Quiz 1 Solution **Winter 2025**

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1. (8 pts) State the definition of uniform continuous and give an example of a continuous function that is not uniformly continuous.

   Solution. A function f on a set A is uniformly continuous if for every $\varepsilon > 0$, there is a $\delta > 0$ such that for all $x, y in A$ satisfying $x, y \in A$, $|f(x) - f(y)| < \varepsilon$.

   Let $f(x) = 1/x$ and $A = (0, \infty)$. Then f is continuous on A but not uniformly continuous. ☐

2. (8 pts) Let $h(x) = x^3 - 4x + 2$. Show all three zeros of h are in $(-3, 3)$.

   Solution. We use the intermediate value theorem: If f is continuous on $[a, b]$ and $f(a)f(b) < 0$, then there is $c \in (a, b)$ such that $f(x) = 0$.

   A quick computation shows $h(-3) = -13 < 0$, $h(-2) = 2 > 0$, $h(0) = 2$, $h(1) = -1 < 0$ and $h(2) = 2 > 0$. Hence, h has a zero in $(-3, -2)$, $(0, 1)$, and $(1, 2)$. ☐

3. (9 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and c and assume $f(c) > 0$. Prove that there exists a number $A > 0$ and a neighborhood $V(c)$ of c such that $f(x) \geq A$ for all $x \in V(c)$

   Solution. Let $\varepsilon = f(c)/2$. There is a $\delta > 0$ such that for all x satisfying $|x - c| < \delta$, or $x \in V_\delta(c)$,
   $$|f(x) - f(c)| \leq \frac{f(c)}{2} \qquad \Longrightarrow f(x) \geq f(c) - \frac{f(c)}{2} = \frac{f(c)}{2} > 0.$$
   This completes the proof with $A = f(c)/2$ and $V(c) = V_\delta(c)$. ☐