Summary of logic routines

For each routine we give a sample showing how it is used.

Modus Ponens (MP):

1.	P	
2.	$P \Rightarrow Q$	
3.	Q	MP, For 2, For 1

Conjunctive Inference (CI):

1.	P	
2.	Q	
3.	$P \wedge Q$	CI, For 1, For 2

Left/Right Conjunctive Simplification (LCS and RCS):

1.	$P \wedge Q$	
2.	P	LCS, For 1
3.	Q	RCS, For 1.

Syllogistic Inference (SI):

1.	$P \Rightarrow Q$	
2.	$Q \Rightarrow R$	
3.	$P \Rightarrow R$	SI, For 1, For 2.

Modus Tollens (MT):

1.	$P \Rightarrow Q$	
2.	$\sim Q$	
3.	$\sim P$	MT, For 1, For 2

Modus Ponens for the Biconditional (MPB):

1.	$P \Leftrightarrow Q$			1.	$P \Leftrightarrow Q$	
2.	P		OR	2.	Q	
3.	Q	MPB, For 1, For 2		3.	P	MPB, For 1, For 2

Commutativity for the Biconditional (CB):

1.	$P \Leftrightarrow Q$	
2.	$Q \Leftrightarrow P$	CB, For 1

Transitivity for the Biconditional (TPB):

1.	$P \Leftrightarrow Q$	
2.	$Q \Leftrightarrow R$	
3.	$P \Leftrightarrow R$	TPB, For 1, For 2

Inference by Cases (IC):

1.	$P \Rightarrow Q$	
2.	$S \Rightarrow Q$	
3.	$[P \vee S] \Rightarrow Q$	IC, For 1, For 2

Deduction Theorem (DT):

1.	P	dis. hyp. (dischargeable hypothesis)
2.		
$\vdots$		
n.	Q	
(n+1).	$P \Rightarrow Q$	DT, discharge For 1 [(1)–(n) unusable]

Indirect Inference (II):

1.	P	dis. hyp. (dischargeable hypothesis)
2.		
$\vdots$		
n.	$A \wedge \sim A$	(this step could be any other contradiction)
(n+1).	$\sim P$	II, discharge For 1 [(1)–(n) unusable]

Disjunctive Inference (DI):

1.	$P \vee Q$			1.	$P \vee Q$	
2.	$\sim P$		OR	2.	$\sim Q$	
3.	Q	DI, For 1, For 2		3.	P	DI, For 1, For 2

Modus Tollens for the Biconditional (MTB):

1.	$P \Leftrightarrow Q$			1.	$P \Leftrightarrow Q$	
2.	$\sim P$		OR	2.	$\sim Q$	
3.	$\sim Q$	MTB, For 1, For 2		3.	$\sim P$	MTB, For 1, For 2

General Substitution Principle (GSP):

If $X(P)$ is a proposition in which the proposition P appears (perhaps multiple times), let $X(Q)$ denote the proposition obtained by subtituting every occurence of P with Q. For example, if $X(P)$ is

$$P \wedge (R \vee (S \Rightarrow \sim P))$$

then $X(Q)$ is

$$Q \wedge (R \vee (S \Rightarrow \sim Q)).$$

GSP can be used in two different ways:

1.	$P \Leftrightarrow Q$			1.	$X(P)$	
2.	$X(P)$		OR	2.	$X(Q)$	GSP, For 1, $P \Leftrightarrow Q$ (taut.)
3.	$X(Q)$	GSP, For 2, For 1				

Note that the second form is only used in the case where $P \Leftrightarrow Q$ is a tautology. You must obey the usual rules regarding tautologies, meaning that if it is not a common tautology we have used multiple times in the course then you must include a truth table. When it *is* a common tautology, it is okay if you write the minimum amount needed to identify it. For example, you could write

1.	$\sim (P \wedge Q)$	
2.	$(\sim P \vee \sim Q)$	GSP, For 1, de Morgan

and this would be perfectly clear. Note that we can even do several tautology-GSP steps at once, like this:

1.	$\sim ((A \vee B) \wedge (P \Rightarrow W))$	
2.	$(\sim A \wedge \sim B) \vee (P \wedge \sim W)$	GSP, For 1, de Morgan

Rules for Quantifiers

At the beginning of a proof, all variables are "available". Picture a bank containing all of these variables. Every time you write a hypothesis, any variables from those hypotheses are removed from the bank and are no longer "available".

Inference to a Universal (IU): If proposition $P(x)$ appears in your proof, and no hypotheses involve the variable x (so x is "available") then you can write $(\forall x)[P(x)]$. (Note that if a hypothesis is made and then discharged, it is gone from the proof and so does not count against you when you try to apply IU). An example is:

1. Assume $x \in \mathbb{Z}$ — dischargeable hyp.
2. $1|x$
3. $x \in \mathbb{Z} \Rightarrow 1|x$ — DT
4. $(\forall x)[x \in \mathbb{Z} \Rightarrow 1|x]$.

Existential Inference (EI): If proposition $P(z)$ appears in your proof, and y is an available variable, then you can write $(\exists y)[P(y)]$. Here we are going *from* an "instance" to an existential statement. An example is:

1. 5 is odd
2. $(\exists x)(x \text{ is odd})$ IE

Instance of an Existential (IE): If $(\exists x)[P(x)]$ appears in your proof, and u is an available variable, then you can write "$P(x)$ is true for some value of x". The cost of doing this is that x is then removed from your bank of available variables. Example:

1. $(\exists x)(x \in \mathbb{Z} \wedge x \text{ is prime})$
2. $u \in \mathbb{Z}$ and u is prime, for some u. IE

Sometimes we alter the phrasing in step 2. It is more common to say

1. $(\exists x)(x \in \mathbb{Z} \wedge x \text{ is prime})$
2. u is prime, for some $u \in \mathbb{Z}$. IE

Often the variable that appears in the existential *is* available, and so we can write

1. $(\exists x)(x \in \mathbb{Z} \wedge x \text{ is prime})$
2. x is prime, for some $x \in \mathbb{Z}$. IE

Be warned that the issues around available variables are tricky, and will take some time to get used to. There are more rules about them than we have indicated here, but it is best to figure them out as you go along.