6, x3), (0.8, x4), (1, x5)},
obtain L(A),
and provide all the distinct alpha-cuts of A.

$A = \{(0.2, x_1), (0.4, x_2), (0.6, x_3), (0.8, x_4), (1, x_5)\}$

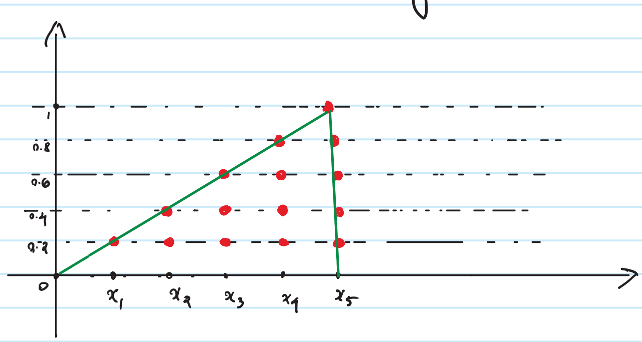
from A, L(A) can be defined as

$L(A) = \{0.2, 0.4, 0.6, 0.8, 1\}$

Alpha cuts of each α in L(A),

$A_{0.2} = \{x_1, x_2, x_3, x_4, x_5\}$
$A_{0.4} = \{x_2, x_3, x_4, x_5\}$
$A_{0.6} = \{x_3, x_4, x_5\}$
$A_{0.8} = \{x_4, x_5\}$
$A_1 = \{x_5\}$

So, MF can be plotted like given below,



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3. Show that (3) is verified from the discrete set A given in item 2.

(3) is given below,

In other words:

$$\forall x \in X: \quad \mu_A(x) = \sup_{\alpha \in [0,1]} \alpha A_\alpha(x), \quad \text{where} \quad \alpha A_\alpha(x) = \begin{cases} \alpha & x \in A_\alpha \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

We have,

Alpha cuts : $A_{0.2}$, $A_{0.4}$, $A_{0.6}$, $A_{0.8}$ and A_1

We need to find A

Membership function value of each x in A is calculated with equation (3)

Based on the presence of x values in alpha cuts, which have been created in the above solution (1.b),

$\mu_A(x_1) = \max\{0.2*1,\ 0.4*0,\ 0.6*0,\ 0.8*0,\ 1*0\}$
$= \max\{0.2\}$
$= \underline{0.2}$

$\mu_A(x_2) = \max\{0.2*1,\ 0.4*1,\ 0.6*0,\ 0.8*0,\ 1*0\} = \max\{0.2, 0.4\} = \underline{0.4}$

$\mu_A(x_3) = \max\{0.2*1,\ 0.4*1,\ 0.6*1,\ 0.8*0,\ 1*0\} = \underline{0.6}$

$\mu_A(x_4) = \max\{0.2*1,\ 0.4*1,\ 0.6*1,\ 0.8*1,\ 1*0\} = 0.8$ //

$\mu_A(x_5) = \max\{0.2*1,\ 0.4*1,\ 0.6*1,\ 0.8*1,\ 1*1\} = 1$ //

This indicate that for each x one, a membership value exist, which is same associated with the fuzzy set A.

So (3) has been verified

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Exercise 9. *Prove using the extension principle, to extend function $f(x) = -x$, that the opposite of a fuzzy number N on $\mathbb{R}$, $-N$ has membership function $\mu_{-N}(x) = \mu_N(-x)$, where μ_N is the membership function of N.*

In this case, we have $f: R \to R$ defined as $f(x) = -x$, and our aim here to extend this from real numbers to the case for fuzzy numbers.

So if we have one fuzzy number N on R, the extension principle states that $f(N)$ is a fuzzy number on R which we denote '$-N$' with membership function.

$$\mu_{-N}(x) = \begin{cases} \sup_{x \to f^{-1}(x)} \{\mu_N(x)\} & , \text{ if } f^{-1}(x) \neq 0 \\ 0 & , \text{ otherwise} \end{cases}$$

where, $f^{-1}(x) = \{x \in R \mid y - f(x)\}$

since $f(x) = -x$, then is,

$$f^{-1}(x) = \{x \in R \mid y = -x\} = \{-x\}$$

$$\mu_{-N}(x) = \mu_N(-x)$$

This shows with $f(x) = -x$, for fuzzy number N on R, with a membership function $\mu_N(x)$, equal to the fuzzy number $-N$, opposite to N, has a membership function $\mu_{-N}(x)$.

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Exercise 14. *For the fuzzy rule of exercise 13 compute the fuzzy outputs when the inputs are $x' = 50$, $x' = 60$, $x' = 75$, $x' = 80$ and $x' = 100$, respectively.*

Exercise 13. *For the IF–THEN rules below*

IF distance is big, THEN acceleration is medium

Model 'big' as the triangular fuzzy number with parameters $(50, 75, 100)$ and 'medium' as the fuzzy number with parameters $(2, 5, 8)$. Give the expression of the membership degree of the fuzzy rule using the four different implication membership functions given above.

The given rule:
If distance is big then acceleration is medium.

