Advanced Agribusiness Management

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Readings

- Stigler, George J. The Cost of Subsistence. *Journal of Farm Economics*, vol. 27, no. 2, 1945, pp. 303?314
- Dantzig, G. (1990). The Diet Problem. *Interfaces*, 20(4), 43-47. Retrieved from http://www.jstor.org/stable/25061369
- El-Nazer, T., McCarl, B. A., 1986. The choice of crop rotation: A modeling approach and case study. *American Journal of Agricultural Economics* 68 (1), 127?136. (available on `http://chla.mannlib.cornell.edu/`).
- Throsby, C.D. (1967) STATIONARY-STATE SOLUTIONS IN MULTI-PERIOD LINEAR PROGRAMMING PROBLEMS, Australian Journal of Agricultural Economics¿Volume 11, Number 02, December 1967

The Diet Problem

Stigler's 1939 Diet		
Food Annual	Quantities	Annual Cost
Wheat Flour	370 lb.	$13.33
Evaporated Milk	57 cans	$3.84
Cabbage	111 lb.	$4.11
Spinach	23 lb.	$1.85
Dried Navy Beans	285 lb.	$16.80
Total Annual Cost		$ 39.93

The Diet Problem

Table of nutrients considered in Stigler's diet

Nutrient	Daily Recommended Intake
Calories	3,000 Calories
Protein	70 grams
Calcium	.8 grams
Iron	12 milligrams
Vitamin A	5,000 IU
Thiamine (Vitamin B1)	1.8 milligrams
Riboflavin (Vitamin B2)	2.7 milligrams
Niacin	18 milligrams
Ascorbic Acid (Vitamin C)	75 milligrams

How to mix feed at minimum cost

Nutritive content and price of ingredients

Ingredient	Calcium (kg/kg)	Protein (kg/kg)	Fiber (kg/kg)	Unit Cost (cents/kg)
Limestone	0.38	0.0	0.0	10.0
Corn	0.001	0.09	0.02	30.5
Soybean meal	0.002	0.50	0.08	90.0

The mixture must meet the following restrictions:

- Calcium at least 0.8% but not more than 1.2%.
- Protein at least 22%.
- Fiber at most 5%.

http://www.me.utexas.edu/~jensen/or_site/models/unit/lp_model/blending/blend1.html

LP formulation

Minimize $Z = 10L + 30.5C + 90S$

Minimum calcium:	0.38L	+ 0.001C	+ 0.002S	> 0.008
Maximum calcium:	0.38L	+ 0.001C	+ 0.002S	< 0.012
Minimum protein:		+ 0.09C	+ 0.50S	> 0.22
Maximum fiber:		+ 0.02C	+ 0.08S	< 0.05
Conservation:	L	+ C	+ S	= 1

Approaches to modelling crop rotations

- Multi-period linear programming
- Dynamic programming
- Repeated (annual timeless) cropping cycle (using LP)

Crop rotations: Stationary Linear Programming

Problem a farm grows N crops and crop yield depends on what was grown on the farm in the previous three years. Current year is i, previous years are j, k, r.

$$\max \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \sum_{r=1}^{N} C_{ijkr} X_{ijkr}$$

subject to

$$\sum_{i=1} \sum_{j=1} \sum_{k=1} \sum_{r} X_{ijkr} \leq TA$$

$$\sum_{i=1}^{N} X_{ijkr} - \sum_{m=1}^{N} X_{jkrm} \leq 0, i, j, k = 1, \ldots, N$$

$$X_{ijkr} \geq 0$$

The second constraint is a rotation constraint that is equal to zero when continuous cropping occurs, i.e. no rotation takes place.

Example: Corn and Potatoes

Possible rotations plans

C	C	C	C	C	C	C	C	P	P	P	P	P	P	P	P
C	C	C	C	P	P	P	P	C	C	C	C	P	P	P	P
C	C	P	P	C	C	P	P	C	C	P	P	C	C	P	P
C	P	C	P	C	P	C	P	C	P	C	P	C	P	C	P

Constraints

Land	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$\leq$ TA
CCC	0	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	≤ 0
CCP	0	1	-1	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤ 0
CPC	0	0	1	0	-1	-1	0	0	0	0	1	0	0	0	0	0	≤ 0
CPP	0	0	0	1	0	0	-1	-1	0	0	0	1	0	0	0	0	≤ 0
PCC	0	0	0	0	1	0	0	0	-1	-1	0	0	1	0	0	0	≤ 0
PCP	0	0	0	0	0	1	0	0	0	0	-1	-1	0	1	0	0	≤ 0
PPC	0	0	0	0	0	0	1	0	0	0	0	0	-1	-1	1	0	≤ 0
PPP	0	0	0	0	0	0	0	1	0	0	0	0	0	0	-1	0	≤ 0

Solution and discussion

- Gives area of land to be planted to each crop for each sequence. so X_{CCCP} is the area of land to be planted to corn following two corn crops and one potato crop.
- i have not provided gross margins for this problem but nor do the authors.
- Think about where gross margins C_{ijkr} might be obtained from.
- How might one infer the rotation sequence in an area from aggregate data?
- How would you go about setting this model up in Jupyter using SciPy?

