ECE 461P: Homework 2 Scratch work

Sidharth Babu and Laith Altarabishi

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1 Problem 1

1.1 Bias-Variance Decomposition

Target: $E[(y(x|D) - h(x))^2]$

$$\begin{aligned}(y(x|D) - h(x))^2 &= (y(x|D) - E_d[y(x|D)] + E_d[y(x|D)] - h(x))^2 \\ &= (y(x|D) - E_d[y(x|D)])^2 + (E_d[y(x|D)] - h(x))^2 + 2(y(x|D) - E_d[y(x|D)])(E_d[y(x|D)] - h(x))\end{aligned}$$

Let $y(x|D) = y$ and $h(x) = h$
Taking Expectation of both sides:

$$\begin{aligned}E[(y-h)^2] &= E[(y - E[y])^2] + E[E[y]^2 - 2E[y] + h^2] - 2E[(y - E[y])(E[y] - h)] \\ &= Var(y) + E[y]^2 - 2E[y]E[h] + E[h]^2 - 2E[(y - E[y])(E[y] - h)] \\ &= Var(y) + E[y]^2 - 2E[y]E[h] + E[h]^2 - 2E[y]^2 + 2E[y][h] + 2E[y]^2 - 2E[h]E[y] \\ (&= Var(y) + E[y]^2 + E[h]^2 - 2E[y]E[h] \\ &= Var(y) + (E[y] - E[h])^2\end{aligned}$$

For a given X:

$$E[(y(x|D) - h(x))^2] = Var(y(x|D)) + (E[y(x|D)] - h(x))^2$$