1. Classify by Jordan Canonical form (i.e. up to similarity and up to the order of the Jordan blocks) for all 6×6 matrices which have characteristic polynomial $(x-2)^2(x+3)^4$.

2. Find the Jordan canonical form of the matrix $\begin{pmatrix} a & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 1 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & a & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & a \end{pmatrix}$.

3. Let $A = \begin{bmatrix} 11 & -4 & -5 \\ 21 & -8 & -11 \\ 3 & -1 & 0 \end{bmatrix}$.

1). Find a basis for each generalized eigenspace of A consisting of a union of disjoint cycles of generalized eigenvectors.
2). Find a Jordan canonical form J of A using your basis in Part 1).
3). Find the minimal polynomial of A.

4. Let D be the differential linear operator on the vector space $V = \operatorname{span}\{1, t, t^2, e^t, te^t\}$, i.e.

$$D(f(x)) = \frac{d}{dx}(f(x)), \quad \text{for any} \quad f(x) \in V.$$

1). Let $\mathcal{B} = \{1, t, t^2, e^t, te^t\}$. Find the matrix representation $A = [D]_{\mathcal{B}}$.
2). Find a basis for each generalized eigenspace of D consisting of a union of disjoint cycles of generalized eigenvectors.
3). Find a Jordan canonical form J of D using your basis in Part 2).
4). Find the minimal polynomial of D.

5. Let $A \in \mathbb{C}^{n\times n}$. Assume $(A + I_n)^m = 0$. Prove that A is invertible and find $\det(A)$.

6. Let V be the vector space of all polynomials over $\mathbb{R}$. Let D be the differentiation operator on the vector space V. Find the minimal polynomial of D on V or prove that D has no minimal polynomial on V.

7. Suppose $A \in \mathbb{C}^{n\times n}$ satisfies $A^2 = A$. Prove that A is diagonalizable.

8. Give an example of two matrices who have the same characteristic polynomial but distinct minimal polynomials.