§12.6: Quadric Surfaces

Sketch surfaces in $\mathbb{R}^3$ by examining its traces

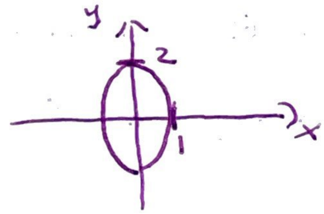
A <u>trace</u> is the intersection of the surface with a plane parallel to ~~the~~ a coordinate plane.

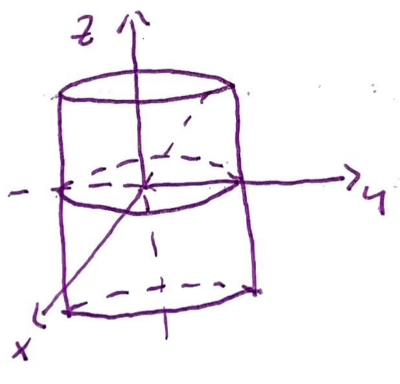
<u>Cylinders</u>

Ex: $4x^2 + y^2 = 4$

Horizontal Trace: $z = k$ $k \in \mathbb{R}$

$4x^2+y^2=4$: Ellipse





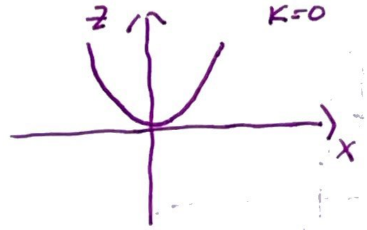
Since the trace parallel to xy-plane is constant for all z, the surface is a cylinder.

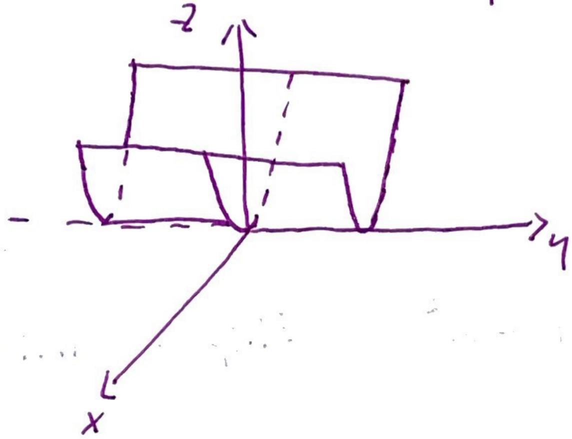
Since the trace is an ellipse, the surface is an elliptic cylinder.

