Constraint Handling Techniques

Static Penalty

- The penalty function method can be written as

$$P(x, R) = f(x) + \sum_{k=1}^{K} R_k \{h(x)\}^{\gamma} + \sum_{j=1}^{J} R_j \ \langle g_j(x) \rangle^{\beta}, \tag{1}$$

where R_k and R_j are the penalty parameters. Normally, β and γ are kept 1 and 2, respectively.

- The penalty parameters remain constant throughout evolutionary process.
- We consider same R value for all constraints for simplification.

Dynamic Penalty

- The penalty factors include the current generation counter (t) in its computation.

$$P(x,R) = f(x) + (C \times t)^{\alpha} \left[\sum_{k=1}^{K} \{h(x)\}^{\gamma} + \sum_{j=1}^{J} \langle g_j(x) \rangle^{\beta} \right]. \tag{2}$$

  where C, α, β and γ are the user defined constants.
- It suggests that the penalty term $((C \times t)^{\alpha})$ is increasing with the generation counter (t).
- Joines and Houck [1994] used $C = 0.5$, $\alpha = 1$, β and γ are kept 1 and 2, respectively.

Deb's Approach

- The fitness is assigned as

$$F(x) = \begin{cases} f(x), & \text{if } x \text{ is feasible;} \\ f_{max} + \sum_{j=1}^{J} |\langle g_j(x) \rangle| + \sum_{k=1}^{K} |h_k(x)|, & \text{Otherwise.} \end{cases} \tag{3}$$

- Here, f_{max} is the objective function value of the worst feasible solution in the population.
- Therefore, this approach is considered as penalty parameter-less approach.

Case Studies

Real-Coded Genetic Algorithm

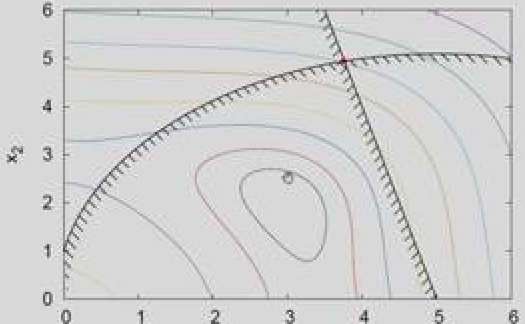
RGA Parameters

- Population size: $N = 100$
- No. of generations: $T = 200$
- Binary tournament selection operator
- SBX crossover operator
  - Probability of crossover: $p_c = 1.0$
  - SBX crossover operator: $\eta_c = 15$
- Polynomial mutation operator
  - Probability of mutation: $p_m = 1/n$
  - Polynomial mutation operator: $\eta_m = 20$
- $(\mu + \lambda)$−strategy

Constrained Himmelblau Function

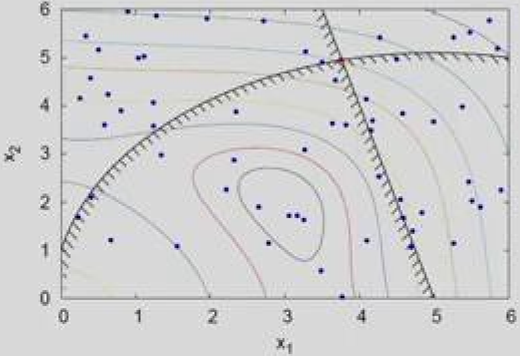
Himmelblau Function

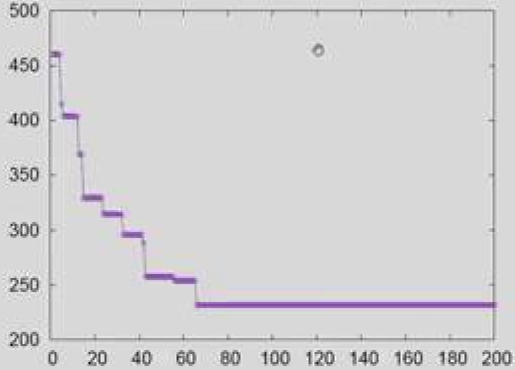
$$
\begin{aligned}
\text{Minimize} \quad & f(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2, \\
\text{subject to} \quad & (x_1 - 5)^2 + x_2^2 \geq 26, \\
& 4x_1 + x_2 \geq 20, \\
& x_1, x_2 \geq 0.
\end{aligned}
$$



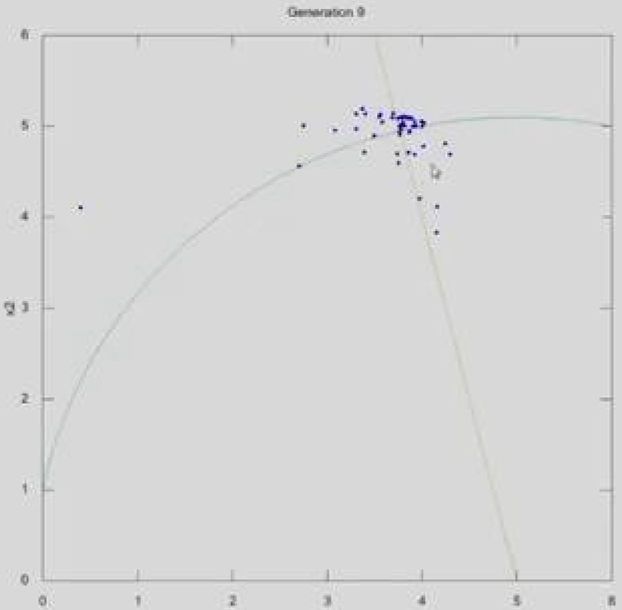
- Optimal solution is $x^* = (3.763, 4.947)^T$ and $f(x^*) = 517.063$

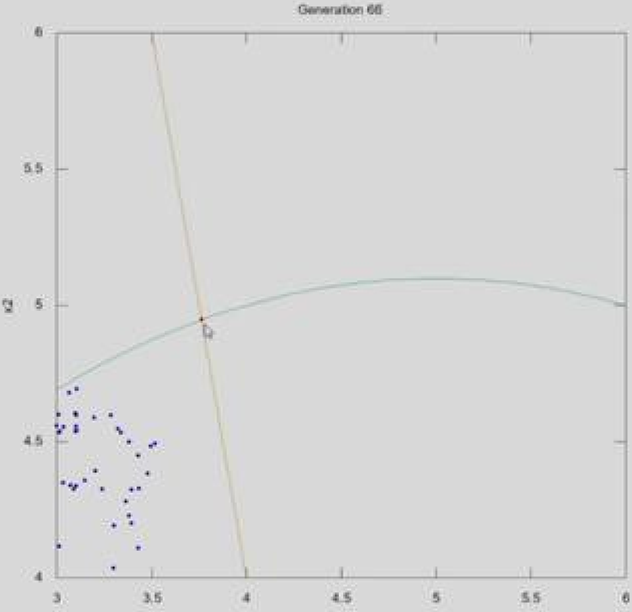
Static Penalty Approach with $R = 2$



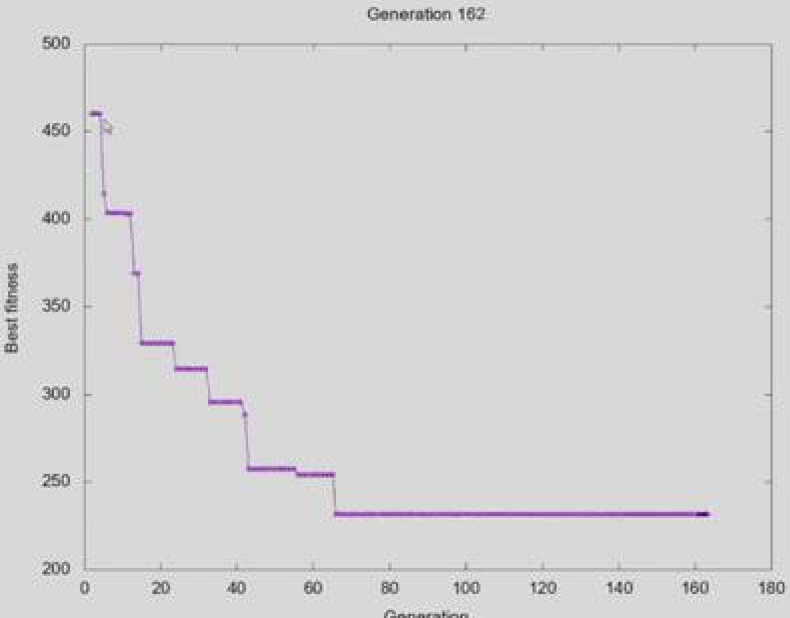


- Simulation Link
- Progress Link

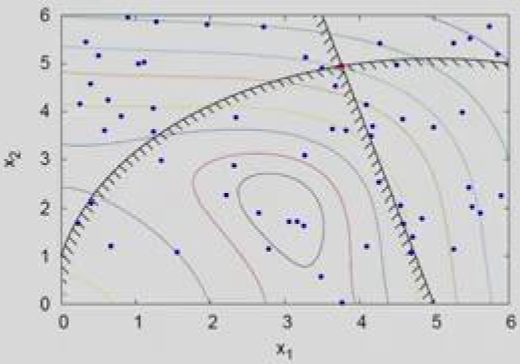

Generation 9
0
1
2
3
4
5
6
x2

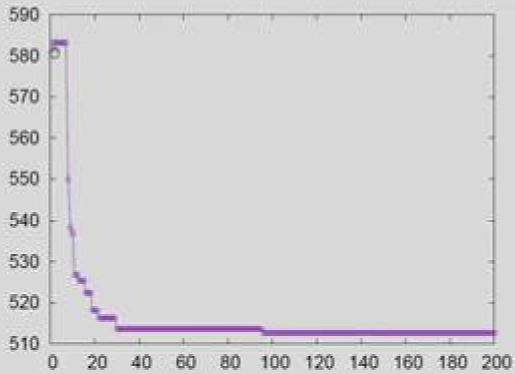


Generation 68
6
5.5
5
4.5
4
x2
3
3.5
4
4.5
5
5.5
6

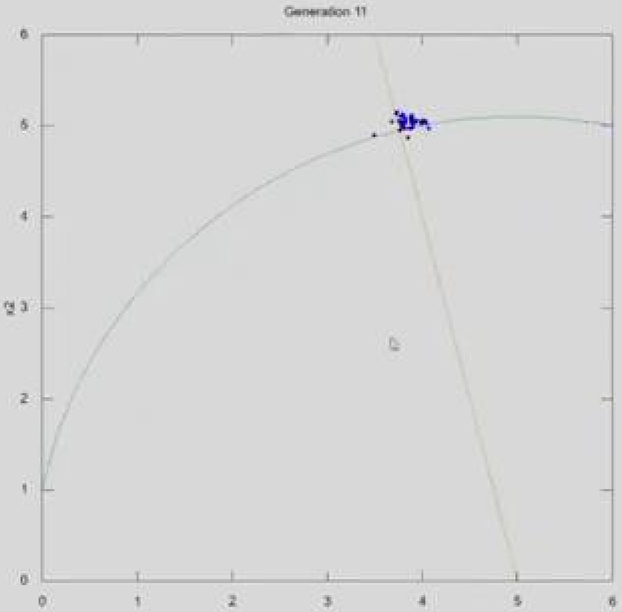

Generation 162
Best fitness
Generation
500
450
400
350
300
250
200
0
20
40
60
80
100
120
140
160
180

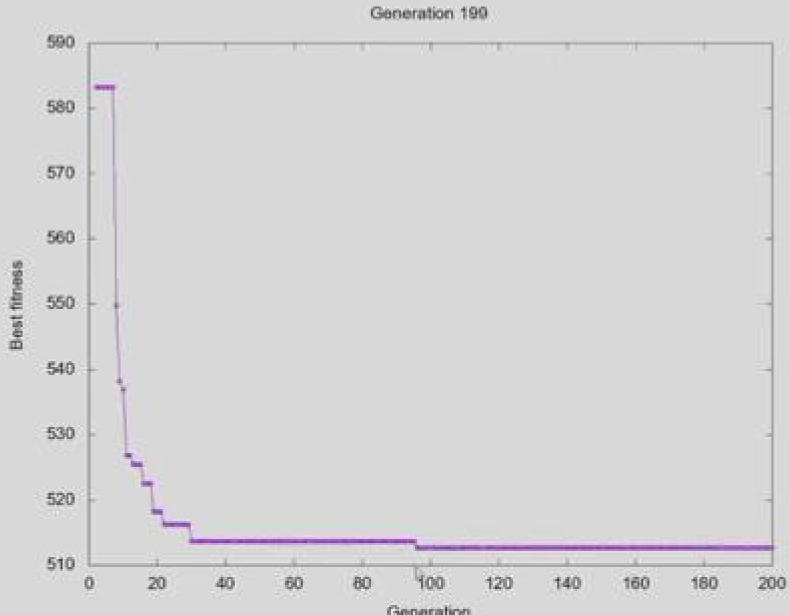
Static Penalty Approach with $R = 100$





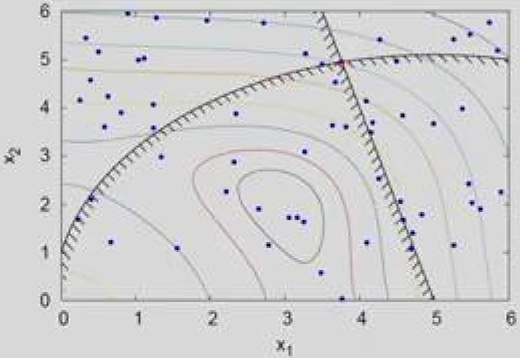
- Simulation Link
- Progress Link

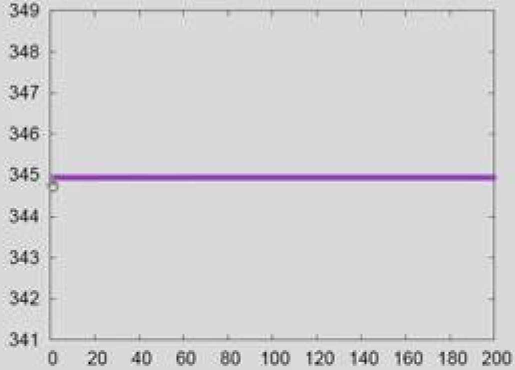

Generation 11
x2
0
1
2
3
4
5
6



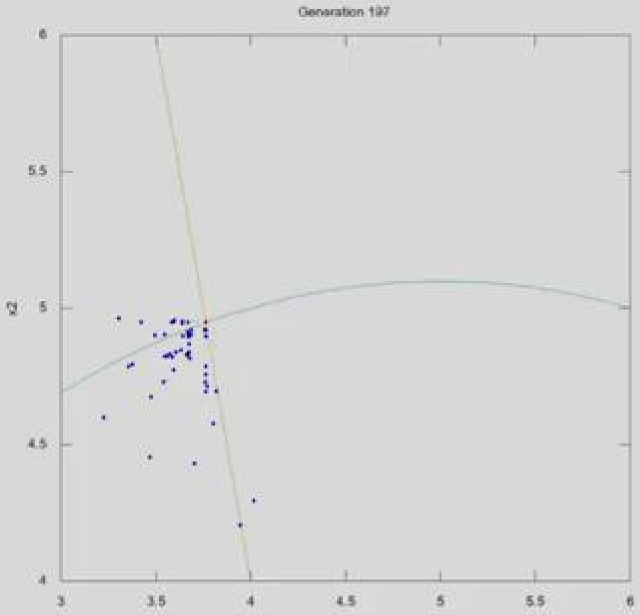
Generation 199
Best fitness
590
580
570
560
550
540
530
520
510
0
20
40
60
80
100
120
140
160
180
200
Generation

Dynamic Penalty Approach



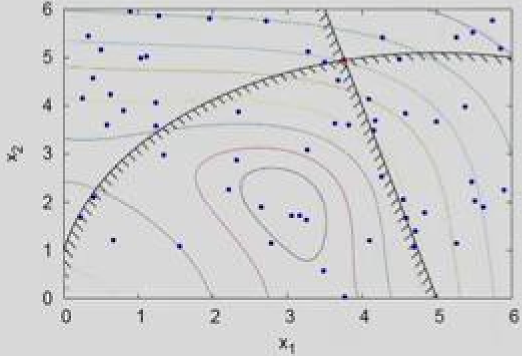


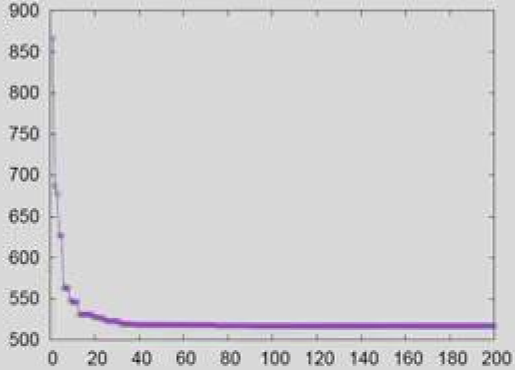
- Simulation Link
- Progress Link



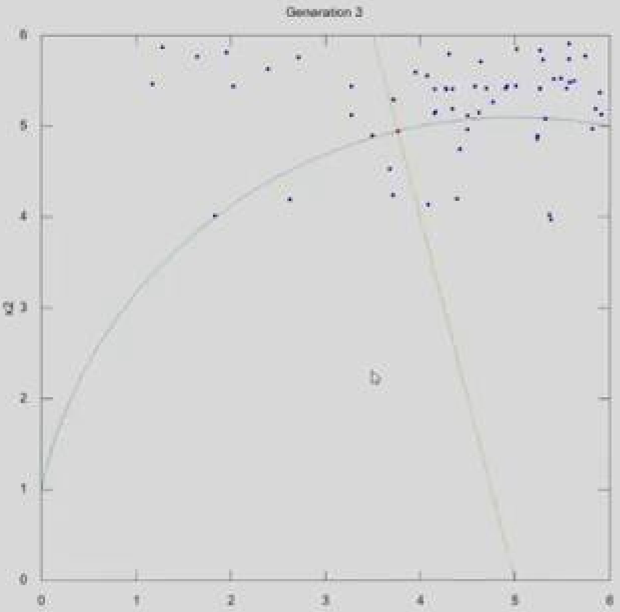
Generation 197
6
5.5
5
4.5
4
3
3.5
4
4.5
5
5.5
6

Deb's Approach

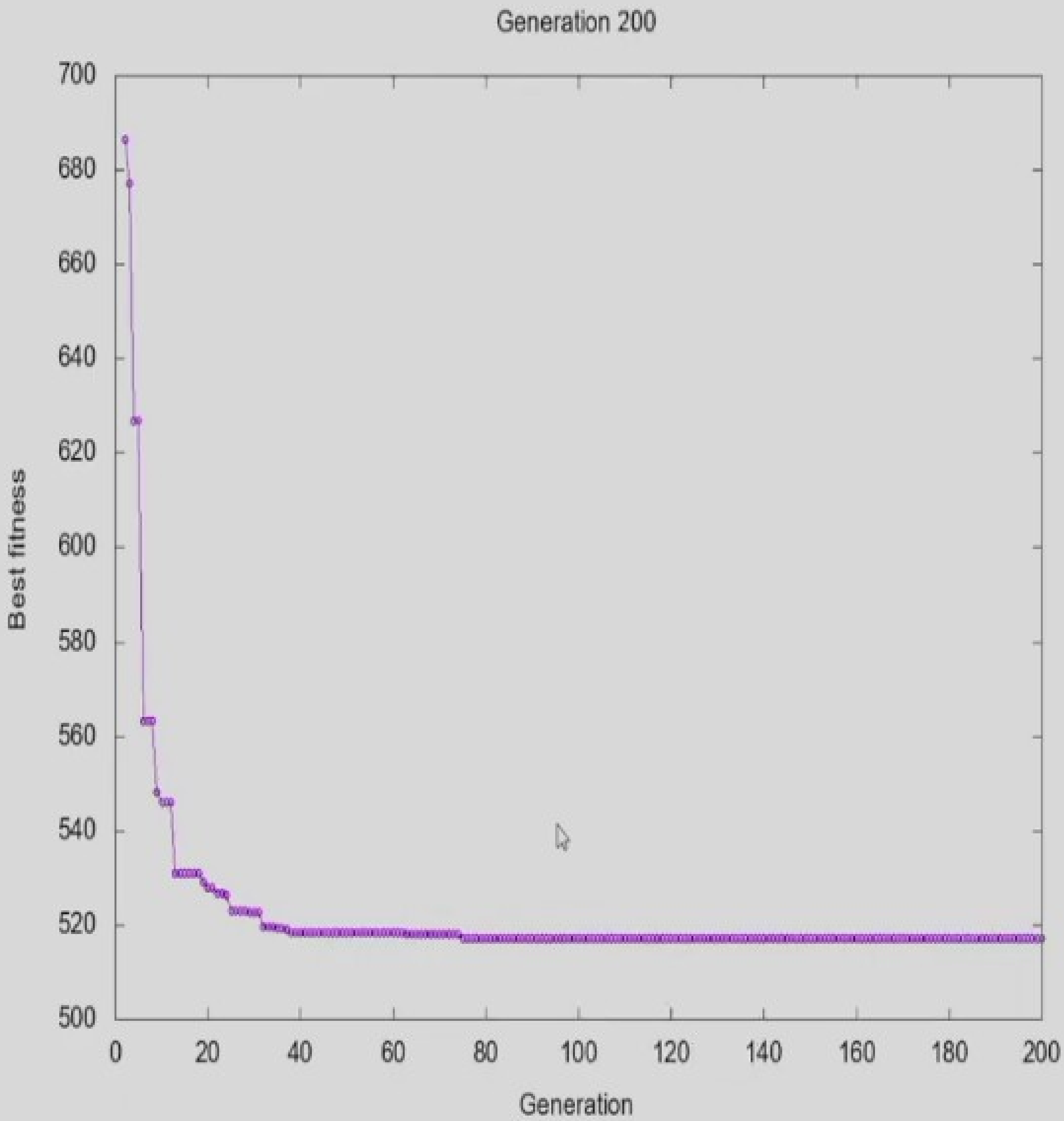




- Simulation Link
- Progress Link



Generation 3
0
1
2
3
4
5
6
0
1
2
3
4
5
6


Generation 200
700
680
660
640
620
600
580
560
540
520
500
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation

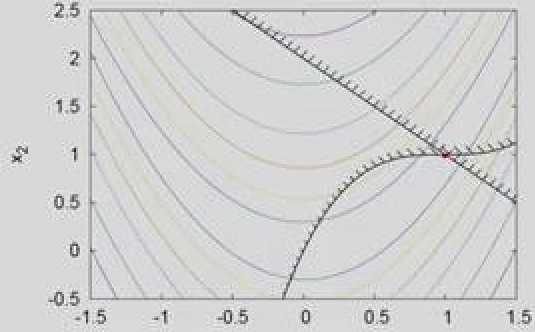
Comparison

Approaches	$f(x)$	$g_1(x)$	$g_2(x)$	x
Static Penalty($R = 2$)	127.381	-4.043	-5.994	$(2.509, 3.969)^T$
Static Penalty($R = 100$)	503.003	-0.310	-0.033	$(3.763, 4.915)^T$
Static Penalty($R = 1000$)	513.983	-0.0502	-0.0119	$(3.761, 4.941)^T$
Dynamic Penalty	106.467	-19.090	-0.441	$(4.259, 2.522)^T$
Deb's Approach	517.063	0.011	0.000	$(3.763, 4.948)^T$

Constrained Rosenbrock Function

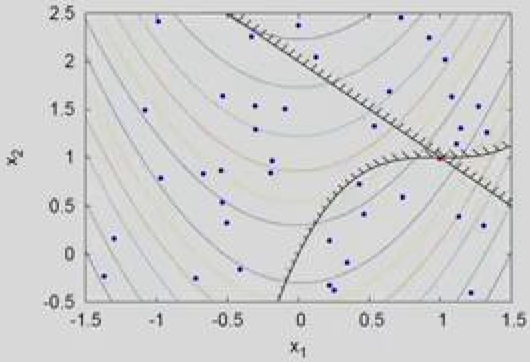
Rosenbrock Function

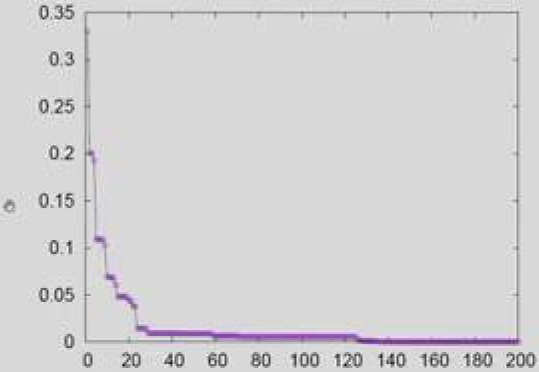
$$
\begin{array}{rl}
\text{Minimize} & f(x_1, x_2) = \left(100(x_2 - x_1^2)^2 + (1 - x_1)^2\right), \\
\text{subject to} & -(x_1 - 1)^3 + x_2 - 1 \geq 0, \\
& 2 - x_1 - x_2 \geq 0, \\
& -1.5 \leq x_1 \leq 1.5, -0.5 \leq x_2 \leq 2.5.
\end{array}
$$



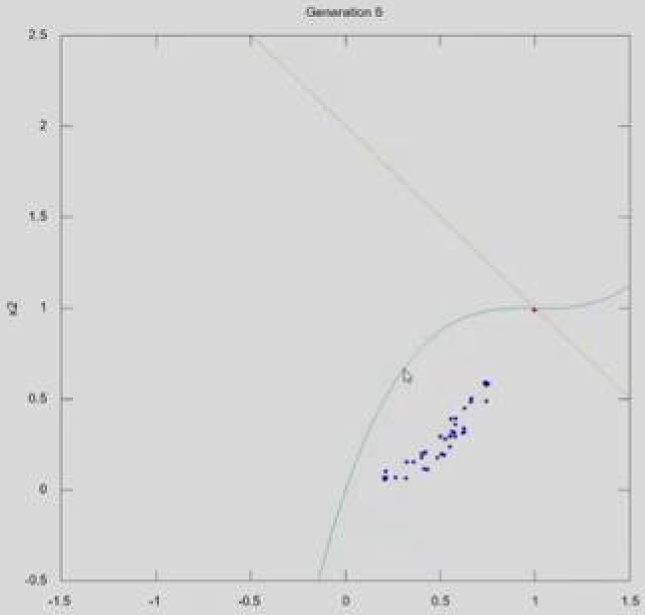
- Optimal solution is $x^* = (1, 1)^T$ and $f(x^*) = 0$

