Math 281 Homework 4 Name: ______________________

Complete the following problems on your own paper. If you use notebook paper, please remove the jagged edges of the paper before submitting your homework. Your solutions must be numbered and submitted in the order the problems were given, legibly written using correct notation and including all mathematical details. If you submit work that is messy, disorganized, or lacking detail, you should expect to receive little credit regardless of having the correct final answer.

Due: 8:00am on Tuesday, October 29

1. Reparametrize the curve $\vec{r}(t) =< 1, t^2, t^3 >$ with respect to arc length measured from the point $(1, 0, 0)$ in the direction of increasing t.

2. Find the curvature of $\vec{r}(t) =< t^2, \ln t, t \ln t >$ at the point $(1, 0, 0)$.

3. Show that for a smooth curve, $\dfrac{d\vec{T}}{ds} = \kappa \vec{N}(t)$. Therefore, at each point along the curve, $\dfrac{d\vec{T}}{ds}$ and $\vec{N}(t)$ are parallel and $\vec{N}$ points in the direction of curvature along the curve.

4. Given a space curve with smooth parametrization, $\vec{r}(t)$, the binormal vector is $\hat{B} = \hat{T} \times \hat{N}$. By properties of the cross product, it is a unit vector orthogonal to both $\hat{T}$ and $\hat{N}$.

   Note : Given $\hat{B} = \hat{B(s)}$ where $s(t)$ is arc length, then by Chain Rule, $\dfrac{d\hat{B}}{dt} = \dfrac{d\hat{B}}{ds} |\vec{r}\,'(t)|$.

   (a) Compute and simplify $\dfrac{d\hat{B}}{ds}$

   (b) Show that $\dfrac{d\hat{B}}{ds}$ is orthogonal to $\hat{B}$.

   (c) Show that $\dfrac{d\hat{B}}{ds}$ is orthogonal to $\hat{T}$.

   (d) Explain why $\dfrac{d\hat{B}}{ds}$ is therefore parallel to $\hat{N}$.

   Note : Since $\dfrac{d\hat{B}}{ds}$ is parallel to $\hat{N}$, it has the form $\dfrac{d\hat{B}}{ds} = -\tau \hat{N}$. The scalar function τ is called the torsion of the space curve. It measures the rate at which the curve is twisting out of the osculating plane toward or away from the binormal vector, $\hat{B}$. This is similar to how curvature measures the rate at which the curve is bending towards the unit normal vector.
   The dot product can be used to solve for τ.

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\begin{aligned}
-\tau \hat{N} &= \frac{d\hat{B}}{ds} \\
-\tau \hat{N} \cdot \hat{N} &= \frac{d\hat{B}}{ds} \cdot \hat{N} \\
\tau &= -\frac{d\hat{B}}{ds} \cdot \hat{N} \qquad \text{since } \hat{N} \cdot \hat{N} = 1
\end{aligned}
$$

   Similar to curvature, this is an unpleasant computation.