Farm and Agribusiness Management Lecture 4

Rodney Beard

October 17, 2016

Introduction

Technologists dilemma

Profit maximization

Equal marginal principle

Input substitutes

Least-cost combination

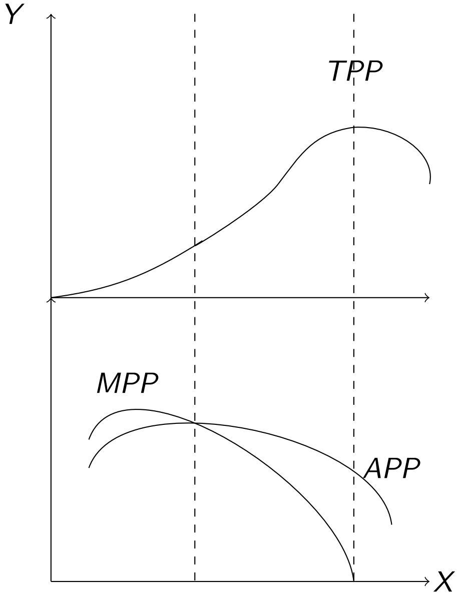
Average Physical Product

$$APP = \frac{totalphysicalproduct}{inputlevel}$$

Marginal Physical Product

$$MPP = \frac{\Delta totalphysicalproduct}{\Delta inputlevel}$$

Law of diminishing marginal returns



Technologists dilemma

Introduction
Technologists dilemma
Profit maximization
Equal marginal principle
Input substitutes
Least-cost combination

What should an Agricultural Scientist or Agronomist maximize?

- Total (physical) product?
- Marginal (physical) product?
- Average (physical) product?

Total cost, total revenue and profit

Input level	N applied	TPP	TC	TR	Profit
0	0	155	750	650	(100)
1	25	148	762.50	740	(22.50)
2	50	162	775.0	810	35
3	75	170	787.50	850	62.50
4	100	177	800	885	85
5	125	180	812.50	900	87.50
6	150	182	825	910	85
7	175	183	837.5	915	77.50
8	200	183	850	915	65

Marginal concepts

Marginal physical product

$$MR = \frac{\Delta \text{total revenue}}{\Delta total \text{ physical product}} = \frac{\Delta \text{Total revenue}}{MPP}$$

Marginal cost

$$MC = \frac{\Delta \text{total cost}}{\Delta \text{total physical product}} = \frac{\Delta \text{total cost}}{MPP}$$

Marginal revenue and marginal cost

Input	N	TPP	MPP	TR	TC	MR		MC
0	0	130		650	750			
			18			5	$>$	0.69
1	25	148		740	762.50			
			14			5	$>$	0.89
2	50	162		810	775			
			8			5	$>$	1.56
3	75	170		850	787.5			
			7			5	$>$	1.79
4	100	177		885	800			
			3			5	$>$	4.17
5	125	180		900	812.5			
			2			5	$<$	6.25
6	150	182		910	825			
			1			5	$<$	12.5
7	175	183		915	837.5			
			0			5	$<$	Infinite
8	200	183		915	850			

Price ratios and profit maximization

$$MPP = \frac{P_I}{P_o}$$

Why?

$$\text{Profit} = \Pi = P_o Y(X) - P_I X$$

$$\frac{d\Pi}{dX} = P_o Y^{'}(X) - P_I X$$

$$= P_o MPP - P_I = 0$$

This implies

$$MPP = \frac{P_I}{P_o}$$

Example

$$\max \Pi = PY(X) - WX$$

$$\frac{d\Pi}{dX} = P\frac{dY}{dX} - W = 0$$

Use $Y = 6\sqrt{X},\ P = \$20,\ W = \$12$

$$\frac{d\Pi}{dX} = 20(\frac{3}{\sqrt{X}} - 12 = 0$$

$$\frac{60}{\sqrt{X}} = 12$$

$5 = X^{1/2}$ or $X = 25$

Marginal value product and Marginal Input cost

Input	N	Y	MPP	TR	TC	MVP		MIC
0	0	130		650	750			
			18			3.6	>	0.5
1	25	148		740.00	762.50			
			14			2.8	>	0.5
2	50	162		810	775			
			8			1.6	>	0.5
3	75	170		850	787.5			
			7			1.4	>	0.5
4	100	177		885	800			
			3			0.6	>	0.5
5	125	180		900	812.50			
			2			0.4	<	0.5
6	150	182		910	825			
			1			0.2	<	0.5
7	175	183		915	837.5			
			0			0	<	0.5
8	200	183		915	850			

Introduction

Technologists dilemma

Profit maximization

Equal marginal principle

Input substitutes

Least-cost combination

Equal Marginal Principle

A limited output should be allocated among alternative uses in such a way that the marginal value products of the last unit used in on each alternative are equal.