Static Penalty Approach with $R = 2$

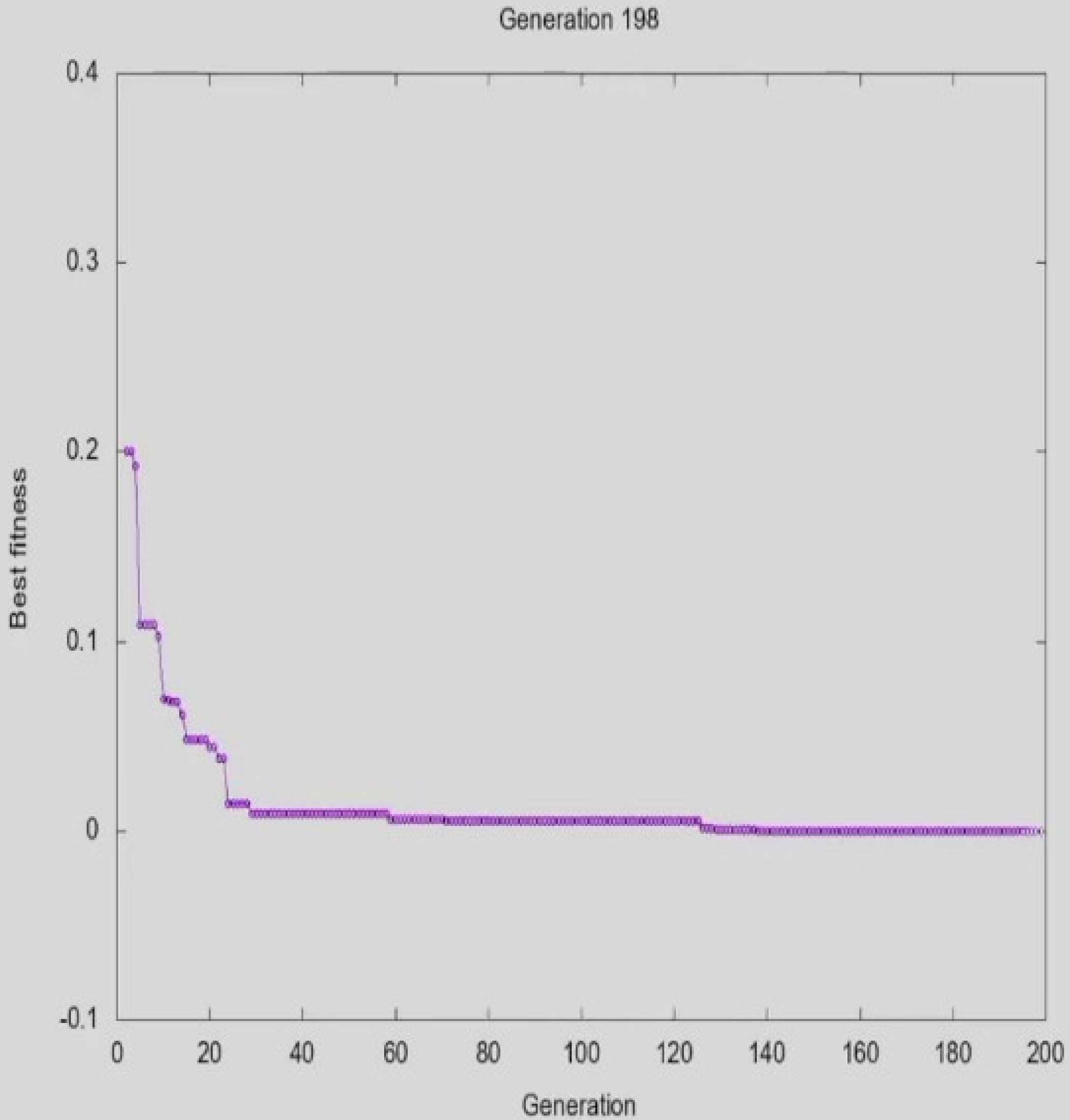




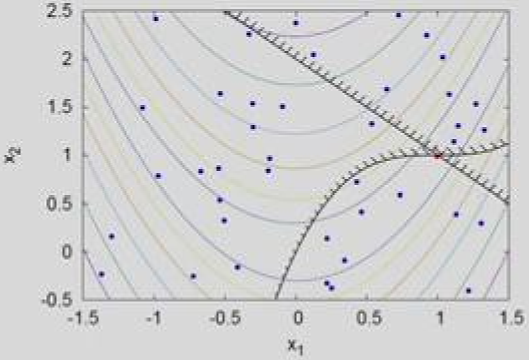
- Simulation Link
- Progress Link

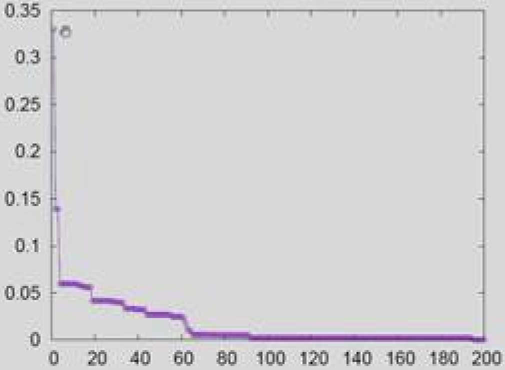


Generation 6
2.5
2
1.5
1
0.5
0
-0.5
-1.5
-1
-0.5
0
0.5
1
1.5

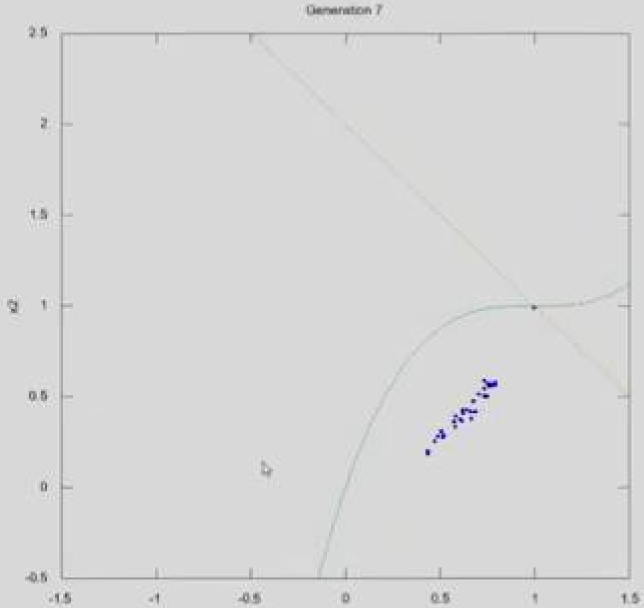

Generation 198
Best fitness
Generation
0.4
0.3
0.2
0.1
0
-0.1
0
20
40
60
80
100
120
140
160
180
200

Static Penalty Approach with $R = 100$

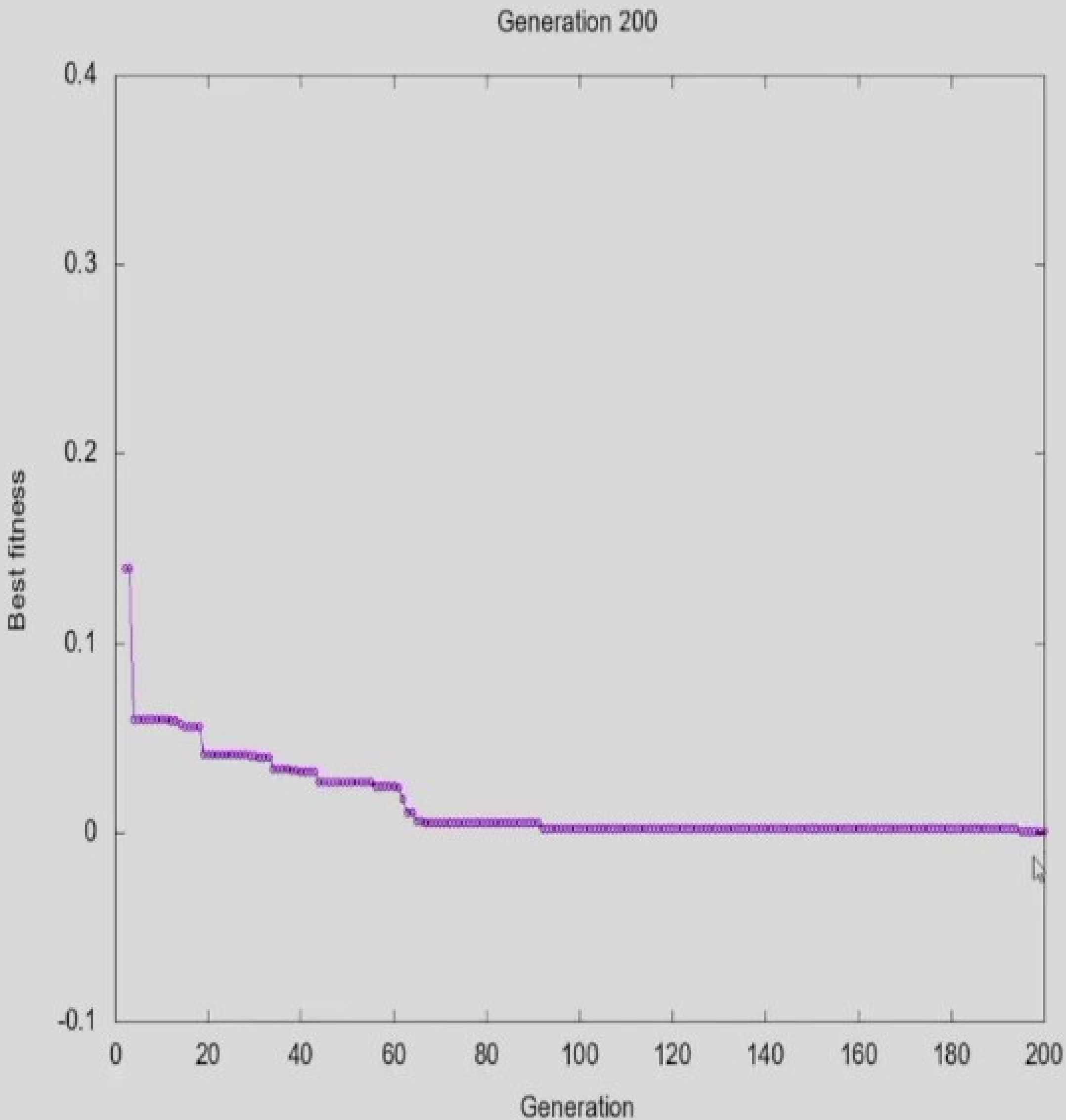




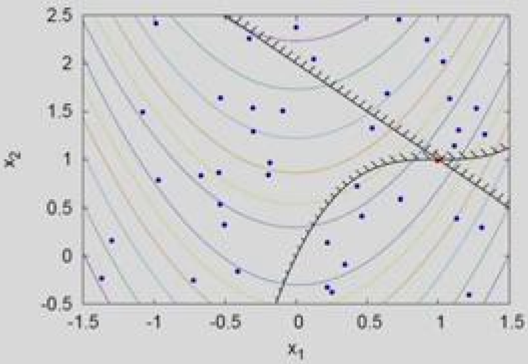
- Simulation Link
- Progress Link

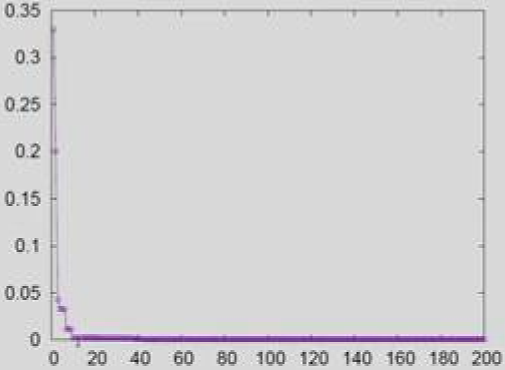


Generation 7
2.5
2
1.5
1
0.5
0
-0.5
-1.5
-1
-0.5
0
0.5
1
1.5

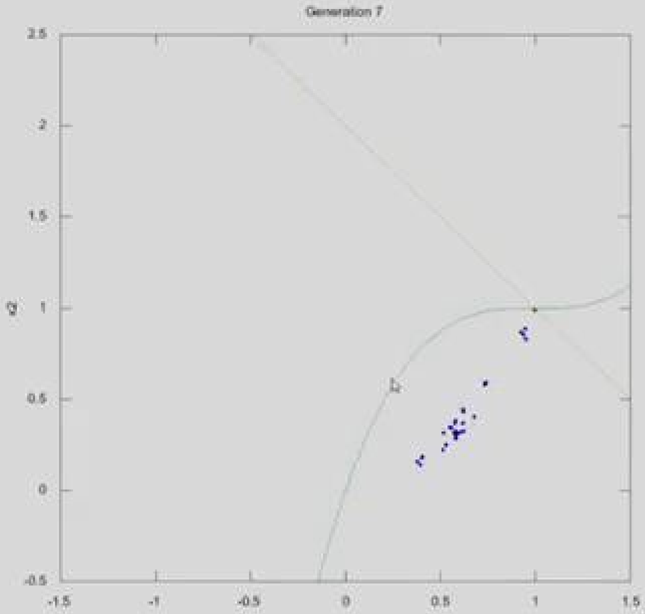

Generation 200
Best fitness
0.4
0.3
0.2
0.1
0
-0.1
0
20
40
60
80
100
120
140
160
180
200
Generation

Dynamic Penalty Approach

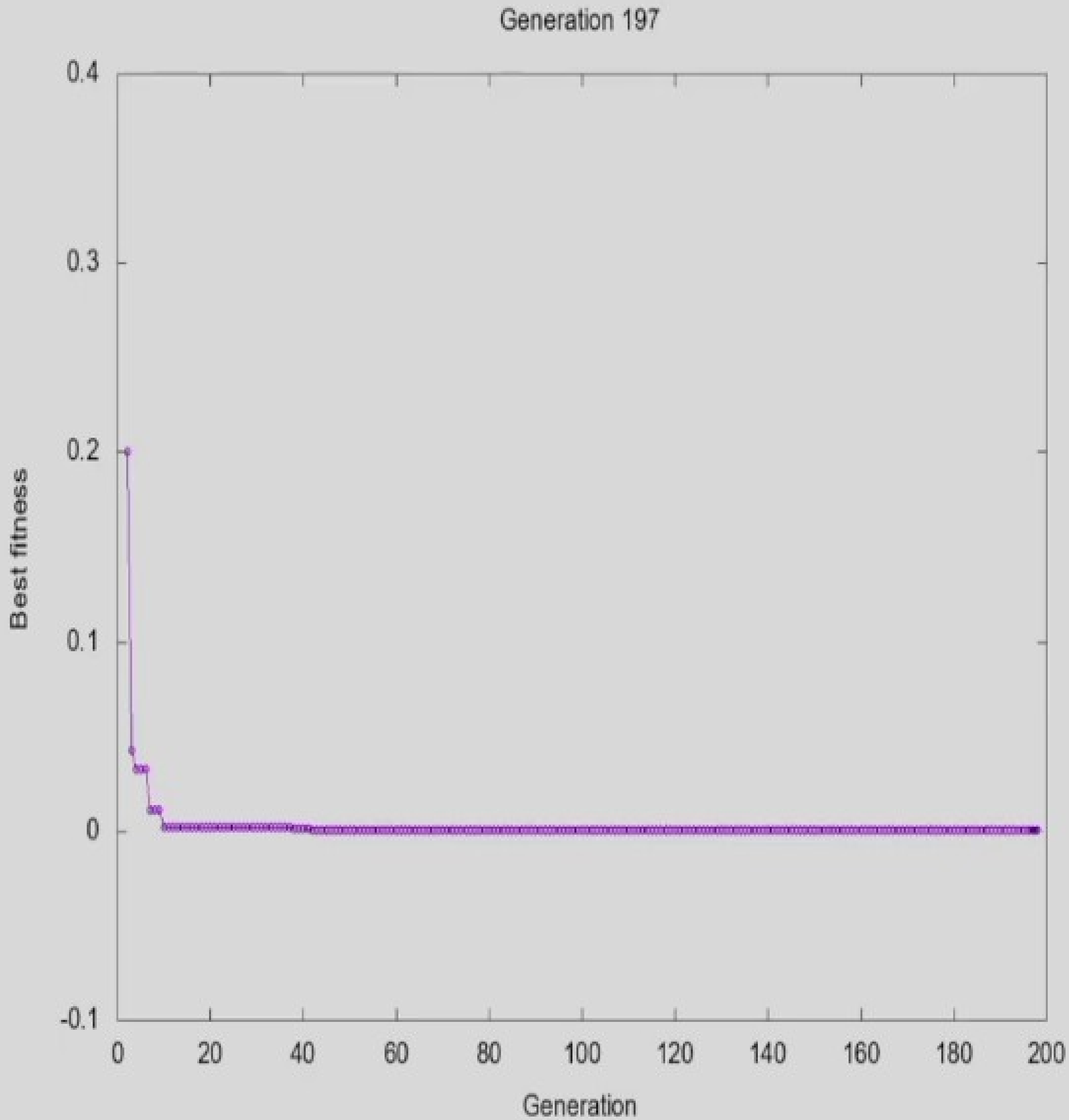




- Simulation Link
- Progress Link

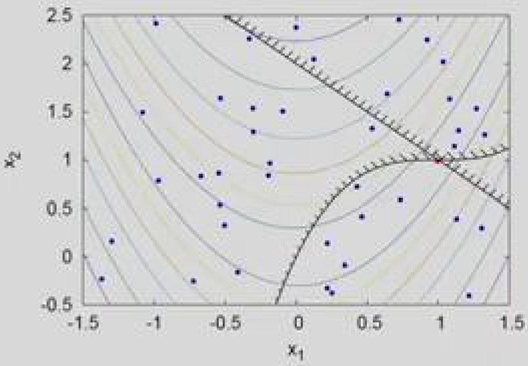


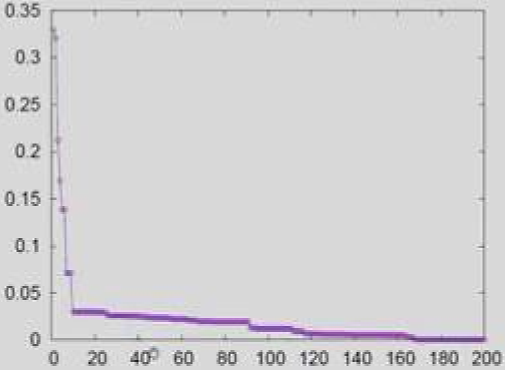
Generation 7
2.5
2
1.5
1
0.5
0
-0.5
-1.5
-1
-0.5
0
0.5
1
1.5



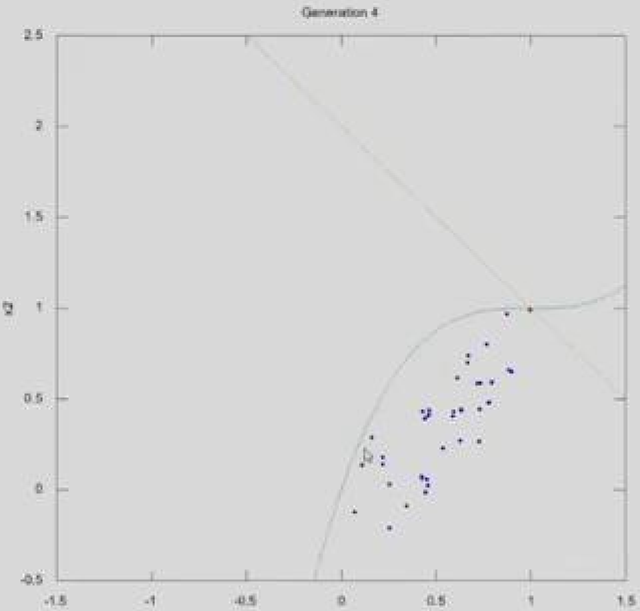
Generation 197
Best fitness
0.4
0.3
0.2
0.1
0
-0.1
0
20
40
60
80
100
120
140
160
180
200
Generation

Deb's Approach

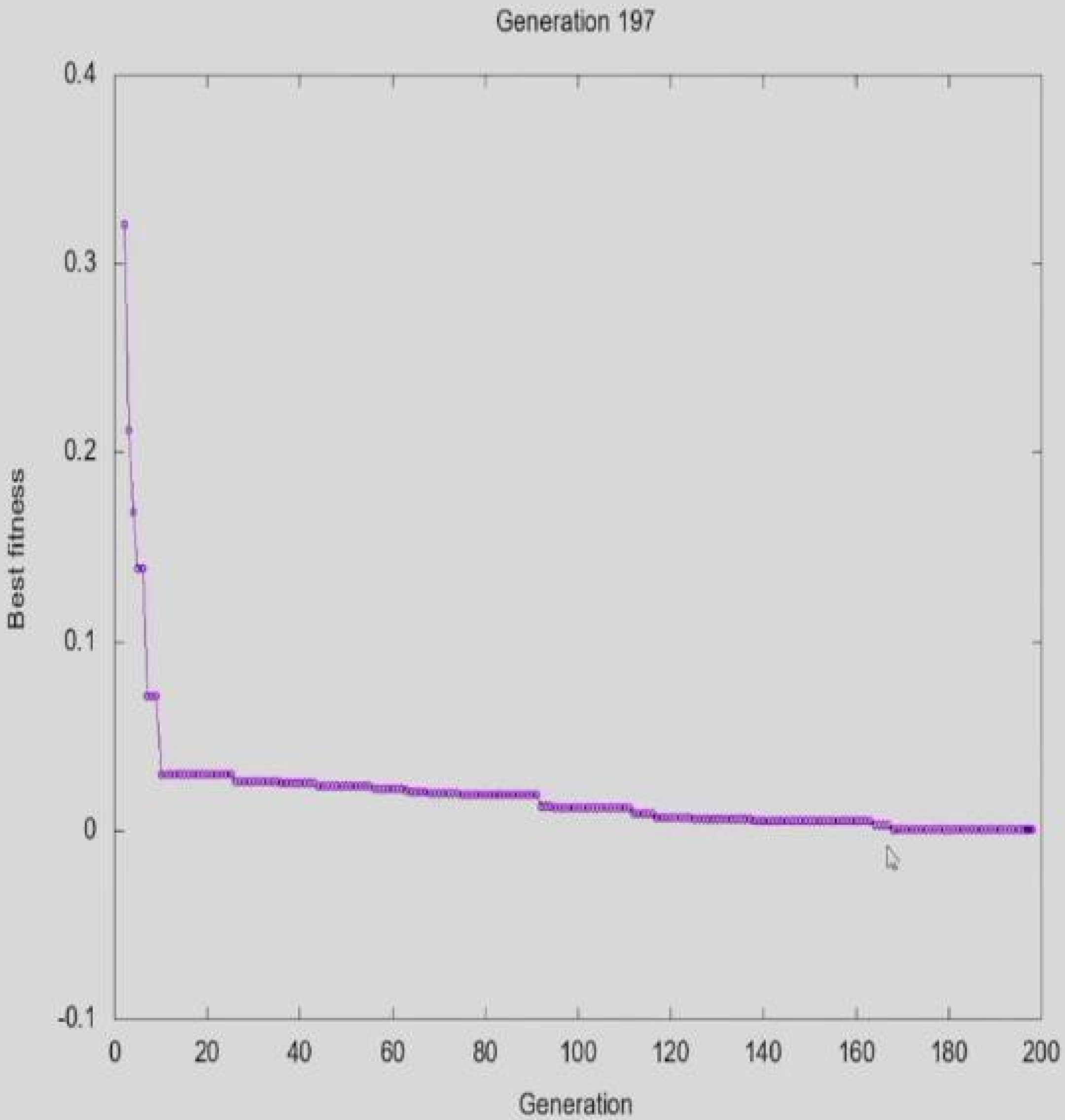




- Simulation Link
- Progress Link



Generation 4
2.5
2
1.5
1
0.5
0
-0.5
-1.5
-1
-0.5
0
0.5
1
1.5


Generation 197
Best fitness
0.4
0.3
0.2
0.1
0
-0.1
0
20
40
60
80
100
120
140
160
180
200
Generation

Comparison

Approaches	$f(x)$	$g_1(x)$	$g_2(x)$	x
Static Penalty($R = 2$)	0.0004366116	0.041	0.062	$(0.979, 0.959)^T$
Static Penalty($R = 100$)	0.00004868493	0.014	0.021	$(0.993, 0.986)^T$
Dynamic Penalty	0.00008667416	0.019	0.028	$(0.991, 0.981)^T$
Deb's Approach	0.0002379794	0.031	0.046	$(0.985, 0.969)^T$

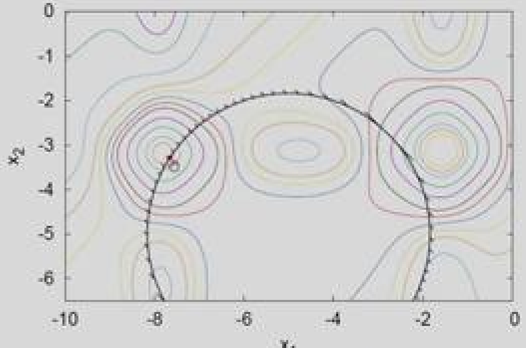
Constrained Bird Function

Bird Function

Minimize $f(x_1, x_2) = \sin(x_2)\exp((1-\cos(x_1))^2) + \cos(x_1)\exp((1-\sin(x_2))^2) + (x_1 - x_2)^2$

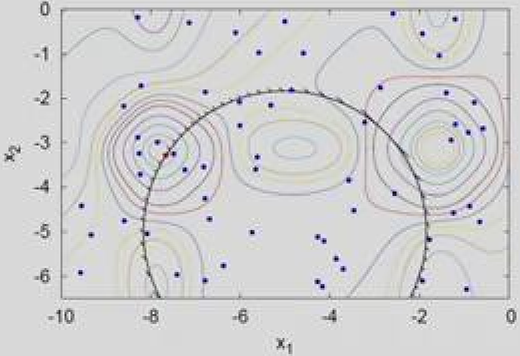
subject to $(x_1+1)^2 + (x_2+5)^2 \leq 10,$

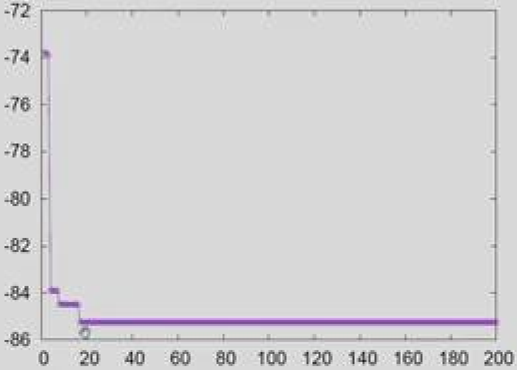
$-10 \leq x_1 \leq 0, -6.5 \leq x_2 \leq 0.$



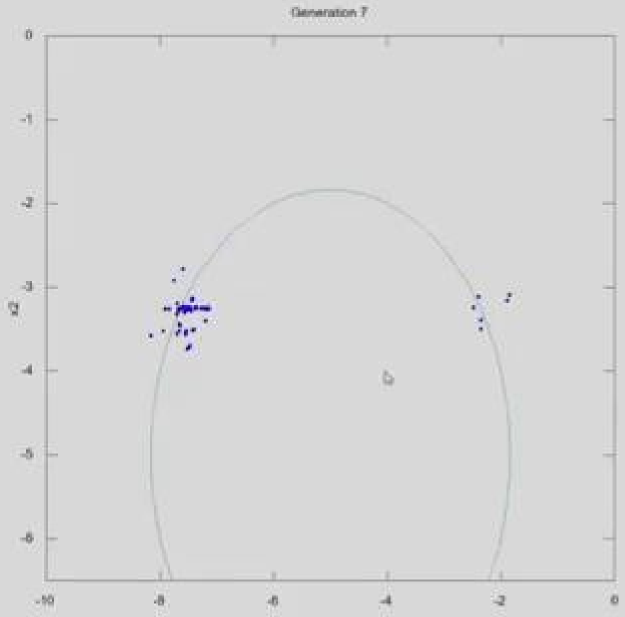
- Optimal solution is $x^* = (-7.653, -3.279)^T$ and $f(x^*) = -82.308$