Lets define the fuzzy set,

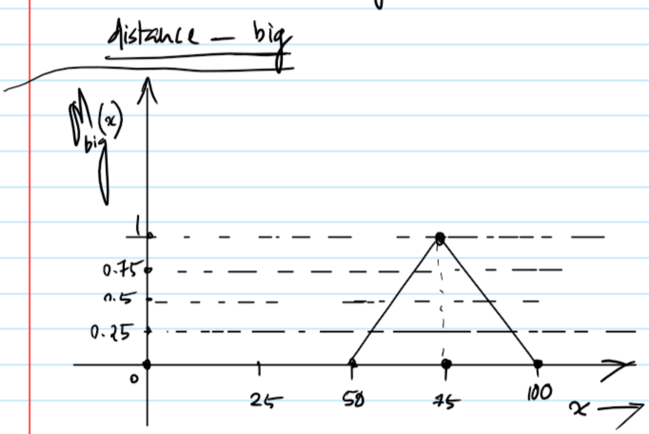
Big : Triangular MF with parameters (50, 75, 100)
Medium: Triangular MF with parameters (2, 5, 8)

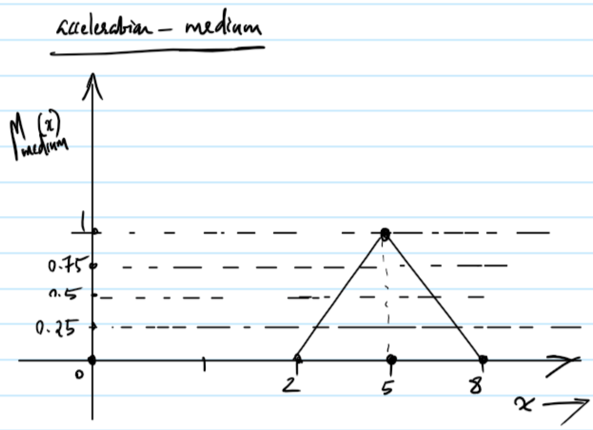
While applying triangular MF, for each of the above,

$$\mu_{big}(x) = \begin{cases} 0, & \text{for } x \leq 50 \\ \frac{x-50}{75-50}, & \text{for } 50 < x \leq 75 \\ \frac{100-x}{100-75}, & \text{for } 75 < x \leq 100 \\ 0, & \text{for } x > 100 \end{cases} \longrightarrow (1)$$

$$\mu_{medium}(x) = \begin{cases} 0, & \text{for } x < 2 \\ \frac{x-2}{5-2}, & \text{for } 2 \leq x \leq 5 \\ \frac{8-x}{8-5}, & \text{for } 5 < x \leq 8 \\ 0, & \text{for } x > 8 \end{cases} \longrightarrow (2)$$

The graph for $\mu_{big}(x)$ and $\mu_{medium}(x)$ can be plotted as given below





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Lets define the fuzzy sets for both big and medium

A - big and B - medium

$A = \{ (50,0), (60, 0.4), (75, 1), (80, 0.8), (100, 0) \} \longrightarrow (3)$

$B = \{ (2,0), (3, 0.33), (4, 0.66), (5,1), (6, 0.66), (7, 0.33), (8,0) \} \longrightarrow (4)$

The formula for evaluation are as follows:

Mamdani : $\mu_{p\to q}(x,y) = \min\{\mu_p(x), \mu_q(y)\}$

Larsen : $\mu_{p\to q}(x,y) = \mu_p(x)\mu_q(y)$

for the given rule?

If X is A THEN Y in B

where,

→ A' and B are fuzzy sets

The output would be fuzzy, in which

minimum : the result is truncated B at the level $\mu_A(x')$, at point x'

product : the result is product of MF values of both $\mu_A(x)$ & $\mu_B(x)$

Consider different inputs of x, the distance

with acceleration at medium-peak, ie. at 5 $\mu_B(x) = 1$,

$x' = 50 \Rightarrow$

$\mu_A(50) = 0$, from (3) $\mu_B(5) = 1$, from (4)

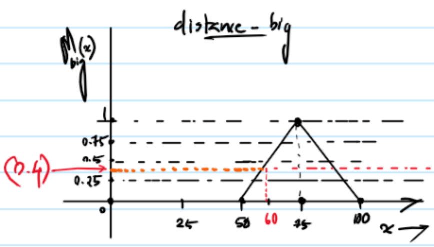
Mamdani minimum = $\min\{0, 1\} = 0$

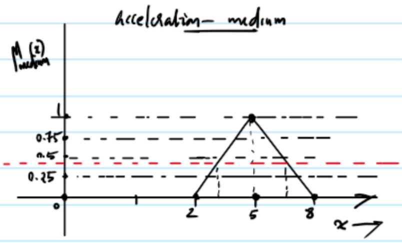
Larsen product = $0 \times 1 = 0$

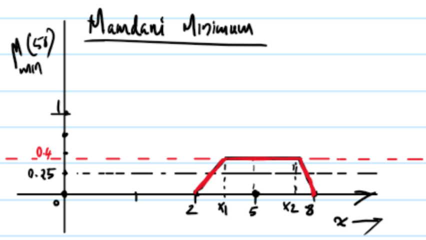
$x' = 60 \Rightarrow$ $\mu_A(60) = 0.4$ from (3) $\mu_B(5) = 1$ from (4)

