Agricultural finance Lecture 6 Valuing investment under risk and uncertainty

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Introduction

Frank Knight's distinction between risk and uncertainty:

- Risk known probabilities
- Uncertainty probabilities are not known

Example

Choice between applying 90 pounds of nitrogen per acre or 110 pounds of nitrogen per acre. Regardless of the choice made by the farmer, one of two possible events (E) will occur (either event A or B). The probability The probability that event A will occure is given by $P(A)$ and the probability that event B will occur is given by $P(B)$. The combination of the farmer's actions an events results in four possible outcomes $O(a_1|A)$. with is the outcome of action a_1 given event A. Assume event A is the case where 30 inches of rain occurs while event B is the case where 35 inches of rain occurs.

Payoffs

The actions are the choice of nitrogen to apply in each case. The payoffs to corn yield and profit per acre are given in the following table:

Nitrogen per acre	Rainfall (inches per season)	
	30	35
	corn yield (bushels per acre)	
90	41.71	46.46
110	44.30	49.35
	Profit per acre	
90	103.95	116.31
110	109.68	122.80

How does the producer decide between the two alternatives?

Expected value of profit is one option

Assume $P[E = A] = 0.6$ and $P[E = B] = 0.40$ then

$$E[\pi|a_1] = P[E = a]116.31 + P[E = B]103.95 = 111.37$$

$$E[\pi|a_2] = P[E = A]122.80 + P[E = B]109.68 = 117.55$$

So because $E[\pi|a_2] > E[\pi|a_1]$ decision-maker should choose alternative a_2. But this alternative also has higher risks involved.

$$\sigma_2^2 = 41.31 \text{ and } \sigma_1^2 = 36.66$$

Decision Tree

put tree diagram here

Expected Utility

$$U(Y) = \frac{Y^{1-r}}{1-r}$$

Moss, calls this the power utility function. correct name is iso-elastic utility function of CARA

Differentiating

$$\frac{dU}{dY} = (1-r)\frac{Y^{-r}}{1-r} = Y^{-r}$$

The elasticity is

$$\frac{dU}{dY}\frac{Y}{U(Y)} = \frac{Y^{-r}Y}{\frac{Y^{1-r}}{1-r}} = 1-r$$

which can be see n to be constant

Risk aversion

Arrow-Pratt measure of relative risk aversion

$$\rho = -Y \frac{U^{''}(Y)}{U^{'}(Y)}$$

Substituting first-derivative from previous slide we get:

$$= -rY \frac{Y^{-r-1}}{Y^{-r}}\} = rY^{-r}Y^{-r} = r$$

So this is constant and is the rate of relative risk aversion.

- risk averse $r > 0$
- risk neutral $r = 0$
- risk averse $r < 0$

Graph of utilities for different degrees of risk aversion

Milleron-Mityushin-Polterovich Theorem

The demand curve slopes down if and only if the rate of relative risk aversion is less than or equal to four (4)

St Petersburg Paradox

Player bets on coin tosses. Pays fixed bet and wins reward 2^n on the $n-th$ toss.
After one toss expected payoff is

$$\frac{1}{2}2$$

with two tosses it is

$$\frac{1}{2}2+\frac{1}{4}4$$

and so on

$$\frac{1}{2}2+\frac{1}{4}4+\ldots+\frac{1}{2^n}2^n+\cdots=1+1+1+\cdots=\infty$$

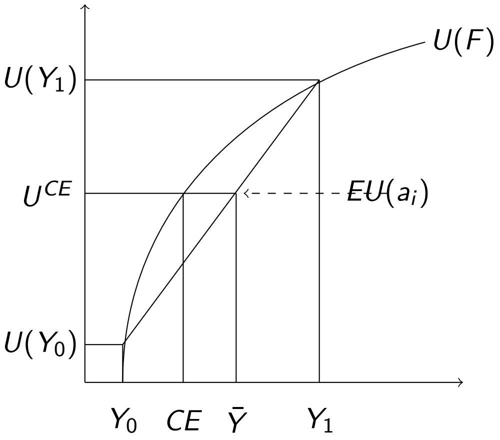
This motivated the introduction of expected utility theory so that the payoff remained bounded.