Ex: $z = x^2$

Vertical trace: $y = k$

$z = x^2$ is a parabola





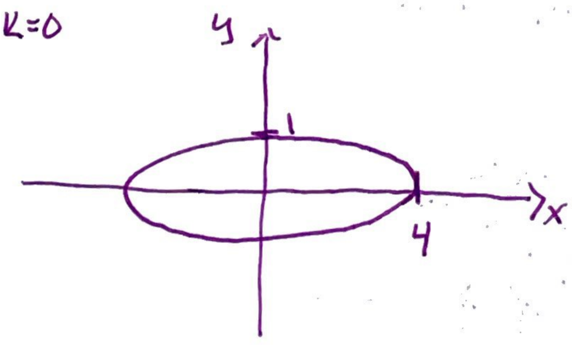
Parabolic Cylinder

Ex: $\frac{x^2}{16} + y^2 + \frac{z^2}{9} = 1$

Horizontal Trace: $z = k$ $\qquad \frac{x^2}{16} + y^2 = 1 - \frac{k^2}{9}$

Ellipse for $|k| \le 3$

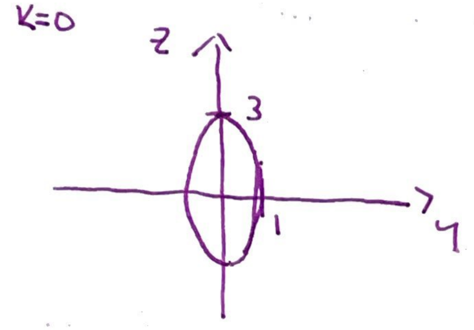
$k=0$



$\frac{x^2}{16} + y^2 = 1$

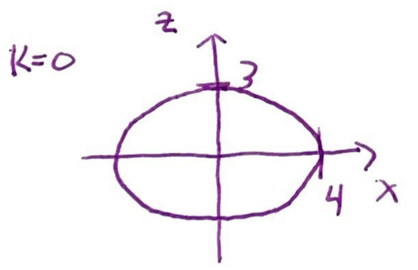
Vertical Trace:

$x = k$: $y^2 + \frac{z^2}{9} = 1 - \frac{k^2}{16}$ Ellipse $|k| \leq 4$

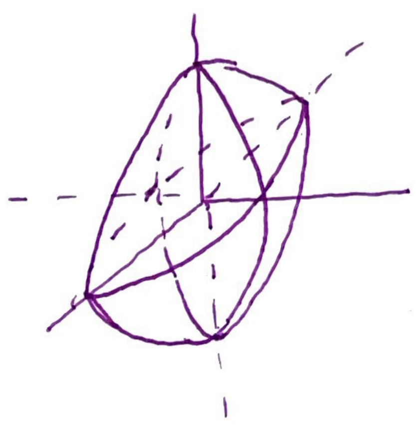


$$y^2 + \frac{z^2}{9} = 1$$

$y = k$: $\frac{x^2}{16} + \frac{z^2}{9} = 1 - k^2$ Ellipse $|k| \leq 1$



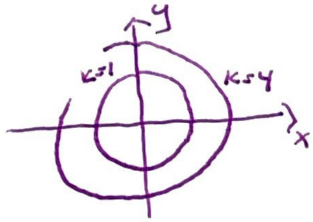
All three traces are ellipses. The surface is an ellipsoid.



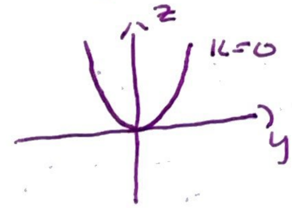
Ex: $z = x^2 + y^2$

$z = k$: $x^2 + y^2 = k$

circle with radius $r = \sqrt{k}$

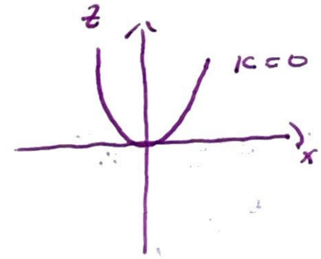


$x = k$: $z = y^2 + k^2$

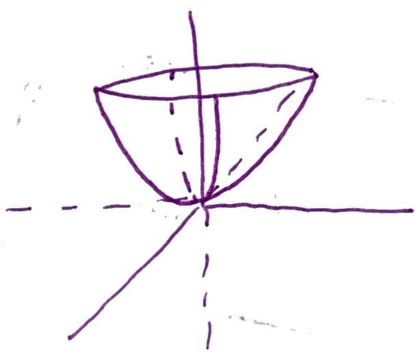


Parabolas

$y = k$: $z = x^2 + k^2$



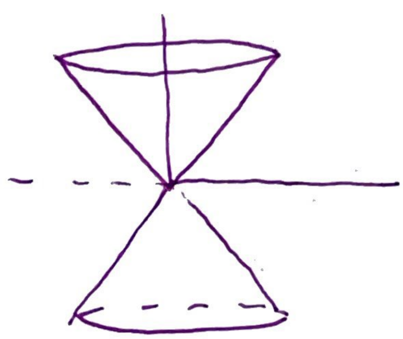
Two of three traces are parabolas therefore the surface is a paraboloid



Circular Paraboloid.

Ex: $z^2 = x^2 + y^2$

$z = k$: $x^2 + y^2 = k^2$ Circle with radius $r = |k|$



Cone