

Complete the following problems and submit your solutions at the beginning of class on Tuesday. If you use notebook paper, please remove the jagged edges of the paper before submitting your homework. Your solutions must be numbered and submitted in the order the problems were given, legibly written using correct notation and including all mathematical details. If you submit work that is disorganized, messy, or lacking detail, you should expect to receive little credit regardless of having the correct final answer.

Due: 9:00am on Tuesday, January 21

For each of the following double integrals, you must sketch the region of integration and indicate the order of integration.

1. Use polar coordinates to evaluate the following integrals.

$$(a) \int_0^1 \int_x^{\sqrt{2-x^2}} \frac{y^2}{x^2 + y^2} dy dx$$

$$(b) \int_0^4 \int_0^{\sqrt{4x-x^2}} \sqrt{x^2 + y^2} dy dx$$

2. Use polar coordinates to rewrite the sum as a single iterated integral and then evaluate the integral.

$$\int_{\frac{1}{\sqrt{2}}}^1 \int_{\sqrt{1-x^2}}^x \frac{1}{x^2 + y^2} dy dx + \int_{\sqrt[3]{2}}^1 \int_0^x \frac{1}{x^2 + y^2} dy dx + \int_{\frac{3}{\sqrt{2}}}^3 \int_0^{\sqrt{9-x^2}} \frac{1}{x^2 + y^2} dy dx$$

3. Use a double integral to find the volume of the following solids.

- (a) The solid that is inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the circular cylinder $x^2 + y^2 = 4$.
- (b) The solid that is bounded by the elliptic paraboloids $z = x^2 + 3y^2$ and $z = 16 - 3x^2 - y^2$.

4. Find the center of mass of the triangular region with vertices $(0, 0)$, $(1, 1)$, and $(0, 2)$ with density $\rho(x, y) = 3x + 2y$.