AGEC 3333 Agricultural Marketing and Price Analysis

Lecture 2 - Prices over space

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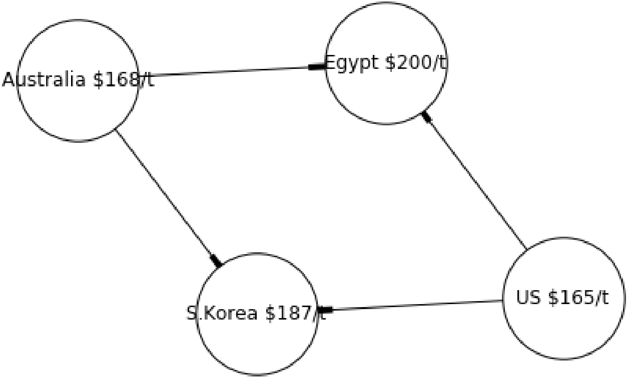
Lecture: Notebook:

Objective

Explain how the set of prices is connected over space?

Ocean freight rates for grain, select ports, 23 No. 2005

US Dollars per Tonne					
From/To	Algeria	Egypt	Iran	Korea	Morocco
Australia	na	32	29	19	na
EU	24	26	na	na	22
US Gulf	na	35	Na	45	32


Australia $168/t
Egypt $200/t
S.Korea $187/t
US $165/t

The Law of One Price (LOP)

Example

Consider a US exporter selling wheat to Egypt. The price of wheat in the US is \$ 165/tonne and the cost of transporting wheat from the US to Egypt is \$ 35/tonne. The break even price for wheat in Egypt for the US exporter to make a profit is therefore \$ 200/tonne.

Mathematical Excursion

We will review some necessary mathematics for understanding non-linear programming. The topics covered will include:

- unconstrained optimization
- optimization with equality constraints
- linear optimization with inequality and posssibly also equality constraints (linear programming)
- non-linear optimization with inequality and posssibly also equality constraints (non-linear programming)

Unconstrained optimization

This includes both maximization and minimization. To convert a maximization to a minimization function and vice versa multiply the function to be maximized or minimized by -1. Let's consider the problem of a monopolist as this is closest in structure to the net aggregate welfare maximization problem that we are interested in.
Profit = Revenue - Costs:
$\Pi = p(Q)Q - C(Q)$
we will assume linear demand $p(Q) = a - bQ$ and linear constant marginal costs $C(Q) = cQ$ then our profit function becomes quadratic (verify this for yourself!).
We will use the Python module SymPy to analyze this problem. First we need to import SymPy:

Optimization with equality constraints

To maximize or minimize a function subject to equality constraints we introduce what is called a Lagrangian.
Our problem is to maximize for example utility $U(x, y)$ subject to a budget constraint $p_x x + p_y y = M$
To do this we construct the Lagrangian function
$L(x, y, \lambda) = U(x, y) + \lambda[M - p_x x - p_y y]$
Then differentiate with respect to x, y, λ
$\frac{\partial L}{\partial x} = \frac{\partial U}{\partial x} - \lambda p_x = 0$
$\frac{\partial L}{\partial y} = \frac{\partial U}{\partial y} - \lambda p_y = 0$
$\frac{\partial L}{\partial \lambda} = M - p_x x - p_y y = 0$

Example

$U = xy$
$L(x, y, \lambda) = xy + \lambda[M - p_x x - p_y y]$
Now we solve this using SymPy

Linear programming

This is just a brief introduction to linear programming. We will do more in the farm and agribusiness management class. Consider the following problem:

$\max 3x_1 + 2x_2$

subject to

$2\ x_1 + 3\ x_2 \leq 100$

$x_1 + x_2 \leq 80$

$x_1 \leq 40$

$x_1 \geq 0$

$x_2 \geq 0$

We can solve this using Pulp.

Example: Transportation problem

min $\sum_i \sum_j c_{ij} x_{ij}$
subject to
$\sum_i x_{ij} = D_j, \ \forall j$ Demands
$\sum_j x_{ij} = S_i, \ \forall i$ Supplies
$x_{ij} \geq 0$

Exercise

Using the demand and supply estimates in Table 2.Q p. 33 of the Textbook solve for the equilibrium quantities in each country and then use Pulp to solve a transportation problem.

Non-linear programming

With non-linear programming we wish to maximimize (minimize) a possibly linear objective with either linear or non-linear constraints.

