# Advanced Agribusiness Management

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April 11, 2017

# Readings

# Introduction to Inventory Management

- Static deterministic inventory models
- Dynamic inventory models
- Stochastic inventory models

# Applications

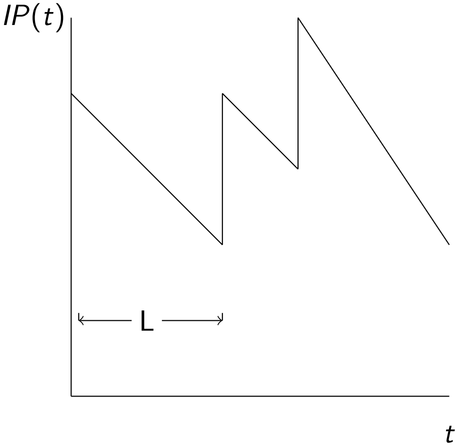
- On-farm storage
- Agribusiness
- Food processing
- Retail

# The Economic Order quantity Model



- $I(t)$ inventory at time $t$ (after arrival).
- $IO(t)$ inventory on order (not yet delivered) at time $t$.
- $IP(t) = I(t) + IO(t)$ inventory in position at time $t$
- $\lambda$ demand rate (quanity/time)
- $D = \lambda L$ Demand with leaditme $L$

# Inventory graph



# Inventory dynamics



$$I(t+L) = I(t) + IO(t) - D$$

$$I(t+L) = IP(t) - D$$

$IP(t^{-})$ is inventory position at instant before $t$

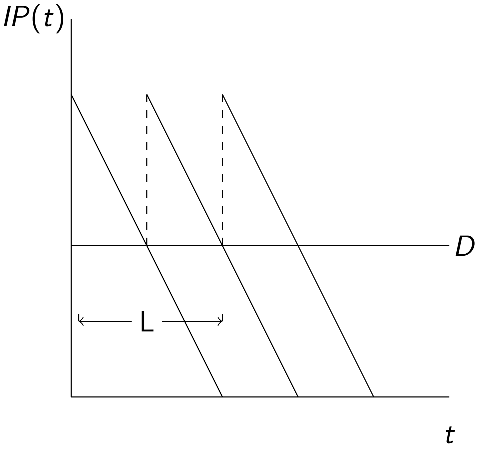
# When to order inventory?



- Business needs to maintain stocks and avoid stock-outs.
- When should they order (parts, products)
- Example: when should a supermarket order new products from suppliers?

# When to order inventory?

when $IP(t^-) = D$ place a new order at time $t$. This allows order to be filled within the lead-time.



# How much should be ordered?



$$q = \text{order or batch size}$$

A decision needs to be made on this we will develop a model to answer this question.

# Optimal order quantity



- Should we order a large amount or should we or more frequently?
- Think about when you go shopping, do you buy a lot and store it or do do you buy small amounts but shop more frequently?
- What factors determine your decision?

# Average Inventory and Order Frequency



$$\bar{I} = \lim_{T \to \infty} \frac{1}{T} \int_0^T I(t) dt \text{ergodic criteria}$$

$$\bar{OF} = \lim_{T \to \infty} \sum \mathbb{1}_{IP(t^-)=D}$$

One order per cycle, cycle length is $u = \frac{q}{\lambda}$ so $\bar{OF} = \frac{1}{\frac{q}{\lambda}} = \frac{\lambda}{q}$

Average inventory is $\frac{1}{2}q$.

# Relevant Costs



Relevant costs are:

- fixed costs of placing an order $k$
- variable costs of placing an order $c$ (these depend on batch size)
- costs of storing $I(t)$ goods $h$ per period of time

# Storage costs



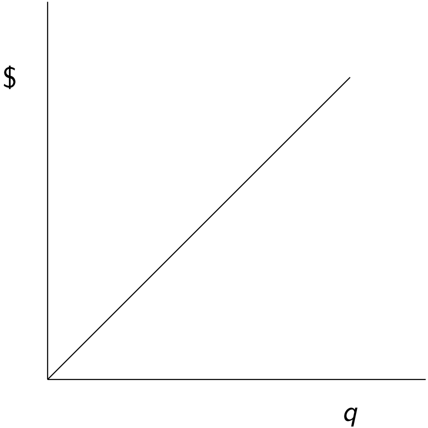
- costs accumulate at rate $hI(t)$.
- units of h are $\$/(units \times time)$

# Fixed procurement costs $k$

- administrative costs of procurement
- costs of receiving and processing goods
- set-up costs (manufacturing)

# Variable Costs

Unit costs $c$ of purchasing $q$ units.
Total variable costs are $cq$ (linear)



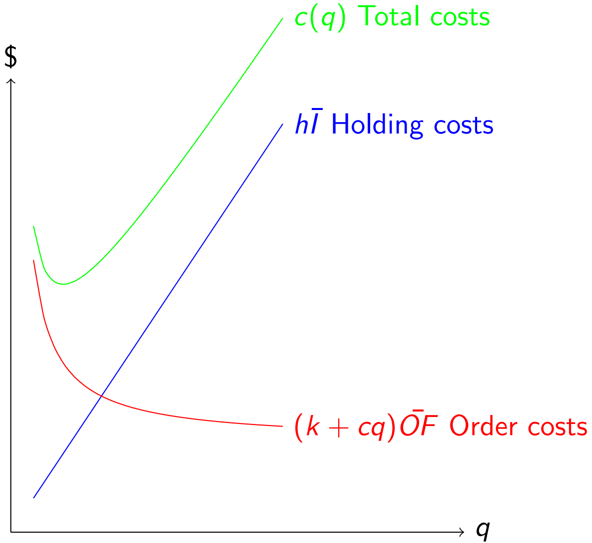
# Total average cost

total average cost

$$c(q) = (k + cq)\bar{O}F + h\bar{I}$$
$$= c\lambda + k\frac{\lambda}{q} + \frac{h}{2}q$$

This is the EOQ cost function

# EOQ -model



# Minimize costs of order and storing



$$C'(q) = \frac{h}{2} - k\lambda q^{-2} = 0$$

$$q^* = \sqrt{\frac{2k\lambda}{h}}$$

some people approximate $h$ with the opportunity costs of capital (inventory is capital). This is just interest rate times the capital value of the inventory, however there are also labor costs associated with storage so this is not the only source of storage costs. another cost would be electricity for refrigeration units.

# Optimal Values



$$\bar{I} = \sqrt{\frac{k\lambda}{2h}}$$

$$\bar{OF} = \sqrt{\frac{\lambda h}{2k}}$$

$$h\bar{I} = k\bar{OF} = \sqrt{\frac{k\lambda h}{2}}$$

substituting

$$C(q^*) = c\lambda + \sqrt{2k\lambda h}$$

# Notes on perishable items

- To deal with perishable items simply place an upper limit $u_{max}$ on the cycle time $u$ or constrain $q \leq \lambda u_{max}$ [Best before date perishable date].
- Then resolve the cost minimization with this constraint
- Non-linear programming problem
- If there is no due date , then model product decay

$$\frac{dI}{dt} = -\delta I(t) - \lambda, I(0) = q \text{ Initial value problem}$$

Solution is

$$I(t) = qe^{-\delta t} - (\frac{\lambda}{\delta})(1 - e^{-\delta t})$$

- Cycle time $u$ is $t$ where $I(t) = 0$ use this to solve for average inventory