Math 307, Homework #2
Due Wednesday, October 16

1. In each of the following give a disjunction that is equivalent to the given proposition:
   (a) $P \Rightarrow Q$
   (b) $\sim P \Rightarrow Q$
   (c) $P \Rightarrow \sim Q$

2. Translate the following into a symbolic logic problem, then provide a proof:

   Given: If Smith wins the nomination, he will be happy, and if he is happy, he is not a good campaigner. But if he loses the nomination, he will lose the confidence of the party. He is not a good campaigner if he loses the confidence of the party. If he is not a good campaigner, then he should resign from the party. Either Smith wins the nomination or he loses it.

   Prove: Smith should resign from the party.

For questions #2 and #3, copy the whole proof onto a sheet of your homework. For all of the logic proofs from now on, you can use any of the routines on the page of "Logic Rules" from the course website.

3. Fill in the blanks to give a proof of $R \vee [P \wedge Q]$, $\sim Q \vdash R$.

	Statement	Explanation
1.	$R \vee [P \wedge Q]$	??
2.	$\sim R$	??
3.	??	taut.
4.	$\sim R \Rightarrow [P \wedge Q]$	??
5.	$P \wedge Q$	??
6.	??	RCS, ??
7.	??	hyp.
8.	$Q \wedge \sim Q$	??
9.	R	??

4. Fill in the blanks to give a proof of $(P \wedge \sim Q) \Rightarrow (R \Rightarrow Q) \vdash P \Rightarrow [Q \vee \sim R]$. [Note: This proof uses both DT and Indirect Inference].

	Statement	Explanation
1.	$(P \wedge \sim Q) \Rightarrow (R \Rightarrow Q)$	??
2.	P	dis. hyp.
3.	$\sim [Q \vee \sim R]$	dis. hyp.
4.	??	taut.
5.	$\sim Q \wedge R$	??
6.	$\sim Q$	??
7.	$P \wedge \sim Q$	??
8.	$R \Rightarrow Q$	??
9.	$\sim R$	??
10.	R	??
11.	$R \wedge \sim R$	??
12.	$Q \vee \sim R$	??
13.	$P \Rightarrow [Q \vee \sim R]$	??

5. Show that $[P \wedge Q] \Rightarrow R$, $\sim R$, $P \vdash \sim Q$.

6. Show that $P \Rightarrow Q$, R, $R \Rightarrow [Q \Rightarrow P] \vdash P \Leftrightarrow Q$.

7. Show that $P \Rightarrow \sim Q$, $\sim R \Rightarrow Q \vdash P \Rightarrow R$.

8. Show that $\sim P \Rightarrow Q$, $T \Rightarrow \sim P$, $\sim [Q \vee R] \vdash \sim T$.

9. Show that $\sim P \Rightarrow Q$, $Q \Rightarrow [R \Rightarrow S]$, $\sim S \vdash R \Rightarrow P$.

10. Show that $P \Rightarrow T$, $Q \Rightarrow T$, $R \Leftrightarrow [P \vee Q]$, $R \vdash T$.

11. Show that $S \Rightarrow P$, $Q \Rightarrow R$, $S \vdash [P \Rightarrow Q] \Rightarrow R$.

12. Show that $R \Rightarrow T$, $\sim T \Leftrightarrow S$, $[R \wedge \sim S] \Rightarrow \sim Q \vdash R \Rightarrow \sim Q$.

13. Show that $\sim P \Rightarrow Q$, $[R \Rightarrow Q] \Rightarrow S$, $\sim S \vee T$, $R \Rightarrow \sim P \vdash T \vee V$.

14. Show that $[R \wedge \sim Q] \Rightarrow P$, $[T \Rightarrow S] \Leftrightarrow [R \Rightarrow Q]$, $R \vdash [\sim P \vee [T \Rightarrow S]] \Rightarrow Q$.

15. Use the Euclidean Algorithm to find integers a and b such that $37a + 100b = 1$. Use this information to solve $37x + 42 = 15$ in $\mathbb{Z}_{100}$.

16. For what primes p is the element $p - 1$ a perfect square in $\mathbb{Z}_p$? Investigate this question by working out the cases $p = 2$, $p = 3$, $p = 5$, $p = 7$, $p = 11$, $p = 13$, $p = 17$, and $p = 19$. See if you notice any patterns, and try to make a conjecture.

17. Find 2^{1000} in $\mathbb{Z}_7$. Then find 3^{1000} in $\mathbb{Z}_7$. Explain how you got your answers.

18. Consider a sum of three consecutive squares (like $7^2 + 8^2 + 9^2$). What do you get when you reduce this mod 3 (that is, when you compute the remainder when divided by 3)? Pick another sum of three consecutive squares and try it again. Try it one more time. State a conjecture, and see if you can prove it.

19. The following proof has a mistake. Find what is wrong, and explain.

    $(R \vee \sim S) \Rightarrow \sim P$, $Q \Rightarrow R$, $S \Rightarrow T \vdash (P \Rightarrow \sim R) \wedge (Q \Rightarrow T)$.

	Statement	Explanation
1.	$(R \vee \sim S) \Rightarrow \sim P$	hyp.
2.	$Q \Rightarrow R$	hyp.
3.	P	dis. hyp.
4.	$\sim (R \vee \sim S)$	MT, For 1, For 3
5.	$\sim (R \vee \sim S) \Leftrightarrow (\sim R \wedge S)$	taut.
6.	$\sim R \wedge S$	MPB, For 5, For 4
7.	S	RCS, For 6
8.	$\sim R$	LCS, For 6
9.	$P \Rightarrow \sim R$	DT, discharge For 3
10.	Q	dis. hyp.
11.	$S \Rightarrow T$	hyp.
12.	T	MP, For 11, For 7
13.	$Q \Rightarrow T$	DT, discharge For 10
14	$(P \Rightarrow \sim R) \wedge (Q \Rightarrow T)$	CI, For 9, For 13.

20. Show that $(R \vee \sim S) \Rightarrow \sim P$, $Q \Rightarrow R$, $S \Rightarrow T \vdash (P \Rightarrow S) \wedge (Q \Rightarrow (\sim P \wedge R))$.