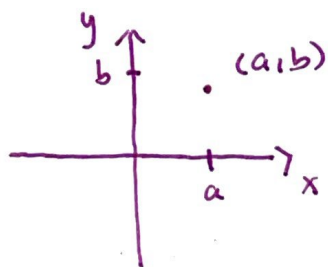


## §12.1 : Three Dimension Coordinate System

Two Dimension:  $xy$  plane

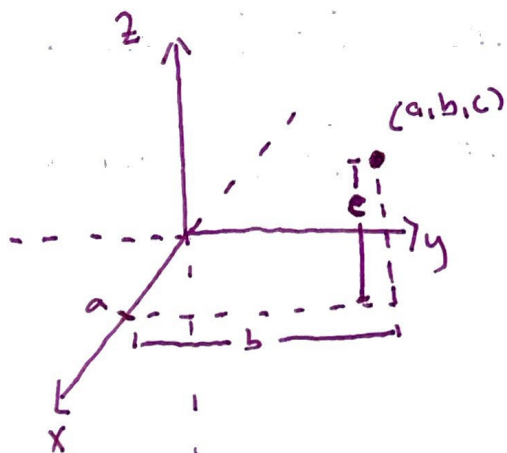


$x$ -axis and  $y$ -axis  
are perpendicular

Any point in the plane can be represented  
by an ordered pair,  $(a, b)$ , of real numbers.

$$\mathbb{R}^2 = \{ (x, y) : x, y \in \mathbb{R} \}$$

Three Dimension:  $xyz$  space

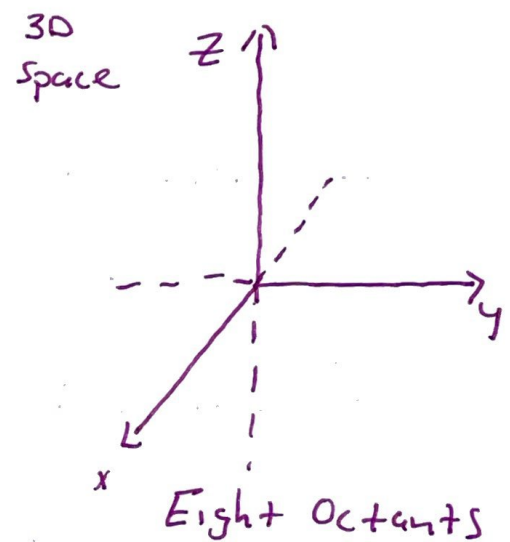
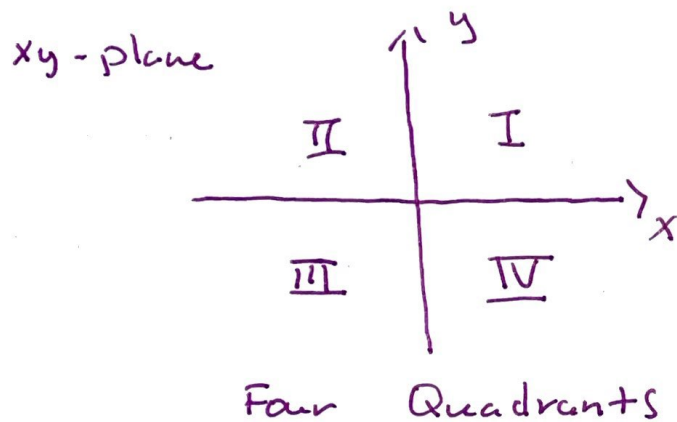


$x$ -axis,  $y$ -axis, and  
 $z$ -axis are pairwise  
perpendicular

Any point in three dimensional space can be  
represented by an ordered triple  $(a, b, c)$  of  
real numbers.

$$\mathbb{R}^3 = \{ (x, y, z) : x, y, z \in \mathbb{R} \}$$

Right Hand Rule is used to determine the direction of the positive  $z$ -axis. : Using your right hand, curl fingers from positive  $x$ -axis to positive  $y$ -axis. Then your thumb points in the direction of positive  $z$ -axis.



Octants I - IV are quadrants I - IV with  $z > 0$ ,  
 Octants V - VIII are quadrants I - IV with  $z < 0$

Coordinate Planes: Axes system determine coordinate planes.

