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Math 307  
Review for Final Exam

The final exam will cover the material from all homeworks. The best thing you can do to prepare for the exam is to study these homeworks and make sure you know how to do all of the problems. Also take a look at the midterms solutions.

On the exam you are allowed to use a notecard: one standard sheet (both sides) or printout of Logic Rules (with one page used for your notes).

The following is an overview of what we have been talking about in this class:

- (1) Symbolic logic proofs. You should know the basic rules (MP, MT, RCS, LCS, CI, MPB, SI, etc.) and be able to use them in proofs. You should know how to construct truth tables, and know how to identify and verify tautologies. You should be able to correctly use the Deduction Theorem and Indirect Inference. Finally, you should know (and be able to use) the following tautologies:

$$\begin{aligned}\sim [P \wedge Q] &\iff [\sim P \vee \sim Q] \\ \sim [P \vee Q] &\iff [\sim P \wedge \sim Q] \\ \sim [P \Rightarrow Q] &\iff [P \wedge \sim Q], \\ [P \vee Q] &\iff [\sim P \Rightarrow Q], \\ P \Rightarrow [P \vee Q].\end{aligned}$$

- (2) Quantifiers. You should be able to construct expressions involving the two quantifiers “for all” and “there exists”, and you should be able to identify whether such expressions are true or false. You should know the rules for negating statements with quantifiers. You should be able to identify and use basic routines with quantifiers (IU, EI, IE).
- (3) Set theory. You should be able to use and analyze set-builder notation (e.g.,  $\{x \mid P(x)\}$ ), set roster notation (e.g.,  $\{5, 3, 2, a, 7, x\}$ ), set intersections, set unions, and subset relations. You should know how to analyze relations (such as being a subset) between sets built using set operations (unions, intersections, complements).
- (4) You should understand how to do basic arithmetic and algebra in  $\mathbb{Z}_n$ . Given  $n$ , you should be able to identify which elements in  $\mathbb{Z}_n$  have multiplicative inverses, and use the Euclidean Algorithm to find these inverses.
- (5) Functions and their properties. You should know definitions of properties of functions (injective, surjective, bijective) and of sets related with functions (image, pre-image). You should be able to prove statements involving functions, their compositions, and set theoretical operations.
- (6) Proofs of arithmetic statements. You should know basic definitions and results in Number Theory (congruences, property P etc). You should be able to give line proofs of statements involving divisibility and congruences.
- (7) Mathematical induction. You should know how to use the Principle of Mathematical Induction for proofs of statements involving natural numbers. There are some extra examples on class website!
- (8) Cartesian products of sets. You should know how to prove set-theoretical statements involving Cartesian products of sets.