Static Penalty Approach with $R = 2$

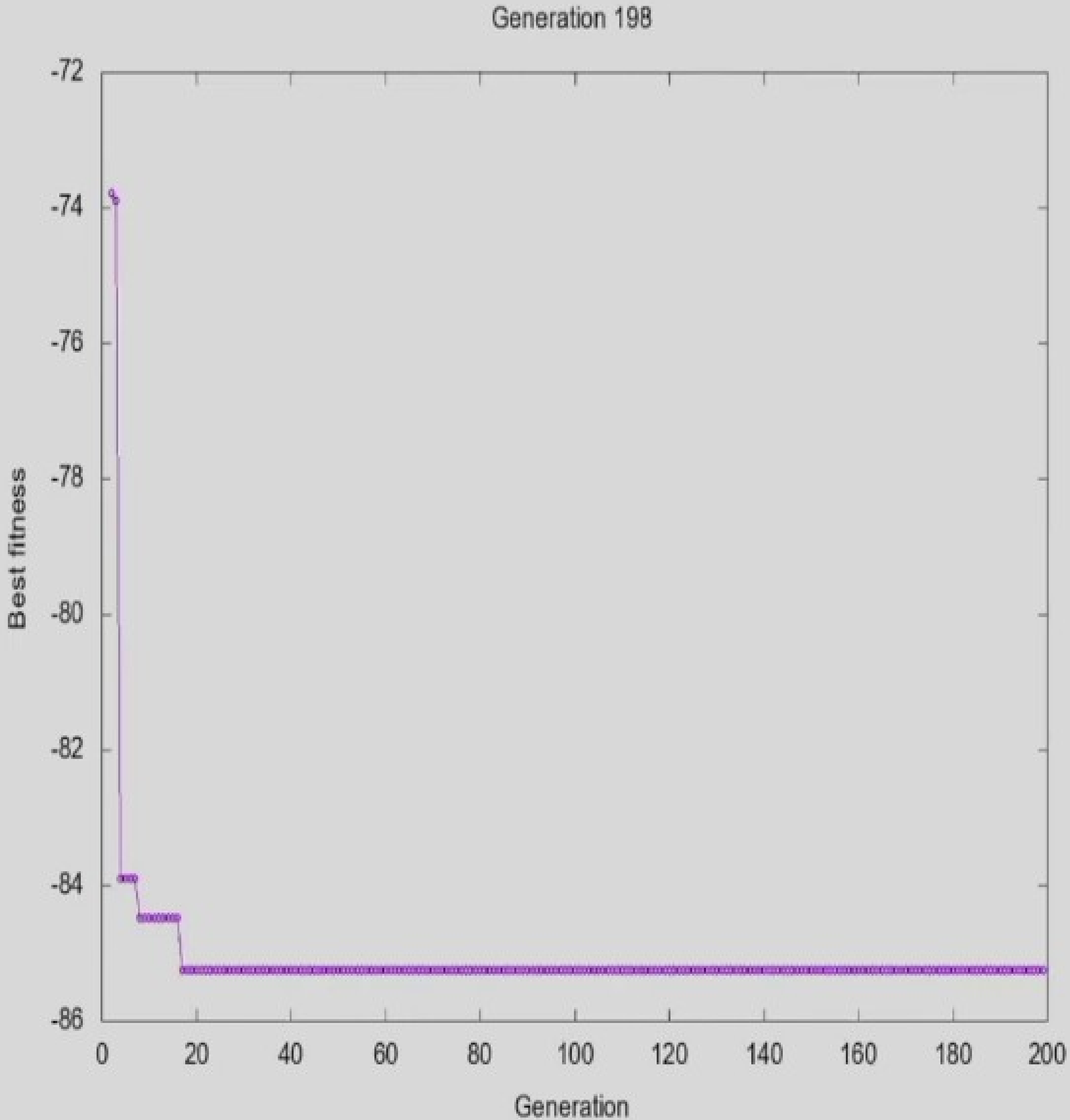




- Simulation Link
- Progress Link

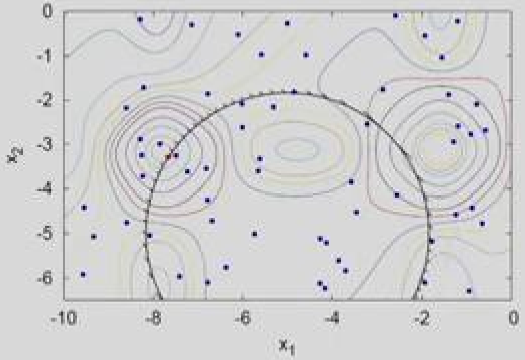


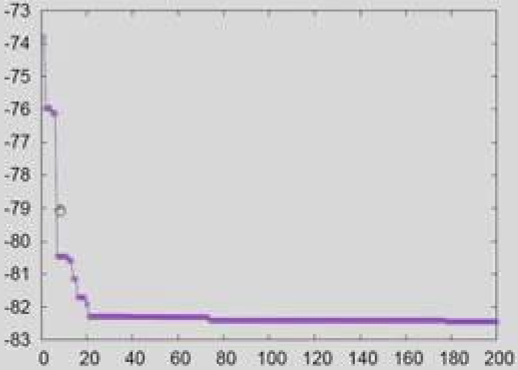
Generation 7
0
-1
-2
-3
-4
-5
-6
-10
-8
-6
-4
-2
0



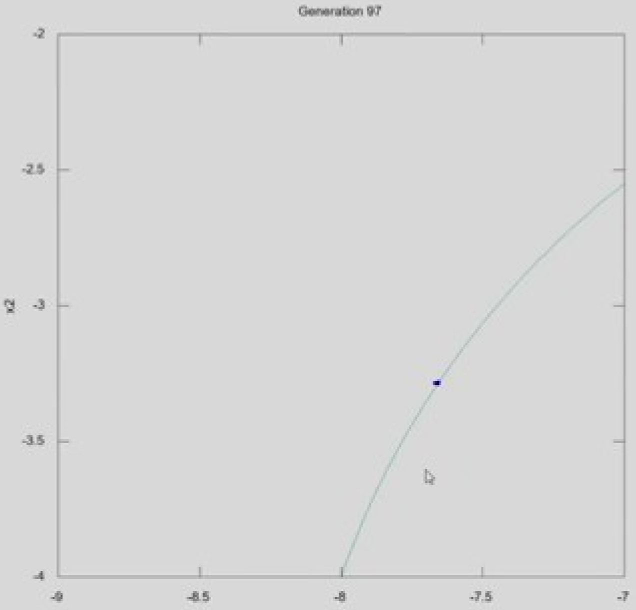
Generation 198
Best fitness
Generation
-72
-74
-76
-78
-80
-82
-84
-86
0
20
40
60
80
100
120
140
160
180
200

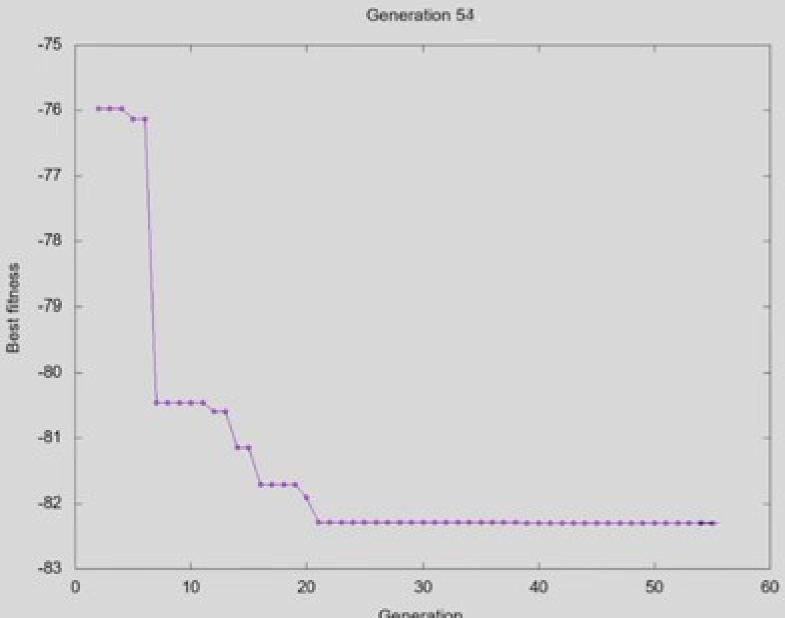
Static Penalty Approach with $R = 100$



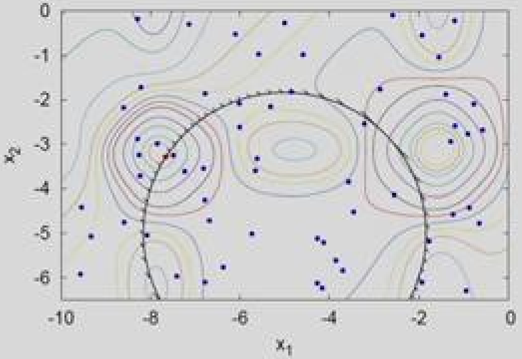


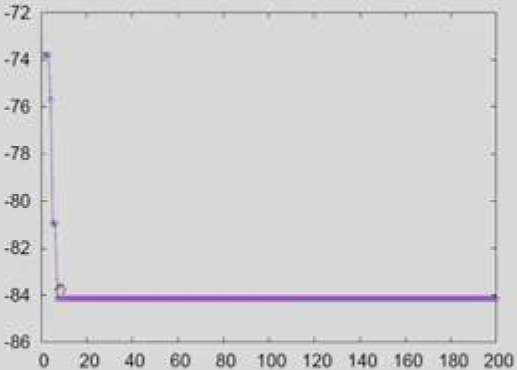
- Simulation Link
- Progress Link


Generation 97
x2
-2
-2.5
-3
-3.5
-4
-9
-8.5
-8
-7.5
-7

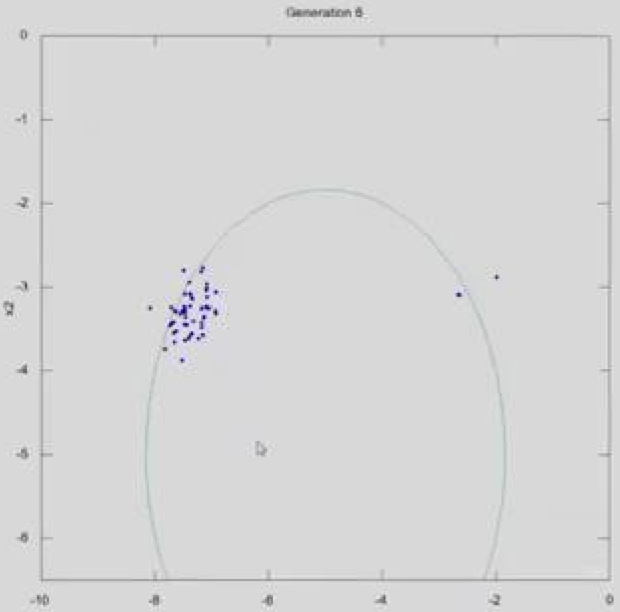

Generation 54
Best fitness
Generation
-75
-76
-77
-78
-79
-80
-81
-82
-83
0
10
20
30
40
50
60

Dynamic Penalty Approach

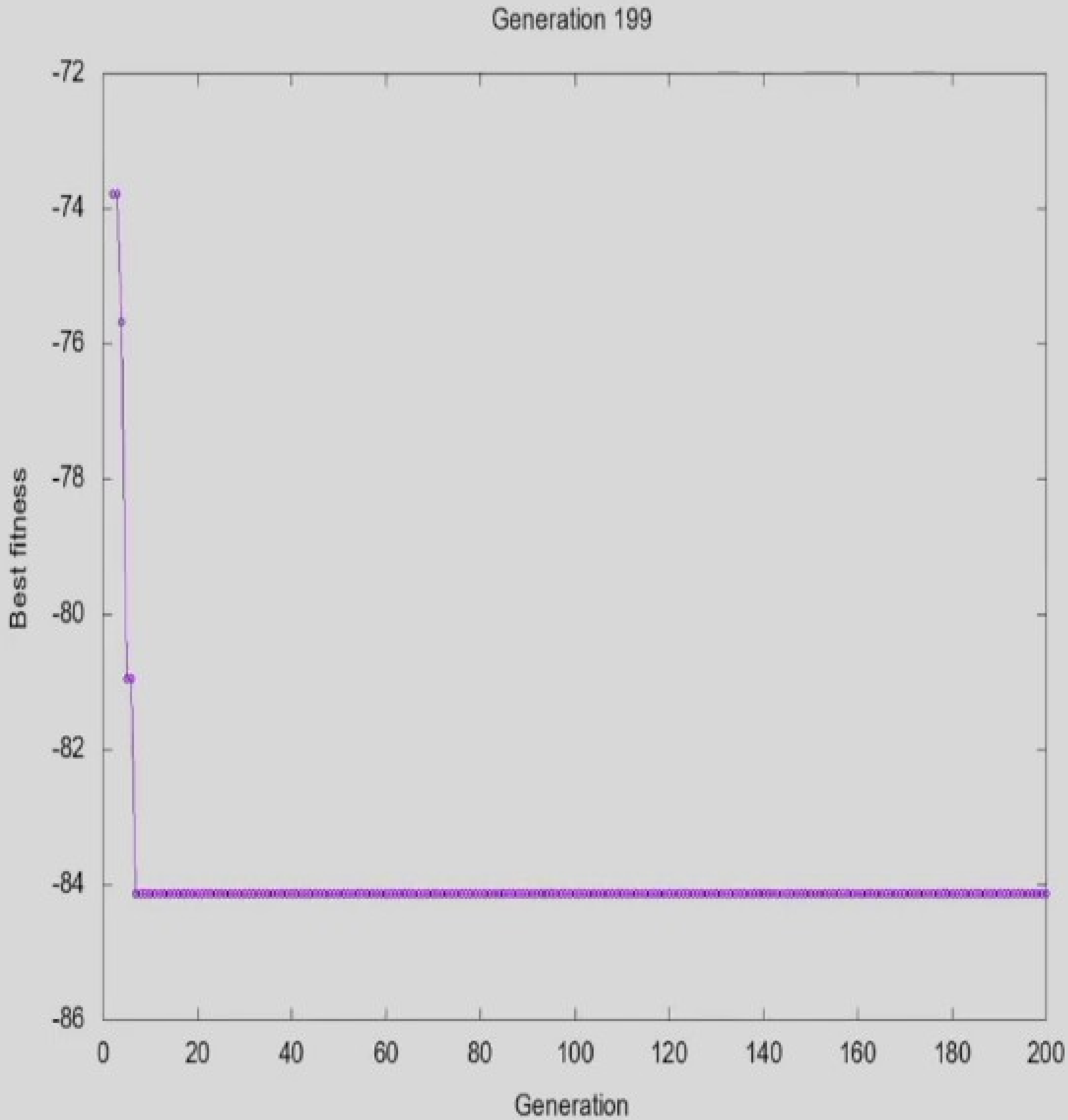




- Simulation Link
- Progress Link

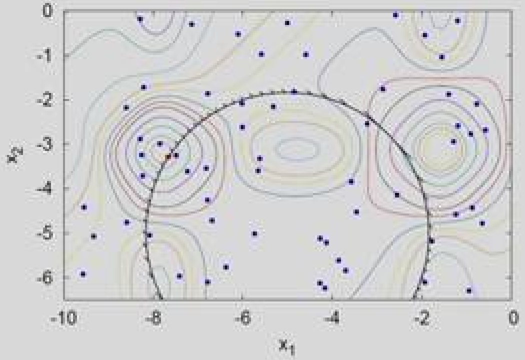


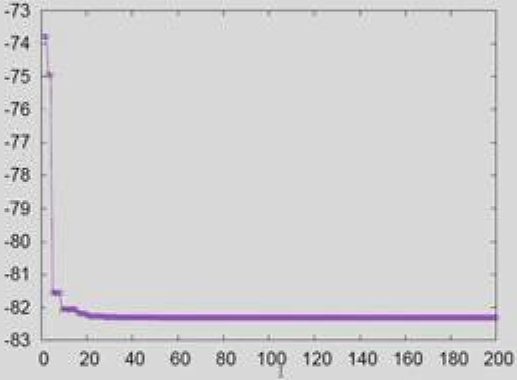
Generation 6
0
-1
-2
-3
-4
-5
-6
-10
-8
-6
-4
-2
0



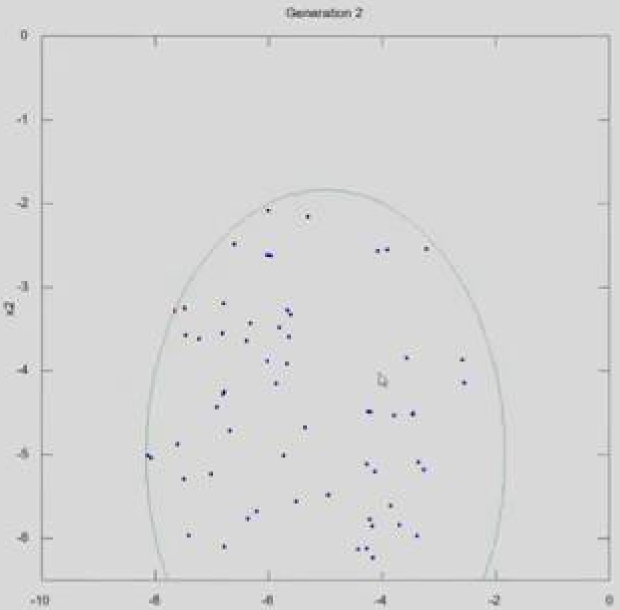
Generation 199
-72
-74
-76
-78
-80
-82
-84
-86
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation

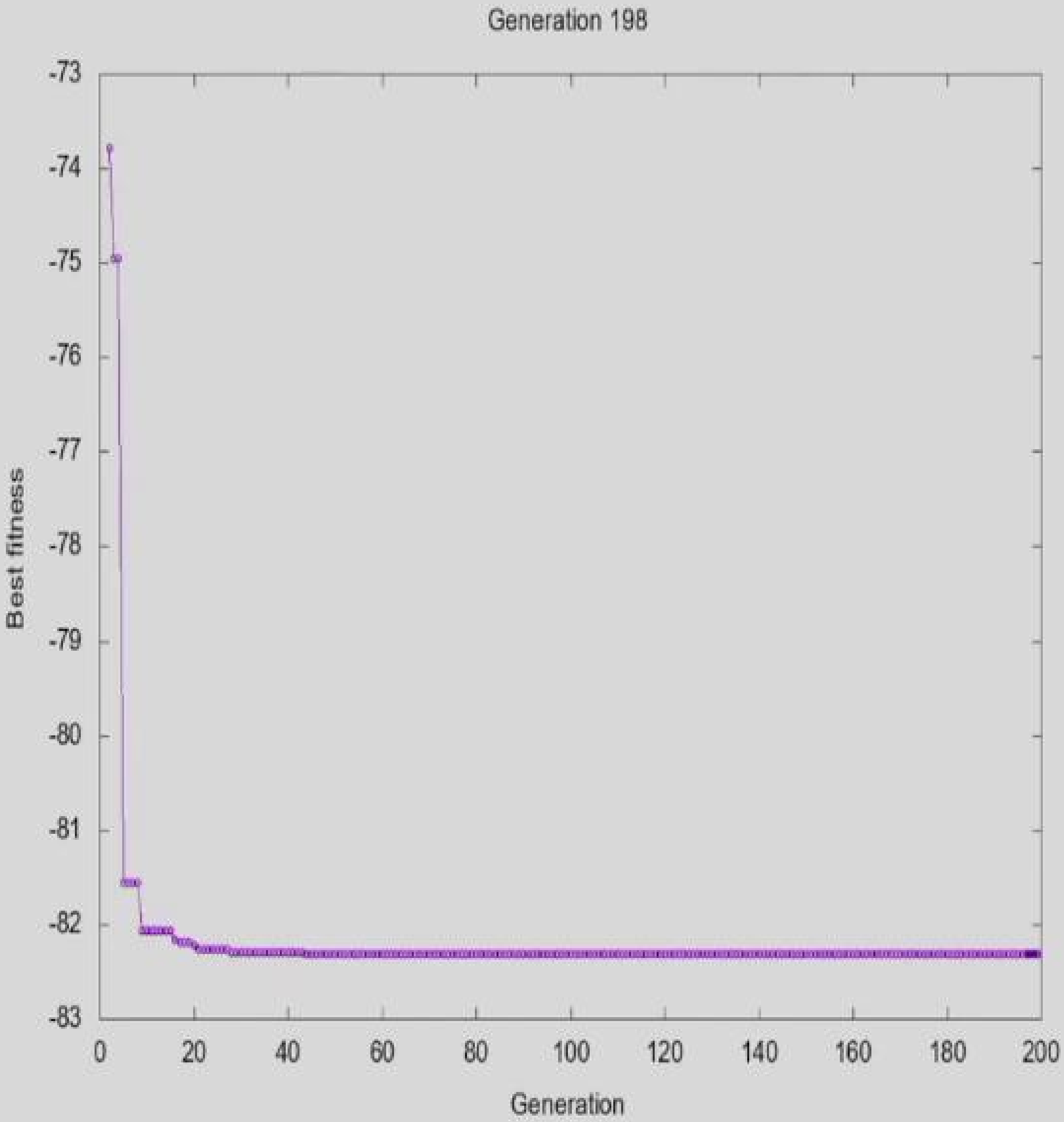
Deb's Approach





- Simulation Link
- Progress Link


Generation 2
0
-1
-2
-3
-4
-5
-6
x2
-10
-8
-6
-4
-2
0



Generation 198
Best fitness
Generation
-73
-74
-75
-76
-77
-78
-79
-80
-81
-82
-83
0
20
40
60
80
100
120
140
160
180
200

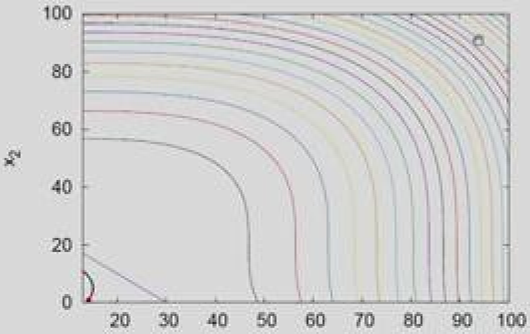
Comparison

Approaches	$f(x)$	$g_1(x)$	x
Static Penalty($R = 2$)	-86.369	-0.750	$(-7.766, -3.240)^T$
Static Penalty($R = 100$)	-82.569	-0.041	$(-7.649, -3.260)^T$
Dynamic Penalty	-84.335	-0.313	$(-7.680, -3.230)^T$
Deb's Approach	-82.308	0	$(-7.653, -3.279)^T$

Constrained g06 CEC 2006 Function

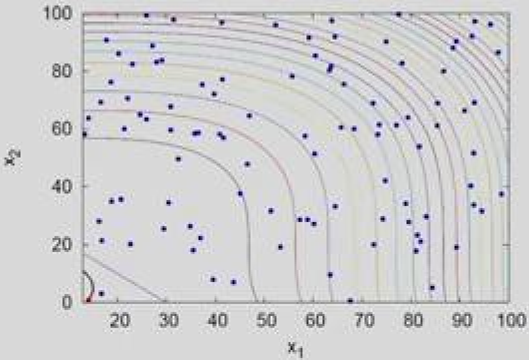
g06 CEC 2006 function

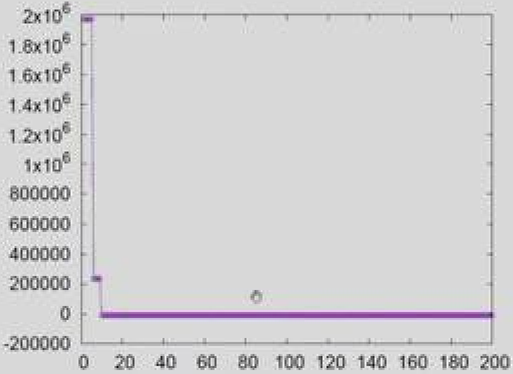
$$\begin{aligned} \text{Minimize} \quad & f(x_1, x_2) = (x_1 - 10)^3 + (x_2 - 20)^3, \\ \text{subject to} \quad & (x_1 - 1)^2 + (x_2 - 5)^2 - 100 \geq 0, \\ & 82.81 - (x_1 - 6)^2 - (x_2 - 5)^2 \geq 0, \\ & 13 \leq x_1 \leq 100, 0 \leq x_2 \leq 100. \end{aligned}$$



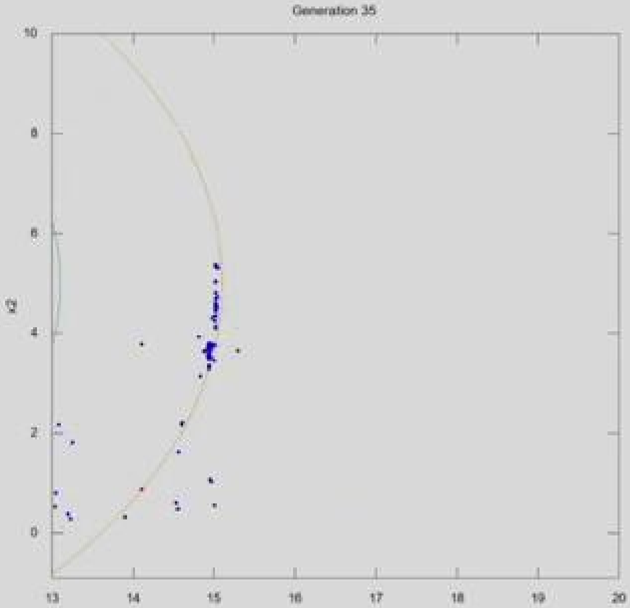
- Optimal solution is $x^* = (1.4111, 0.8792)^T$ and $f(x^*) = -6921.072$

Static Penalty Approach with $R = 2$

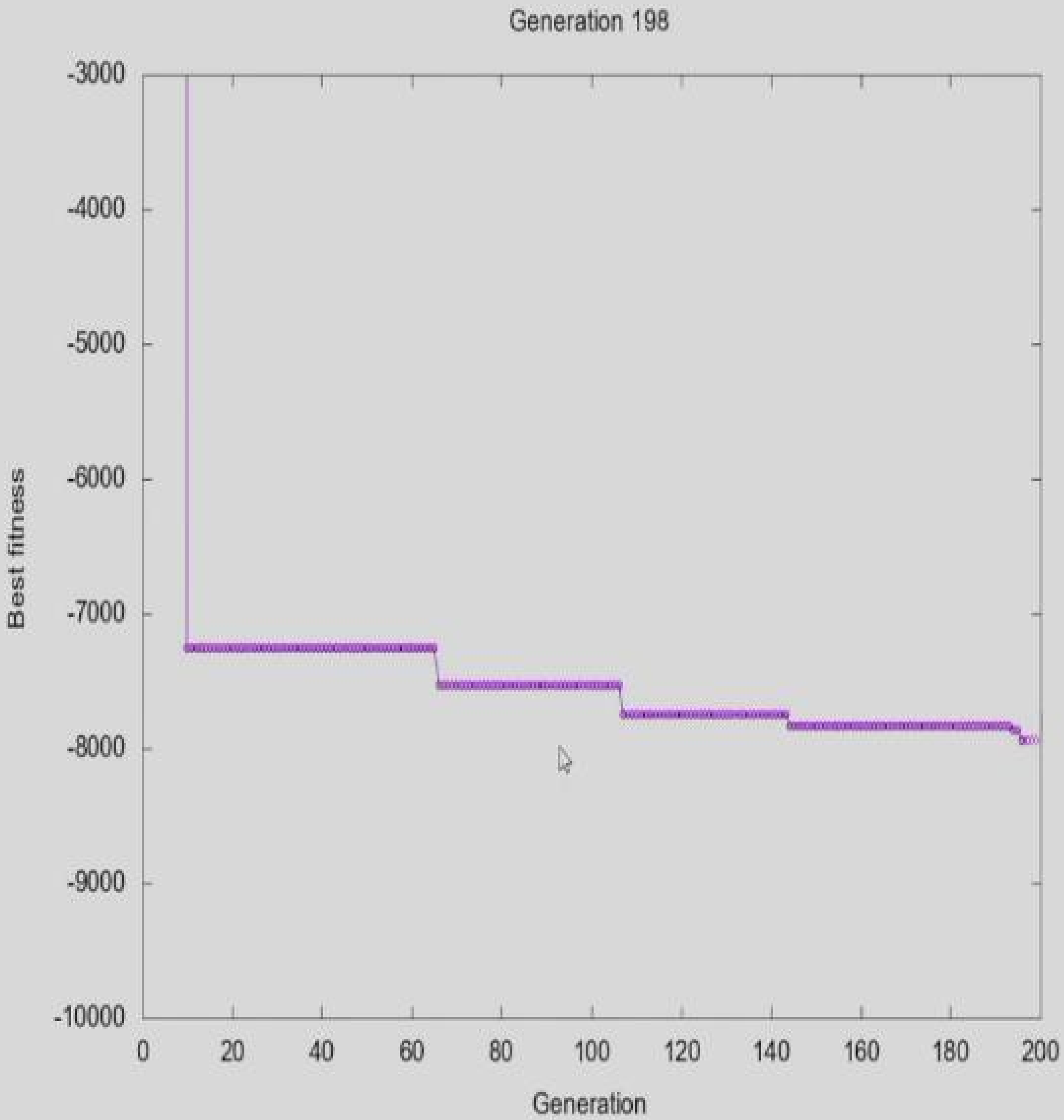




- Simulation Link
- Progress Link

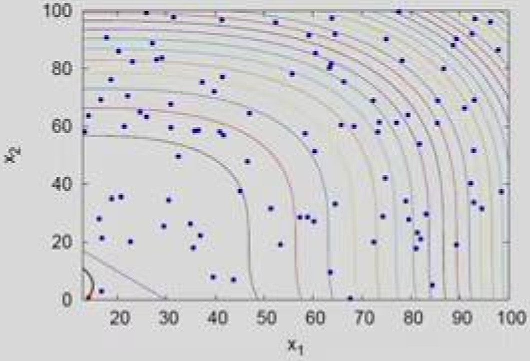


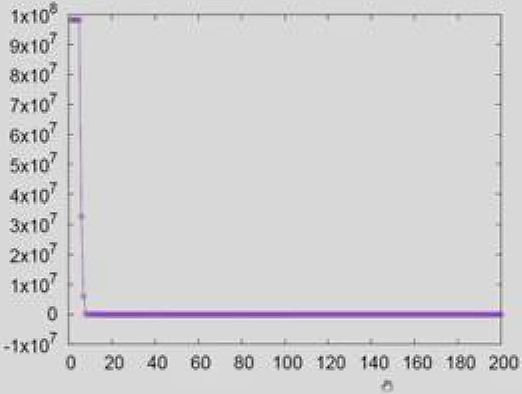
Generation 35
10
8
6
4
2
0
x2
13
14
15
16
17
18
19
20



