Ex: Find the tangent plane to $x^2z + 4xy^2 + 2xyz = 2$

at $(1,-1,2)$

Define $F = x^2z + 4xy^2 + 2xyz$

$$\nabla F = \langle 2xz + 4y^2 + 2yz,\ 8xy + 2xz,\ x^2 + 2xy \rangle$$

$$\nabla F(1,-1,2) = \langle 4, -4, -1 \rangle$$

The tangent plane is $4(x-1) - 4(y+1) - (z-2) = 0$

Propertities of Gradient.

1) For $f(x,y)$ (or $f(x,y,z)$), ∇f is orthogonal to level curve $f = K$ at each point along the curve.

2) At each point, the maximum rate of change of f with respect to distance is $|\nabla f|$ and it occurs in direction of ∇f.

Ex: Suppose the temperature in a ball centered at $(0,0,0)$ is inversely proportional to the distance from the center to (x,y,z). Suppose the temp at $(1,2,2)$ is $120°F$. Find the rate of change of temp at $(1,2,2)$ toward $(2,1,3)$.

Let (x,y,z) be a point in ball. Distance from $(0,0,0)$ to (x,y,z) is $d = \sqrt{x^2+y^2+z^2}$

$$T(x,y,z) = \frac{K}{d} = \frac{K}{\sqrt{x^2+y^2+z^2}} \quad \text{for a constant } K.$$

$$T(1,2,2) = \frac{K}{\sqrt{9}} = 120 \qquad K = 360$$

$$T = \frac{360}{\sqrt{x^2+y^2+z^2}}$$

Vector from $(1,2,2)$ is $(2,1,3)$ is

$\vec{u} = \langle 1,-1,1\rangle$

$\hat{u} = \frac{1}{\sqrt{3}}\langle 1,-1,1\rangle$

$$\nabla T = \langle -360x(x^2+y^2+z^2)^{-3/2}, -360y(x^2+y^2+z^2)^{-3/2}, -360z(x^2+y^2+z^2)^{-3/2} \rangle$$

$$= \frac{-360}{(x^2+y^2+z^2)^{3/2}} \langle x, y, z \rangle$$

$$\nabla T(1,2,2) = \frac{-360}{27} \langle 1,2,2 \rangle = \frac{-40}{3} \langle 1,2,2 \rangle$$

$$D_{\hat{u}} T(1,2,2) = \hat{u} \cdot \nabla T(1,2,2)$$

$$= \frac{1}{\sqrt{3}} \langle 1,-1,1 \rangle \cdot \frac{-40}{3} \langle 1,2,2 \rangle$$

$$= \frac{-40}{3\sqrt{3}} \langle 1,-1,1 \rangle \cdot \langle 1,2,2 \rangle$$

$$= \frac{-40}{3\sqrt{3}}$$