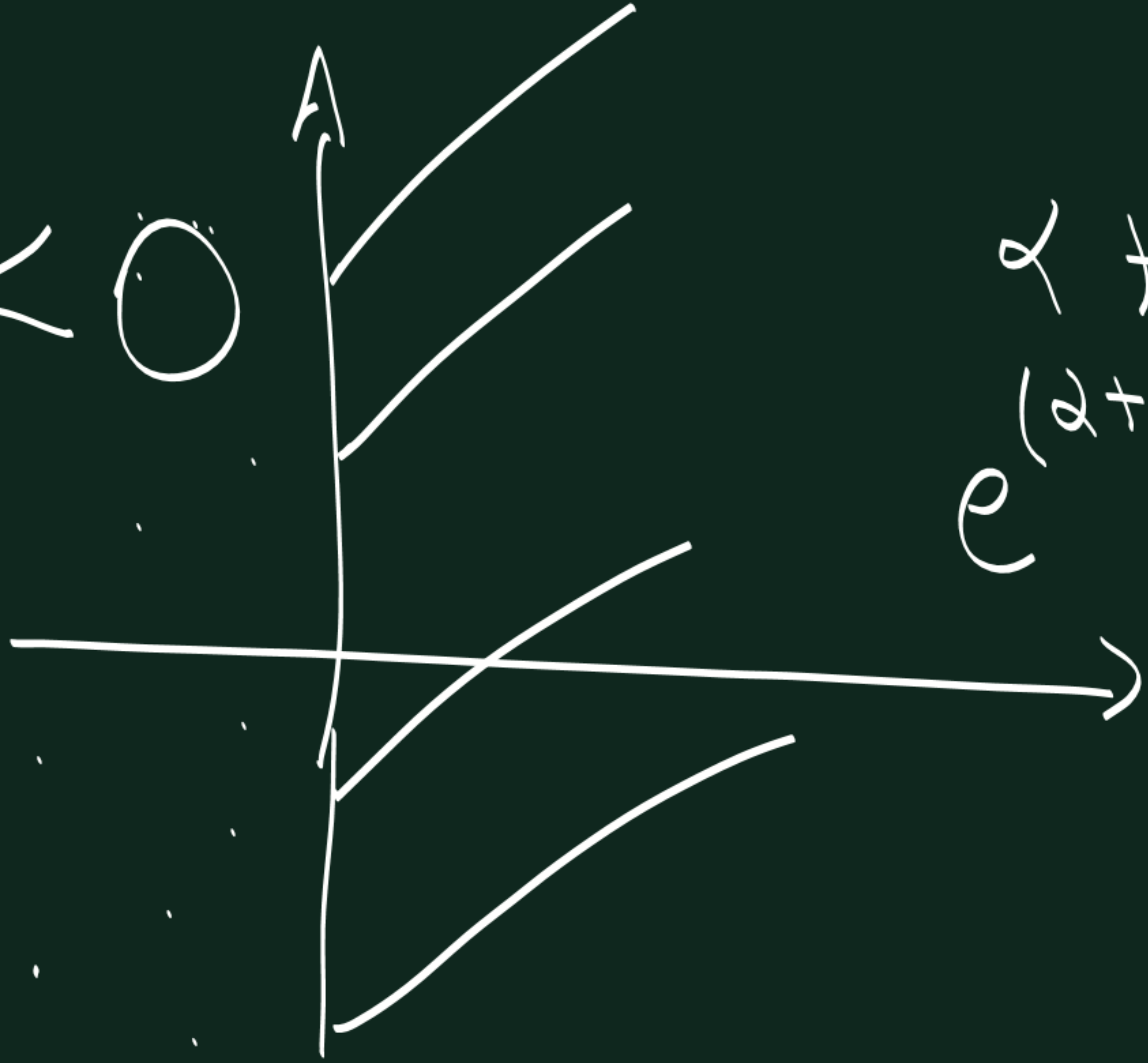
$\mathrm{Re}\,\lambda < 0$



$\alpha + i\beta$

$e^{(\alpha+i\beta)t} = e^{\alpha t}\, e^{i\beta t}$

λ

$\alpha < 0$

Heat equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u = f \\ u(x,0) = u_0(x) \end{cases}$$

well-posed equation

Backwards heat equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} - Du = f \\ u(x,T) = u_T(x) \end{cases} \qquad t' = T - t \qquad dt' = -dt$$

becomes:

$$\begin{cases} -\dfrac{\partial u}{\partial t'} - Du = f \\ u(x, t'=0) = u_T(x) \end{cases}$$

$$\begin{cases} \dfrac{\partial u}{\partial t} + \Delta u = f \\ u(x, t=0) = u_T(x) \end{cases}$$

ill-posed
(eigenvalues
in the right
half-space)

Partial differential equations

· Elliptic pde

EDO $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$x^2 + y^2$

Laplace equation

$\Delta u = 0$ Laplace

$\Delta u = f$ Poisson

Parabolic pde:

$$\frac{\partial u}{\partial t} - Du = s$$

1D $$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = s$$

$$T - X^2$$ parabola

$[0,T]\,[T,0]\,[0,T]\,[T,0]\ldots$

$k=1 \qquad\qquad k=2$

Hyperbolic equation:

Wave equation:

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = 0$$

$$T^2 - X^2 = C \quad \text{hyperbola}$$