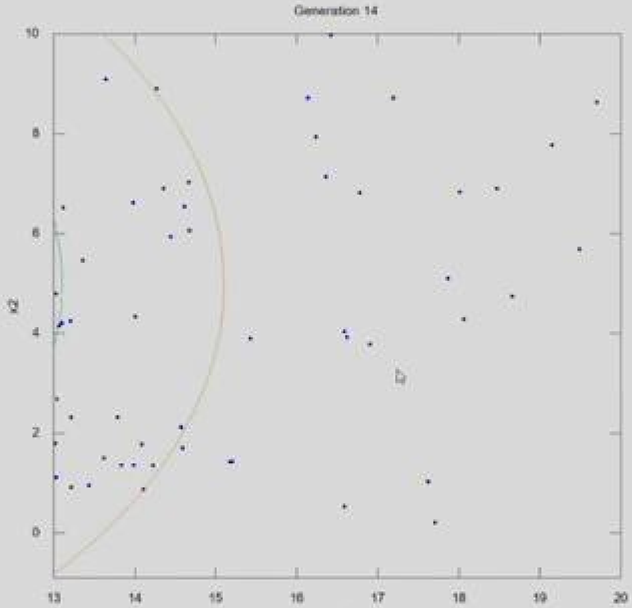
Generation 198
-3000
-4000
-5000
-6000
-7000
-8000
-9000
-10000
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation

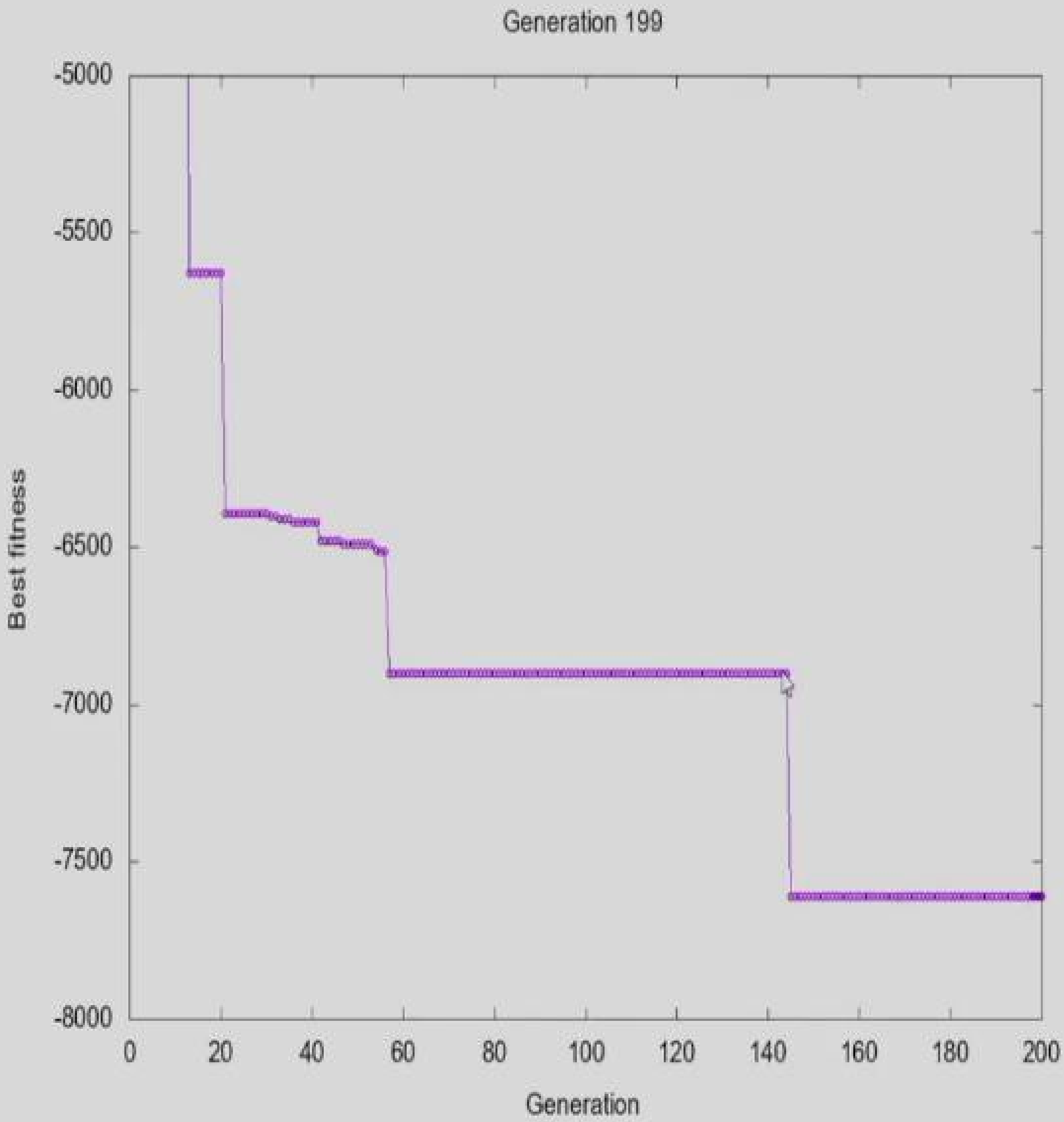
Static Penalty Approach with $R = 100$



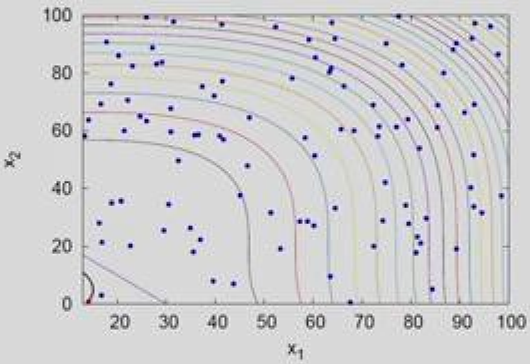


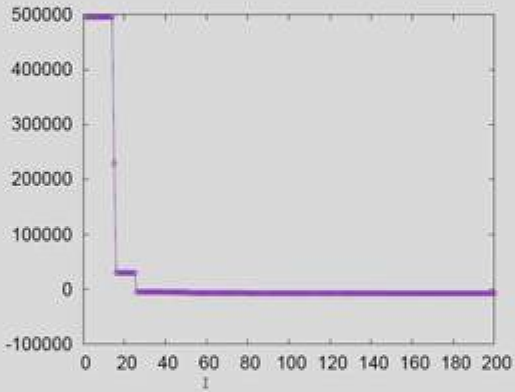
- Simulation Link
- Progress Link


Generation 14
10
8
6
4
2
0
13
14
15
16
17
18
19
20

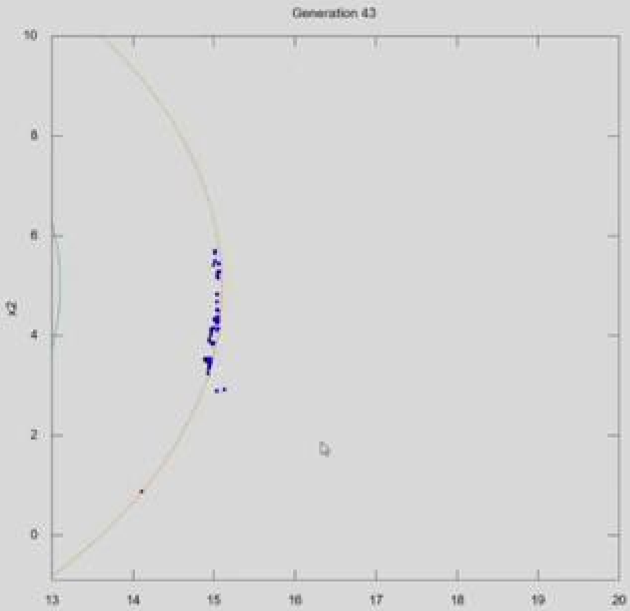

Generation 199
Best fitness
Generation
-5000
-5500
-6000
-6500
-7000
-7500
-8000
0
20
40
60
80
100
120
140
160
180
200

Dynamic Penalty Approach

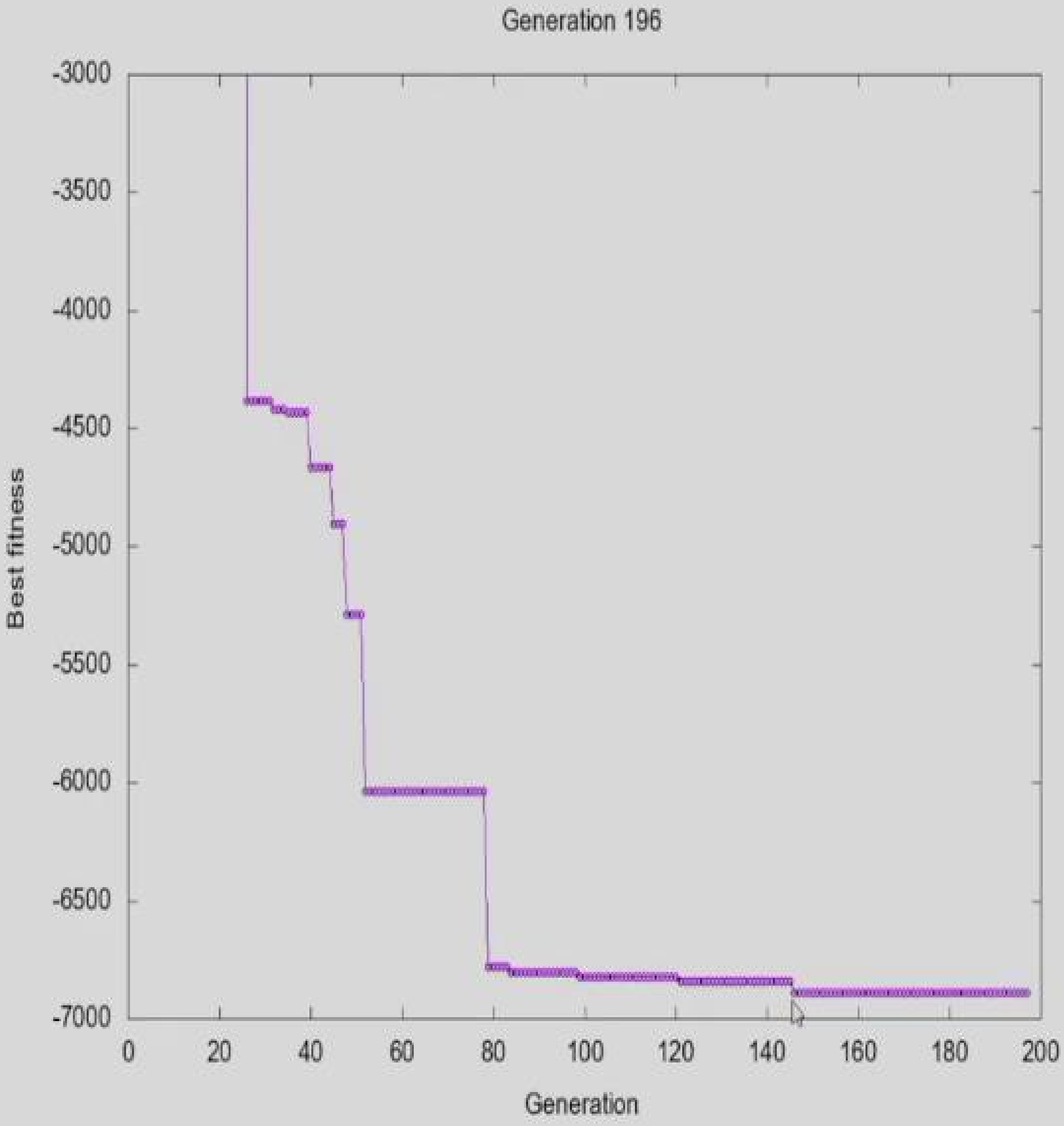




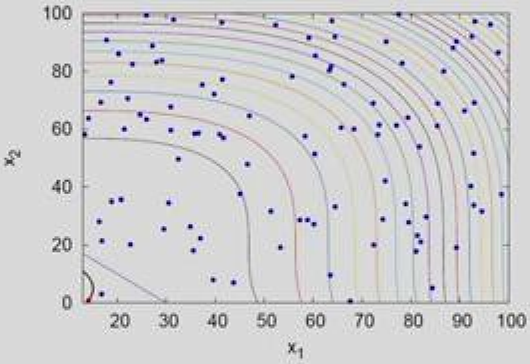
- Simulation Link
- Progress Link

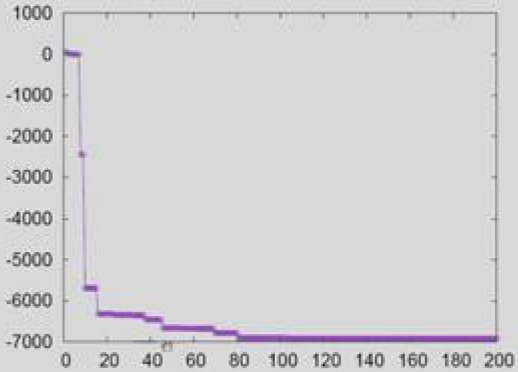


Generation 43
10
8
6
4
2
0
13
14
15
16
17
18
19
20

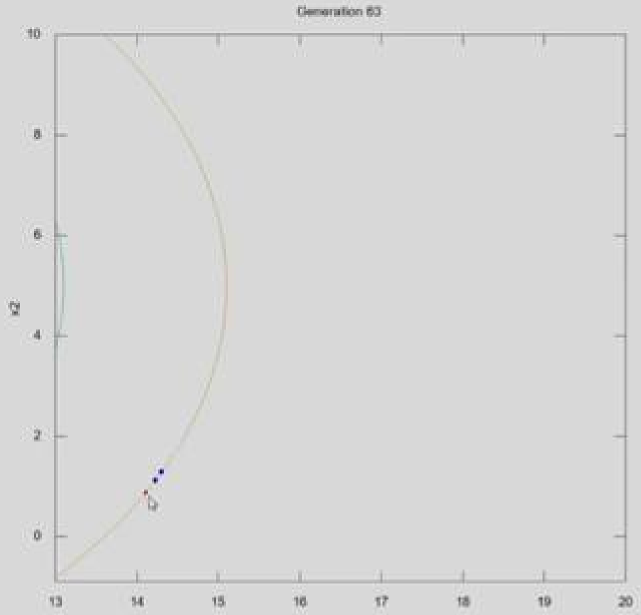

Generation 196
Best fitness
-3000
-3500
-4000
-4500
-5000
-5500
-6000
-6500
-7000
0
20
40
60
80
100
120
140
160
180
200
Generation

Deb's Approach

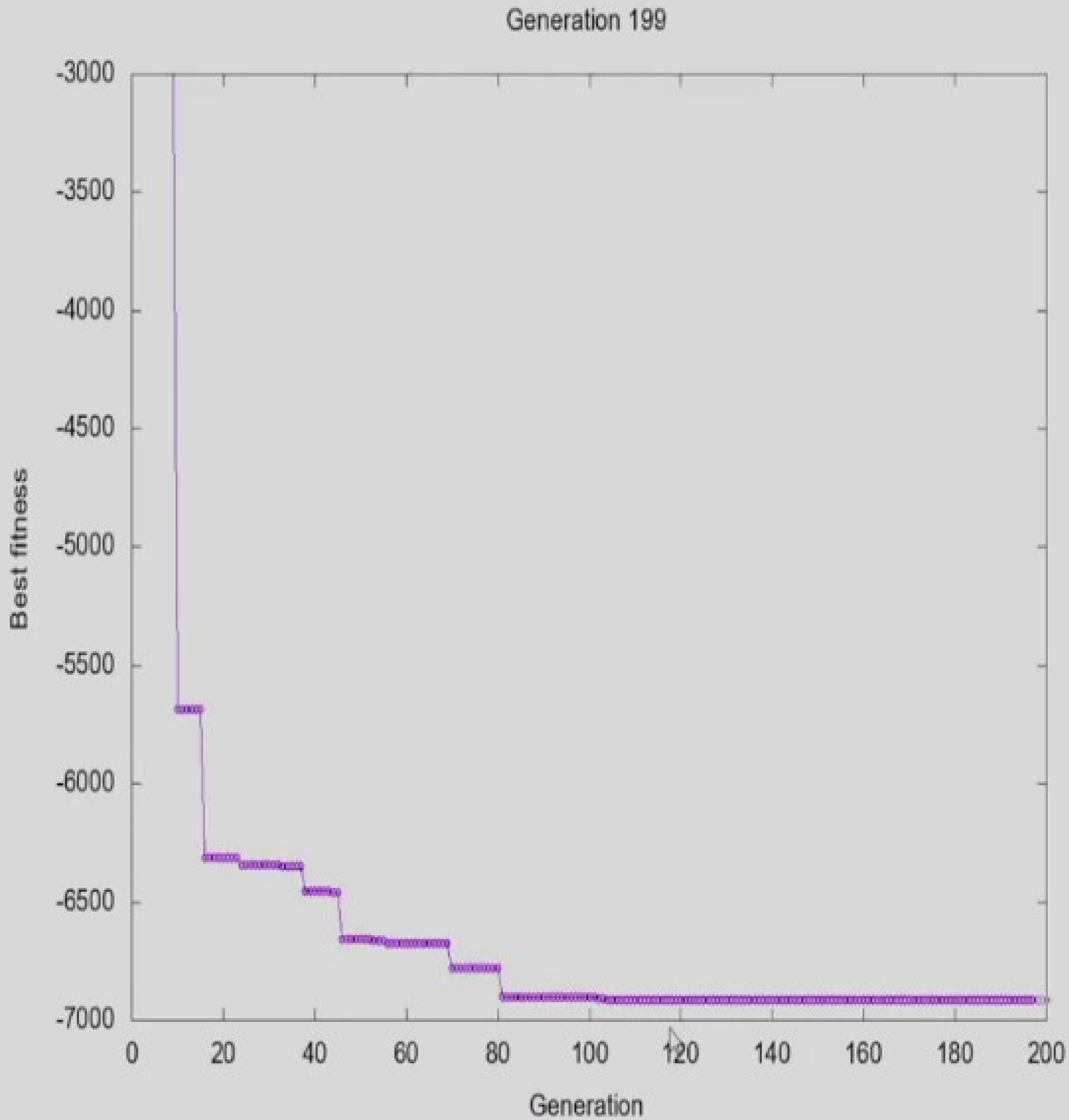




- Simulation Link
- Progress Link



Generation 63
x2
10
8
6
4
2
0
13
14
15
16
17
18
19
20


Generation 199
-3000
-3500
-4000
-4500
-5000
-5500
-6000
-6500
-7000
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation

Comparison

Approaches	$f(x)$	$g_1(x)$	$g_2(x)$	x
Static Penalty($R = 2$)	-7945.858	-1.344	0.326	$(13.586, 0.007)^T$
Static Penalty($R = 100$)	-7356.572	1.825	-1.958	$(14.029, 0.494)^T$
Dynamic Penalty	-6982.838	1.269	-1.150	$(14.155, 0.821)^T$
Deb's Approach	-6914.469	0.040	0.003	$(14.116, 0.885)^T$

Constrained g08 CEC 2006 Function

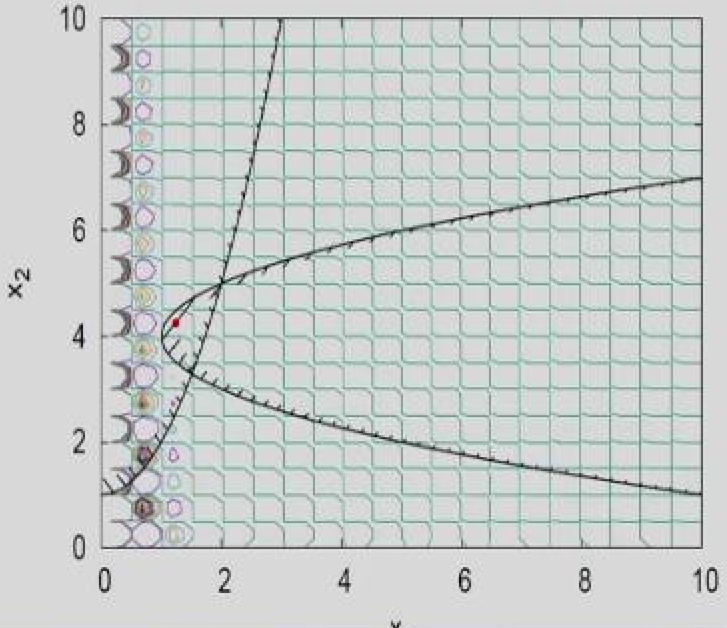
g08 CEC 2006 function

Minimize $f(x_1, x_2) = -\frac{(\sin(2\pi x_1))^3 \sin(2\pi x_2)}{x_1^3(x_1+x_2)}$,

subject to $-x_1^2 + x_2 + 1 \geq 0$,

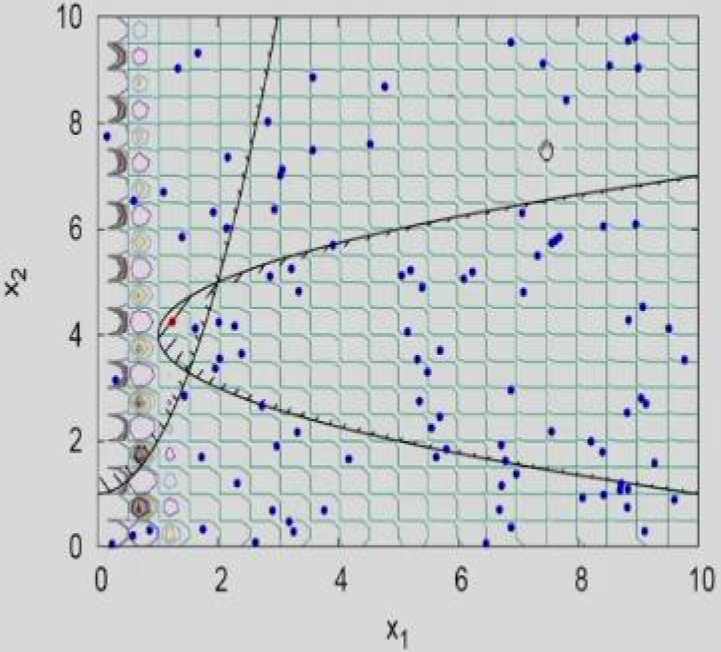
$-1 + x_1 + (x_2 - 4)^2 \geq 0$,

$0 \leq x_1 \leq 10, 0 \leq x_2 \leq 10$.

