Agricultural Marketing and Price Analysis Lecture 7 Prices over form (quality)

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Introduction

- Product hererogeneity (products are different)
- differences in genetics
- (farm) management practices
- post-farm handling
- weather, pests, diseases

Grading

- Grading to create standardized subsets (categories) (also see category management in the management literature)
- Standards without grades, e.g. free-range eggs
- Standards and grades can be enforced by government or industry associations

Grading example: French wheat

- No.1. grade French wheat: Minimum weight of 76 kilograms per hectoliter, a maximum moisture content of 15.5 percent and a maximum of 4 percent broken kernels.
- No 2. grade French wheat: 75 kg per hectoliter, 16 percent moisture, and 5 percent broken kernels.

Blending of products (grains, coffee, beans and livestock) allows traders to achieve pre-defined grading standards at minimum cost.

Exercise:

Given what you know how might a trader go about "designing" a blend?

Canadian Wheat Board Case

- single-desk seller
- 5 primary grades
- # 1CWAD (Candian western amber durum) to # 5 CWAD
- #1 and # 2 subdivided by protein content

Example

Suppose importer A is willing to pay $ 300/tonne for wheat with a minimum protein level of 14.5 percentand importer B is willing to pay $ 250 /tonne for wheat with any protein content. Wheat stocks in a country C consist of 750 tonnes.of ungraded wheat with 15 percent protein , 600 tonnes of ungraded wheat with 14 percent protein and 800 tonnes of ungraded wheat with 13 percent protein. This wheat can be purchased and blended by competitive exporters in country C and then resold to importers in countries A and B. In the absence of blending costs, the value of the three classes of wheat is maximized if 50 tonnes of the 13 percent wheat is blended with all of the 14 and 15 percent wheat to create a single blend with 14.5 percent protein. An additional tone of 15 percent wheat will allow an additional one-third of a tonne of 13 percent wheat to be added to the blend, which in turn will raise the aggregate value of the 13 percent wheat by $\frac{300-250}{3} = \$16.67/tonne$. *Consequently a competitive exporter will bid 300+16.67 = $ 316.67/tonne for the marginal unit of 15 percent wheat. this bid will establish the market price for this wheat.*

Source: Vercammen, p. 86

LOP model of blending and grading

Consider two grades A and B with prices P_A and P_B the cost of blending is m
Blend 1 unit of high quality product (grain) with γ units of low quality. What is the profit?

$$\pi = (1+\gamma)(P_A - m) - (P_H + \gamma P_L)$$

What are the equilibrium prices for the high and low quality varieties P_H and P_L in an LOP equilibrium?
Solution approaches:

1. direct solution via zero profit condition
2. indirectly by solving the social planners problem

Zero arbitrage profits - The direct approach

$$\pi = (1+\gamma)(P_A - m) - (P_H + \gamma P_L) = 0$$

Has two solutions depending on whether low quality product is in surplus or not.

- $$X_L \geq \gamma X_H$$

  then γX_H units of X_L will be incorporated into the blend and $X_L - \gamma X_H$ units of surplus X_L sold unblended on market at price P_B.

- $$X_L < \gamma X_H$$

  then the high quality commodity is in surplus so $\frac{X_L}{\gamma}$ units of X_H will be incorporated into the blend. $X_H - \frac{X_L}{\gamma}$ units of surplus X_H will be sold on the market unblended at price P_A.

Three scenarios

- Case I: $X_L > \gamma X_H$ low quality is in surplus

$$P_L = P_B \text{and} \begin{cases} P_a & \text{if } P_A - P_B \leq \left(\frac{1+\gamma}{\gamma}\right) m \\ P_A + \gamma(P_A - P_B) - (1+\gamma)m & \text{if } P_A - P_B > \left(\frac{1+\gamma}{\gamma}\right) m \end{cases}$$

- Case 2: $X_L \geq \gamma X_H$ High quality is in surplus

$$P_H = P_A \text{ and } P_L = \begin{cases} P_B & \text{if } P_A - P_B \leq \left(\frac{1+\gamma}{\gamma}\right) m \\ P_A - \left(\frac{1+\gamma}{\gamma}\right) m & \text{if } P_A - P_B > \left(\frac{1+\gamma}{\gamma}\right) m \end{cases}$$

- First line in each case is corner solution (marginal cost of blending $>$ is greater than the marginal revenue from blending
- Bottom line in each case corresponds to positive level of blending
- positive blending obtained by setting $P_H \geq P_A$ and $P_L \geq P_B$

Interpretation

- If the high quality commodity is scarce then the gap between P_H^* and P_L^* will be comparatively large.
- If the low quality commodity is in scarce supply then the gap between P_H^* and P_L^* will be comparatively small.
- This implies that for fixed P_A, P_B the prices for P_L, P_H are expected to be negatively correlated over time.
- Extreme case: When the unit cost of blending is zero then $P_H = P_L = P_A$ when the low quality product is scarce.

The Social Planner's Problem

Another way to obtain the same results is via a social planners problem. Let Q be the quality of X_L to be added to the blend.

- Avaialabiility restiction: $0 \leq Q \leq X_L$
- Grading restriction: $0 \leq Q \leq \gamma X_H$
- Total supply of grade A commodity is $X_H + Q$
- The total supply of grade B commodity is $X_L - Q$

The Social Planner's Problem

Net aggregate market surplus

$$V(Q) = (X_H + Q)P_A + (X_L - Q)P_B - m(1 + \frac{1}{\gamma})Q$$

Note 1 unit of low quality commodity results in $1 + \frac{1}{\gamma}$ units of high quality commodity.

