Math 281 Homework 1 Name: ____________________________

Complete the following problems on your own paper. If you use notebook paper, please remove the jagged edges of the paper before submitting your homework. Your solutions must be numbered and submitted in the order the problems were given, legibly written using correct notation and including all mathematical details. If you submit work that is messy, disorganized, or lacking detail, you should expect to receive little credit regardless of having the correct final answer.

Due: 8:00am on Tuesday, October 8

1. Consider the following regions of $\mathbb{R}^3$ described by an inequality. Describe each region in words and sketch it to the best of your ability. Note : Inequalities define a solid region in space rather than a surface.
   (a) $4 < x^2 + y^2 + z^2$
   (b) $x^2 + z^2 \leq 9$
2. Consider the vectors $\vec{a} =< -2, 5, 4 >$ and $\vec{b} =< 4, 8, 1 >$.
   (a) Find $2\vec{a} - 3\vec{b}$
   (b) Find the length of the vectors $\vec{a}$ and $\vec{b}$.
   (c) Find the angle between the vectors $\vec{a}$ and $\vec{b}$.
   (d) Find a unit vector in the direction of $\vec{a}$.
   (e) Find $\text{comp}_{\vec{a}}\vec{b}$ and $\text{proj}_{\vec{a}}\vec{b}$
3. Find the values of x such that the vectors $< 6, 3x, x >$ and $< -3, 1, x >$ are orthogonal.
4. If $\vec{r} =< x, y, z >$ is an arbitrary position vector and $\vec{u} =< u_1, u_2, u_3 >$ and $\vec{v} =< v_1, v_2, v_3 >$ are constant position vectors, show that $(\vec{r} - \vec{u}) \cdot (\vec{r} - \vec{v}) = 0$ defines a sphere. Find the center and radius of the sphere.
5. Show that if $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal, then $\vec{u}$ and $\vec{v}$ have the same length.