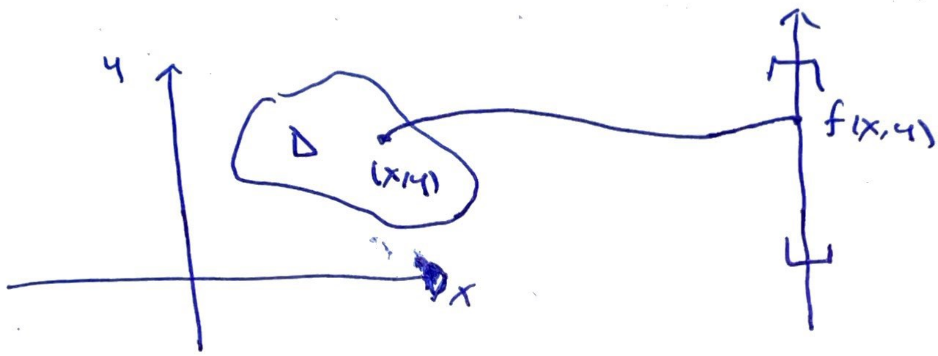
Chapter 14: Partial Derivatives

§14.1: Functions of Several Variables

Defn: A function, f, of two variables assigns each (x,y) in a set D to a unique number $f(x,y)$. The set D is a subset of $\mathbb{R}^2$ and is the domain of f. The range of f is a subset of $\mathbb{R}$.



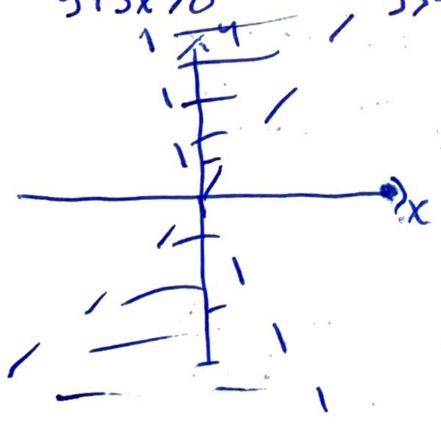
Ex: Find the domain and range of $f(x,y) = \ln(y^2 + 2xy - 3x^2)$

Domain: $D = \{(x,y) \in \mathbb{R}^2 : y^2 + 2xy - 3x^2 > 0\}$

$y^2 + 2xy - 3x^2 = (y-x)(y+3x)$

① $y - x > 0$ $\quad y > x$ $\quad$ OR $\quad$ ② $y - x < 0$ $\quad y < x$

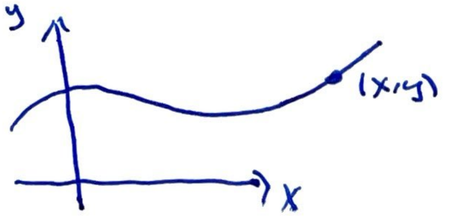
$y + 3x > 0$ $\quad y > -3x$ $\quad\quad$ $y + 3x < 0$ $\quad y < -3x$



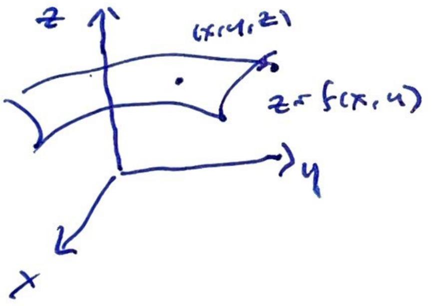
Range: The range of f is the range of log function.
Range is $\mathbb{R}$.

Graphs

The graph of $y=f(x)$ is the set of all ordered pairs (x,y) such that $y=f(x)$. It is a curve if the xy-plane.



Defn: The graph of $f(x,y)$ is the set of all ordered triples (x,y,z) such that $z=f(x,y)$ for $(x,y) \in D$ where D is the domain of f. The graph is a surface in 3D-space.



