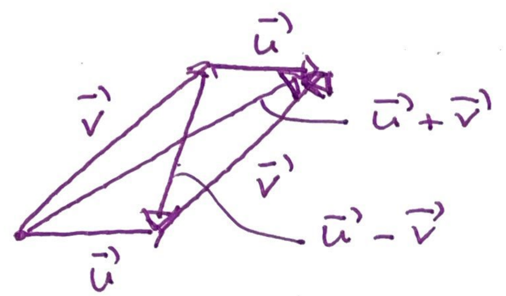
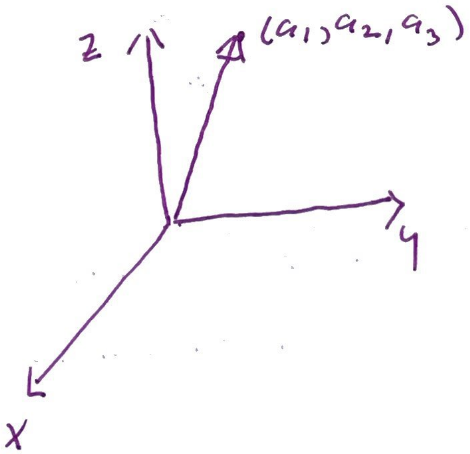
Parallelogram: Use $\vec{u}$ and $\vec{v}$ to form a parallelogram.



② Algebraically

Place the initial point of a vector, $\vec{a}$, in $\mathbb{R}^3$ at the origin. Then the terminal point has coordinates (a_1, a_2, a_3) in space



The coordinates of the terminal point define the <u>components</u> of the vector $\vec{a}$.

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

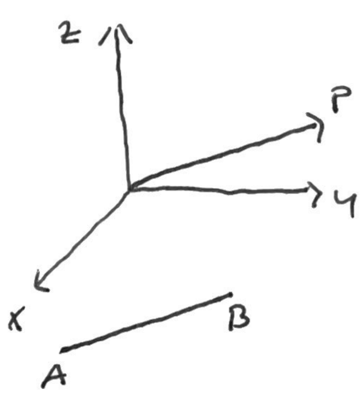
Note: Parantheses are used to notate a point while brackets are used to notate a vector in $\mathbb{R}^n$.

(a_1, a_2, a_3) : Point

$\langle a_1, a_2, a_3 \rangle$: Vector

Let $O(0,0,0)$ be the origin and $P(a_1, a_2, a_3)$ be a point in $\mathbb{R}^3$. Then $\vec{OP} = \langle a_1, a_2, a_3 \rangle$ is called a position vector that initiates at the origin and terminates at P.

Suppose $\vec{OP} = \langle a_1, a_2, a_3 \rangle$ and $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are points such that $\vec{OP}$ and $\vec{AB}$ are equivalent.



The components of $\vec{OP}$ define the displacement along coordinate axes from origin to point P. Applying the displacements to coordinates of A produces coordinates of B.

$$\vec{OP} = \vec{AB}$$

$$= \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

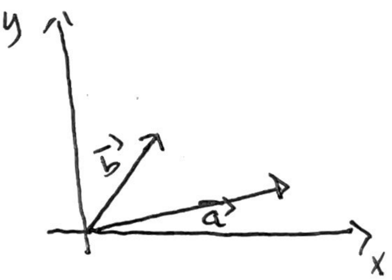
$$x_1 + a_1 = x_2$$

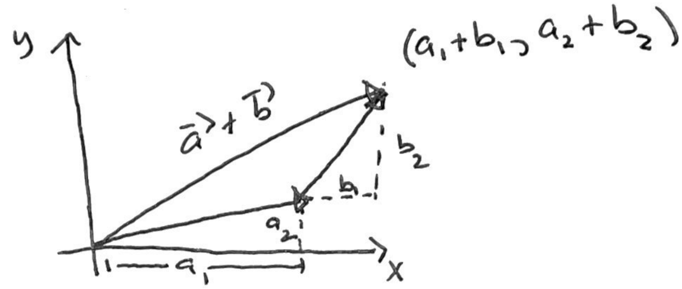
$$y_1 + a_2 = y_2$$

$$z_1 + a_3 = z_2$$

Consider $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$ in $\mathbb{R}^2$.

Vector addition using components must be consistent with triangle law.





$$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$$

Vector Operations: Let $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$ and let c be a scalar.

$$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

$$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$

$$c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$$

Ex: $\vec{a} = \langle 1, 0, -3 \rangle \qquad \vec{b} = \langle 2, -3, 5 \rangle$

$$4\vec{a} - 3\vec{b} = \langle 4, 0, -12 \rangle - \langle 6, -9, 15 \rangle$$

$$= \langle -2, 9, -27 \rangle$$

Defn: The <u>magnitude</u> or <u>length</u> of a vector is the distance between its initial and terminal points.

