Math 432/532: Introduction to Topology II

HW #2

1. Describe a simplicial complex in $\mathbb{R}^{n+1}$ whose realization is homeomorphic to the n-dimensional sphere S^n.

2. Describe a simplicial complex whose realization is homeomorphic to the projective plane $\mathbb{R}P^2$. You can be loosey goosey about where it lives, so that your answer is in the same form as the triangulation of the torus given in Section 6.1 of the text book.

3. Let X be any topological space, and let $f : X \to S^n$ be a map that is not surjective. Prove that f is homotopic to a map that takes all of X to a single point.

4. Let $f : S^4 \to S^7$ be any map. Use simplicial approximation to prove that f is homotopic to a map that takes all of S^4 to a single point.

5. Let K and L be simplicial complexes. Use the simplicial approximation theorem to show that the set of homotopy classes of maps from $|K|$ to $|L|$ is countable.