

1. [1 pt] True or False: Any subset of a linearly dependent set is linearly dependent. **F**

2. [3 pts] Give the definition of a basis of a vector space.

Let  $V$  be a vector space. A subset  $S \subseteq V$  is a basis of  $V$  if

- (1)  $S$  is linearly independent, and
- (2)  $\text{Span } S = V$ .

3. [6 pts] Let  $V = \{(a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$ . For any  $(a_1, a_2), (b_1, b_2) \in V$  and  $c \in \mathbb{R}$ , define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2), \quad \text{and} \quad c(a_1, a_2) = (ca_1, a_2).$$

Is  $V$  a vector space over  $\mathbb{R}$  with these operations? Justify your answer.

Answer:  $V$  is not a vector space.

- Axiom VII is not satisfied:  $(c+d)(a_1, a_2) = ((c+d)a_1, a_2)$

$$\begin{aligned} c(a_1, a_2) + d(a_1, a_2) &= (ca_1, a_2) + (da_1, a_2) \\ &= ((c+d)a_1, a_2) \end{aligned}$$

Take any  $a_2 \neq 0$ , and  $a_2 \neq 1$ , then  $a_2 \neq a_2^2$

$$\Rightarrow (c+d)(a_1, a_2) \neq c(a_1, a_2) + d(a_1, a_2).$$

- Axiom IV is not satisfied:

Note the zero vector of addition is  $(0, 1)$ , b/c

$$(a_1, a_2) + (0, 1) = (a_1 + 0, a_2 \cdot 1) = (a_1, a_2)$$

$$(0, 1) + (a_1, a_2) = (0 + a_1, 1 \cdot a_2) = (a_1, a_2).$$

However, take vectors  $(a_1, 0) \in V$ . Then for any  $(b_1, b_2) \in V$

$$(a_1, 0) + (b_1, b_2) = (a_1 + b_1, 0) \neq (0, 1)$$

$\Rightarrow (a_1, 0)$  does not have additive inverse in  $V$ .