①  $xy$  plane contains  $x$  and  $y$  axes. ( $z=0$ )  
Horizontal Plane

②  $yz$  plane contains  $y$  and  $z$  axes ( $x=0$ )  
Vertical Plane

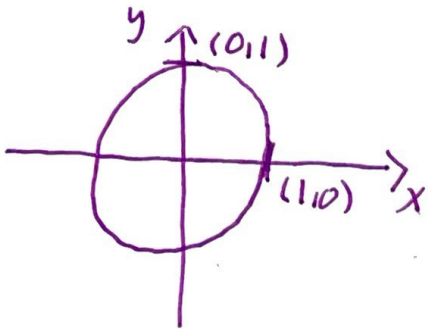
③  $xz$  plane contains  $x$  and  $z$  axes ( $y=0$ )  
Vertical Plane

Note: In  $\mathbb{R}^2$ , the equation  $f(x, y) = c$  defines a curve in the  $xy$  plane.

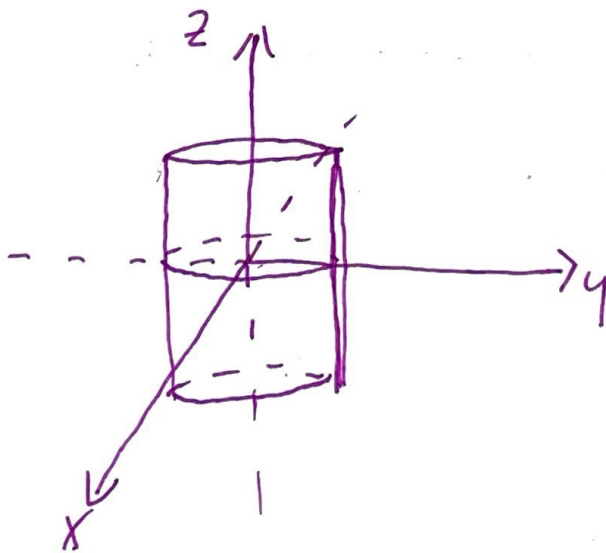
In  $\mathbb{R}^3$ , the equation  $g(x, y, z) = c$  defines a surface in space.

$$\text{Ex', } x^2 + y^2 = 1$$

a) In  $\mathbb{R}^2$ ,  $x^2 + y^2 = 1$  is a circle of radius 1 centered at  $(0,0)$ .

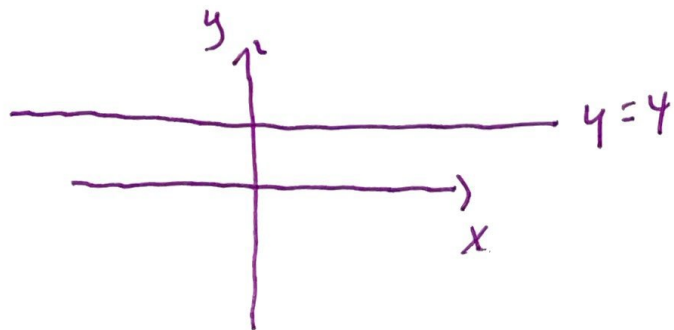


b) In  $\mathbb{R}^3$ ,  $x^2 + y^2 = 1$  places no restriction on  $z$  so the  $z$ -coordinate of the surface can be any real number. Surface defines a circular cylinder.

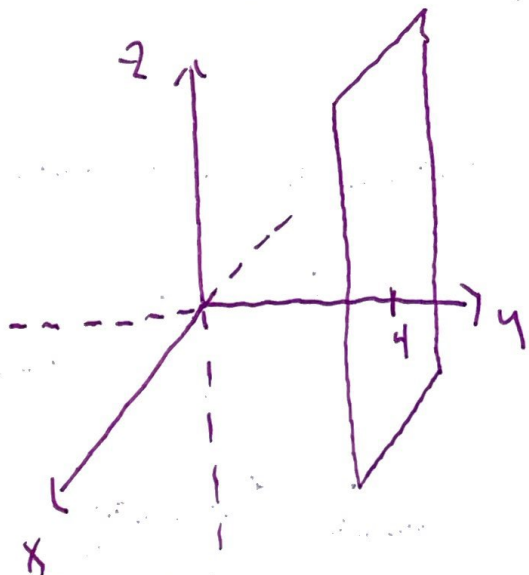


Ex',  $y=4$

- a) In  $\mathbb{R}^2$ ,  $y=4$  is a horizontal line parallel to  $x$ -axis,



- b) In  $\mathbb{R}^3$ ,  $y=4$  is a vertical plane parallel to  $xz$  coordinate plane ( $y=0$ ).



In general,  $x=k$  is a vertical plane parallel to  $yz$  plane,  $y=k$  is a vertical plane parallel to  $xz$  plane, and  $z=k$  is a horizontal plane parallel to  $xy$  plane.