Lagrangian

$$L(Q) = (X_H + Q)P_A + (X_L - Q)P_B - m(1 + \frac{1}{\gamma})Q +$$
$$\lambda_1(X_L - Q) + \lambda_2(\gamma X_H - Q) + \lambda_3 Q$$

The Karush-Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial Q} = P_A - P_B - \left(1 + \frac{1}{\gamma}\right) m - \lambda_1 - \lambda_2 + \lambda_3 \leq 0$$
$$\frac{\partial L}{\partial Q} Q = 0$$

Complementary slackness conditions:

$$\lambda_1(X_L - Q) = 0$$
$$\lambda_2(\gamma X_H - Q) = 0$$
$$\lambda_3 Q = 0$$

Inequality conditions:

$$0 \leq Q \leq X_H$$

Solution: Case I (low quality in surplus

If $\gamma X_H < X_L$ then $Q \leq \gamma X_H$ implies $Q < X_L$ using complementary slackness $\lambda_1(Q - X_L) = 0$ we get $\lambda_1 = 0$. Now either $Q = 0$ or $Q > 0$. If $Q = 0$ then $\lambda_2(\gamma X_H - Q) = 0$ implies $\lambda_2(\gamma X_H) = 0$ from which follows $\lambda_2 = 0$ or $\gamma X_H = Q$ implying $\lambda_3 = 0$ becuse $\lambda_3 Q = 0$. The first-order condition implies implies

$$\lambda_3 = -\left[P_A - P_B - \left(1 + \frac{1}{\gamma}\right) m\right] \geq 0$$

when $Q = 0$ and $\lambda_1 = \lambda_2 = 0$.

$$\lambda_2 = \left[P_A - P_B - \left(1 + \frac{1}{\gamma}\right) m\right] \geq 0$$

when $Q = \gamma X_H$ and $\lambda_1 = \lambda_3 = 0$. Summary

$$Q^* = \begin{cases} 0 & \text{if} P_A - P_B \leq \left(\frac{1+\gamma}{\gamma} m\right) \\ \gamma X_H & \text{if} P_A - P_B \geq \left(\frac{1+\gamma}{\gamma}\right) m \end{cases}$$

Case II: High quality in surplus

$\gamma X_H > X_L$ in this case $Q \leq X_L$ which implies $\lambda_2 = 0$ because $\lambda_2(Q - \gamma X_L) = 0$. There are two possibilities: either $Q = 0$ which implies $\lambda_1 = 0$. because $\lambda_1(Q - X_L) = 0$ or $Q = X_L$ this implies $\lambda_3 = 0$ because $\lambda_3 Q = 0$. From the first-order condition one sees

$$\lambda_3 = -\left[P_A - P_B - \left(1 + \frac{1}{\gamma}\right) m\right] \geq 0$$

when $Q = 0$ and $\lambda_1 = \lambda_2 = 0$

$$\lambda_1 = \left[P_A - P_B - \left(1 + \frac{1}{\gamma}\right) m\right] \geq 0$$

when $Q = X_L$ and $\lambda_2 = \lambda_3 = 0$

Summary:

$$Q^* = \begin{cases} 0 & \text{if} P_A - P_B \leq \left(\frac{1+\gamma}{\gamma} m\right) \\ X_L & \text{if} P_A - P_B \geq \left(\frac{1+\gamma}{\gamma}\right) m \end{cases}$$

Summary

Case I: $\gamma X_H < X_L$ low quality in surplus and $Q^* > 0$

$$Q^* = \gamma X_H, \lambda_1 = \lambda_3 = 0 \text{and} \lambda_2 = \left[P_A - P_B - \left(1 + \frac{1}{\gamma}\right) m\right] \geq 0$$

Case 2: $\gamma X_H > X_L$ high quality in surplus and $Q^* > 0$

$$Q^* = X_L, \lambda_2 = \lambda_3 = 0, \lambda_1 = \left[P_A - P_B - (1 + \frac{1}{\gamma} m)\right] \geq 0$$

- Case 1 all available high quality commodity is used in blend.
- Case 2 all availabl low quality commodity is used in blend.

Shadow prices: Excerpts from a half-finished paper

Shadow prices (The concept has a long history):

- The first reference to the concept of shadow price is in J.R. Hicks? Value and Capital (Hicks, 1939: 111).
- First journal mention is in a review article by J.M. Clark in (1940)
- The first serious mention in the Journal literature is by Martin Beckmann (1952)
- The first mention of it in the context of linear programming is by John Chipman (1953)
- It is also used in environmental economics where it dates to Francis Bator (1958)

Shadow prices

Shadow prices can be determined by differentiating the Lagrangian:

$$P_L = \frac{\partial L}{\partial X_L} = P_B + \lambda_1$$

$$P_H = \frac{\partial L}{\partial X_H} = P_A + \gamma\lambda_2$$

The Beta distribution

Beta distribution is used to model the protein distribution. x is the percent protein in the wheat. pdf is

$$f(x) = \frac{(x - min)^{\alpha-1}()^{\beta-1}}{(\beta(\alpha, \beta)(max - min)^{\alpha+\beta-1}}$$

whew α is a location parameter and β is a shape parameter.

$$\mu = min + \frac{\alpha}{\alpha + \beta}(max - min)$$

Linear programming and duality

Consider a linear programming problem:

$$\min c^T x$$

subject to

$$Ax \geq b$$

$$x \geq 0$$

The dual of this problem is

$$\max \lambda^T b$$

subject to

$$\lambda^T A \leq c^T$$

$$\lambda \leq 0$$

λ can be interpreted as a shadow price of the resource b.
Remember we can solve this type of problem with scipy.optimize.
See `https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.linprog.html`

Duality Theorem

DualityTheorem of Linear Programming.If either of the problems has a finite optimal solution, so does the other,and the corresponding values of the objective functions are equal. If either problem has an unbounded objective,the other problem has no feasible solution.