Ex: Graph $f(x,y) = 8-2x-4y$

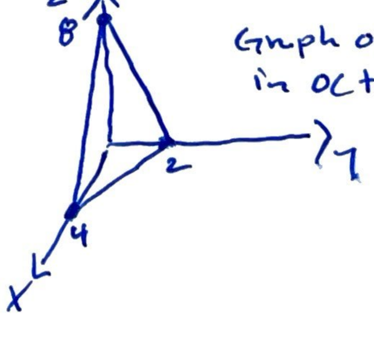
$z = 8-2x-4y$

Plane $2x+4y+z=8$ with normal $\vec{n} = \langle 2,4,1\rangle$

Interepts: $(4,0,0)$

$(0,2,0)$

$(0,0,8)$



Graph of f in octant I

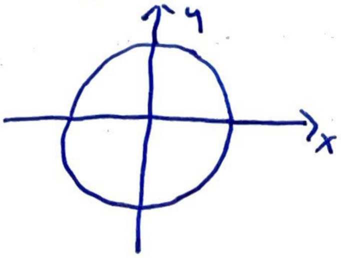
Ex: Find the domain of $f(x,y) = \sqrt{4-x^2-y^2}$.

Sketch its graph.

Domain: $D = \{(x,y) : 4-x^2-y^2 \geq 0\}$

$4-x^2-y^2 \geq 0$

$x^2+y^2 \leq 4$



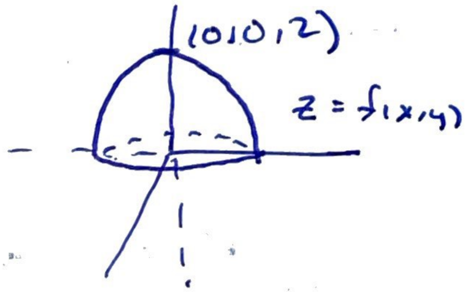
Domain is closed circular disk of radius 2.

Range: $[0,2]$

$z = \sqrt{4-x^2-y^2}$: Upper half sphere

$z^2 = 4-x^2-y^2$

$x^2+y^2+z^2=4$: Sphere



Level Curves

Defn: The level curves of $f(x,y)$ are the curves with equation $f(x,y) = K$ where K is a constant in the range of f.

It is a plot of all domain points that produce a range value.

The collection of level curves is called a contour map of f.

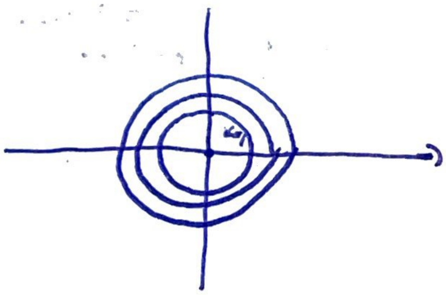
If $F(x,y)$ defines the elevation at (x,y), then the level curve $F(x,y) = K$ describes all points (x,y) with elevation K.

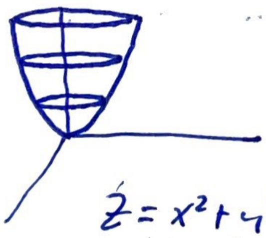
If $T(x,y)$ defines the temperature at (x,y), then the level curve $T(x,y) = K$ describes all locations with temperature K.

Ex: $f(x,y) = x^2+y^2$

Domain : $\mathbb{R}^2$

Range : $[0,\infty)$





$z = x^2+y^2$
Circular
Paraboloid

Level curves $x^2+y^2 = K$

Circle with radius $\sqrt{K}$

$K=0$: $x^2+y^2=0$: $(0,0)$

$K=1$: $x^2+y^2=1$

$K=2$: $x^2+y^2=2$

$K=3$: $x^2+y^2=3$

As K increases, the level curves are closer together.
Given a unit increase in f, the change in radii approaches 0

$$\lim_{K\to\infty} \sqrt{K} - \sqrt{K-1} = 0$$

Function changes rapidly for large values.