1. Give the definition of *isomorphic* vector spaces.

Let V and W be vector spaces. If there exists an isomorphism $\phi: V \to W$, Then V and W are called isomorphic vector spaces.

2. Give an example of a vector space V and a subspace W of V such that $\dim W = \dim V$, but $W \neq V$. *Briefly explain why your example satisfies the required conditions.*

Example1: $V = \mathbb{R}^\infty = \{(a_1, a_2, \cdots) : a_i \in \mathbb{R}\} \Rightarrow \dim V = \infty$

$W = \{(0, a_1, a_2, \cdots) : a_i \in \mathbb{R}\} \Rightarrow \dim W = \infty$

And as $(1, a_1, a_2, \cdots) \in V$ but $(1, a_1, a_2, \cdots) \notin W \Rightarrow W \neq V$.

Example 2: $V = \mathbb{P}^\infty(\mathbb{R})$ = the set of all Polynomials $\Rightarrow \dim V = \infty$

$W = \text{Span}\{1, x^2, x^4, x^6, \cdots, x^{2n}, \cdots\} \Rightarrow \dim W = \infty$

but $W \neq V$ since $p(x) = x \in V$ but $p(x) = x \notin W$

3. Give an example of a linear transformation from a vector space V to V that is onto but not one-to-one. *Briefly explain why your example satisfies the required condition.*

Let $T: \mathbb{R}^\infty \to \mathbb{R}^\infty$

$(a_1, a_2, a_3, \cdots) \longmapsto (a_2, a_3, \cdots)$

T is onto b/c $\forall (b_1, b_2, b_3, \cdots) \in \mathbb{R}^\infty$, $\exists (1, b_1, b_2, b_3, \cdots) \in \mathbb{R}^\infty$ such that $T(1, b_1, b_2, \cdots, b_3) = (b_1, b_2, b_3, \cdots)$

T is not one to one b/c $(1, a_1, a_2, \cdots) \neq (2, a_1, a_2, \cdots)$

but $T((1, a_1, a_2, \cdots)) = (a_1, a_2, \cdots)$

$T(2, a_1, a_2, \cdots) = (a_1, a_2, \cdots)$