Agricultural Marketing and Price Analysis Lecture 4

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Prices over time

Assumptions:

- single homogeneous product (commodity)
- single location
- Sold at different points in time (intertemporal analysis)
- No centralized market

Sale at different points in time implies storage which implies dynamic analysis

Incentive to store a commodity

Expected discounted future sales price - Current price
(Marginal Benefits) $>$ Marginal physical costs of storage

Intertemporal law of one price

- Speculative storage implies: Expected discounted future sales price - Current price (Marginal Benefits) $=$ Marginal physical costs of storage (cf. Hotelling's rule in resource economics)
- If traders hold no speculative stocks, then Expected discounted future sales price - Current price (Marginal Benefits) $<$ Marginal physical costs of storage (stock-out property)

“Saw-tooth” pricing

Adjacent time periods should be characterized by rising (current prices) prices (until Δ marginal price = marginal cost of storage. Followed by stockout (after the new harvest) and price collapse. resulting in price cycles and a saw -tooth pattern.

- Rarely observed in reality due to trade (e.g. spatial effects)

Storage and Price Stabilization

- pipeline stocks
- speculative stocks (storage in excess of pipeline requirements)
- investment in storage responds to price differential
- Government may store commodities for other purposes

A Two-period Storage Model

- Production h_1 and h_2, (harvest in periods 1 and period 2)
- Inverse market demand $P(X)$ and $P^{'}(X) < 0$.
- S units of production will be stored in period 1 and sold in period 2
- Equilibrium prices in each period are $P(h_1 - S)$ and $P(h_2 + S)$
- $C(S)$ is the cost of storing S units for one period. $C^{'}(S) > 0$ and $C^{''}(S) \geq 0$

Consumer surplus

$$\int_0^{h_1-S} P(x)dx - P(h_1 - S)(h_1 - S)\text{period } 1$$

$$\int_0^{h_2+S} P(x)dx - P(h_2 + S)(h_2 + S)\text{period } 2$$

Producer surplus is just the revenue, production costs can be ignored because they do not impact the optimal storage rule (due to fixed marginal costs)

Net Aggregate Welfare

$$V(S) = \int_0^{h_1 - S} P(x)dx + \delta \int_0^{h_2 + S} P(x)dx - C(S)$$

storage is paid for "up-front".

Maximizing net aggregate welfare

$$L(S, \lambda) = V(S) + \lambda S$$

Karrush-Kuhn-Tucker conditions:

$$\frac{\partial L}{\partial S} = V^{'}(S) + \lambda \leq 0$$

$$\lambda S = 0 \text{ complementary slackness}$$

$V'(S)$

To differentiate this we use Leibniz' rule http://mathworld.wolfram.com/LeibnizIntegralRule.html

$$\frac{\partial}{\partial z}\int_{a(z)}^{b(z)} f(x,z)dx = \int_{a(z)}^{b(z)} \frac{\partial f}{\partial z}dx +$$
$$f(b(z),x)b'(z) - f(a(z),x)a'(z)$$

Applying this rule to $V(S)$:

$V'(S) = P(h_1 - S)(-1) + \delta P(h_2 + S)(+1) - C'(S)$ please verify

So KKT implies

$$-(P(h_1 - S) + C'(S)) + \delta P(h_2 + S) + \lambda \leq 0$$
$$\lambda S = 0$$

Case I: $\lambda > 0, S = 0$

$$-(P(h_1) + C^{'}(0)) + \delta P(h_2) + \lambda \leq 0$$
$$\lambda S = 0$$

This implies

$$P(h_1 - S) + C^{'}(S)) > \delta P(h_2)$$

and that no storage would take place

Case II: $\lambda = 0, S > 0$

$$-(P(h_1 - S^*) + C^{'}(S^*)) + \delta P(h_2 + S^*) \leq 0$$
$$\lambda S^* = 0$$

This implies

$$P(h_1 - S^*) + C^{'}(S^*) = \delta P(h_2 + S^*)$$

This is just our LOP storage rule from the introduction.

Summary

- Note $P^{'}(x) < 0$ and the optimal storage rule imply $h_1 > h_2$
- Intuition: Why store goods if you "know" there will be a good harvest.
- Optimal storage rule can be interpreted as arbitrage conditions within a market.
- tow work out how much to store we would need to know details of the inverse demand function and the cost function

T-period model

- 2-period models are not always sufficient
- What about a T-period storage model?
- In this case optimal storage period t depends on the equilibrium price in the $t+1$ period (Why?).
- This problem is dynamic because decisions in different periods are coupled together and not separable

T-period model

- Assume production h is fixed and exogenous
- Assume market demand is linear
- Assume marginal physical cost of storage is a constant m

The LOP equilibrium

Choose the level of storage to maximize the sum of consumer and producer revenue minus the cost of storage over a T-period time horizon

