F **1.** True or False: If $A \in \mathbb{R}^{3\times 4}$, then Rank($A$) + Nullity($A$) = 3. (No need to explain).

2. Give the definition of a Hermitian matrix.

A matrix $A \in \mathbb{C}^{n\times n}$ is Hermitian if $A^* = A$.

3. Given that the reduced echelon form of $A = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 3 & 2 \\ -2 & 3 & 5 & 1 \\ 1 & 0 & 2 & 1 \end{bmatrix}$ is $B = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, find a basis for Null($A$) and a basis for Range($A$).

- A basis for Range(A) is given by $\left\{ \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix} \right\}$

- $A\vec{x} = \vec{0} \iff \begin{matrix} x_1 + 2x_3 = 0 \\ x_2 + 3x_3 = 0 \\ x_4 = 0 \end{matrix} \iff \begin{matrix} x_1 = -2x_3 \\ x_2 = -3x_3 \\ x_3 = x_3 \\ x_4 = 0 \end{matrix} \iff \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_3 \begin{pmatrix} -2 \\ -3 \\ 1 \\ 0 \end{pmatrix}$ for any $x_3 \in \mathbb{R}$

$\Rightarrow$ A basis for Null(A) is given by $\left\{ \begin{pmatrix} -2 \\ -3 \\ 1 \\ 0 \end{pmatrix} \right\}$.