Given $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, $\vec{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$ and magnitude $|\vec{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

In general, if $\vec{v} = \langle v_1, v_2, v_3 \rangle$, then $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

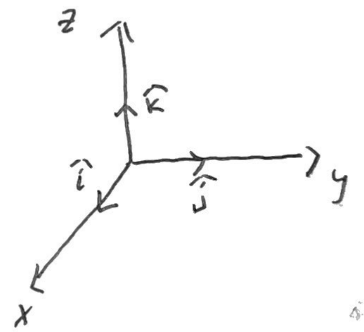
Defn: A <u>unit vector</u> is a vector whose length is 1.

Standard unit vectors in $\mathbb{R}^3$

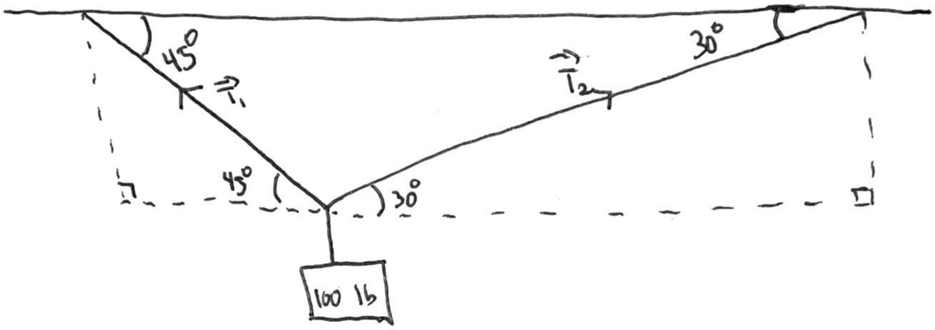
$\hat{i} = \langle 1, 0, 0 \rangle$

$\hat{j} = \langle 0, 1, 0 \rangle$

$\hat{k} = \langle 0, 0, 1 \rangle$



If $\vec{a} \neq \vec{0}$, then the unit vector with the same direction as $\vec{a}$ is $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$. $-\hat{a}$ has opposite direction as $\vec{a}$, Ex: A 100 lb weight hangs from two wires as in picture. Find the tensions $\vec{T}_1$ and $\vec{T}_2$ in both wires.



The tension vectors can be viewed as the hypotenuse of two right triangles.

$$\vec{T}_1 = -|\vec{T}_1|\cos(45°)\hat{i} + |\vec{T}_1|\sin(45°)\hat{j} = \frac{-|\vec{T}_1|}{\sqrt{2}}\hat{i} + \frac{|\vec{T}_1|}{\sqrt{2}}\hat{j}$$

$$\vec{T}_2 = |\vec{T}_2|\cos(30°)\hat{i} + |\vec{T}_2|\sin(30°)\hat{j} = \frac{\sqrt{3}}{2}|\vec{T}_2|\hat{i} + \frac{1}{2}|\vec{T}_2|\hat{j}$$

Forces are in balance therefore $\vec{T}_1 + \vec{T}_2 + (-100)\hat{j} = \vec{0}$

$$\left(\frac{-|\vec{T}_1|}{\sqrt{2}} + \frac{\sqrt{3}|\vec{T}_2|}{2}\right)\hat{i} + \left(\frac{|\vec{T}_1|}{\sqrt{2}} + \frac{|\vec{T}_2|}{2} - 100\right)\hat{j} = 0$$

$$\left.\begin{aligned} \frac{-|\vec{T}_1|}{\sqrt{2}} + \frac{\sqrt{3}|\vec{T}_2|}{2} &= 0 \\ \frac{|\vec{T}_1|}{\sqrt{2}} + \frac{|\vec{T}_2|}{2} &= 100 \end{aligned}\right\}$$

Solve system to get

$$|\vec{T}_1| = \frac{100\sqrt{6}}{\sqrt{3}+1}$$

$$|\vec{T}_2| = \frac{200}{\sqrt{3}+1}$$

$$\vec{T}_1 = \frac{-100\sqrt{3}}{\sqrt{3}+1}\hat{i} + \frac{100\sqrt{3}}{\sqrt{3}+1}\hat{j}$$

$$\vec{T}_2 = \frac{100\sqrt{3}}{\sqrt{3}+1}\hat{i} + \frac{100}{\sqrt{3}+1}\hat{j}$$