Complex variable and contour integrations, Spring 2025

Instructor: Weiyong He

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- office hours: MW: 1:00pm-1:50pm, or by appointments.

Lecture: MWF 12:00pm - 12:50pm, Columbia 45 Mar 31, 2025 - Jun 08, 2025
Textbook: *Complex Variables and applications*, 8th edition, James Ward Brown, Ruel V. Churchill.
Prerequisite: Math 281 and the bridge requirement (math 232 or math 307). Math 282 and Math 316 are highly recommended.
Gradings:

(1) Homework, 30%. There would be weekly homework due on Wednesday (1st HW is due on Wednesday, Week 2).
Homework assignment and submission is through Canvas. You may scan your HW into a pdf file and upload it (it should be one pdf file). Your homework should be legible, containing sufficient work logically leading to the answer. Only a numerical answer without sufficient supporting work would receive few credit. You may work on HW together, and discussing among classmates is encouraged. On the other hand, your homework must be your own work and written by yourself.
(2) Midterm, 40%. There would be one in-class midterm, May 9th. (Week 6, Friday)
(3) Final exam, 30%. The final exam would be take-home exam.

Course description:

The theory of complex functions and contour integration is a true gem of mathematics which every math major should see. From calculus, you will be familiar with the idea of a differentiable function, and know that some functions are smoother than others. For example, the function

$$f(x) = \begin{cases} x^2, \text{if } x \geq 0 \\ 0, \text{if } x < 0 \end{cases}$$

is differentiable everywhere, but its derivative is not differentiable at $x = 0$. Surprisingly, the situation for functions $f(z)$ of a complex variable $z \in \mathbb{C}$ is completely different: a complex function that is one time differentiable is automatically infinitely differentiable. Such functions are called holomorphic, and include many special functions like trig functions, the exponential function and the mystery logarithm function. This is also the right context to properly understand the identity

$$e^{iz} = \cos z + i \sin z$$

for a complex number z. In this one quarter class, you will be introduced to complex functions and the remarkable Cauchy-Riemann equations, certain differential equations which capture what it is to be holomorphic. Holomorphic functions are also analytic, meaning that they are given locally by formal power series. This point of view leads to a revolution in the methods of calculus when you work with complex rather than real functions. After that, the course focusses on Cauchy's Theorem and the powerful method of contour integration. This can be used to compute many difficult integrals involving real functions by embedding the real line into the complex plane—shockingly, certain seemingly impossible calculations become rather easy from the new viewpoint!

Course material:

- Complex numbers, polar form
- Complex functions and complex derivative
- Cauchy-Riemann equation
- Elementary functions: exponential, trig, and log functions; Logarithm and branches
- Contour integrals, Cauchy's theorem, Cauchy's integral formula
- Power series
- More contour integrals, singularities and residue
- Residue theorem and its applications
- Laurent series.