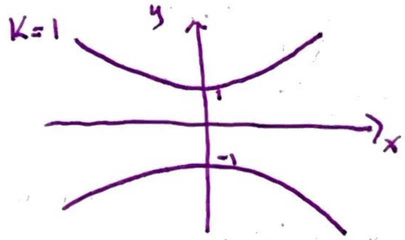
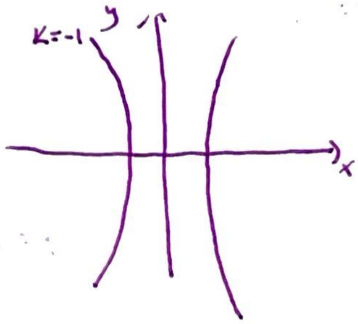
Ex: $z = y^2 - x^2$

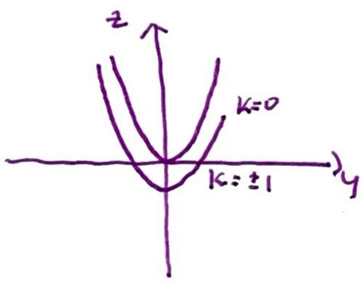
$z = k$: $y^2 - x^2 = k$ Hyperbola



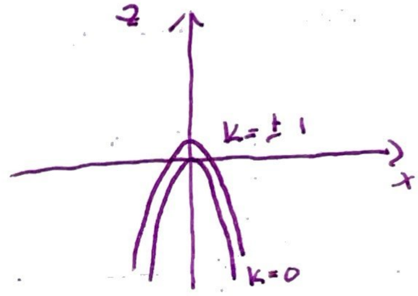


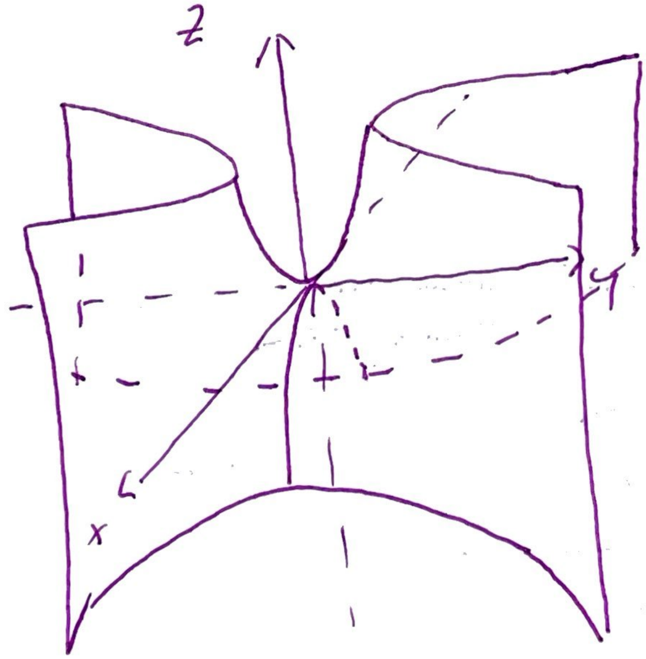
$x = k$: $z = y^2 - k^2$

$y = k$: $z = -x^2 + k^2$



Parabolas





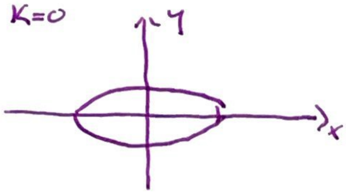
Hyperbolic Paraboloid

Ex: $\frac{x^2}{4} + y^2 - z^2 = 1$

$z = k$: $\frac{x^2}{4} + y^2 = 1 + k^2$

Ellipse for all k

$k=0$

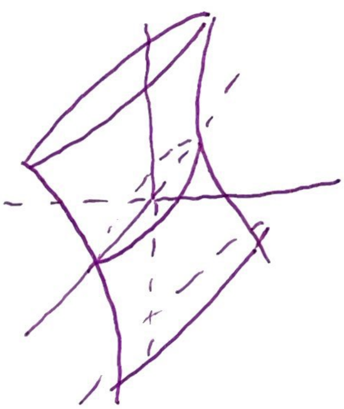


$x = k$: $y^2 - z^2 = 1 - \frac{k^2}{4}$

$y = k$: $\frac{x^2}{4} - z^2 = 1 - k^2$

Hyperbolas

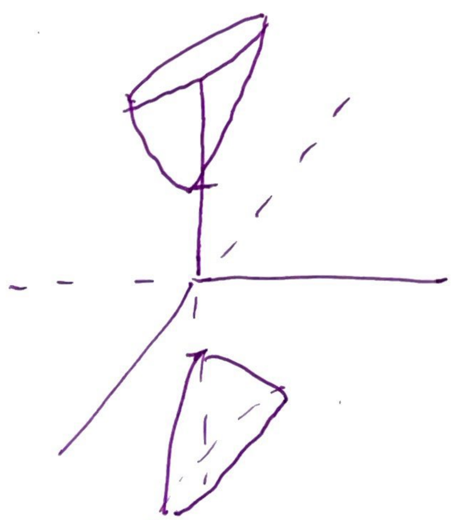
Two of the three traces are hyperbolas therefore surface is a Hyperboloid. Extends along entire z-axis so Hyperboloid of One Sheet.



