

1. State the rank-nullity theorem for an $m \times n$ matrix with real entries.

2. Given that $A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$ is row equivalent to $B = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, find a basis for $\text{Null}(A)$ and a basis of $\text{Range}(A)$.

3. Let A be an $m \times n$ matrix with real entries. Prove that $A = 0$ if and only if $\text{Tr}(A^T A) = 0$.

4. True or False. No explanation needed.

- 1). If A is a 3×3 matrix, then $\det(3A) = 9 \det A$.
- 2). If A, B are invertible $n \times n$ matrices, then $[(AB)^T]^{-1} = (B^T)^{-1}(A^T)^{-1}$.