Math 281 Homework 7 Name: ____________________________

Complete the following problems on your own paper. If you use notebook paper, please remove the jagged edges of the paper before submitting your homework. Your solutions must be numbered and submitted in the order the problems were given, legibly written using correct notation and including all mathematical details. If you submit work that is messy, disorganized, or lacking detail, you should expect to receive little credit regardless of having the correct final answer.

Due: 8:00am on Tuesday, November 19

1. Use the chain rule to find $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$ for $u(x,y,z) = x^2 \ln(y^2+z^2) + e^{-xy^3}$ where $x(s,t) = \frac{t^2}{1-3s}$, $y(s,t) = t^2\cos(3s)$ and $z(s,t) = t^5 s^{-4}$.

   Note : Please leave your answer in terms of x, y, z, s, and t.

2. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ assuming $z = f(x,y)$ for the implicitly defined surface.

$$\tan(x^2 - 3yz) = xz^2 - y^2$$

3. Let $f(x,y)$ be an arbitrary function of (x,y) where $x(r,\theta) = r\cos\theta$ and $y(r,\theta) = r\sin\theta$. Assume that $f(x,y)$ and all of its derivatives are continuous.

   (a) Use the chain rule to find f_r, f_θ, f_{rr}, and $f_{\theta\theta}$.

   Note : By using the chain rule, f_r and f_θ are sums of products. Therefore when computing the second derivatives, product rule is required. Also, f and all of its derivatives are composite functions of (r,θ). Therefore chain rule will be needed to differentiate f_x and f_y with respect to r or θ.

   (b) Evaluate $f_{rr} + \frac{1}{r^2} f_{\theta\theta} + \frac{1}{r} f_r$ to show that

$$f_{xx} + f_{yy} = f_{rr} + \frac{1}{r^2} f_{\theta\theta} + \frac{1}{r} f_r$$

4. Consider the function $f(x,y) = 2xy^3 + \ln(3y - x)$.

   (a) Find the directional derivative of f in the direction of $\vec{u} =< 3, 4 >$ at the point $(2,1)$.

   (b) What is the maximum rate of change of f at $(2,1)$? In what direction does the maximum rate of change occur?

   (c) Find all directions in which the directional derivative of f at $(2,1)$ has the value 1.