$$\max f(\vec{x})$$

$$s.t. g(\vec{x}) \leq 0$$

$$h(\vec{x}) = 0$$

Example

$$\max_{x \in \mathcal{R}^2} \sqrt{x_2}$$

$$x_2 \geq (a_1 x_1 + b_1)^3$$

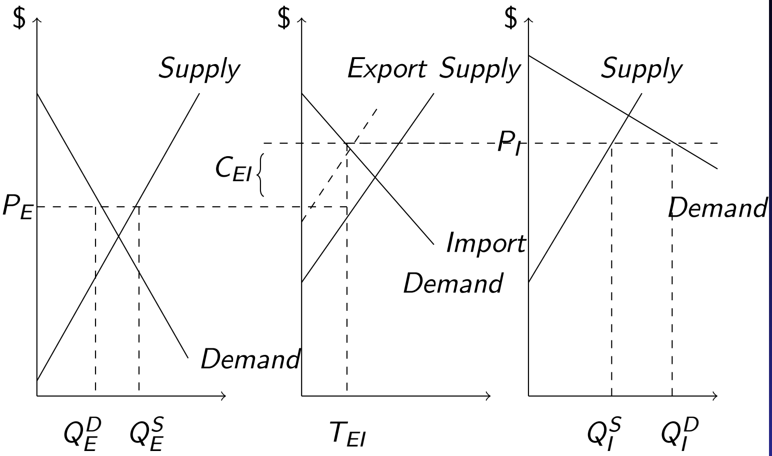
$$x_2 \geq (a_2 x_1 + b_2)^3$$

$$x_2 \geq 0, a_1 = 2, b_1 = 0, a_2 = -1, b_2 = 1$$

source: `http://ab-initio.mit.edu/wiki/index.php/NLopt_Tutorial`

Application: Spatial equilibrium pricing

- Assume a linear inverse demand for region i, $P_i = a_i - b_i Q_i^D$
- Area under the demand curve for a given price is the area of the consumer welfare triangle $\frac{1}{2}(a_i - P_i)Q_i^D$
- Plus the area of the producer revenue $P_i Q_i^D$
- Simplifying this is $(a_i - 0.5 b_i Q_i^D) Q_i^D$


$
Supply
Demand
P_E
Q_E^D
Q_E^S
$
Export Supply
Import
Demand
C_{EI}
T_{EI}
$
Supply
Demand
P_I
Q_I^S
Q_I^D

Net Aggregate Welfare

We need to maximize either one of the following

$$NAW_V = \sum_{i=1}^{n}(a_i - 0.5b_iQ_i^D)Q_i^D - \sum_{i=1}^{n}(\alpha_i + 0.5\beta_iQ_i^S)Q_i^S$$

$$-\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}T_{ij} \text{ with vertical axis intercept}$$

$$NAW_H = \sum_{i=1}^{n}(a_i - 0.5b_iQ_i^D)Q_i^D - \sum_{i=1}^{n}0.5(\alpha_i + \beta_iQ_i^S)(Q_i^S + \frac{\alpha_i}{\beta_i})$$

$$-\sum_{j=1}^{n}\sum_{j=1}^{n} C_{ij}T_{ij} \text{ with horizontal axis intercept}$$

Constraints

subject to

$$\sum_{j=1}^{n} T_{ji} \geq Q_i^D, \ \forall i$$

and

$$\sum_{j=1}^{n} T_{ij} \leq Q_i^S \ \forall i$$

and

$$T_{ij}, Q_i^D, Q_i^S \geq 0, \forall i$$

Lagrangian

$$L = \sum_{i=1}^{n} \left[(a_i - 0.5 b_i Q_i^D) Q_i^D - (\alpha_i + 0.5 \beta_i Q_i^S) Q_i^S \right] - \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} T_{ij}$$
$$+ \sum_{i=1}^{n} \lambda_i^D \left(\sum_{j=1}^{n} T_{ji} - Q_i^D \right) + \sum_{i=1}^{n} \lambda_i^S \left(Q_i^S - \sum_{j=1}^{n} T_{ij} \right) - \sum_{i=1} \sum_{j=1} \lambda_{ij}^T T_{ij}$$

Karush-Kuhn-Tucker conditions (KKT)

$$\frac{\partial L}{\partial Q_i^D} = a_i - b_i Q_i^D - \lambda_i^D \leq 0$$

$$\frac{\partial L}{\partial Q_i^D} Q_i^D = 0$$

non-negativity condition

$$\lambda_i^D \left(\sum_{j=1}^{n} T_{ji} - Q_i^D \right) = 0$$

complementary slackness condition

$$\frac{\partial L}{\partial Q_i^S} = -(a_i + b_i Q_i^D) + \lambda_i^S \leq 0$$

$$\frac{\partial L}{\partial Q_i^S} Q_i^S = 0$$

non-negativity condition

Karush-Kuhn-Tucker conditions (KKT)

$$\lambda_i^D(\sum_{j=1}^{n} T_{ij} - Q_i^S) = 0$$

complementary slackness condition

$$\frac{\partial L}{\partial T_{ij}} = -C_{ij} + \lambda_j^D - \lambda_i^S - \lambda_{ij}^T \leq 0$$

$$\frac{\partial L}{\partial T_{ij}} T_{ij} = 0$$

non-negativity condition

$$\lambda_{ij}^T T_{ij} = 0, j = 1, 2, \ldots, n$$

complementary slackness condition

Spatial pricing case study

Pre-scaled parameters for tomato case study: Supply and demand intercept and slope parameters

(a)

	Intercept parameters		Slope parameters	
Region	Supply	Demand	Supply	Demand
Mexico	- 2532	8732.3	0.00146	0.00578
US	- 1279	2217.1	0.00021	0.00011
Canada	-2128	5,131.1	0.0059	0.00581
EU	-5337	4258.7	0.00043	0.00022
L. Amer.	3306	2806.5	0.00059	0.00036

Pre-scaled parameters for tomato case study: Transportation cost parameters

(b)					
	US Dollars per ton				
Region	Mexico	US	Canada	EU	L. Amer.
Mexico	0.00	58.50	96.63	155.55	161.88
US	58.50	0.00	42.21	106.98	142.05
Canada	96.63	42.21	0.00	106.47	164.43
EU	155.55	106.98	106.47	0.00	137.37
L. Amer.	161.88	142.05	164.43	137.37	0.00

Free-flow equilibrium

- Determining the free-flow equilibrium can be used to find starting values for the NLP
- Find the scaled demands and supplies across all regions
- Then solve for the equilibrium price
- The equilibrium price of the aggregate demand and supplies is the "free-flow" equilibrium price

Other alternative approaches

- Gravity models (Econometrics)
- Spatial general Equilibrium models (GTAP, Monash type models)
- Regional input-output models