Ex: $\frac{x^2}{4} + y^2 - z^2 = -1$

$z = k$: $\frac{x^2}{4} + y^2 = k^2 - 1$

Ellipse if $|k| \geq 1$

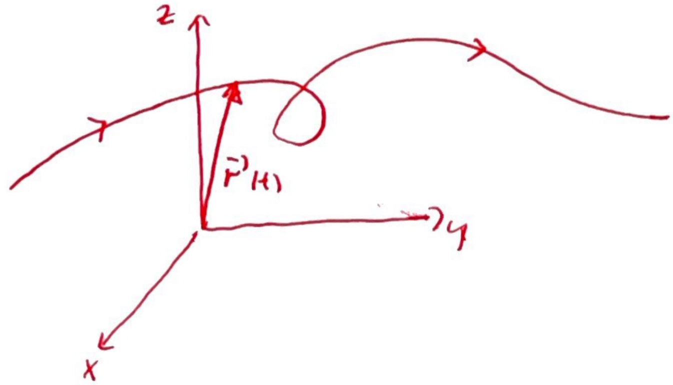


Hyperboloid of Two Sheets

§13.1

Chapter 13: Vector Functions

$\vec{r}(t) = \langle f(t), y(t), z(t) \rangle$ is a vector function whose component functions are scalar functions of t.



For each t in the domain of $\vec{r}(t)$, $\vec{r}(t) = \langle x, y, z \rangle$ is a position vector that terminates at a point $P(x, y, z)$ on a space curve.

$\vec{r}(t) = \langle x, y, z \rangle = \langle f(t), g(t), h(t) \rangle$

Parametric Equations

$$\begin{cases} x(t) = f(t) \\ y(t) = g(t) \\ z(t) = h(t) \end{cases}$$

The domain of $\vec{r}(t)$ is the set of all t where $f(t)$, $g(t)$, and $h(t)$ are defined at the same time.

Ex: Find domain of $\vec{r}(t) = \left\langle \frac{1}{t+1},\ \sqrt{1-t},\ e^{t^2} \right\rangle$

$f(t) = \frac{1}{t+1}$ Domain: All $t \neq -1$ $(-\infty,-1) \cup (-1,\infty)$

$g(t) = \sqrt{1-t}$ Domain: All $t \leq 1$ $(-\infty, 1]$

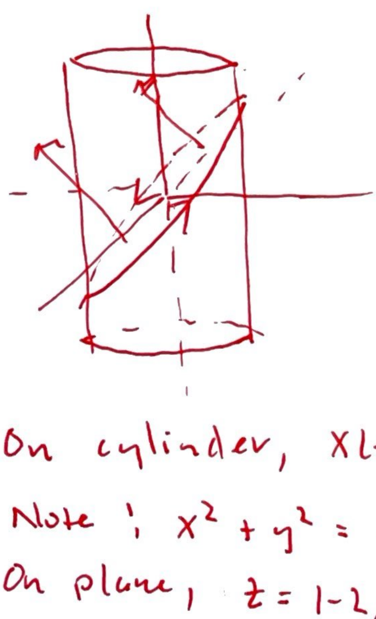
$h(t) = e^{t^2}$ Domain: $\mathbb{R}$

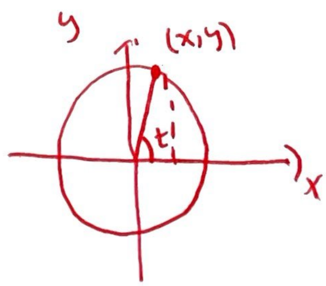
Domain of $\vec{r}(t)$ is $(-\infty,-1) \cup (-1, 1]$

Ex: Find the vector function defining curve of intersection of surfaces

$x^2 + y^2 = 4$: Circular Cylinder

$2x - 3y + z = 1$: Plane with normal $\vec{n} = \langle 2, -3, 1 \rangle$





$\cos t = \frac{x}{2}$

$\sin t = \frac{y}{2}$

On cylinder, $x(t) = 2\cos t$ and $y(t) = 2\sin t$

Note: $x^2 + y^2 = 4\cos^2 t + 4\sin^2 t = 4(\cos^2 t + \sin^2 t) = 4$

On plane, $z = 1 - 2x + 3y = 1 - 4\cos t + 6\sin t$

$\vec{r}(t) = \langle 2\cos t,\ 2\sin t,\ 1 - 4\cos t + 6\sin t \rangle$