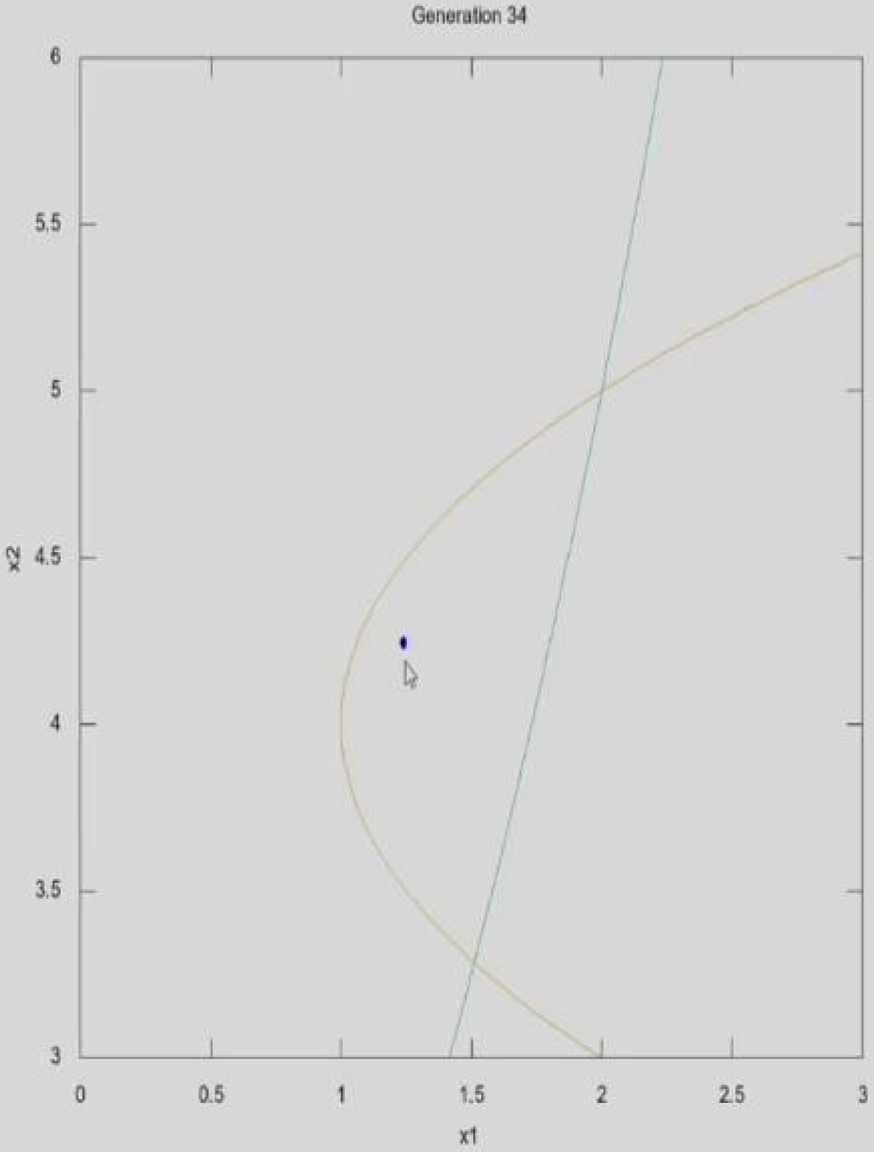


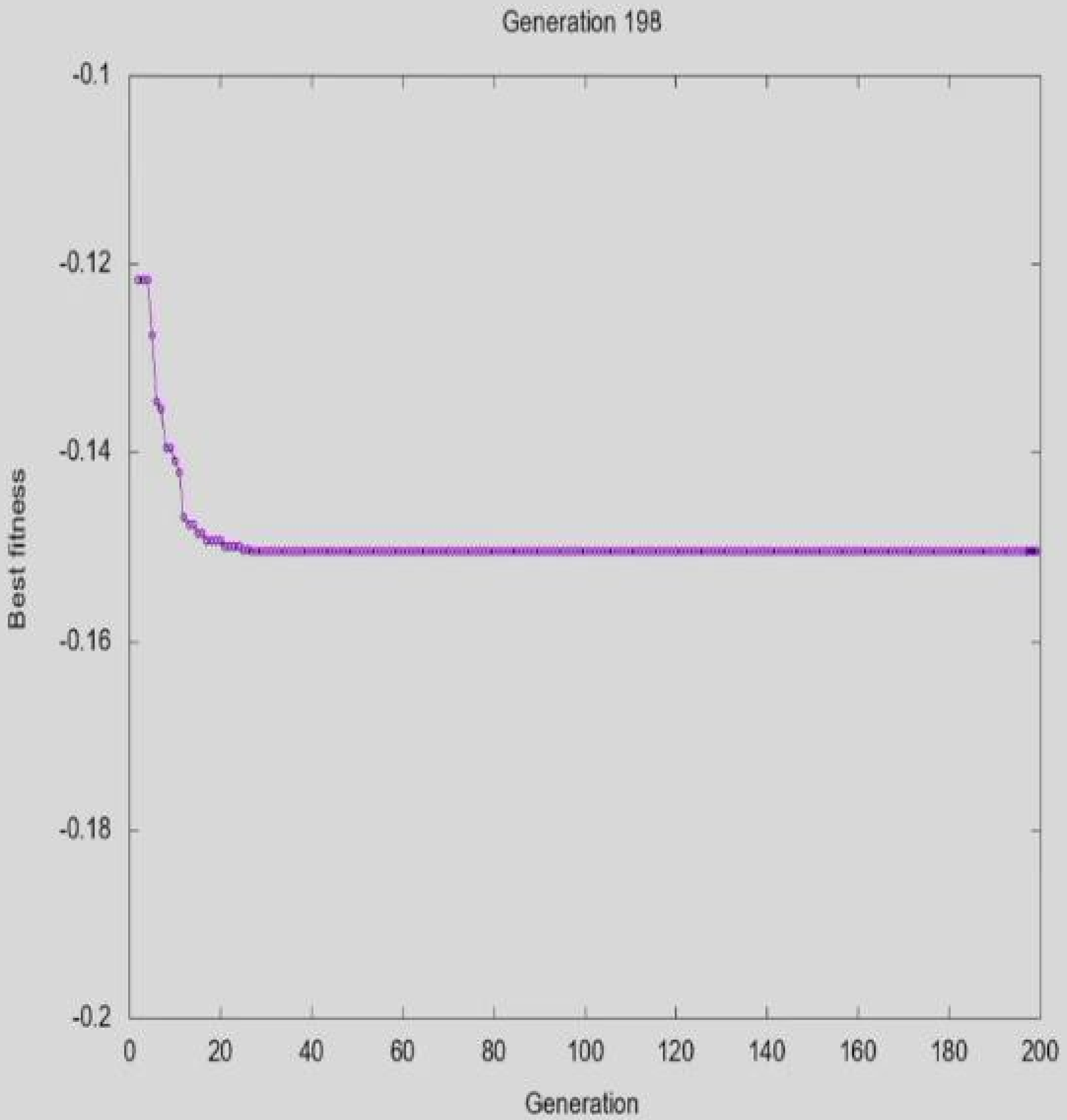
- Optimal solution is $x^* = (1.2402, 4.2462)^T$ and $f(x^*) = -0.1503$

Static Penalty Approach with $R = 2$

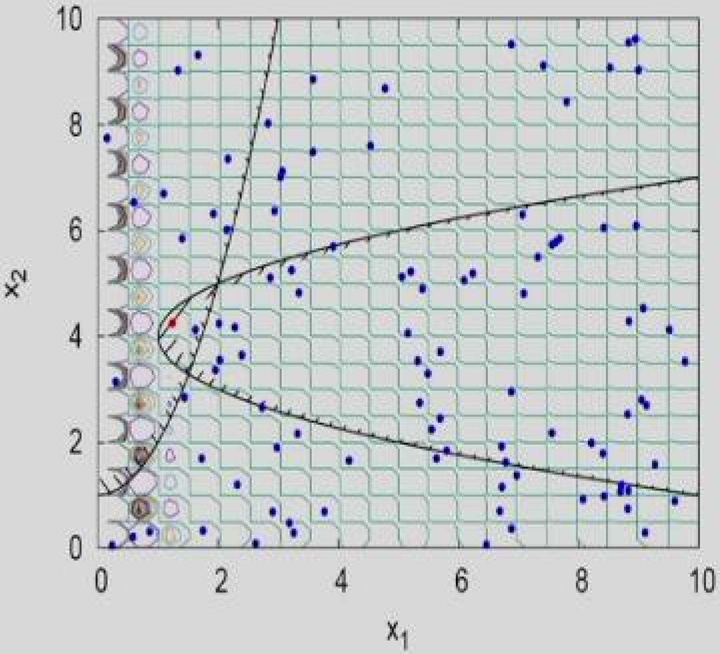


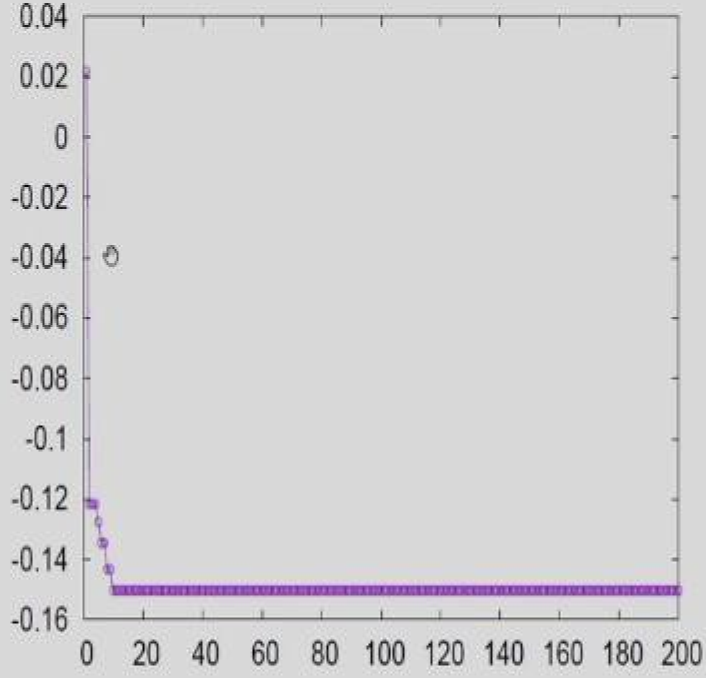
- Simulation ▸ Link


Generation 34
x2
x1
6
5.5
5
4.5
4
3.5
3
0
0.5
1
1.5
2
2.5
3

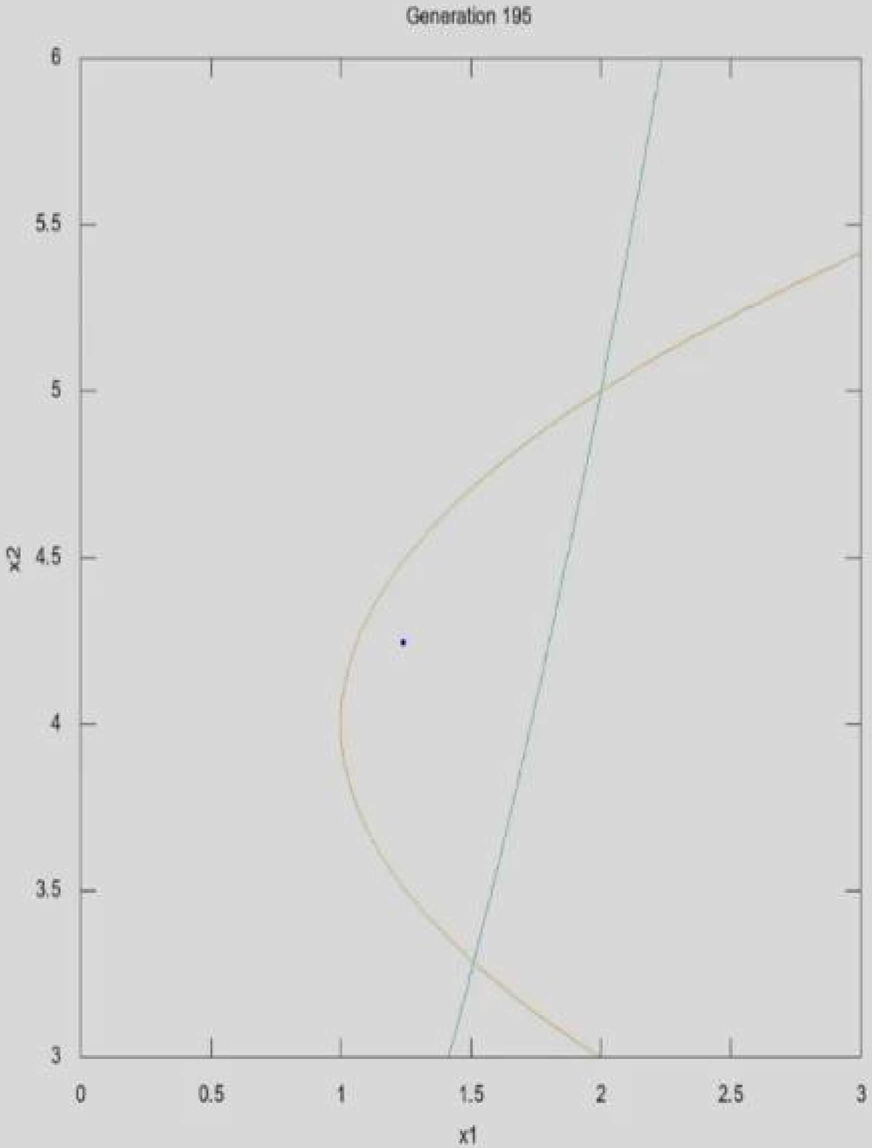

Generation 198
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation

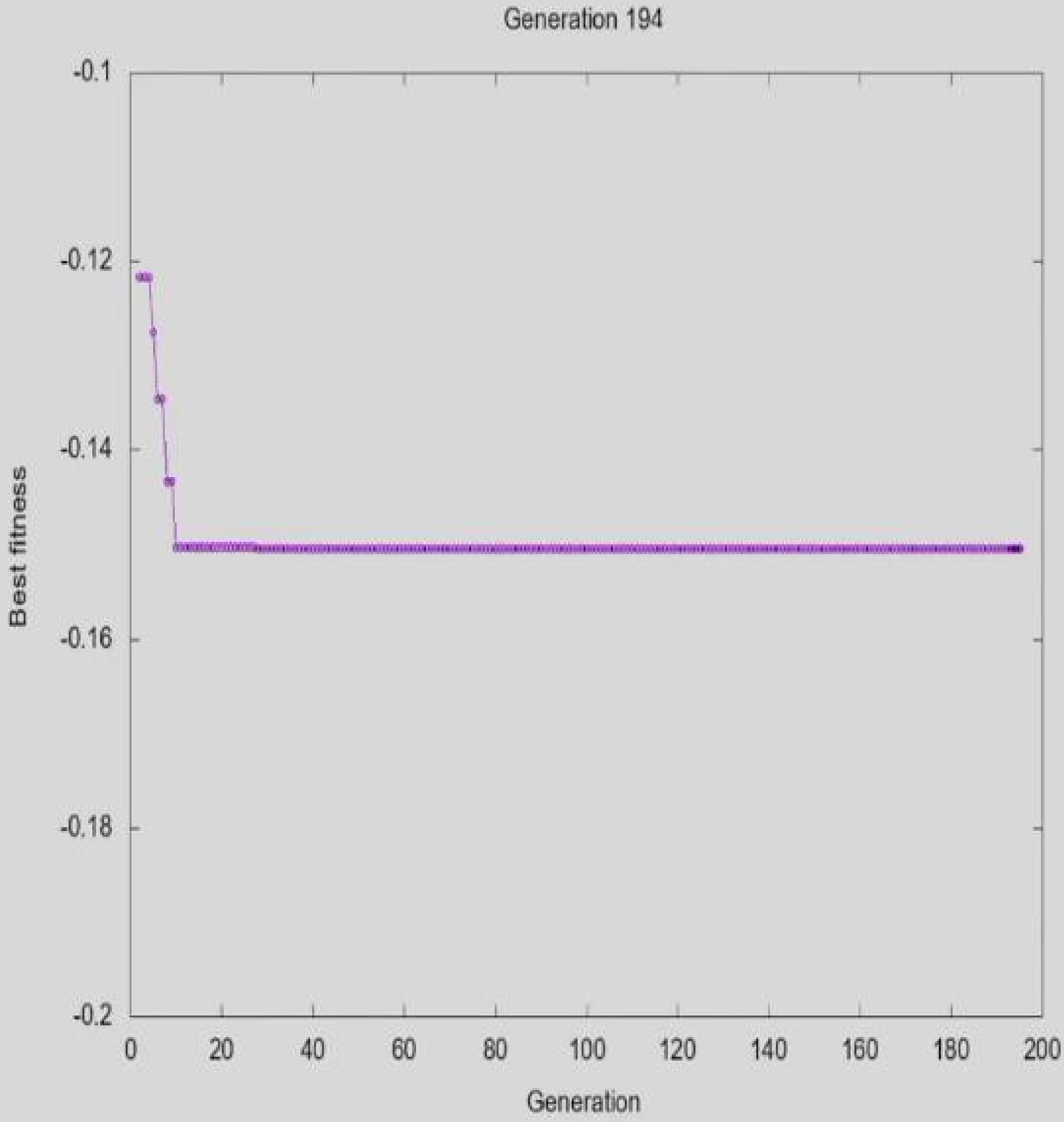
Static Penalty Approach with $R = 100$





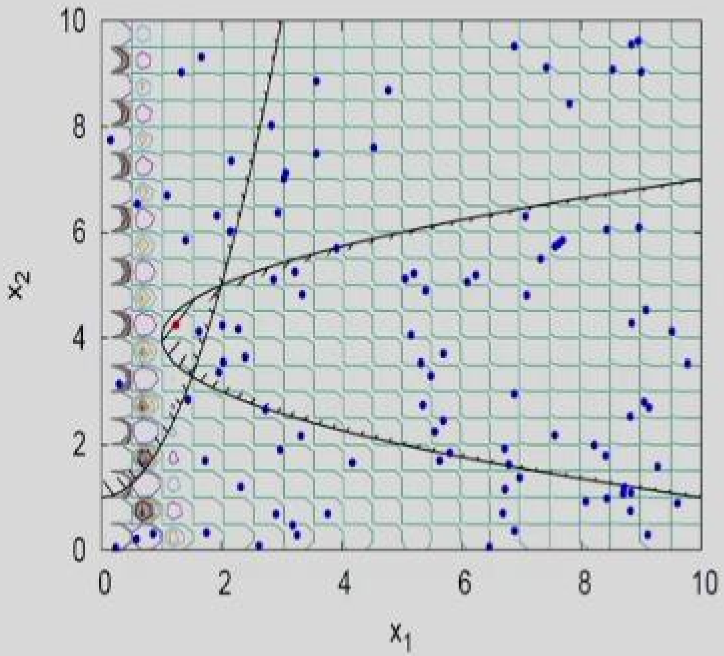
- Simulation ▸ Link
- Progress ▸ Link

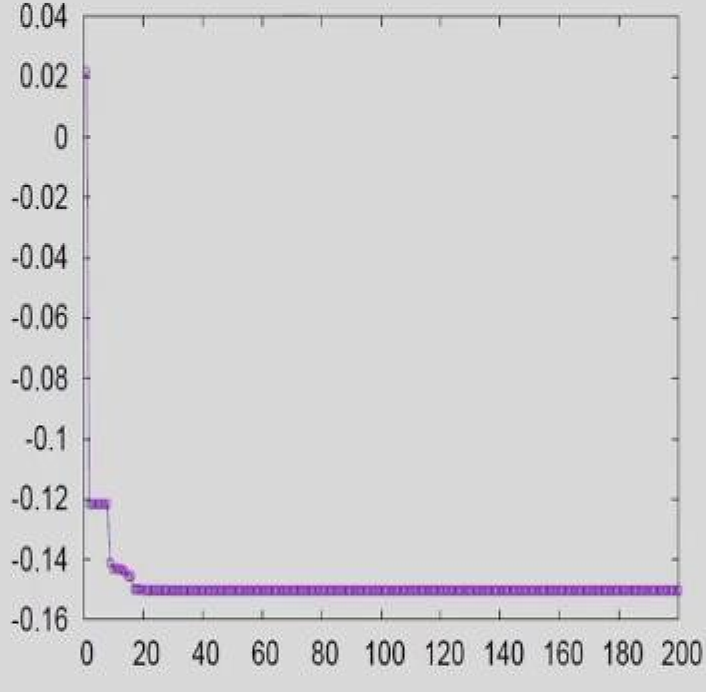

Generation 195
x2
6
5.5
5
4.5
4
3.5
3
0
0.5
1
1.5
2
2.5
3
x1



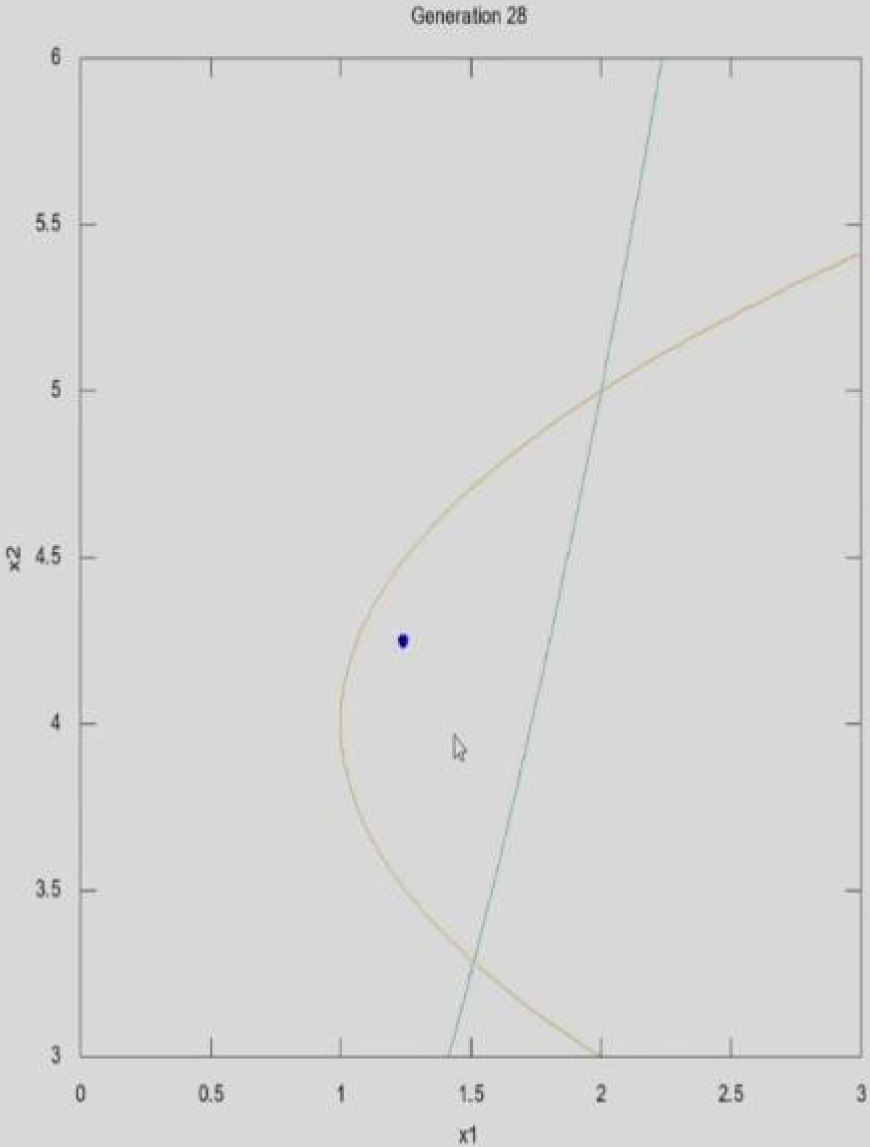
Generation 194
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation

Dynamic Penalty Approach

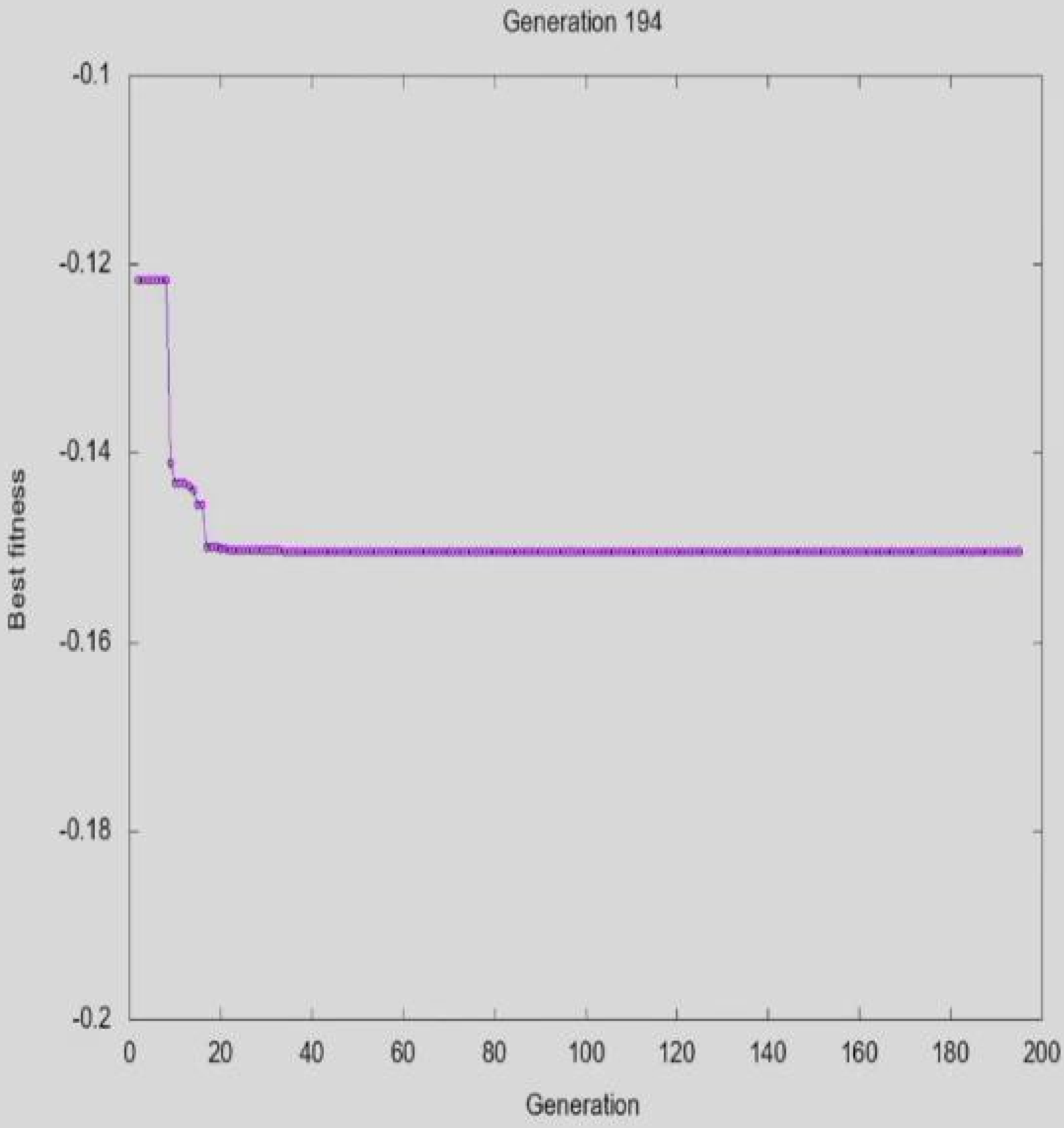




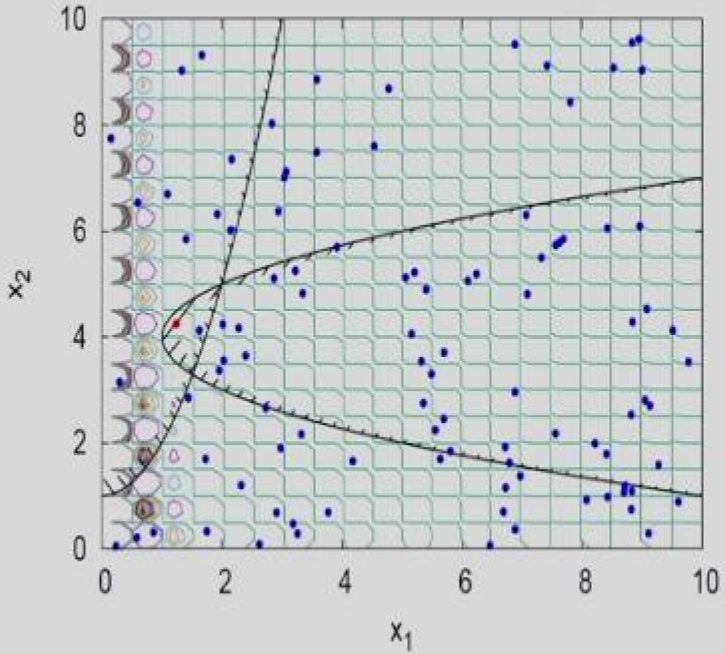
- Simulation ▸ Link
- Progress ▸ Link

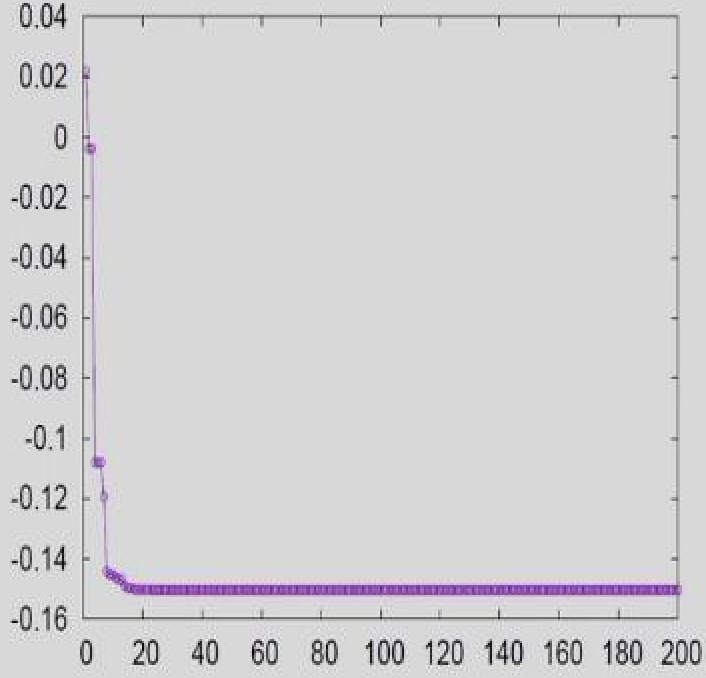


Generation 28
6
5.5
5
4.5
4
3.5
3
x2
0
0.5
1
1.5
2
2.5
3
x1

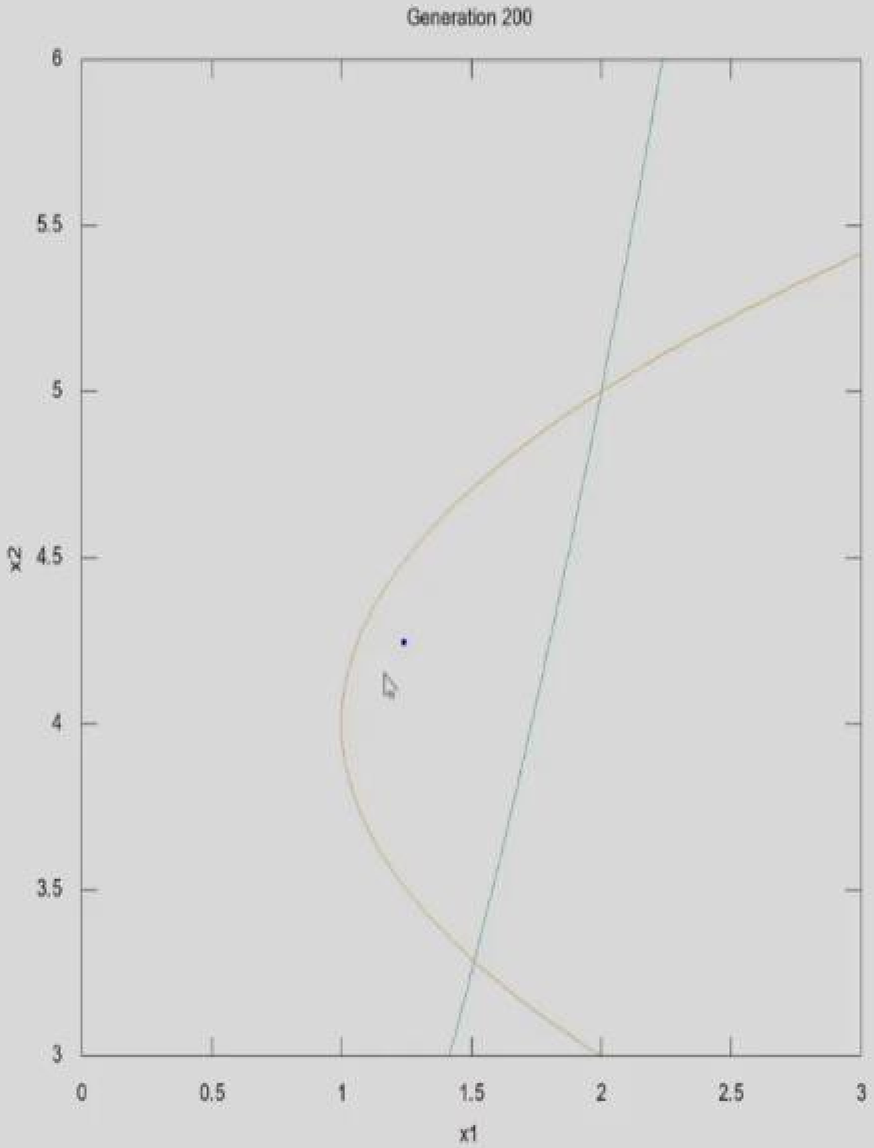

Generation 194
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation

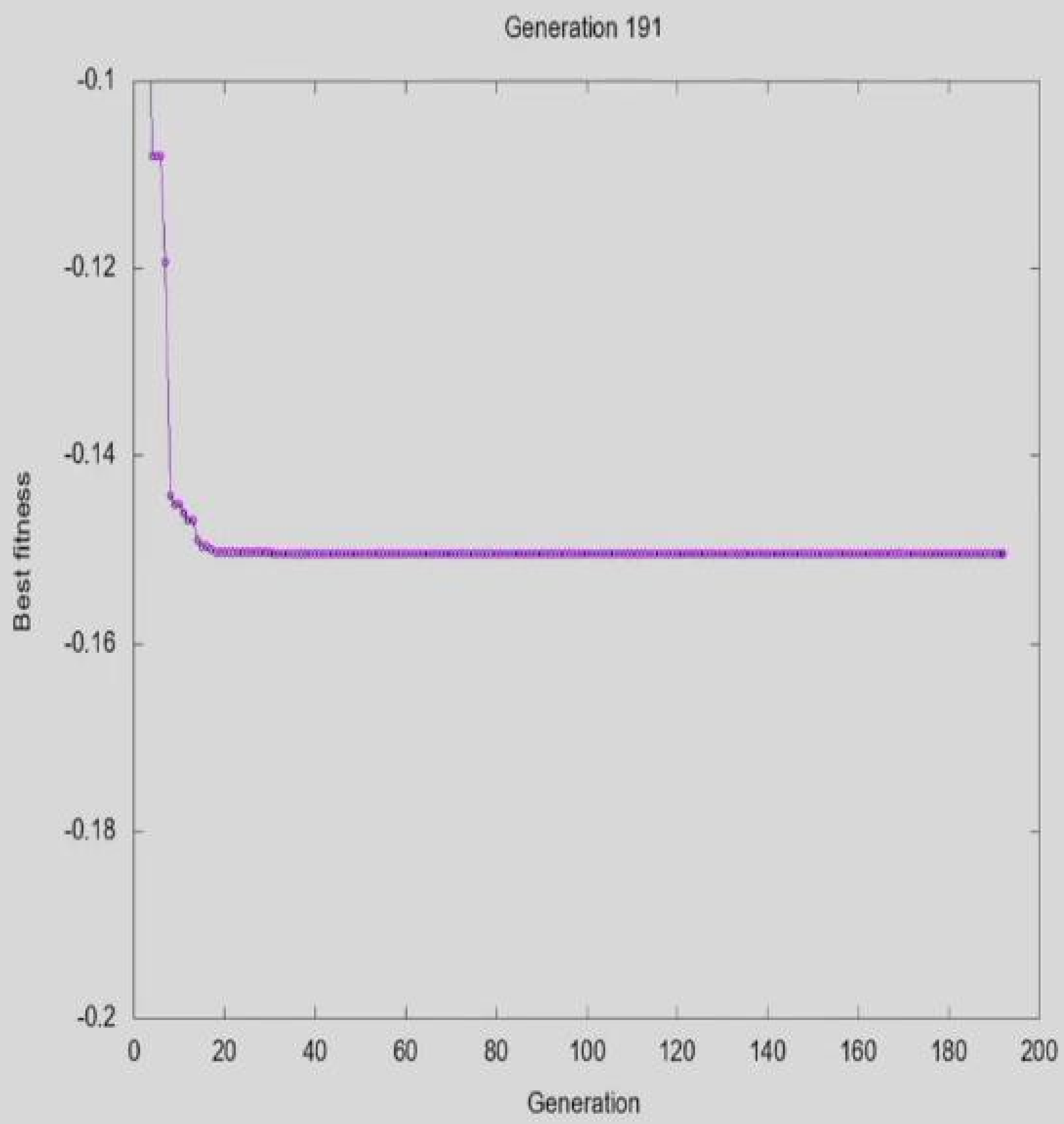
Deb's Approach





- Simulation ▸ Link
- Progress ▸ Link


Generation 200
6
5.5
5
4.5
4
3.5
3
x2
0
0.5
1
1.5
2
2.5
3
x1


Generation 191
Best fitness
Generation
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2
0
20
40
60
80
100
120
140
160
180
200

Comparison

Approaches	$f(x)$	$g_1(x)$	$g_2(x)$	x
Static Penalty($R = 2$)	-0.150	1.708	0.180	$(1.240, 4.246)^T$
Static Penalty($R = 100$)	-0.150	1.708	0.180	$(1.240, 4.246)^T$
Dynamic Penalty	-0.150	1.708	0.180	$(1.240, 4.246)^T$
Deb's Approach	-0.150	1.708	0.180	$(1.240, 4.246)^T$

Milling Process Parameter Optimization

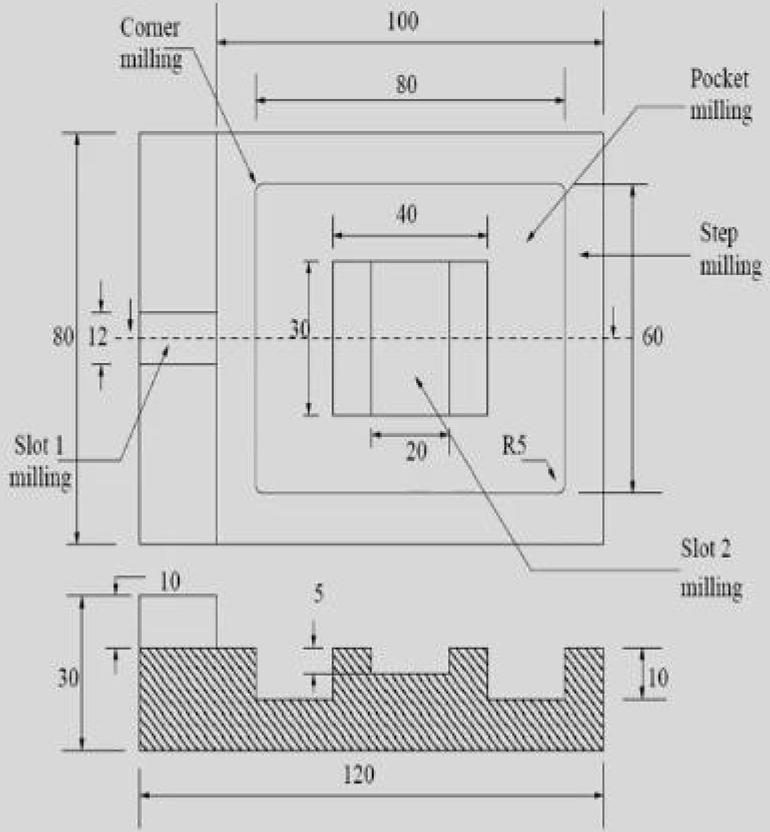


Figure: Final design of workpiece that is manufactured using five milling operations (all dimensions in mm).

Milling Process Parameter Optimization

Problem formulation[1]

$$
\begin{array}{lll}
\text{Minimize:} & C_u & \text{(Unit cost in \$)}, \\
\text{Subject to:} & & \\
& C_5 V f^0.8 \leq 1 & \text{(Power constraint)}, \\
& C_6 f^2 \leq 1 & \text{(Surface finish constraint for end milling)}, \\
& C_7 f \leq 1 & \text{(Surface finish constraint for face milling)}, \\
& C_8 F_C \leq 1 & \text{(Cutting force constraint)}.
\end{array}
\tag{4}
$$

- Ten real continuous variables, four constraints.

[1] Deepak Sharma and Prem Soren, "Infeasibility Driven Approach for Bi-objective Evolutionary Optimization", IEEE Congress on Evolutionary Computation (CEC), June 20-23, 2013, Cancun, Mexico, pp. 868 - 875.

Milling Process Parameter Optimization

- The unit cost is given as $C_u = C_1 + \sum_{i=1}^{m} C_{2i} V_i^{-1} f_i^{-1} + \sum_{i=1}^{m} C_{3i} V_i^{(1/n)-1} f_i^{[(w+g)/n]-1} + \sum_{i=1}^{m} C_{4i}$. It is the sum of material cost, setup cost, machining cost, and tool changing cost.
- In constraints, $C_5 = 0.78 K_p W z a_{rad} (a/60\pi d e P_m)$, $C_6 = 318(4d)^{-1}/R_{a(at)}$, $C_7 = 318[\tan(la) + \cot(ca)]^{-1}/R_{a(at)}$, and $C_8 = 1/F_{C(per)}$.
- Constants used for formulating unit cost: $C_1 = c_{mat} + (c_l + c_0)t_s$, $C_{2i} = (c_l + c_0)K_{1i}$, $K_{1i} = \pi d_i K_i / 1000 V_i f_i z_i$, $C_{3i} = c_{ti} K_{31}$, $K_{3i} = K_{1i}/K_{2i}$, $K_{2i} = 60 Q_i^{-1} C_i^{1/n_i} 5^{-g/n_i} a_i^{(g-w)/n_i}$, $C_{4i} = (c_l + c_o)t_{tci}$.
- Variable bounds: $60 \leq V_1 \leq 120$ (face milling), $40 \leq V_2 \leq 70$ (corner milling), $40 \leq V_3 \leq 70$ (pocket milling), $30 \leq V_4 \leq 50$ (slot milling 1), $30 \leq V_5 \leq 50$ (slot milling 2); $0.05 \leq f_1 \leq 0.4$ (face milling), $0.05 \leq f_2 \leq 0.5$ (corner milling), $0.05 \leq f_3 \leq 0.5$ (pocket milling), $0.05 \leq f_4 \leq 0.5$ (slot milling 1), $0.05 \leq f_5 \leq 0.5$ (slot milling 2).

Milling Process Parameter Optimization

a, a_{rad}: axial depth of cut, radial depth of cut (mm); $C = 33.98, 100.45$: constant in cutting speed for HSS tools and carbide tools, respectively; ca: clearance angle; $c_l = 0.45, c_o = 1.45$: labour and over-head cost (\$/min); $c_m, c_{mat} = 0.50, c_t$: costs of machining, material per part and cutting tool (\$); d: cutter diameter (mm); $e = 95\%$: machine tool efficiency factor; F: feed rate (mm/min); f: feed rate (mm/tooth); $F_C, F_C(per)$: cutting force and permitted cutting force (N); $g = 0.14$: exponent of slenderness ratio; K: distance to be traveled by the tool to perform the operation (mm); $K_p = 2.24$: power constant depending on the workpiece material; la: lead (corner) angle of tool; $m = 5$: number of machining operation required to produce the product; $n = 0.15, 0.3$: tool life exponent for HSS tools and carbide tools, respectively; $P_m = 8.5$: motor power (kW); Q: contact proportion of cutting edge with the workpiece per revolution; $R_a, R_{a(at)}$: arithmetic value of surface finish, and attainable surface finish (μm); $t_m, t_s = 2, t_{tc} = 0.5$: machining time, set-up time, tool changing time (min); V: cutting speed; $w = 0.28$: exponent of chip cross-sectional area; $W = 1.1$: tool wear factor; z: number of cutting teeth of the tool.

Milling Process Parameter Optimization

Table: Required data for milling parameter optimization problem.

Oper. no.	Oper. no.	Tool no.	a	K	R_a	Face for surface roughness	Q	a_{rad}
1	Face milling	1	10	450	2	bottom	0.45	50
2	corner milling	2	5	90	6	bottom	1.0	10
3	pocket milling	2	10	450	5	bottom	0.5	10
4	slot milling	3	10	32	-	-	1.0	12
5	slot milling	3	5	84	1	side	1.0	12

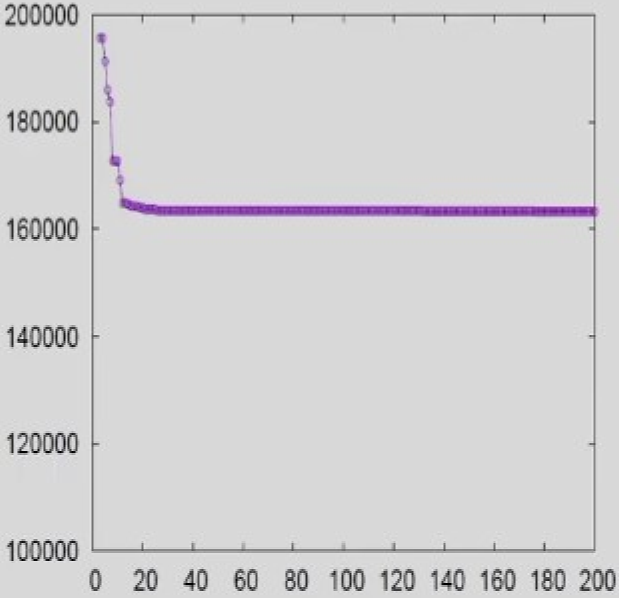
Milling Process Parameter Optimization

Table: Tools data for milling parameter optimization problem.

Tool No.	Tool type	Quality	d	CL	z	Price	SD	Helx angle	la	ca
1	face mill	Carbide	50	20	6	49.50	25	15	45	5
2	end mill	HSS	10	35	4	7.55	10	45	0	5
3	end mill	HSS	12	40	4	7.55	10	45	0	5

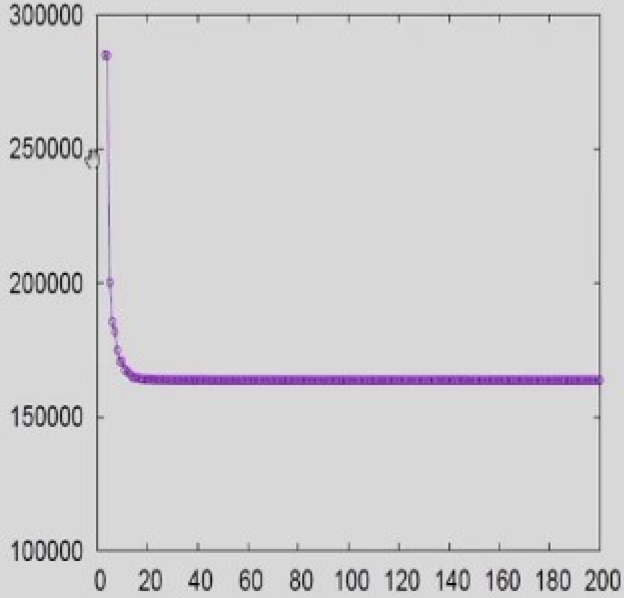
Static Penalty Approach

For $R = 2$

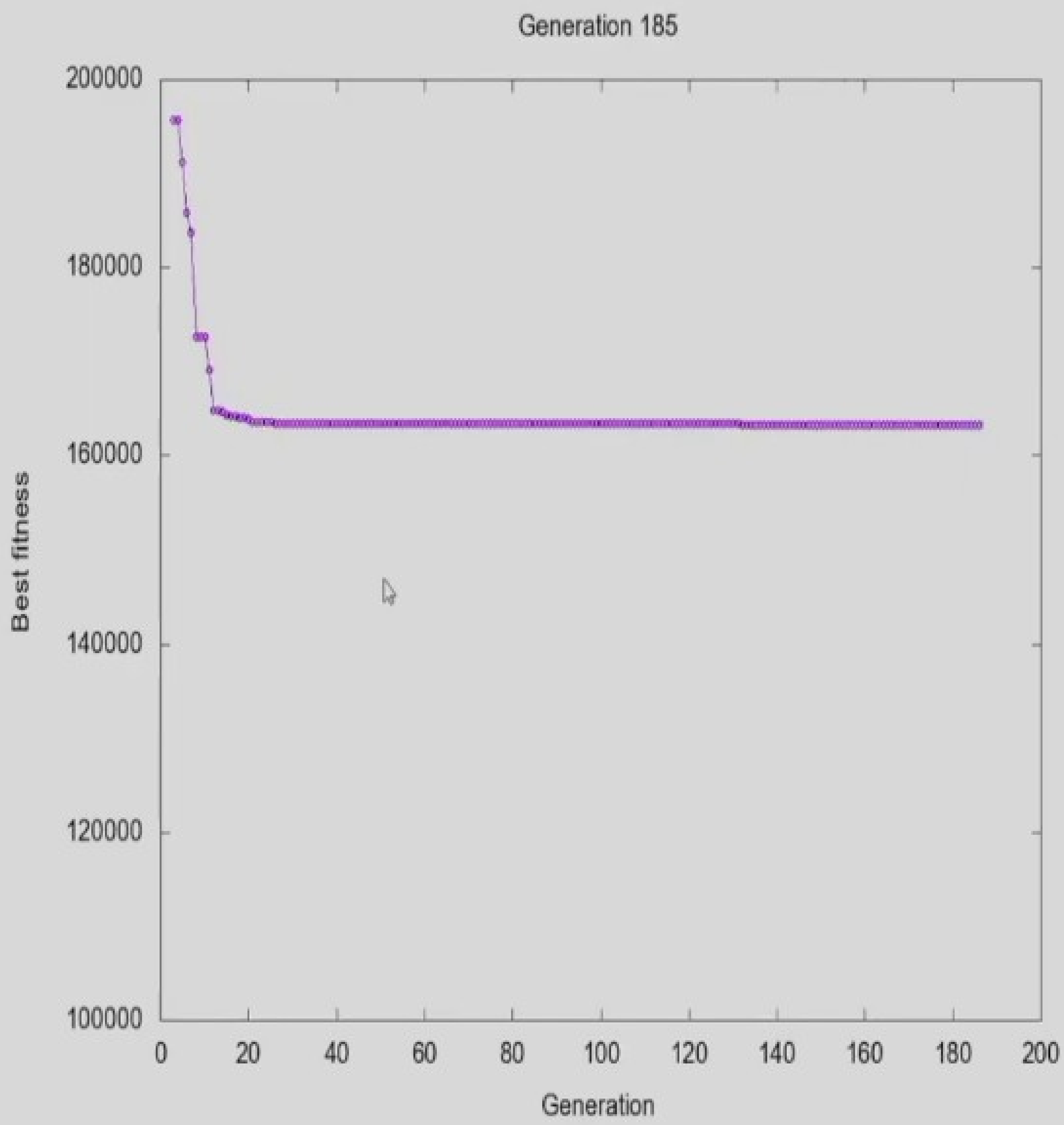


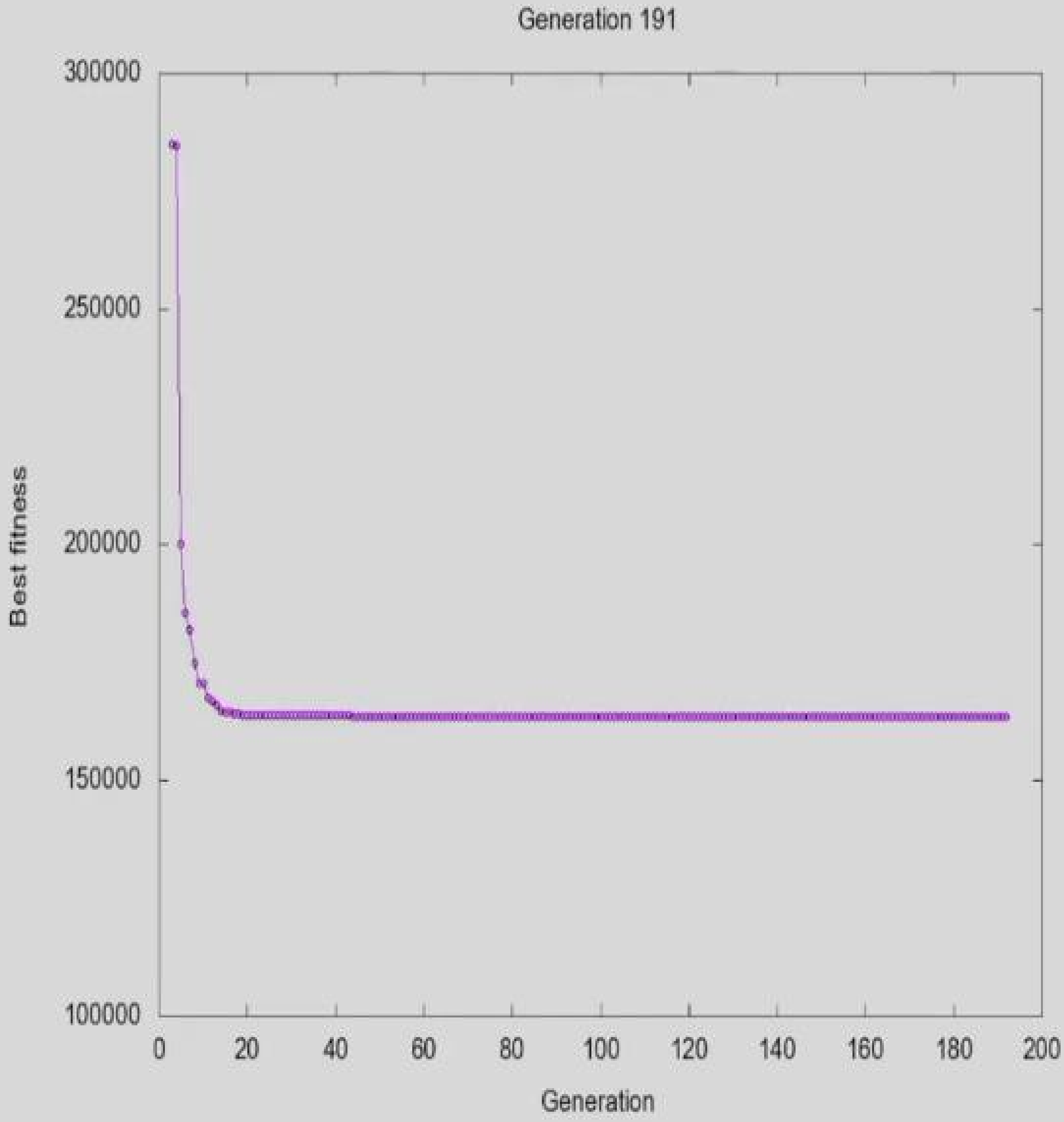
- Progress [Link]

For $R = 100$



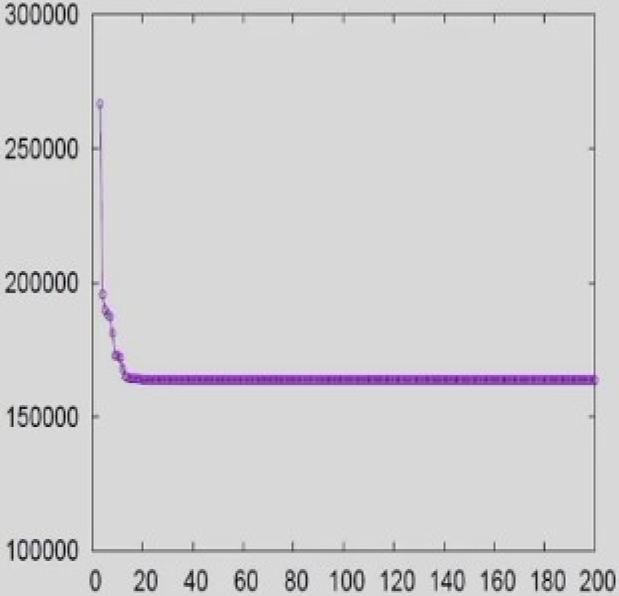
- Progress [Link]


Generation 185
Best fitness
200000
180000
160000
140000
120000
100000
0
20
40
60
80
100
120
140
160
180
200
Generation


Generation 191
300000
250000
200000
150000
100000
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation

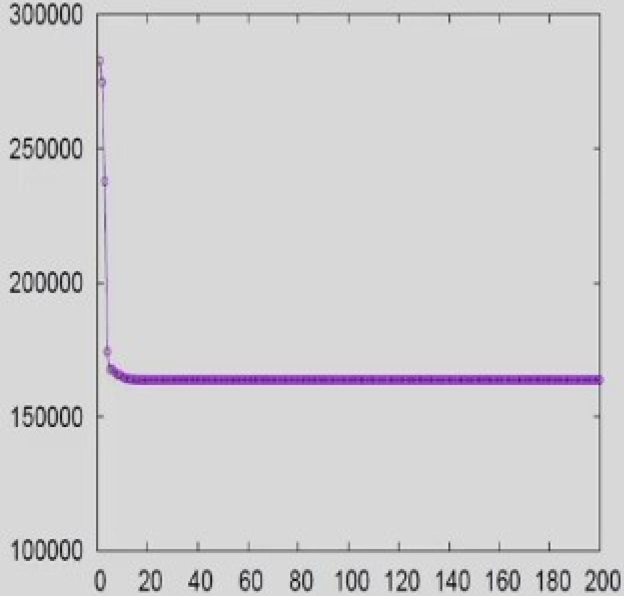
Constraint Handling

Dynamic Penalty Approach

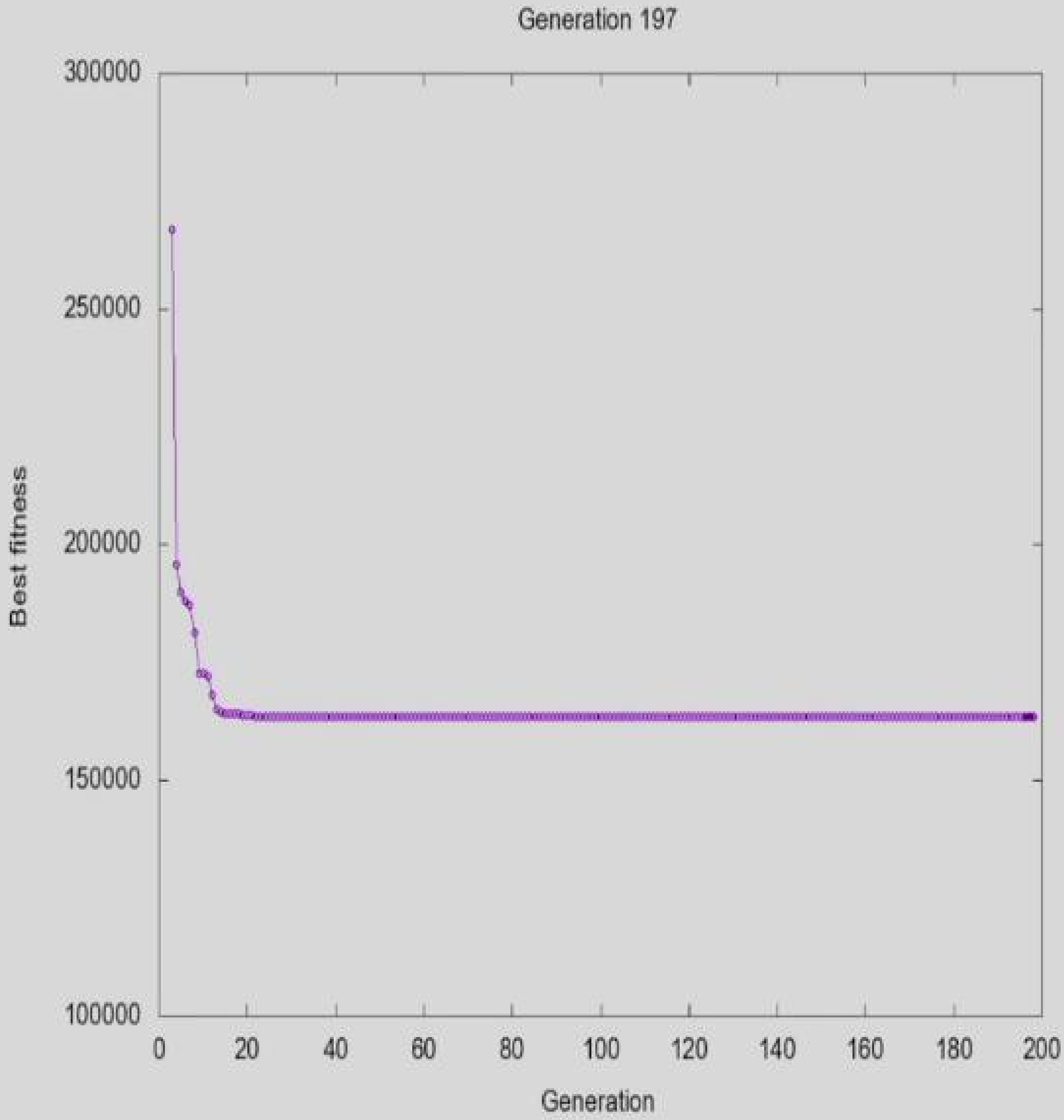


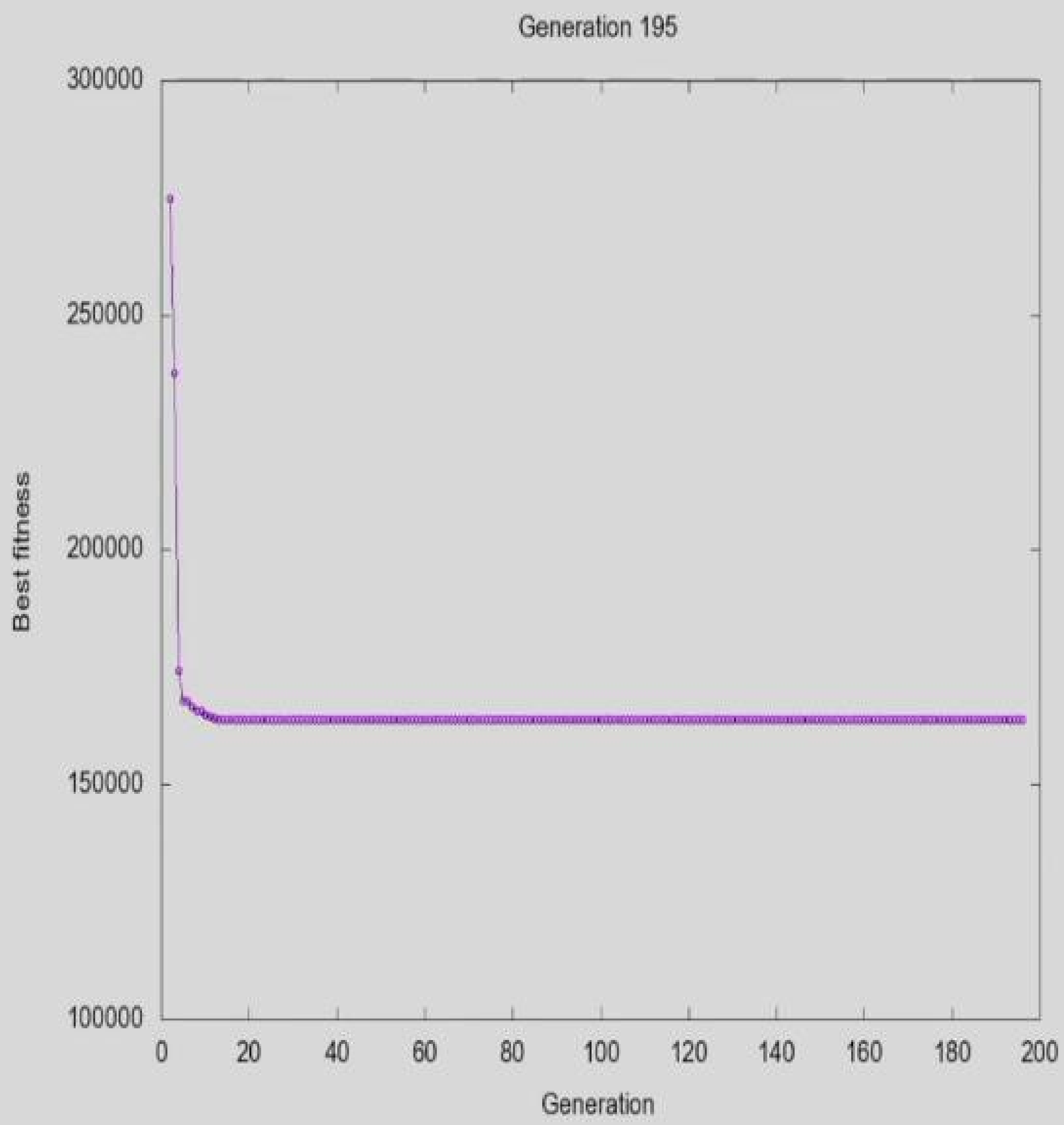
- Progress ▸ Link

Deb's Approach



- Progress ▸ Link


Generation 197
300000
250000
200000
150000
100000
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation


Generation 195
300000
250000
200000
150000
100000
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation

Comparison

Approaches	$f(x)$	$g(x)$	x
Static Penalty($R = 2$)	163097.700	$g_3 = -8.538$	$(60.000, 51.462, 40.002, 30.002, 47.832, 0.379, 0.451, 0.492, 0.059, 0.340)^T$
Static Penalty($R = 100$)	163726.100	$g_3 = -0.365$	$(60.000, 59.635, 40.000, 30.000, 41.359, 0.291, 0.312, 0.337, 0.330, 0.451)^T$
Dynamic Penalty	163582.500	$g_3 = -2.337$	$(60.000, 57.664, 40.001, 30.038, 47.160, 0.332, 0.338, 0.458, 0.106, 0.321)^T$
Deb's Approach	163754.800	all satisfied	$(60.000, 60.003, 40.000, 30.000, 49.897, 0.084, 0.135, 0.163, 0.346, 0.113)^T$

- Active constraints are g_3 and g_5 to g_{14}.

Welded Beam Design

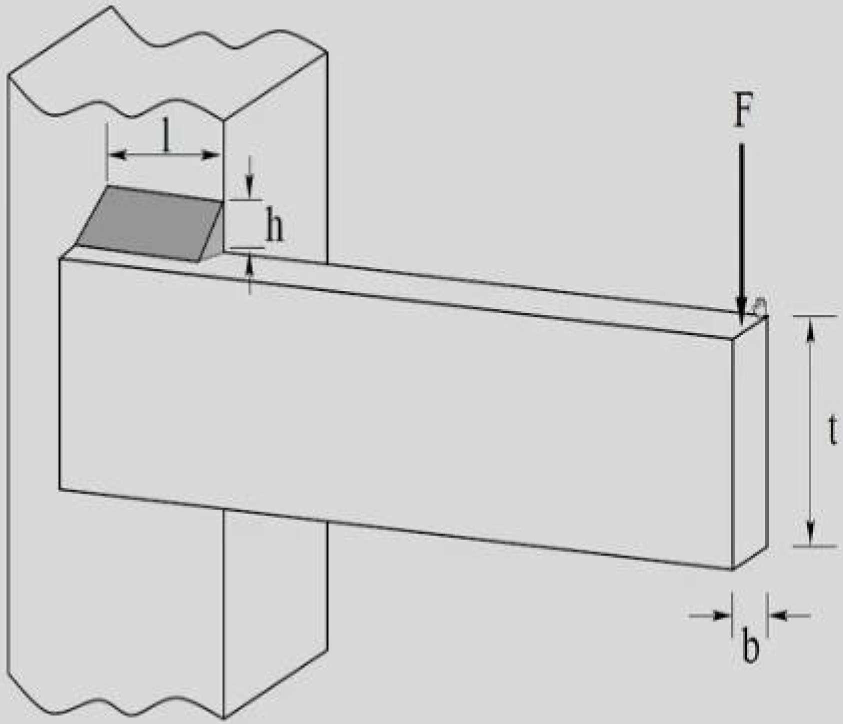


Figure: Welded beam design problem.

Welded Beam Design

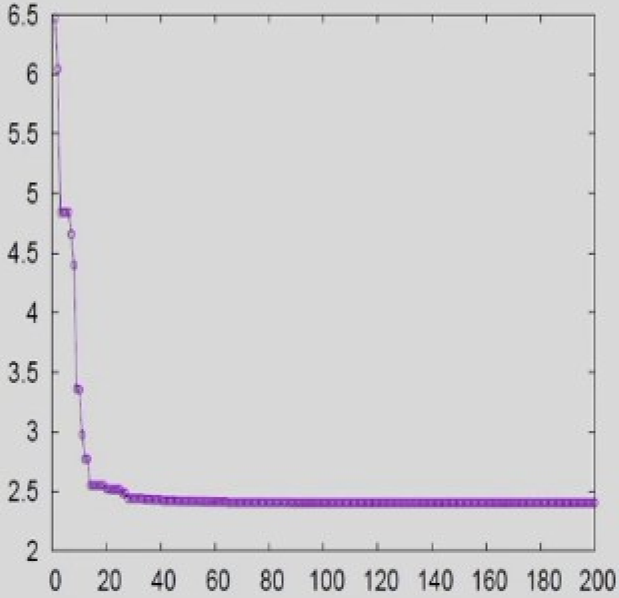
Problem formulation

$$
\begin{aligned}
\text{Minimize:} \quad & 1.10471h^{l} + 0.04811tb(14.0 + l) && \text{(Cost of the beam in \$)},\\
\text{Subjected to:} \quad & \\
& 13,600 - \tau(\vec{x}) \geq 0, \\
& 30,000 - \sigma(\vec{x}) \geq 0, \\
& b - h \geq 0, \\
& P_c(\vec{x}) - 6000 \geq 0, \\
& 0.125 \leq h, b \leq 5 \text{ and } 0.1 \leq l, t \leq 10, \\
\text{where} \quad & \tau(\vec{x}) = \sqrt{\frac{(\tau')^2 + (\tau'')^2 + (l\tau'\tau'')}{\sqrt{0.25(l^2 + (h+t)^2)}}}, \\
& \tau' = 6,000/\sqrt{2}hl, \\
& \tau'' = \frac{6,000(14 + 0.5l)\sqrt{0.25(l^2 + (h+t)^2)}}{2\{0.707hl(l^2/12 + 0.25(h+t)^2)\}}, \\
& \sigma(\vec{x}) = 504,000/t^2 b, \\
& P_c(\vec{x}) = 64,746.022(1 - 0.0282346t)tb^3.
\end{aligned} \tag{5}
$$

- Four decision variables ($\vec{x} = b, t, l, h$), and four constraints.

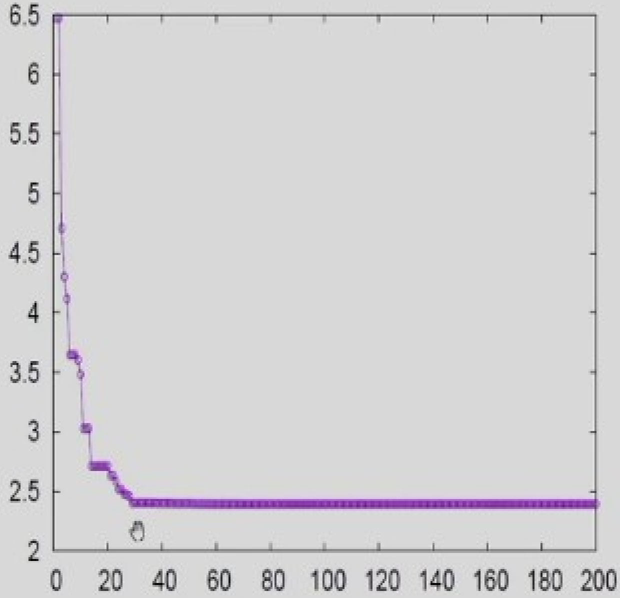
Static Penalty Approach

Fro $R = 2$

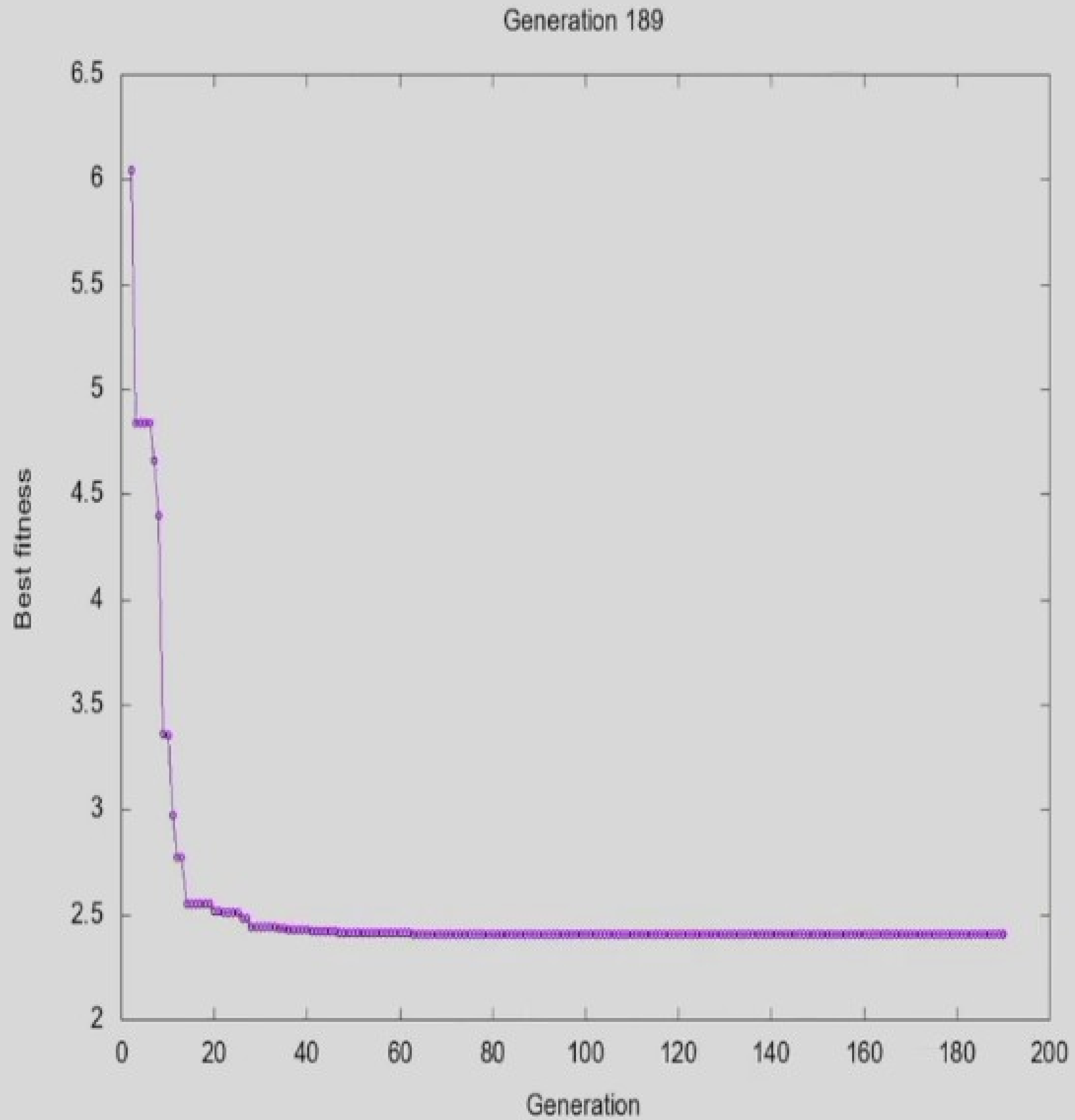


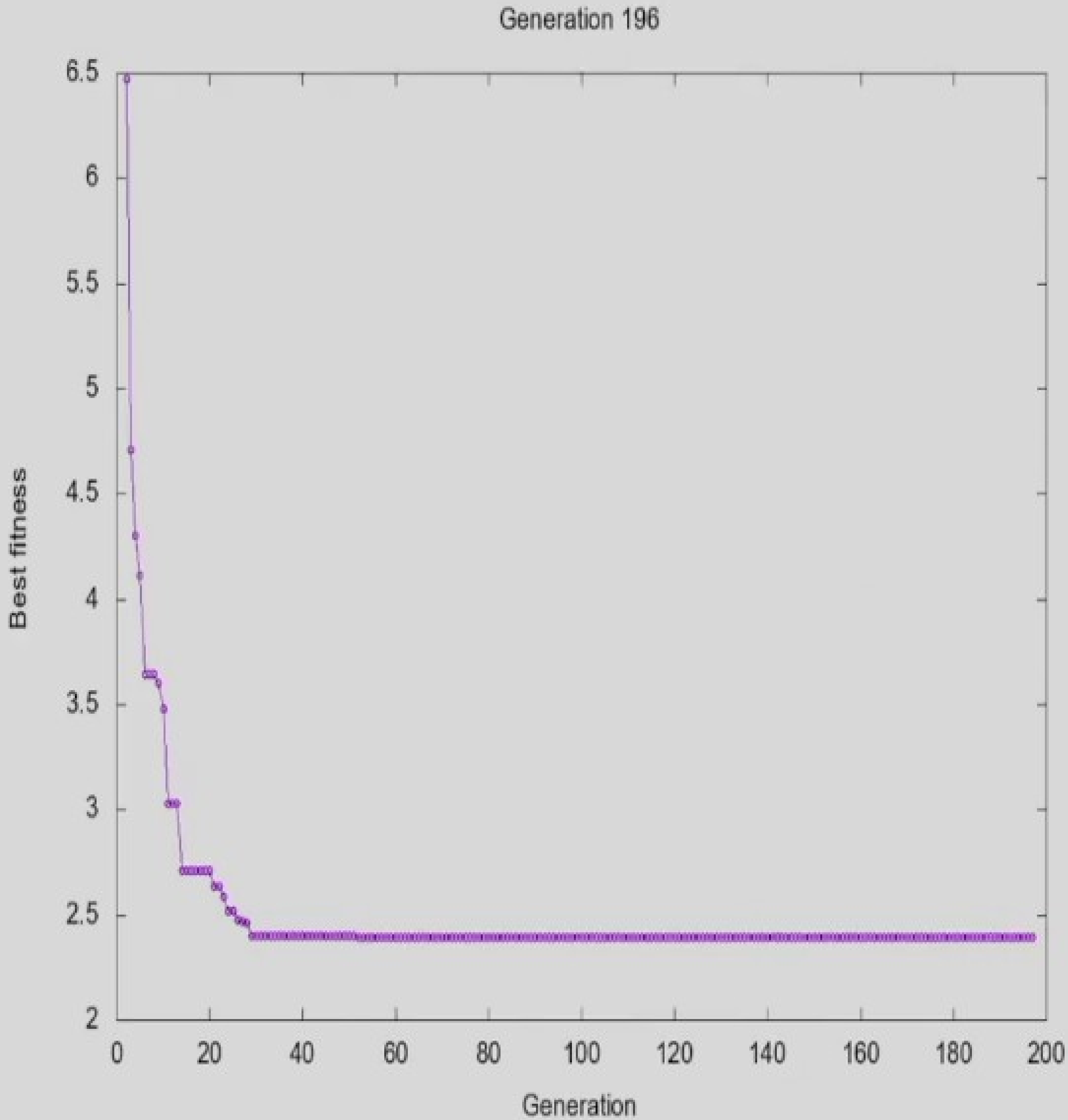
- Progress Link

For $R = 100$



- Progress Link

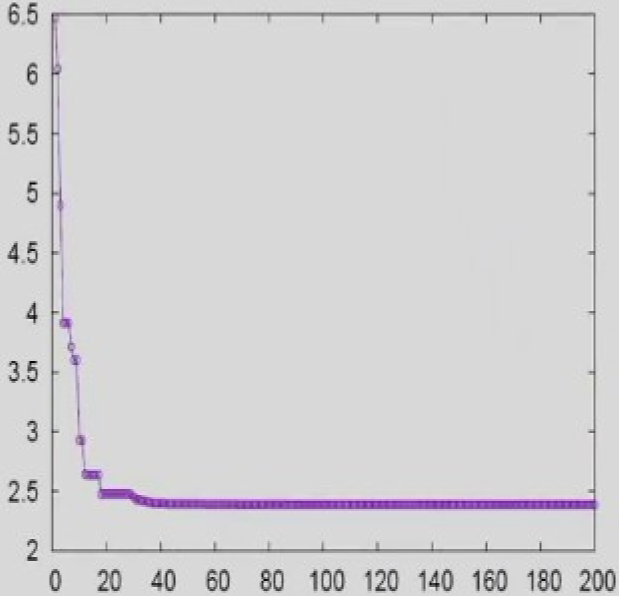

Generation 189
Best fitness
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
0
20
40
60
80
100
120
140
160
180
200
Generation



Generation 196
Best fitness
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
0
20
40
60
80
100
120
140
160
180
200
Generation

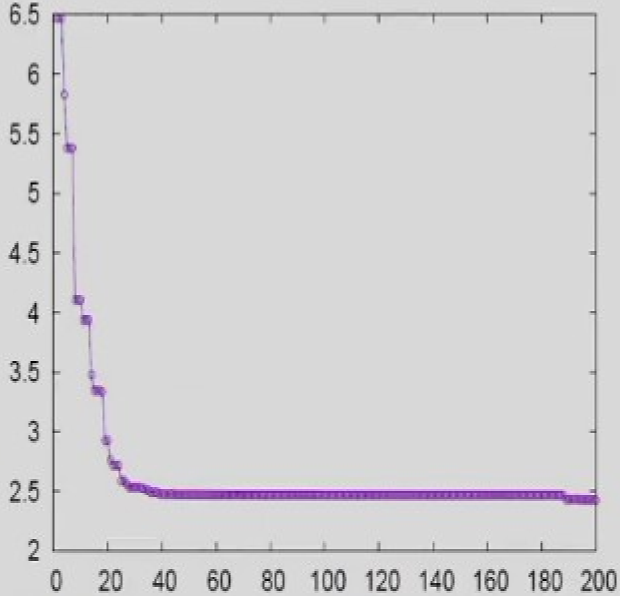
Constraint Handling

Dynamic Penalty Approach

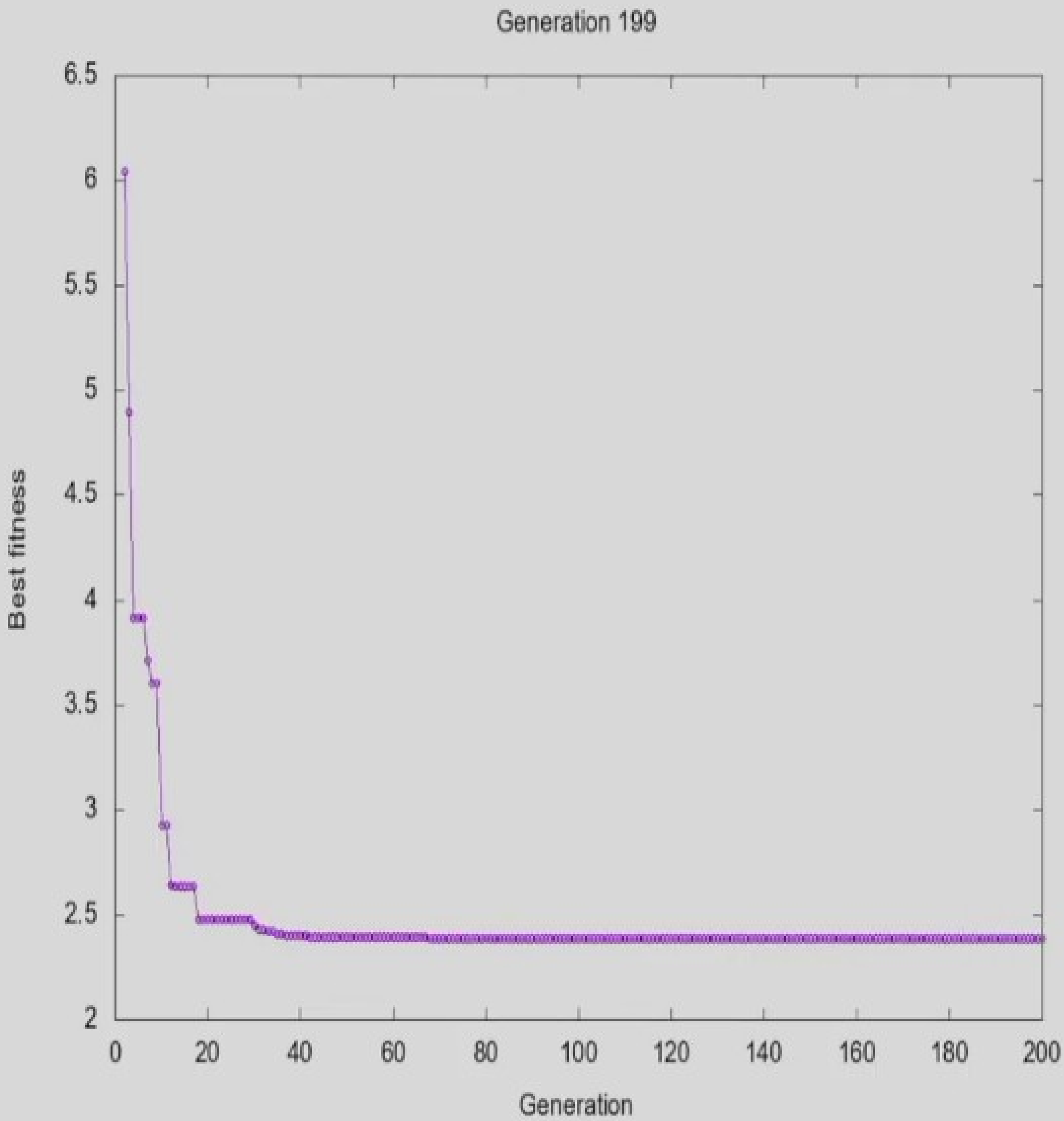


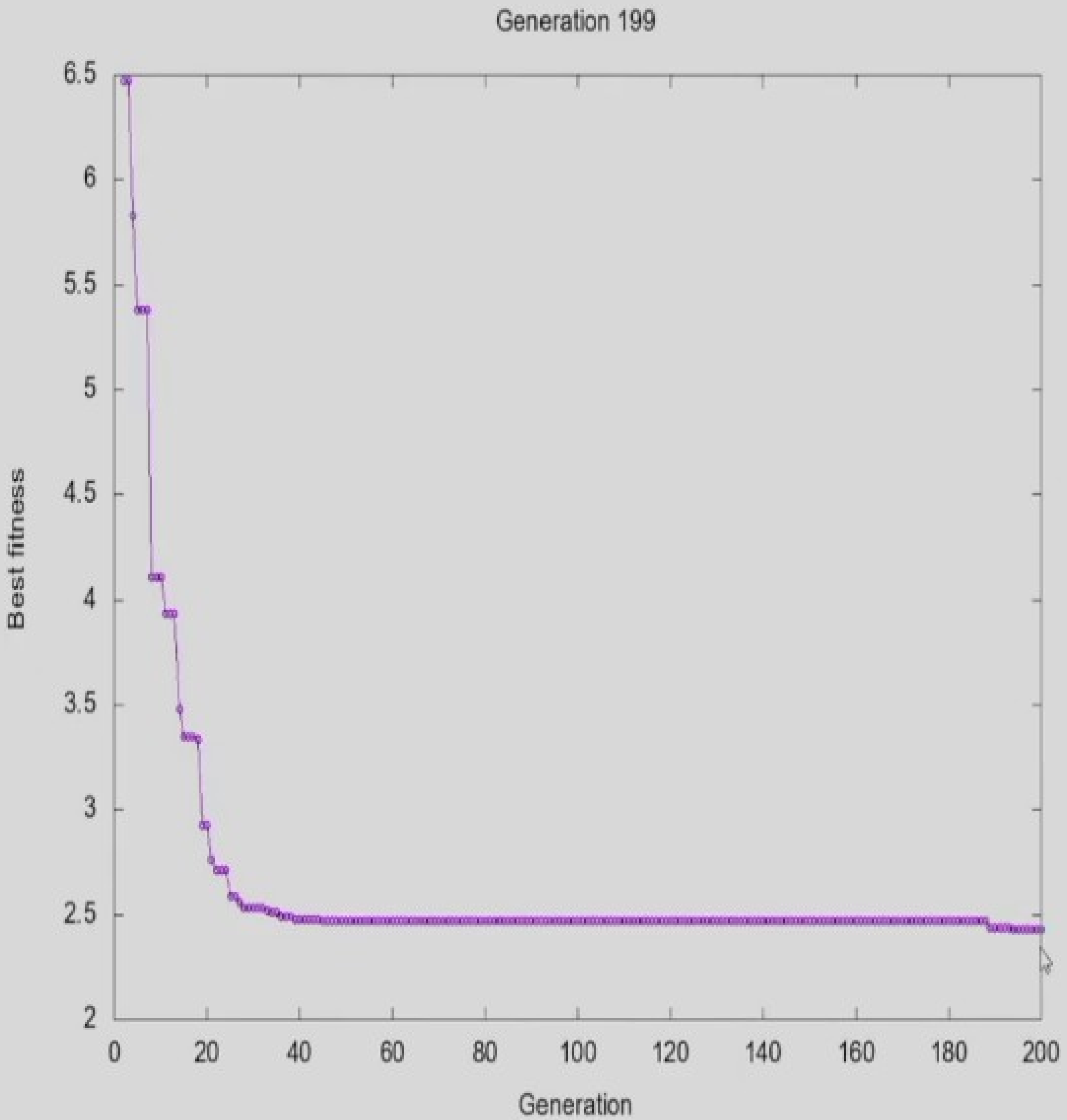
- Progress Link

Deb's Approach



- Progress Link


Generation 199
Best fitness
Generation
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
0
20
40
60
80
100
120
140
160
180
200


Generation 199
Best fitness
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
0
20
40
60
80
100
120
140
160
180
200
Generation

Comparison

Approaches	$f(x)$	$g(x)$	x
Static Penalty($R = 2$)	2.942	$0.128, 3.012, 0.003, 12773.410$	$(0.374, 0.377, 4.533, 6.677)^T$
Static Penalty($R = 100$)	2.783	$0.147, 10.666, 0.008, 8034.875$	$(0.329, 0.337, 5.034, 7.059)^T$
Dynamic Penalty	3.939	$0.162, 22.813, 0.044, 63432.780$	$(0.579, 0.623, 3.346, 5.194)^T$
Deb's Approach	2.426	$16.499, 3552.643, 0.005, 0.099$	$(0.236, 0.240, 5.987, 8.902)^T$

- The active constraint is g_3 and g_4.