Step 1

- Construct an expression for the net present value of net aggregate welfare
- Assume production/harvesting occurs at the beginning of each period and and consumption immediately after harvest.
- S_{t-1} is the amount of stock carried over from period t-1 to period t.
- Period t consumption is $x_t = S_{t-1} - S_t$
- Stocks available for consumption in period t are $q_t = h + S_{t-1}$
- Consumption is $x_t = q_t - S_t$
- q_t is called the state variable

Dynamic programming

- The price of the commodity in period t depends on the consumption $q_t - S_t$
- $P_t = a - b(q_t - S_t)$
- Sum of producer and consumer surplus (aggregate welfare) is
- $a(q_t - S_t) - 0.5b(q_t - S_t)^2$
- Equation of motion $q_{t+1} = q_t - x_t$

Dynamic programming

$$\max_{S_t} \sum_{t=0}^{T-1} \delta^t \left\{ax_t - 0.5bx_t^2 - m(q_t - x_t)\right\} + V_T(S_T)$$

subject to

$$q_{t+1} = q_t - x_t$$

and $x_t = q_t - S_t$ and $S_T = 0$

Bellman's principle - Backward recursion

The Bellman equation:

$$V_t(q) = max_S \left\{ a(q-S) - 0.5b(q-S)^2 - mS + \delta V_{t+1}(h+S) \right\}$$

To solve this, start at t=T with $S_T = 0$ then

$$V_T(q) = max_{S_T} \left\{ a(q_T - S_T) - 0.5b(q_T - S_T)^2 - mS_T \right\}$$

$$= \left\{ a(q_T) - 0.5b(q_T)^2 \right\}$$

$$V_T(h + S_{T-1}) = \left\{ a(h + S_{T-1}) - 0.5b(h + S_{T-1})^2 \right\}$$

Now substitute this back into the Bellman equation and evaluate it (B.E.) at $t = T - 1$.

Bellman's principle

$$V_{T-1}(q) = max_{S_{T-1}} \left\{ a(q_{T-1} - S_{T-1}) - 0.5b(q_{T-1} - S_{T-1})^2 - mS_{T-1} + \delta \left\{ a(h + S_{T-1}) - 0.5b(h + S_{T-1})^2 \right\} \right\}$$

Dfferentiating the RHS gives

$$-a + b(q_{T-1} - S_{T-1}) - m + \delta a - \delta b(h + S_{T-1}) = 0$$

$$\frac{-a + bq_{T-1} - m + \delta a - \delta bh}{b(1+\delta)} = S_{T-1}$$

The textbook additionally defines $A_{T-1} = \delta a$ and $B_{T-1} = \delta b$, these will lead to an additional system of difference (recursive) equations that can be solved.

Deriving the continuation value

Substitute storage S_{T-1} into V_{T-1} and replace q_{T-1} with $h + S_{T-2}$ (we will do this in steps):

$$V_{T-1}(q) = max_{S_{T-1}} \left\{ a(q_{T-1} - \frac{-a + bq_{T-1} - m + \delta a - \delta bh}{b(1+\delta)}) - \right.$$

$$0.5b(q_{T-1} - \frac{-a + bq_{T-1} - m + \delta a - \delta bh}{b(1+\delta)})^2 -$$

$$m\frac{-a + bq_{T-1} - m + \delta a - \delta bh}{b(1+\delta)} +$$

$$\delta \left\{ a(h + \frac{-a + bq_{T-1} - m + \delta a - \delta bh}{b(1+\delta)}) - \right.$$

$$\left. \left. 0.5b(h + \frac{-a + bq_{T-1} - m + \delta a - \delta bh}{b(1+\delta)})^2 \right\} \right\}$$

Now substitute out q_{T-1}

$$V^*_{T-1}(q) = \{a(h + S_{T-2} -$$

$$\frac{-a + b(h + S_{T-2}) - m + \delta a - \delta bh}{b(1+\delta)}) -$$

$$0.5b(h + S_{T-2} - \frac{-a + b(h + S_{T-2}) - m + \delta a - \delta bh}{b(1+\delta)})^2 -$$

$$m \frac{-a + b(h + S_{T-2}) - m + \delta a - \delta bh}{b(1+\delta)} +$$

$$\delta \left\{ a(h + \frac{-a + b(h + S_{T-2}) - m + \delta a - \delta bh}{b(1+\delta)}) - \right.$$

$$\left. 0.5b(h + \frac{-a + b(h + S_{T-2}) - m + \delta a - \delta bh}{b(1+\delta)})^2 \right\} \}$$

All terms should now be in terms of $T-2$ (Check this!!)
This is the continuation value for period $T-2$.
This can be simplified a little by substituting A_{T-1}, B_{T-1}

Step 2: $T-2$

We now return to the Bellman equation:

$$V_{T-2}(q_{T-2}) = max_{S_{T-2}} \left\{ a(q_{T-2} - S_{T-2}) - 0.5b(q_{T-2} - S_{T-2})^2 - mS_{T-2} + \delta V^*_{T-}\right.$$

substitute in the continuation value and repeat the process until all periods are exhausted