E-V portfolio model

Start with a portfolio model (quadratic programming, can we turn it into a linear programming model)

$$\min \sum_{j=1}^{n} \sum_{k=1}^{n} x_j x_k \sigma_{jk}$$

such that

$$\sum_{j=1}^{n} f_j x_j = \lambda$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$$

$$x_j \geq 0$$

Definitions

- x_i level of the i-th farm activity
- f_j the expected or forecast gross margin
- σ_{jk} the covariance of the gross margin between the j-th and k-th farm activity
- a_{ij} how much the j-th activity utilizes of the i-th resource
- b_i availalbility of the i-th resource

Advantages of the E-V model

- consistent with probability if gross margins are normally distributed
- subjective probability values may be used
- consistent with the separation theorem

Derivation

Replace variance with

$$\sum_{j=1}^{n}\sum_{k=1}^{n} x_j x_k [\frac{1}{s-1}\sum (c_{hj}-g_j)(c_{hk}-g_k)$$

s observations in a random sample of gross margins c, $g_j = \frac{1}{s}\sum_{h=1}^{s} c_{hj}$, j indicates the activity out of a total of n activities. Hazell notes that the eman gross margin may differ from the forecast if subjective information is used. Alternatively,

$$\frac{1}{s-1}\sum_{h=1}^{s}\left[\sum_{j=1}^{n} c_{hj}x_j - \sum_{j=1}^{n} g_j x_j\right]^2$$

E-A criterion

$$A = \frac{1}{s}\sum_{h=1}^{s}|\sum_{j=1}^{n}(c_{hj} - g_j)x_j|$$

Define

$$y_h = \sum_{j=1}^{n} c_{hj}x_j - \sum_{j=1} g_j x_j$$

such that $y_h = y_h^+ - y_h^-$

MOTAD model

$$\min sA = \sum_{i=1}^{s}(y_h^+ + y_h^-)$$

subject to

$$\sum_{j=1}^{n}(c_{hj} - g_j)x_j - y_h^+ + y_y^- = 0, h = 1, \ldots, s$$

$$\sum_{j=1}^{n} f_j x_j = \lambda$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, i = 1, \ldots, m$$

$$x_j, y_h^+, y_h^- \geq 0, \forall h, j$$

Alternative formulation

$$\min \sum_{h=1}^{s} y_h^-$$

$$\sum_{j=1}^{n} (c_{hj} - g_j) x_j + y_h^- \geq 0$$

$$\sum_{j=1} f_j x_j = \lambda$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1, \ldots, m$$

$$x_j, y_h^- \geq 0, \forall h, j$$

Numerical example

- Activities: carrots x_1, celery x_2, cucumbers x_3 and peppers x_4
- Resources: acreage of land b_1, hours of labor b_2, rotational and marketing constraint b_3

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 25 & 36 & 27 & 87 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 200 \\ 10,000 \\ 0 \end{bmatrix}$$

Other data time series of gross margins needed.

Gross margins

	dollars			
1	292	-128	420	579
2	179	560	187	639
3	114	648	366	379
4	247	544	249	924
5	426	182	322	5
6	259	850	159	569
Average	253	443	284	516

MOTAD Table

Row and unit	x_1	x_2	x_3	x_4	y_1^-	y_2^-	y_3^-	y_4^-	y_5^-	y_6^-	Constraint
A					1	1	1	1	1	1	Minimize
b_1	1	1	1	1							≤ 200
b_2	25	36	27	87							$\leq 10,000$
b_3	-1	1	-1	1							≤ 0
t_1	39	-571	136	63	1						≥ 0
t_2	-74	117	-97	123		1					≥ 0
t_3	-139	205	82	-137			1				≥ 0
t_4	-6	101	-35	408				1			≥ 0
t_5	173	-261	38	-511					1		≥ 0
t_6	6	407	-125	53						1	≥ 0
E	253	443	284	516							$= \lambda$