Two-Bar Truss Design

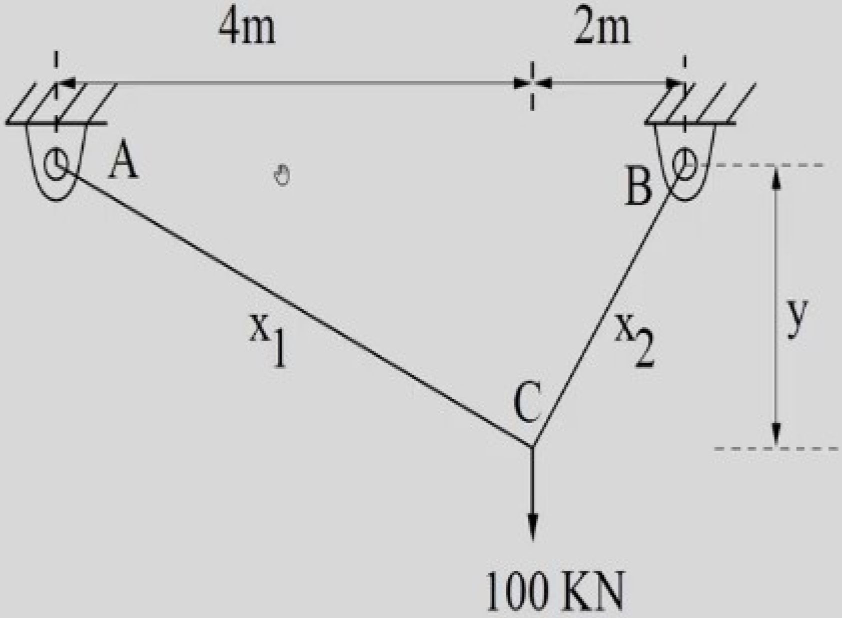


Figure: A two-bar truss.

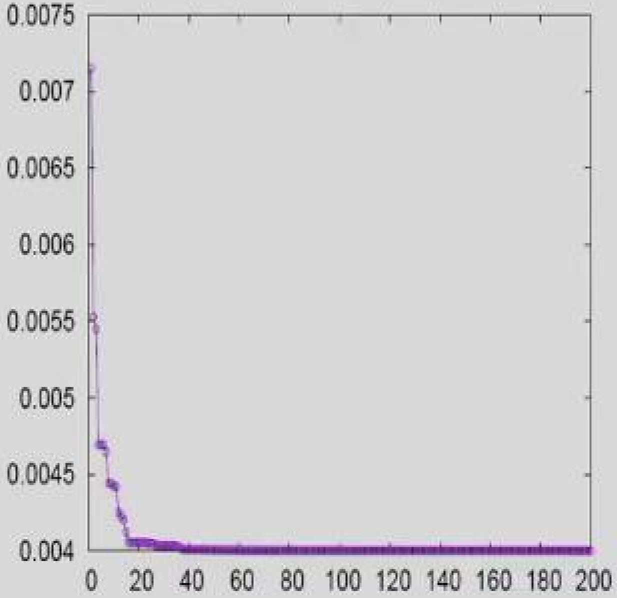
Two-Bar Truss Design

Problem Formulation

$$
\begin{aligned}
&\text{Minimize:} && f_1(x,y) = x_1\sqrt{16+y^2} + x_2\sqrt{1+y^2}, \\
& && \qquad\qquad \text{(Total volume, } m^2\text{)} \\
&\text{Subjected to:} && \\
& && max(\sigma_{AC}, \sigma_{BC}) \le 10^5 \text{ (Stress constraint)} \\
& && 1 \le y \le 3 \text{ and } 0 \le x_1, x_2 \le 0.01 \\
&\text{where} && \sigma_{AC} = \frac{20\sqrt{16+y^2}}{yx_1},\ \sigma_{BC} = \frac{80\sqrt{1+y^2}}{yx_2}
\end{aligned}
\tag{6}
$$

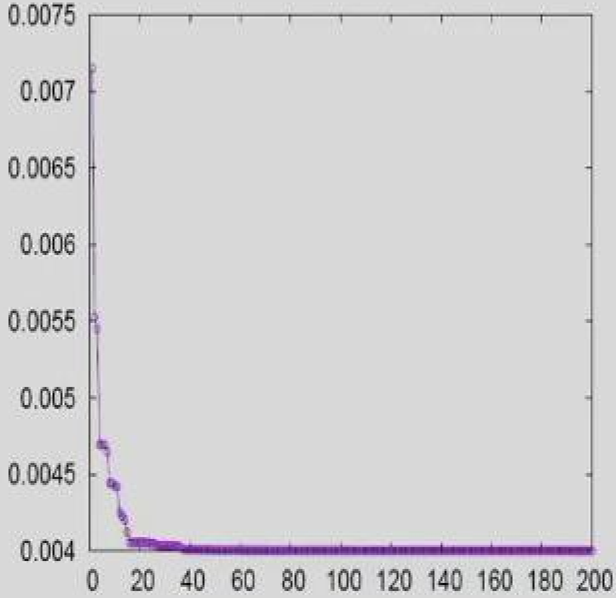
Static Penalty Approach

For $R = 2$

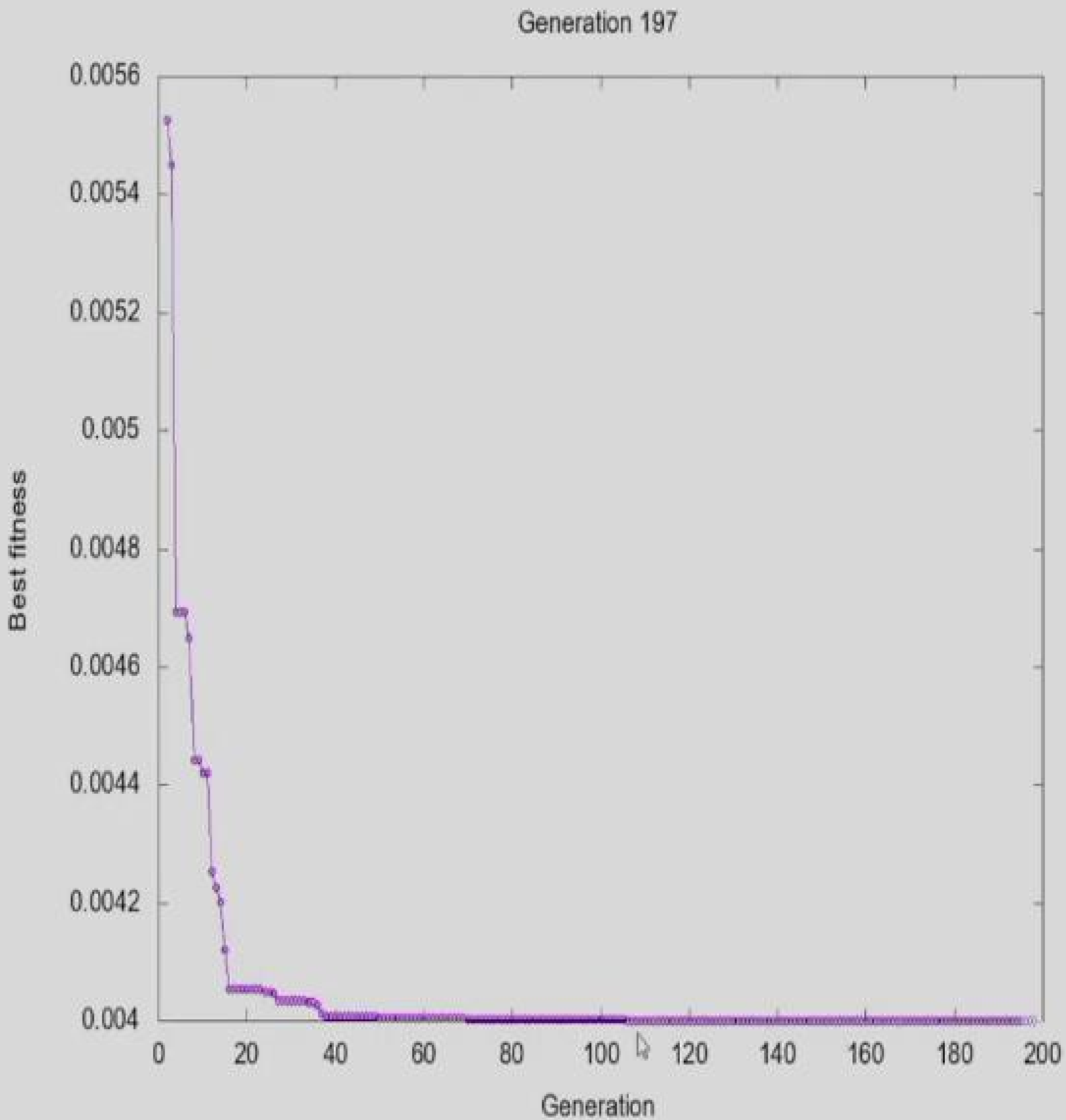


- Progress Link

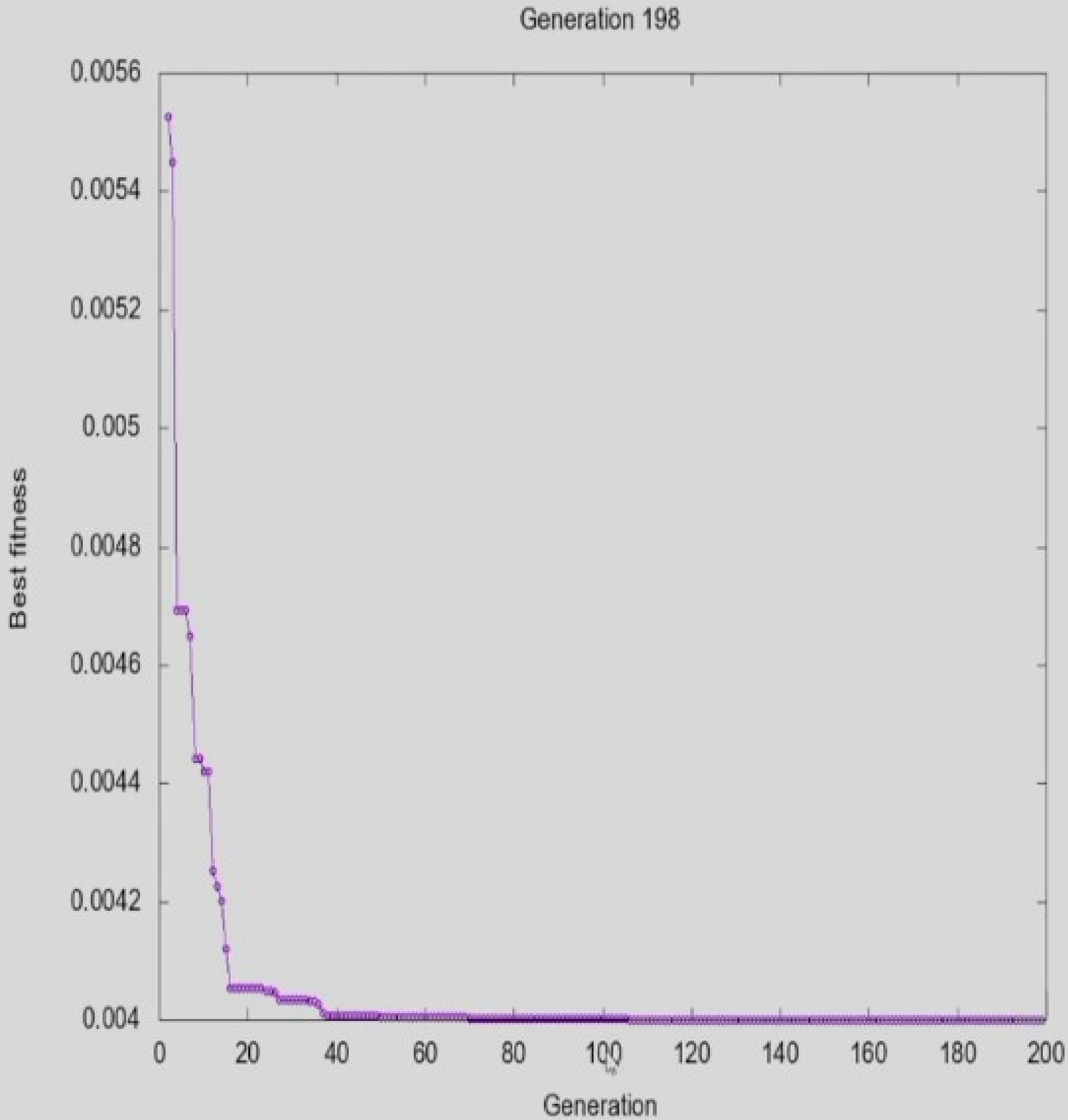
For $R = 100$



- Progress Link

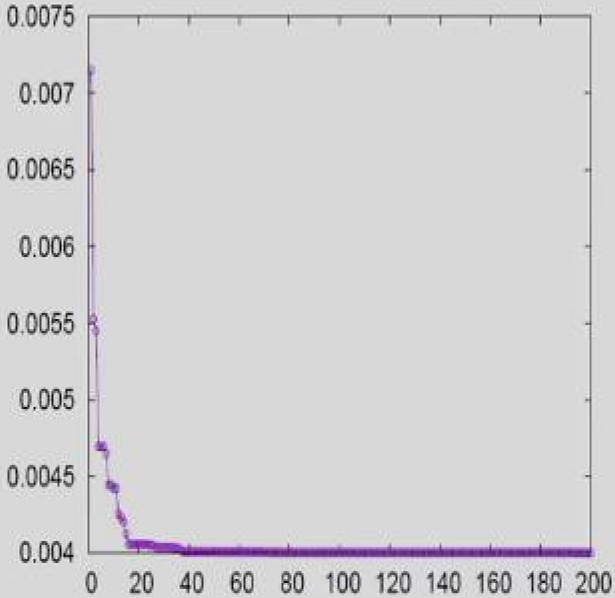


Generation 197
0.0056
0.0054
0.0052
0.005
0.0048
0.0046
0.0044
0.0042
0.004
Best fitness
0
20
40
60
80
100
120
140
160
180
200
Generation


Generation 198
Best fitness
Generation
0.0056
0.0054
0.0052
0.005
0.0048
0.0046
0.0044
0.0042
0.004
0
20
40
60
80
100
120
140
160
180
200

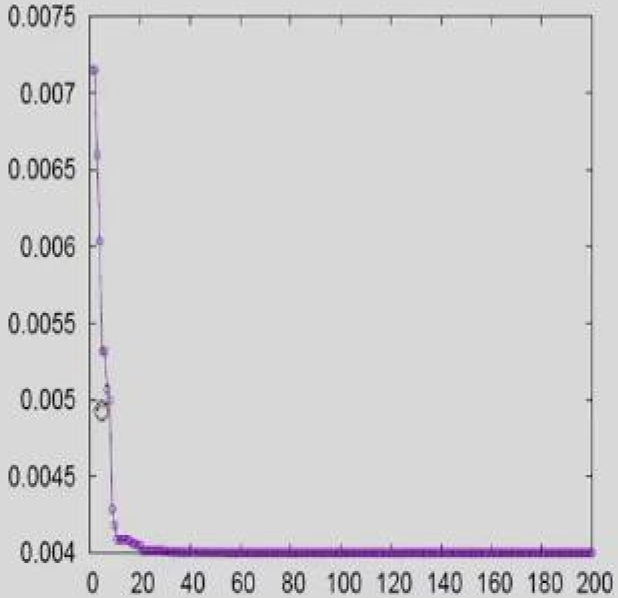
Constraint Handling

Dynamic Penalty Approach

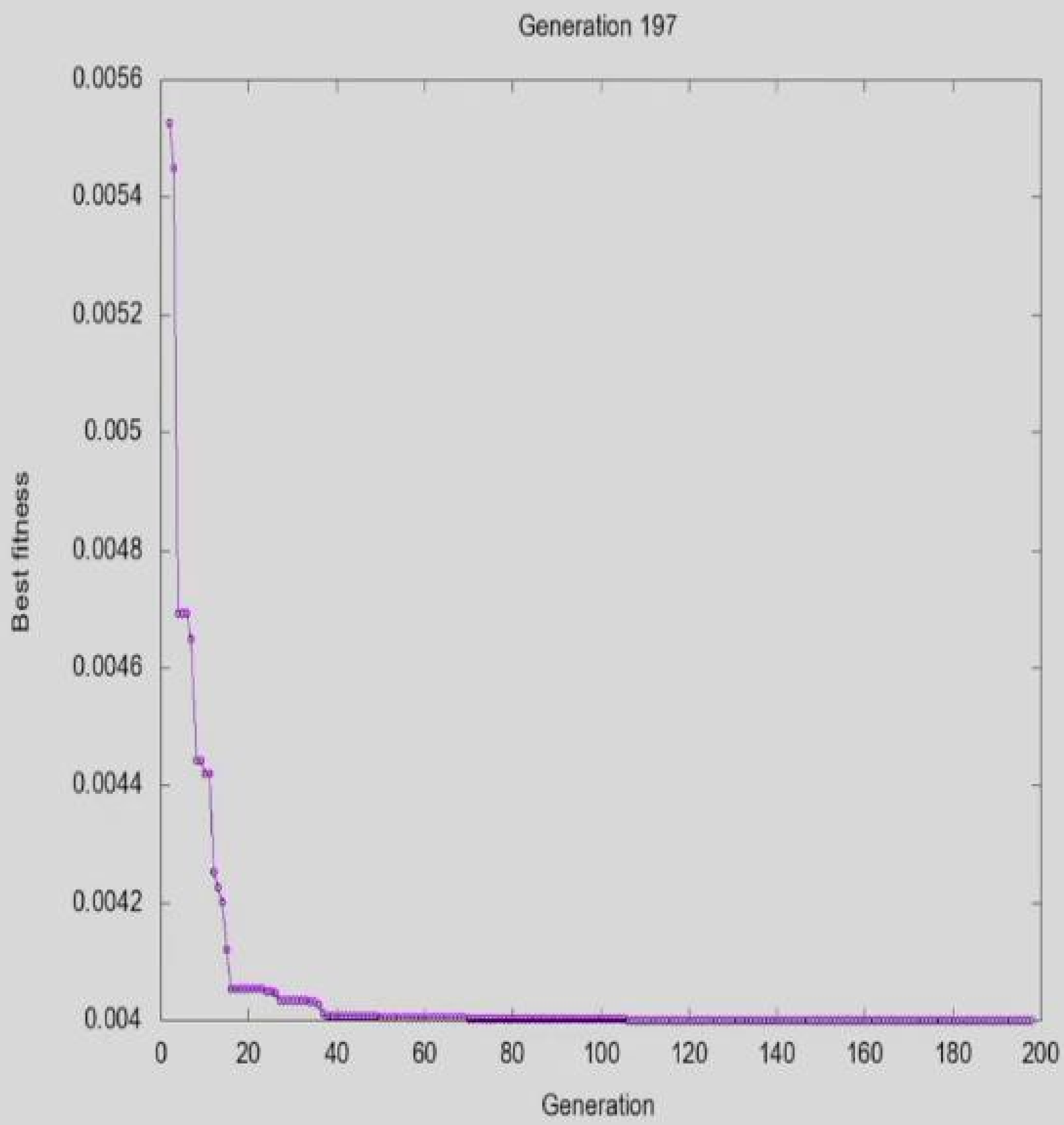


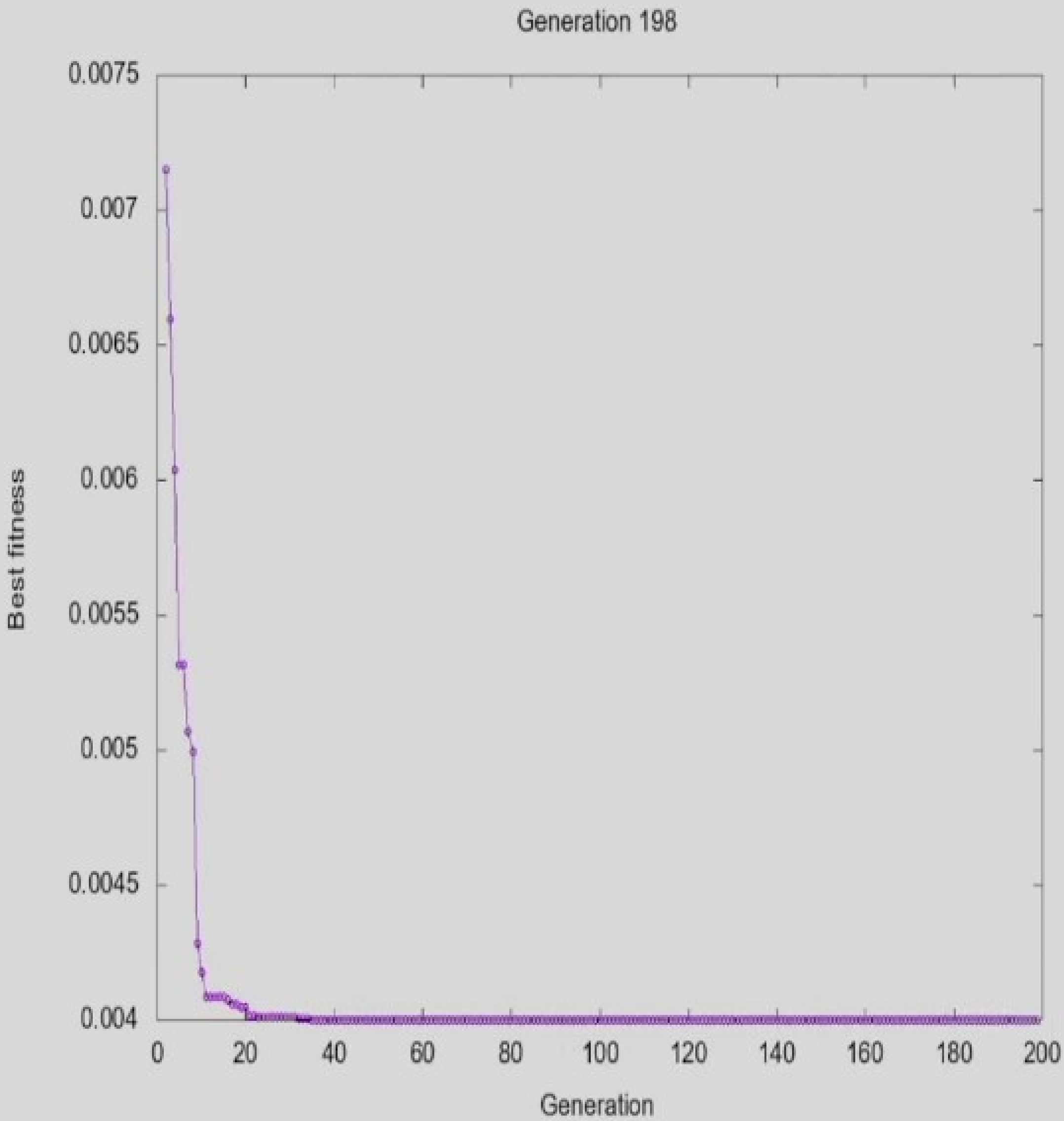
- Progress Link

Deb's Approach



- Progress Link


Generation 197
Best fitness
0.0056
0.0054
0.0052
0.005
0.0048
0.0046
0.0044
0.0042
0.004
0
20
40
60
80
100
120
140
160
180
200
Generation


Generation 198
Best fitness
0.0075
0.007
0.0065
0.006
0.0055
0.005
0.0045
0.004
0
20
40
60
80
100
120
140
160
180
200
Generation

Comparison

Approaches	$f(x)$	$g_1(x)$	x
Static Penalty$(R = 2)$	0.004	0.086	$(0.0004, 0.0009, 2.017)^T$
Static Penalty$(R = 100)$	0.004	0.086	$(0.0004, 0.0009, 2.017)^T$
Dynamic Penalty	0.004	0.086	$(0.0004, 0.0009, 2.017)^T$
Deb's Approach	0.004	0.000	$(0.0004, 0.0009, 1.991)^T$

Closure

- Three constraint handling techniques were used and coupled with real-coded genetic algorithm.
- Five mathematical and three practical constrained optimization problems were used to check the performance of the constraint handling techniques.
- Simulation and progress were shown.
- It was found that the penalty approaches are sensitive to their parameters for different types of problems.
- For some problems, penalty approaches were unable to generate any feasible solution.
- Deb's approach was found to be the most effective among the chosen set of constraint handling techniques.