Expected Utility theory

$$EU = P(A)U(Y) + P(B)U(Y)$$

$$P(A) + P(B) = 1$$

Certainty Equivalents



Certainty Equivalent-Algebra

The amount the decision maker is willing to pay to remain indifferent to a risky gamble.
Set expected utility equal to utility

$$EU = P(A)\frac{Y_2^{1-r}}{1-r} + P(B)\frac{Y_1^{1-r}}{1-r} = \frac{Y_{CE}^{1-r}}{1-r}$$

Then solve for Y_{CE}.
Risk premium is $\bar{Y} - Y^{CE}$.

Solving

$$P(A)\frac{Y_2^{1-r}}{1-r} + P(B)\frac{Y_1^{1-r}}{1-r} = \frac{Y_{CE}^{1-r}}{1-r}$$

$$P(A)Y_2^{1-r} + P(B)Y_1^{1-r} = Y_{CE}^{1-r}$$

$$Y_{CE} = [P(A)Y_2^{1-r} + P(B)Y_1^{1-r}]^{\frac{1}{1-r}}$$

$$RP = P(A)Y_2 + P(B)Y_2 - Y_{CE}$$

A nice exercise now would be to look at how the risk premium varies with r and with the probabilities.

Numerical example

Using this data calculate the certainty equivalent and risk premium

N	Rain			
	30	35	Mean	Standard deviation
	Profit			
90	98056	113877	107548	7751
110	105390	122184	115467	8227

Example

$$EU = P(A)U(Y) + P(B)U(Y)$$

Assume $P[E = A] = 0.6$ and $P[E = B] = 0.40$, $r = 0.5$

$$EU = 0.6U(Y) + 0.4U(Y)$$

$$= 0.6\frac{113877^{0.5}}{0.5} + 0.4\frac{98056^0.5}{0.5} = 655.459$$

$$Y_{CE} = 107,407$$

$$R_P = \bar{Y} - Y_{CE} = 142$$

risk premium is the maximum amount the producer is willing to pay to aoid the gamble (e.g. bu insurance)

Alternative distributions

Above example had two outcomes (Bernoulli distribution) if we use a log-normal distribution and CRRA utility then the certainty equivalent utility is (lot's of handwaving)

$$U^{CE} = \frac{1}{1-r} \exp\left\{(1-r)(\mu + (1-r)\frac{\sigma^2}{2}\right\}$$

Assuming a normal distribution and a CARA (constant absolute risk aversion) (i.e. negative exponential utility) we would get:

$$U^{CE} = -\exp\left\{-\rho(\mu - \frac{\rho}{2}\sigma^2)\right\}$$

We will make use of this later of mean-variance analysis.

Portfolio analysis

Example with four crops with 11 years of data

$$\min zV(\sum_{i=1}^{4} z_i r_r)$$

subject to $\sum_{i=1}^{4} z_i \bar{r}_i \geq \mu = 40000$

$$\sum_{i=1}^{4} z_i = 100$$

$$z_i \geq 0$$

Variance and Mean

$$V(\sum_{i=1}^{4} z_i r_r) = \frac{1}{11}\sum_{i=1}^{11}\left(z_i\left[\sum_{j=1}^{4} r_{ij} - \bar{r}_i\right]\right)^2$$

$$\bar{r}_i = \frac{1}{11}\sum_{j=1}^{11} r_{ij}$$

Then solve for z using a non-linear (quadratic) programming package.

Mean-variance (efficient) frontier

The mean variance approach involves maximizing a linearized utility

$$U(z) = \mu_P(z) - \frac{\rho}{2}\sigma_P^2(z)$$