Agricultural finance Lecture 6 Capital Budgeting

Rodney Beard

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Introduction

- Chapter 5, Moss
- How to apportion a firm's capital expenditure in different planning periods.
- Investment analysis
- Financial mathematics

Indirect utility

Instead of maximizing by choosing (C_t, C_{t+1}) choose monetary values instead

$$\max U(Y_t, Y_{t+1})$$

subject to

$$W \geq Y_t + \frac{1}{1+r} Y_{t+1}$$

MRS= Price Ratio

$$dU = \frac{\partial U}{\partial Y_t} dY_t + \frac{\partial U}{\partial Y_{t+1}} dY_{t+1} = 0$$

Rearranging gives
$\frac{\partial U}{\partial Y_{t+1}} dY_{t+1} = -\frac{\partial U}{\partial Y_t} dY_t$

$$-\frac{dY_{t+1}}{dY_t} = \frac{\frac{\partial U}{\partial Y_t}}{\frac{\partial U}{\partial Y_{t+1}}}$$

$=$ Marginal Rate of substitution

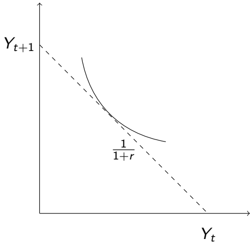
From the budget constraint

$$W = Y_t + \frac{1}{1+r} Y_{t+1}$$

Rearranging we get

$$Y_{t+1} = W(1+r) - (1+r)Y_t$$

Equation MRS = (1+r)



Multiperiod decisions

$$\max U(Y_t, Y_{t+1}, Y_{t+2})$$

subject to

$$W \geq Y_t + \frac{1}{1+r_1} Y_{t+1} + \frac{1}{1+r_2} Y_{t+2}$$

assume $\frac{1}{1+r_1} > \frac{1}{1+r_2}$
assume $\frac{1}{1+r_2} = \frac{1}{1+r_1} \frac{1}{1+\bar{r}_2}$

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Term Structure of Interest Rates

Yield curve

If plotted over time this is is referred to as the yield curve

$$\frac{1}{1+r_1}\frac{1}{1+\bar{r}_2} \overset{\geq}{\underset{<}{=}} \frac{1}{1+r_1}\frac{1}{1+r_1} = \frac{1}{1+r_1}^2$$

Weighted Average Cost of capital

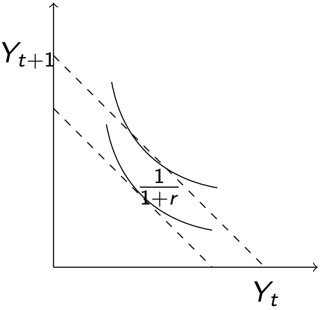
Equity source	Interest rate	Total amount	Share of equity	Res
Non-real estate debt				
Operating debt	15			

Investments

$$\max U(Y_t . Y_{t+1})$$

subject to

$$W + \tilde{W} \geq (Y_t + \tilde{Y}_t) + \frac{1}{1+r}(Y_{t+1} + \tilde{Y}_{t+1})$$



Change in Wealth

$$\tilde{W} = \tilde{Y}_t + \frac{1}{1+r}\tilde{Y}_{t+1}$$

Preferences can be ignored (consequence of Fisher separation).

Net Present Value

$$NPV = -I + \sum_{t=1}^{T} \frac{NCF_t}{(1+i)^t}$$

alternative formular

$$NPV = \sum_{t=0}^{T} \frac{NCF_t}{(1+i)^t}$$

Word of warning: Only works for deterministic and reversible investments

Some Python Code


```
def NPV(r,f,cashflows):
    total = 0.0
    for t,cashflow in enumerate(cashflows):
        delta = 1/(1+r/f)
        df = delta**t
        total += df*cashflow
    return total
```

Usage


```
CF = [-100,60,60,60]
NPV(0.1,365,CF)
```

Alternative discounting models - Exponential versus Hyperbolic Discounting

An alternative to exponential discounting (what we have been using) is hyperbolic discounting:

Hyperbolic discounting

$$NPV^H = \sum_{t=0}^{T} \frac{NCF_t}{(1+it)^t}$$

Quasi-hyperbolic discounting

$$NPV^{QH} = \beta \sum_{t=0}^{T} \frac{NCF_t}{(1+i)^t}, \beta < 1$$

Evidence for hyperbolic discounting from psychology

Exercise graph NPV against discount rate for different discount models

Multiple Projects

- normal investment (one sign reversal)
- calculate difference between cashflows between projects
- $CF_A - CF_B$ then calculate NPV of the differenced cash flow.
- Always subtract the benchmark cashflow stream (opportunity cost idea)

Taxation

$$NPV = -I + \sum_{t=1}^{T} \frac{NCF_t(1-\tau)}{(1+i(1-\tau))^t}$$

If the project produces tax benefits M_t then this becomes

$$NPV = -I + \sum_{t=1}^{T} \frac{NCF_t(1-\tau) + M_t\tau}{(1+i(1-\tau))^t}$$

Payback Method

How long before the project pays off?
Taking constant cashflow each period:

$$NPV = 0 = -I + \frac{CF \times T}{(1+i)^t}$$

$$\frac{I(1+i)^t}{CF} = T$$

Internal rate of return

Set NPV equal to zero and solve for i
Note that the internal rate of return may not have a unique solution.
Assuming constant cashflow then

$$NPV = 0 = -I + CF \times T/(1+i)^t$$

$$i = \sqrt[t]{\frac{CF \times T}{I}} - 1$$

Benefit-cost analysis

NPV is used in Benefit-cost analysis and in dynamic economic models Benefit cost ratio is a frequently used summary measure

$$BCR = \frac{\sum_{t=0}^{T} B_t}{\sum_{t=0}^{T} C_t}$$

This is used for public projects. Main difference between Benefit-Cost analysis and Investment analysis is the inclusion of social benefits and costs in Benefit-Cost analyses that are typically ignored in private investment analyses. Corporate social responsibility is placing increasing pressure on firms to include these in the calculation so that the idfference between these two approaches is narrowing over-time

Summary

To be continued...