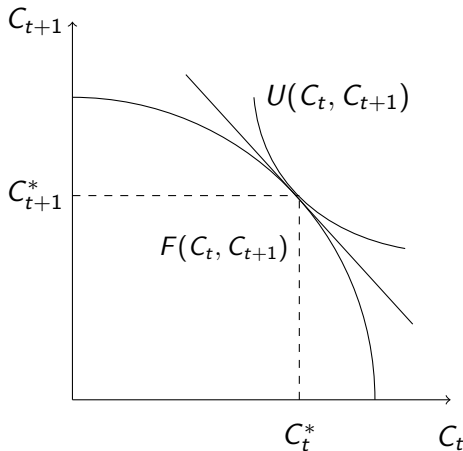
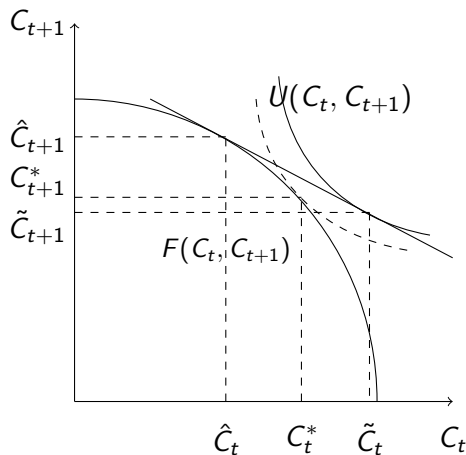


Should an individual consumer now or save and invest in the future?



Borrowing and Lending -The Fisher separation theorem



$\tilde{C}_t - \hat{C}_t$ is borrowed, i.e. in first period he consumes more than he produces

$\hat{C}_{t+1} - \tilde{C}_{t+1}$ is paid back he produces more than he consumes

Interest rates and the intertemporal budget line

The future value of consumption (assume consumption is valued at \$1;

$$C_{t+1} = (1 + r)C_t$$

assume consumption isn't stored but can be deferred at the implied interest rate r .

$$C_{t+1} - (1 + r)C_t = 0$$

Then the slope of the budget line is just $-(1 + r)$.

Fisher separation and net present value

The Fisher separation theorem says the the investment decision:

$$C_t = \frac{1}{1+r} C_{t+1}$$

which is at the pint of tangency between the budget line and the feasible consumption set $F(C_t, C_{t+1})$ is independent of the consumer's preferences.

This is the basis for the use of the net present value (NPV) formula in discounted cashflow (DCF) analyses and in capital budgeting .

Consider the production possibilities frontier:

$$y = (10000 - x^2)^{1/2}$$

x is current consumption.

Rearranging the production possibilities frontier is

$$10000 = x^2 + \frac{y^2}{4}, x \in [0, 100], y \in [0, 200]$$

Plot this using Matplotlib in Jupyter!!

Robinson Crusoes Intertemporal Consumption Problem

$$\max_{x,y} U(x,y) x^{0.6} y^{0.4}$$

s.t

$$10000 \geq x^2 + \frac{y^2}{4}$$

Construct the Lagrangian:

$$L = x^{0.6}y^{0.4} + \lambda(10000 - x^2 - \frac{y^2}{4})$$

$$\frac{\partial L}{\partial x} = 0.6x^{0.6-1}y^{0.4} - \lambda 2x = 0$$

$$\frac{\partial L}{\partial y} = 0.4x^{0.6}y^{0.4-1} - \lambda \frac{y}{2} = 0$$

$$\frac{\partial L}{\partial \lambda} = 10000 - x^2 - \frac{y^2}{4} = 0$$

some algebra

From this we get (Try this using SymPy as well as by hand!!
You could also try this numerical non-linear programming!!)

$$\frac{\frac{\partial L}{\partial y}}{\frac{\partial L}{\partial x}} = \frac{\frac{\partial U}{\partial y}}{\frac{\partial U}{\partial x}} = \frac{\lambda \frac{y}{2}}{\lambda 2x} = \frac{\frac{y}{2}}{2x}$$

substituting in the marginal utilities:

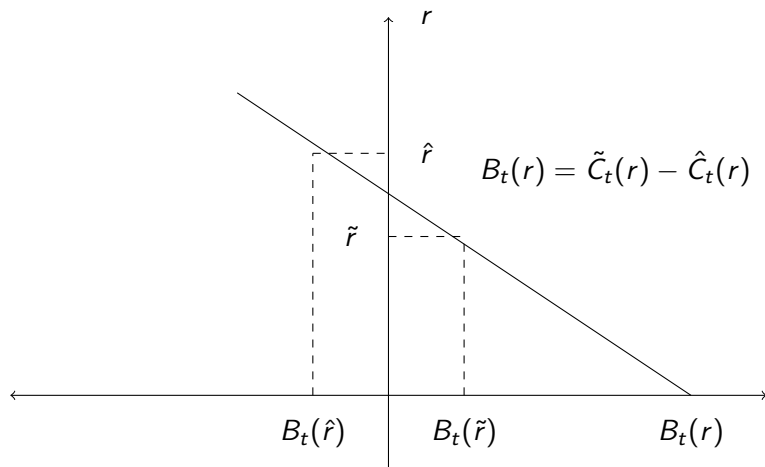
$$\frac{\frac{\partial L}{\partial y}}{\frac{\partial L}{\partial x}} = \frac{0.4x^{0.6}y^{0.4-1}}{0.6x^{0.6-1}y^{0.4}} = \frac{\lambda \frac{y}{2}}{\lambda 2x} = \frac{\frac{y}{2}}{2x}$$

Rearranging we get

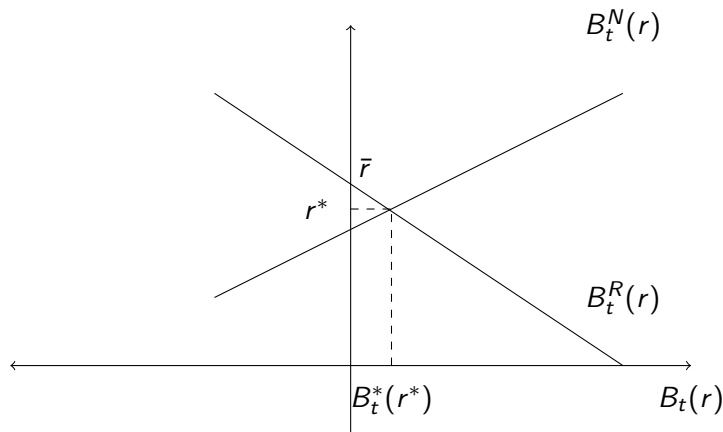
$$y^2 = \frac{8}{3}x^2$$

substitute this into the budget production possibility
equation $10000 - x^2 - \frac{y^2}{4}$ and solve for x to get $x = 77.46$.
then substitute this back in to obtain $y =$

Loan demand



Capital market equilibrium



Bank balance sheets

Assets		Liabilities and equity	
Cash	75000	Deposits	1,910,560
Deposits	50,000	Equity	955,280
Loans			
Non-real estate loans	305,522		
Operating Loans	1,543,295		
Real estate loans			
Accumulation for bad loans	(109,820)		
Building and facilities			
Total	3,240,840	Total	3,240,840

An investment portfolio for three (3) assets (assuming risk is characterized by a distribution in a two-parameter family) is summarized by the mean and variance of returns:

$$\mu_P = \mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3$$

$$\sigma_P^2 = \sigma_{11} + 2\sigma_{12}x_1x_2 + 2\sigma_{13}x_1x_3 + \sigma_{22}x_2^2 + 2\sigma_{23}x_2x_3 + \sigma_{33}x_3^2$$

Question?

If we wished to minimize risk subject to expected returns, how would we do this?

Exercise (Example 2.4

Given the lending returns in table 2.5 (Textbook) , compute the average and variance of the returns on the bank's portfolio. What is the probability that the interest rate on the bank's loan portfolio is higher than 3.50 percent? (Hint: You need to assume a distribution to evaluate the probability, what two-parameter distribution would you choose?).

Motivation:

Base I and Basel II regulatory standardization framework for banking

Residual claim idea: Equity is what is left after liabilities are paid (I have already hinted at this)

$$\text{debt to asset ratio } \frac{D}{A}$$

$$\text{leverage ratio } \frac{D}{E}$$

$$\text{capital adequacy ratio} = \frac{\text{Capital}}{0.00C_1 + 0.20C_2 + 0.50C_3 + 1.00C_4}$$

- ▶ Tier 1 capital (C_1) (core capital)
- ▶ Tier 2 capital (C_2) (less well established claims of the bank)
- ▶ C_3 fully secured loans
- ▶ C_4 standard risk assets.

For definitions (see textbook or Federal reserve's commercial bank examination manual)