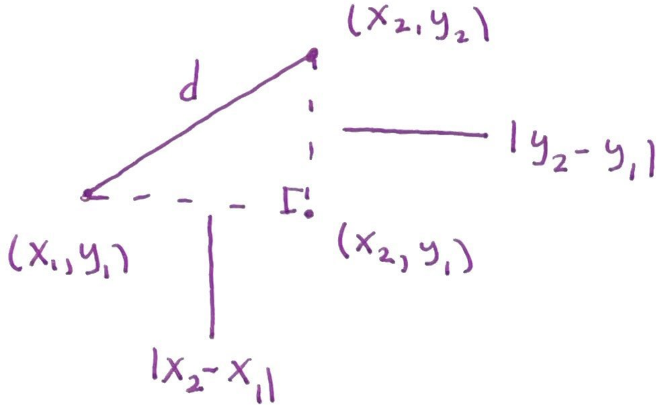
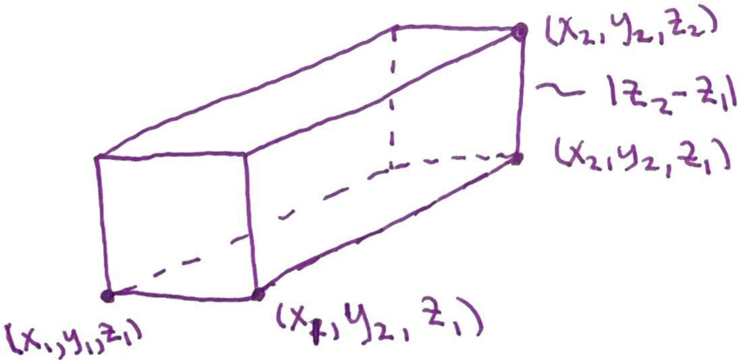
Distance

In $\mathbb{R}^2$, consider points $P(x_1, y_1)$ and $Q(x_2, y_2)$

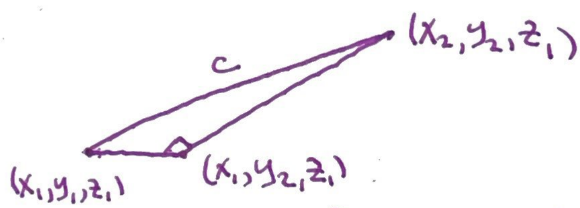


By Pythagorean Theorem, the distance between P and Q is $|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

In $\mathbb{R}^3$, consider points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$



In base of box



$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

P and Q are opposite corners of box

The distance between P and Q is

$$|PQ| = \sqrt{c^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

§12.2: Vectors

In $\mathbb{R}^n$, a vector is used to represent a quantity that has a direction and magnitude

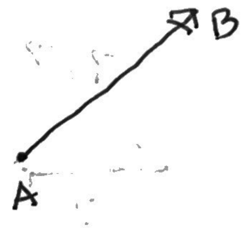
Ex: a) Displacement of a moving object
The direction of vector is direction of travel and magnitude is distance traveled.

b) Velocity of a moving object
The direction of vector is direction of travel and magnitude is speed of object.

c) Force
The direction of vector is direction force is applied and magnitude is strength of force.

Graphically, a vector in $\mathbb{R}^2$ or $\mathbb{R}^3$ is represented by a directed line segment whose length is magnitude of vector.

Notation: $\vec{v}$



Vector $\overrightarrow{AB}$

A is the initial point (tail)

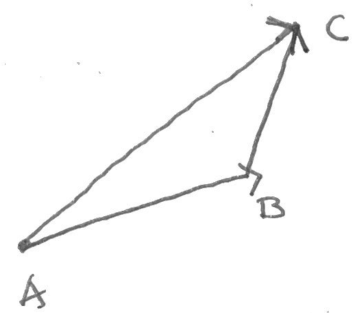
B is the terminal point (tip, head)

Vector Operations in $\mathbb{R}^n$

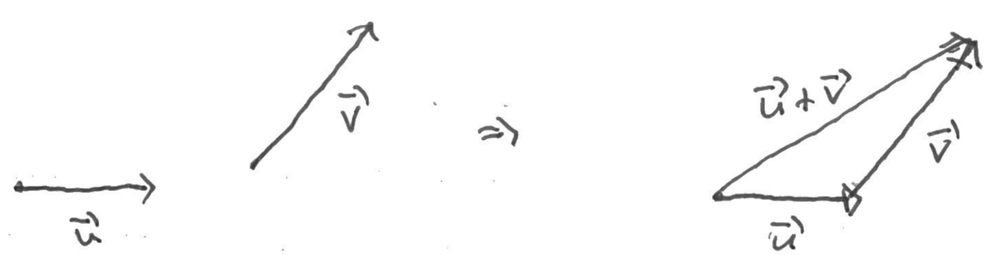
① Geometric

Suppose an object travels from A to D with displacement $\vec{AB}$ and then changes direction ~~from~~ and travels from B to C with displacement $\vec{BC}$. The vector sum $\vec{AB} + \vec{BC}$ is equal to $\vec{AC}$.

$\vec{AC} = \vec{AB} + \vec{BC}$



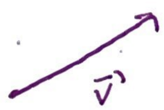
Ⓐ Vector Addition: To sum $\vec{u}$ and $\vec{v}$, position the initial point of $\vec{v}$ at the terminal point of $\vec{u}$. Then $\vec{u} + \vec{v}$ is the vector from initial point of $\vec{u}$ to terminal point of $\vec{v}$.



: Triangle Law for Addition

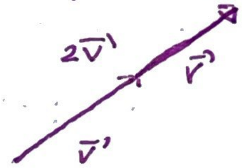
(B) Suppose c is a real-valued constant called a <u>scalar</u> and let $\vec{v}$ be a vector. Then $c\vec{v}$ is a vector with length $|c|$ times the length of $\vec{v}$.

If $c>0$, $\vec{v}$ and $c\vec{v}$ have same direction.
If $c<0$, $\vec{v}$ and $c\vec{v}$ have opposite direction.
If $c=0$, $c\vec{v} = \vec{0}$, zero vector.



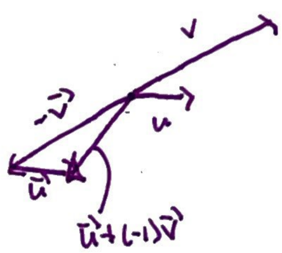
$2\vec{v} = \vec{v} + \vec{v}$

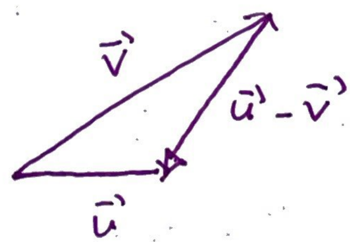
By Triangle Law,



(C) Vector Difference: $\vec{u} - \vec{v} = \vec{u} + (-1)\vec{v}$

Position $\vec{u}$ and $\vec{v}$ to have common initial point.





Triangle Law

$\vec{u}-\vec{v}$ is vector from terminal point of $\vec{v}$ to terminal point of $\vec{u}$.