

## MATH 316: ASSIGNMENT 1

YOUR NAME

**Exercise 1.** Let  $a$  and  $b$  be two real numbers. Prove  $a \leq b$  if and only if, for any  $\varepsilon > 0$ ,  $a < b + \varepsilon$ .

*Solution.* One direction is simple: if  $a \leq b$ , then  $a \leq b < b + \varepsilon$  for any  $\varepsilon > 0$ .

On the other direction, assume  $a < b + \varepsilon$  for any  $\varepsilon > 0$  but  $a > b$ . Then  $\varepsilon_0 = a - b > 0$  and  $a = a - b + b = b + \varepsilon_0$ , which contradicts to  $a < b + \varepsilon$  fails for all  $\varepsilon > 0$ .  $\square$

**Exercise 1.2.5.** Using the triangle inequality to establish the inequalities

- (a)  $|a - b| \leq |a| + |b|$
- (b)  $||a| - |b|| \leq |a - b|$ .

*Solution.* This gives an example on how to type a list.  $\square$

**Example.** Here are example of displayed formulas:

$$f(x) = 1 + x^2 \sin x + \sqrt{x}$$

and (this one has the equation number)

$$(1) \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad -1 < x < 1.$$

Here is an example of multi-lined equations:

$$\begin{aligned} f(x) &= \frac{1}{1-x} \\ &= \sum_{n=0}^{\infty} x^n, \quad -1 < x < 1. \end{aligned}$$