	Marginal value products		
Irrigation water	Wheat	Grain	Cotton
0			
	1200	1600	1800
4			
	800	1200	1500
8			
	600	800	1200
12			
	300	500	800
16			
	50	200	400
20			

Hint: Rank order the MVP's

Feeder calfs

- livestock can be trickier than crop production
- take weaned feeder calves, fatten on pasture or feed ration and finish on feedlot
- Fatter calves need more nutrients to maintain weight
- heavier calves process feed less efficiently this drives up costs
- Sales price falls as calfs get fatter, marginal revenue is not constant

Feeder calf model

My attempt to explain this

$$\Pi(X) = P(W)W(X) - C(W(X))X$$

assume $W = X^2$ and $P = a - bW(X)$ and $C(W) = cW(X)$. Very simple model substituting

$$\Pi(X) = (a - bX^2)X^2 - cX^3 = aX^2 - bX^4 - cX^3$$
$$\frac{d\Pi}{dX} = 2aX - 4bX^3 - 3cX^2 = 0$$

Check the second order conditions

$$2a - 12bX^2 - 6cX < 0$$

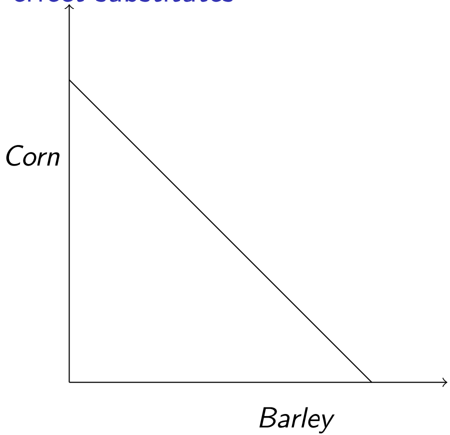
This is a polynomial inequality we can see the second-order conditions will be satisfied if

$$X \leq 16c \pm \frac{\sqrt{36c^2 - 128cb}}{-24b}$$

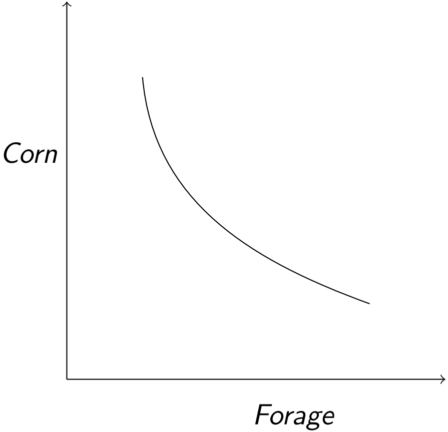
Multiple inputs: choosing input combinations

$$\text{Input substitution ratio} = \frac{\text{amount of input replaced}}{\text{amount of input added}}$$

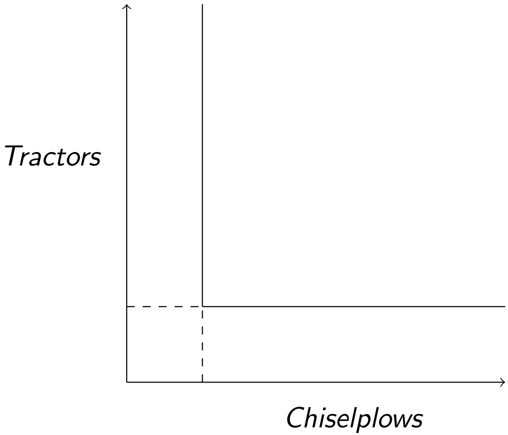
Perfect substitutes



Imperfect substitutes



Perfect complements



Least-cost combination

$$\min p_1 X_1 + p_2 X_2$$

subject to $Y = F(X_1, X_2)$

Lagrangian

$$L = p_1 X_1 + p_2 X_2 + \lambda \left[Y - F(X_1, X_2) \right]$$

$$\frac{\partial L}{\partial p_1} = X_1 - \lambda \frac{\partial F}{\partial X_1} = 0$$
$$\frac{\partial L}{\partial p_2} = X_2 - \lambda \frac{\partial F}{\partial X_2} = 0$$
$$\frac{\partial L}{\partial \lambda} = Y - F(X_1, X_2)$$

From the first two of these we get

$$\frac{p_1}{\frac{\partial F}{\partial X_1}} = \frac{p_2}{\frac{\partial F}{\partial X_2}}$$

or

$$\frac{p_1}{p_2} = \frac{\frac{\partial F}{\partial X_1}}{\frac{\partial F}{\partial X_2}}$$

Slope of iso-quant

To find the slope of the iso-quant we can use the total derivative

$$dY = \frac{\partial F}{\partial X_1} dX_1 + \frac{\partial F}{\partial X_2} dX_2 = 0$$

$$\frac{\partial F}{\partial X_2} dX_2 = -\frac{\partial F}{\partial X_1} dX_1$$

so

$$-\frac{dX_2}{dX_1} = \frac{\frac{\partial F}{\partial X_1}}{\frac{\partial F}{\partial X_2}} = \frac{p_1}{p_2}$$

$$\frac{\frac{\partial F}{\partial X_1}}{\frac{\partial F}{\partial X_2}} = MRTS$$

$MRTS$ = Input substitution ratio (ISR) = input price ratio (IPR)

is the decision rule for determining the least-cost

Least-cost feed ration

Feed ration	Grain	Hay	ISR		IPR	TC
A	825	1350				$155.25
			2.93	$\geq$	1.5	
B	900	1130				$148.80
			2.6	$\geq$	1.5	
C	975	935				$143.85
			2.2	$\geq$	1.5	
D	1050	770				$1450.70
			1.93	$\geq$	1.5	
E	1125	625				$138.75
			1.33	$\geq$	1.5	
F	1200	525				$139.5
			1.07	$\geq$	1.5	
G	1275	445				$141.45