$$J(x) = \frac{1}{2} \langle Ax, x \rangle - \langle b, x \rangle$$

Quadratic cost function

$$\nabla J(x) = J'(x) = Ax - b$$

if A symmetric

$$\nabla J(x) = B^{-1}(x - x_b) - H^T R^{-1}(x_{obs} - Hx)$$

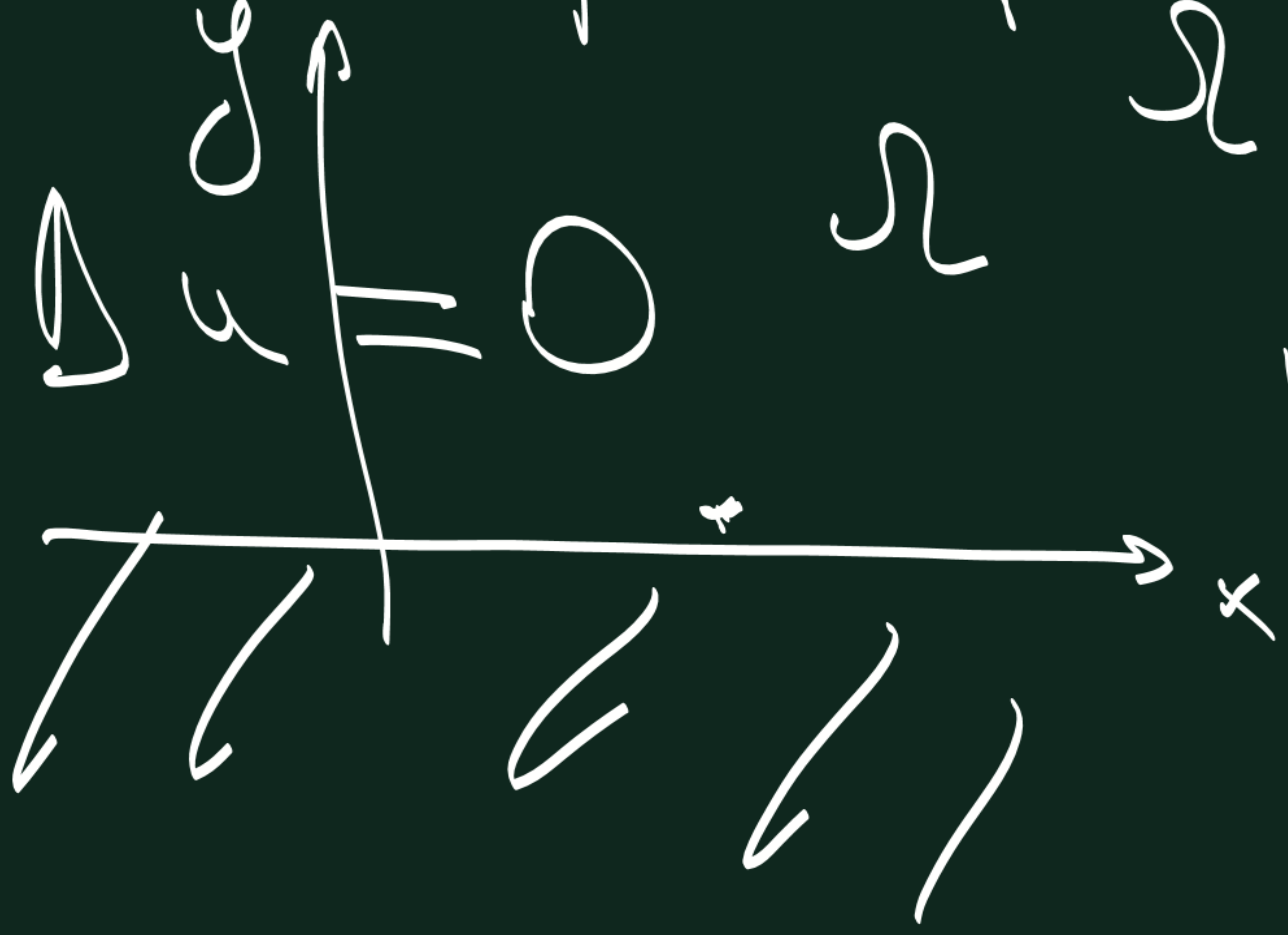
H matrix

Euler equation for x^* :

$$B^{-1}(x^* - x_b) - H^T R^{-1}(x_{obs} - Hx^*) = 0$$

Ill-posed problem:

$$\Delta u = 0$$



$$\Omega = \{(x,y) \text{ s.t. } y>0\}$$

$$u(x,y) = \frac{\sin nx \ \operatorname{sh} ny}{n^2}$$

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Boundary Cauchy condition: (Dirichlet + Neumann)

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial y}(x,y) = \frac{\cancel{n}\sin nx \; \mathrm{ch}\, ny}{n^{\cancel{2}}}$$

$$\frac{\partial u}{\partial y}(x,0) = \frac{\sin nx}{n} \qquad \left|\frac{\partial u}{\partial y}(x,0)\right| \le \frac{1}{n} \xrightarrow[n\to+\infty]{} 0$$

$$u(x,y)\Big|_{n\to+\infty} \to +\infty \quad \text{for } y>0$$

The Cauchy problem in the upper half-plane is ill-posed

Tikhonov regularization $\longrightarrow$ well-posed