

1. [1 pt] True or False: Any subset of a linearly dependent set is linearly dependent. **F**

2. [3 pts] Give the definition of a basis of a vector space.

Let V be a vector space. A subset $S \subseteq V$ is a basis of V if

(1) S is linearly independent, and

(2) $\text{Span } S = V$.

3. [6 pts] Let $V = \{(a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$. For any $(a_1, a_2), (b_1, b_2) \in V$ and $c \in \mathbb{R}$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2), \quad \text{and} \quad c(a_1, a_2) = (ca_1, a_2).$$

Is V a vector space over \mathbb{R} with these operations? Justify your answer.

Answer: V is not a vector space.

— Axiom VII is not satisfied: $(c+d)(a_1, a_2) \neq (ca_1, a_2) + (da_1, a_2)$

$$\begin{aligned} c(a_1, a_2) + d(a_1, a_2) &= (ca_1, a_2) + (da_1, a_2) \\ &= ((c+d)a_1, a_2^2) \end{aligned}$$

Take any $a_2 \neq 0$, and $a_2 \neq 1$, then $a_2 \neq a_2^2$

$$\Rightarrow (c+d)(a_1, a_2) \neq c(a_1, a_2) + d(a_1, a_2).$$

— Axiom IV is not satisfied:

Note the zero vector of addition is $(0, 1)$, b/c

$$(a_1, a_2) + (0, 1) = (a_1 + 0, a_2 \cdot 1) = (a_1, a_2)$$

$$(0, 1) + (a_1, a_2) = (0 + a_1, 1a_2) = (a_1, a_2).$$

However, take vectors $(a_1, 0) \in V$. Then for any $(b_1, b_2) \in V$

$$(a_1, 0) + (b_1, b_2) = (a_1 + b_1, 0) \neq (0, 1)$$

$\Rightarrow (a_1, 0)$ does not have additive inverse in V .