Agricultural Finance - Handout 1
Financial markets

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In this lecture I will introduce you to the the basics of the capital market, borrowing and lending and Fisher separation

This handout is your map to the lecture it will cover the key takeaway messages and the most important graphs will be placed in the right margin with a short explanation.

The aim of the lecture is to introduce you to the motivating forces behind borrowing and lending and the need for capital markets and to introduce you to discounting.

Introduction

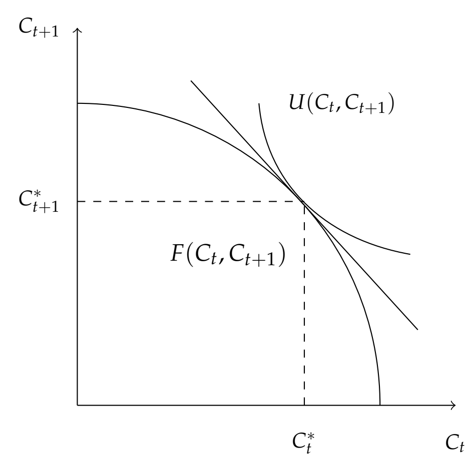
The Fisher Separation Theorem

$$\max_{C_t, C_{t+1}} U(C_t, C_{t+1})$$

$$s.t\ F(C_t, C_{t+1})$$

First write down the Lagrangian

$$L(C_t, C_{t+1}, \lambda_{t+1}) = U(C_t, C_{t+1}) + \lambda_{t+1}[F(C_t, C_{t+1})]$$



Example

Let's use a the utility function

$$U(C_t, C_{t+1}) = C_t^{\alpha} C_{t+1}^{\beta}$$

This is first maximized subject to the production possibilities constraint

$$1000 \geq C_t^2 + \frac{C_{t+1}^2}{4}$$

to find C we construct the Lagrangian:

$$L(C_t, C_{t+1}, \lambda_{t+1}) = C_t^{\alpha} C_{t+1}^{\beta} + \lambda_{t+1}[1000 - C_t^2 - \frac{c_{t+1}^2}{4}]$$

To find the point of tangency between the budget line and the production possibility frontier, maximize expenditure subject to production possibilities

$$\max C_t, C_{t+1} p_t p_t + p_{t+1} C_{t+1}$$

st.

$$1000 \geq c_t^2 + \frac{C_{t+1}^2}{4}$$

The Lagrangian for this is

$$L = p_t C_t + p_{t+1} C_{t+1} + \lambda_{t+1}[1000 - C_t^2 - \frac{C_{t+1}^2}{4}]$$

$$\frac{\partial L}{\partial C_t} = p_t - 2\lambda_{t+1} C_t = 0$$

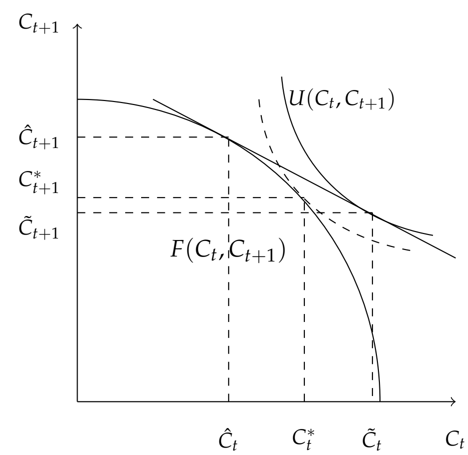
$$\frac{\partial L}{\partial C_{t+1}} = p_{t+1} - \lambda_{t+1} \frac{C_{t+1}}{2} = 0$$

$$\Longrightarrow$$

$$\frac{p_t}{p_{t+1}} = \frac{4C_t}{C_{t+1}}$$

which can be rearranged to obtain

$$C_{t+1} = \frac{p_{t+1}}{p_t} 4C_t \qquad \hat{}$$



Substituting this into the production possibility constraint

$$1000 - C_t^2 - \frac{(\frac{p_{t+1}}{p_t} 4C_t)^2}{4} = 0$$
$$1000 - C_t^2 - \frac{p_{t+1}}{p_t} 4C_t^2 = 0$$
$$1000 = (1 + \frac{p_{t+1}}{p_t} 4)C_t^2$$

or

$$\hat{C}_t = \sqrt{\frac{1000}{1 + \frac{p_{t+1}}{p_t} 4}}$$

and substituting into ˆ we get

$$\hat{C}_{t+1} = \frac{p_{t+1}}{p_t} 4 \sqrt{\frac{1000}{1 + \frac{p_{t+1}}{p_t} 4}}$$

So for the following prices $P_t = 1$ and $p_{t+1} = 0.94$ we obtain

Now to obtain the point of tangency of the indifference curves with the budget constraint, Moss, notes that the this is the same budget line so the income associated with maximizing the expenditure subject to production possibilities and maximizing utility subject to a budget constraint must be the same. In other words, income from production must equal the income of households. So substituting the current and future consumption possibilities found above into the budget constraint we can obtain the implied income.

This can then be used to obtain representative households optimal consumption plan $(\bar{C}_t, \bar{C}_{t+1})$. As follows.

$$\max C_t^{0.4} C_{t+1}^{0.6}$$

subject to

$$p_t C_t + p_{t+1} C_{t+1} = p_t \hat{C}_t + p_{t+1} \hat{C}_{t+1}$$

The Lagrangian for this is

$$L(C_t.C_{t+1}) = C_t^{0.4}C_{t+1}^{0.6} + \lambda\left[p_t\hat{C}_t + p_{t+1}\hat{C}_{t+1} - p_tC_t - p_{t+1}C_{t+1}\right]$$

The first-order conditions are

$$\frac{\partial L}{\partial C_t} = 0.4C_t^{0.4-1}C_{t+1}^{0.6} - \lambda p_t = 0$$

$$\frac{\partial L}{\partial C_t} = 0.6C_t^{0.4}C_{t+1}^{0.6-1} - \lambda p_{t+1} = 0$$

$$\frac{\partial L}{\partial \lambda} = p_t\hat{C}_t + p_{t+1}\hat{C}_{t+1} - p_tC_t - p_{t+1}C_{t+1} \ = 0$$

Solving results in

$$\bar{C}_t = \frac{0.4(p_t\hat{C}_t + p_{t+1}\hat{C}_{t+1})}{p_t}$$

$$\bar{C}_{t+1} = \frac{0.6(p_t\hat{C}_t + p_{t+1}\hat{C}_{t+1})}{p_t}$$

Borrowing is then given by

$$\bar{C} - \hat{C}$$

Another way to approach this, is to start with a general two period model. The intertemporal budget-constraint is

$$(1+r)C_t + C_{t+1} = (1+r)M_t + M_{t+1}$$

start with utility $U(C_t, C_{t+1}$ and maximize subject to this budget constraint (The steps would be the same as above).

If we were to plot this budget constraint in (C_t, C_{t+1}) space we would use the equation

$$C_{t+1} = (1+r)M_t + M_{t+1} - (1+r)C_t$$

The slope of which is $-(1+r)$. One could set the prices (p_t, p_{t+1}) in Moss' version to $(1+r), 1$ and use his method to backsolve for M, for some reason he doesn't do this.