Mamdani minimum = $\min\{0.4, 1\} = 0.4$

Larsen product = $0.4 \times 1 = 0.4$







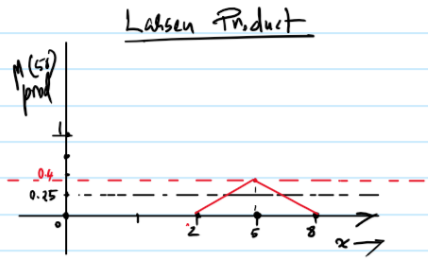
Note that, Mamdani minimum gives trapezoidal curve, MF can be defined for 5 intervals

applying on eqn (2),

$x_1 = 1.2 + 2 = 3.2$

$x_2 = 8 - 1.2 = 6.8$

$$\mu_{min}(x) = \begin{cases} 0 & x < 2 \\ (x-2)/(3.2-2) & 2 \le x \le 3.2 \\ 1 & 3.2 \le x \le 6.8 \\ (8-x)/(8-6.8) & 6.8 \le x \le 8 \\ 0 & x > 8 \end{cases}$$



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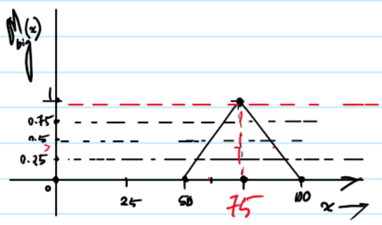
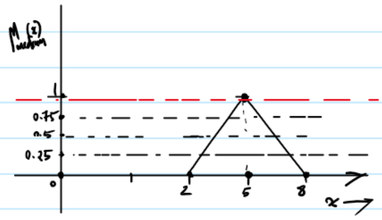
$x' = 75 \Rightarrow \quad \mu_A(75) = 1$ from ③ $\quad \mu_B(5) = 1$ from ④

Mamdani minimum $= \min\{1, 1\} = 1$
Larsen product $= 1 \times 1 = 1$

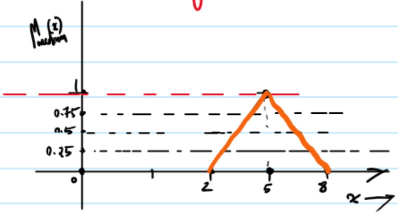
distance - big



Acceleration - medium



Fuzzy Output

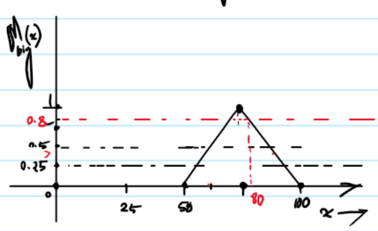


The resultant graph of both product and minimum is same as of MF of $\mu_B(x)$.

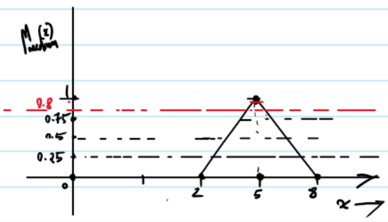
$x' = 80 \Rightarrow \quad \mu_A(80) = 0.8$ from ③ $\quad \mu_B(5) = 1$ from 4

Mamdani minimum $= \min\{0.8, 1\} = 0.8$
Larsen product $= 0.8 \times 1 = 0.8$

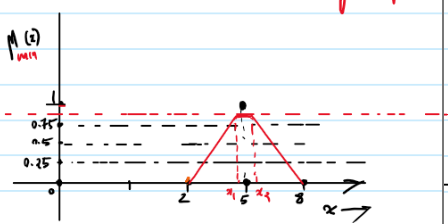
distance - big



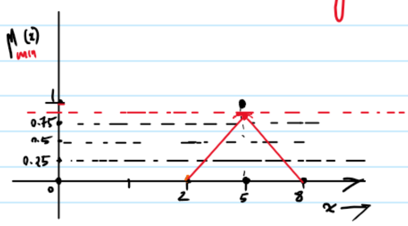
Acceleration - medium



Minimum Operation fuzzy output



Minimum Operation fuzzy output



Note that, Mamdani minimum gives trapezoidal curve, MF can be defined for 5 intervals

from ②, $x_1 = 2.4 + 2 = 4.4$
$x_2 = 8 - 2.4 = 5.6$

$$\mu_{min}(x) = \begin{cases} 0 & x < 2 \\ (x-2)/(4.4-2) & 2 \leq x \leq 4.4 \\ 1 & 4.4 \leq x \leq 5.6 \\ (8-2)/(8-5.6) & 5.6 \leq x \leq 8 \\ 0 & x > 8 \end{cases}$$

$x' = 100 \Rightarrow$

$\mu_A(100) = 0$, from ③ $\quad \mu_B(5) = 1$, from ④

Mamdani minimum $= \min\{0, 1\} = 0$
Larsen product $= \cancel{0 \times 1} = 0$