Agricultural Marketing and Price Analysis
Lecture 9
Supply chains

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Introduction

Previously we have looked at two scenarios:

- Equivalence of market outcome with social planning outcome
- Price relationships (law of one price) for the same or substitute commodities (spatial or temporally separated markets, quality differences, etc.).

Purpose of Lecture

Examine vertical pricing relationships in a farm to retail food supply chain, in which downstream firms with market power purchase raw commodities from upstream farmers and sell processed and semi-processed differentiated products to retail consumers.

Supply chain

Definition

A supply chain consists of all activities and infrastructure whose purpose is to move products from where they are produced to hwere they are consumed. - Snyder and Shen, Fundamentals of Supply Chain Theory.

Marketing margin

A farm to retail marketing margin is the difference between the implicit value of an agricultural commodity when sold at the retail level in processed form versus the explicit value of the unprocessed commodity at the farm level.

Example

If the retail price of milk is \$/liter and the farm price of milk is \$0.35/liter. then the farm to retail marketing margin is \$0.65/liter.

In the case of fixed proportions (product moves through the chain in fixed proportions) the marketing margin is the net selling price of for the firm that supplies the marketing service.

Marketing margin and Average Costs

$$\Pi = (p - c)y$$

$$\frac{\Pi}{y} = (p - c) = \text{Marketing margin}$$

equals
Unit profits.

Product differentiation and marketing margins

- degree of product differentiation as a determinant of the marketing margin
- processing firm that sells more differentiated products will have more market power
- firms with more market power will have higher marketing margins (cf. Lerner markup-pricing rule)

Marketing margins and the competitive supply chain

- Marketing margin should be consistent with LOP.
- Set of prices in a competitive supply chain will be the same as the set of prices chosen by a social planner.
- Price of a commodity in a downstream position is equal tot the upstream price plus the unit cost of shifting the commodity downstream.
- Vertical price transmission measures how quickly and completely a downstream price change causes the upstream price to adjust and vice versa.

Demand for differentiated products

Producers of a homogenous commodity (farmers) sell their output to food manufacturers. These firms produce processed and semi-processed versions of the commodity and then sell their differentiated food products in a downstream retail market. Variable of interest is the marketing margin (difference between retail price of transformed commodity and the producer price of the raw commodity in the upstream market after adjusting for product conversion.).

Consumer problem

Representative consumer with fixed amount of disposable income I to be spent on combination of n processed food products and $\hat{n}$ semi-processed **standard** food products.

- Let q_i denote the quantity consumed and p_i the price of the i-th processed food product.
- Let $\hat{q}_i$ denote the quantity consumed and $\hat{p}_i$ the price of the i-th standard food product.

Utility Maximization

The representative consumer maximizes Utility $U(q, \hat{q})$ subject to the budget constraint:

$$\sum_{j=1}^{n} p_j q_j + \sum_{j=1}^{\hat{n}} \hat{p}_j \hat{q}_j = I$$

The solution of this as you know) will give $n + \hat{n}$ demand curves for each of the processed and semi-processed food products.

Two-stage budgeting

Equivalent to solving a "nested" utility maximization problem.

- Stage 1: Consumer allocates income between q units of processed food basket and $\hat{q}$ units of the unprocessed food basket.
- Stage 2: The budget allocated to processed foods in stage 1 is allocated to each of the n processed food items.

Similar procedure followed for the stand food basket.

Two-stage budgeting

$$\max_{q,\hat{q}} U(q, \hat{q})$$

subject to

$$qP + \hat{q}\hat{P} = I$$

Solving results in $q(P, \hat{P}, I)$ and stage one expenditure is $e(P, \hat{P}, I) = Pq(P, \hat{P}, I)$ for the processed food basket and demand $\hat{q}(P, \hat{P}, I)$ and expenditure $\hat{e}(P, \hat{P}, I) = \hat{P}\hat{q}(P, \hat{P}, I)$ for the standard food basket.

Stage 2: Sub-problems

Consumer maximizes

$$U_2(q)$$

subject to

$$\sum_{j=1}^{n} p_j q_j = e(P, \hat{P}, I)$$

and
Maximizes

$$\hat{U}_2(\hat{q})$$

subject to

$$\sum_{j=1}^{\hat{n}} \hat{p}_j \hat{q}_j = \hat{e}(P, \hat{P}, I)$$

Example

Assume a stage 1 CES-utility function;

$$\omega = (\alpha^{1-\theta} q^{\theta} + (1-\alpha)^{1-\theta} \hat{q}^{\theta})^{\frac{1}{\theta}}$$

- $\theta \in (0,1)$ is a measure of the degree of substitutability.
- $\alpha \in (0,1)$ is a share parameter.
- If α is large more processed food items are consumed and less standard food items.

Example Stage 1

Maximize the following Lagrangian:

$$L_1 = (\alpha^{1-\theta} q^{\theta} + (1-\alpha)^{1-\theta} \hat{q}^{\theta})^{\frac{1}{\theta}} + \lambda_1 (I - qP - \hat{q}\hat{P})$$

The first-order conditions for a maximum are given by

$$\frac{\partial L}{\partial q} = \frac{1}{\theta} (\alpha^{1-\theta} q^{\theta} + (1-\alpha)^{1-\theta} \hat{q}^{\theta})^{\frac{1}{\theta}-1} \alpha^{1-\theta} \theta q^{\theta-1} - \lambda_1 P = 0$$

$$\frac{\partial L}{\partial \hat{q}} = \frac{1}{\theta} (\alpha^{1-\theta} q^{\theta} + (1-\alpha)^{1-\theta} \hat{q}^{\theta})^{\frac{1}{\theta}-1} (1-\alpha)^{1-\theta} \theta \hat{q}^{\theta-1} - \lambda_1 \hat{P} = 0$$

$$\frac{\partial L}{\partial \lambda_1} = I - qP - \hat{q}\hat{P} = 0$$

Example Stage 1

Solving in steps:

$$\frac{\partial L}{\partial q} = \frac{1}{\theta}(\alpha^{1-\theta}q^{\theta} + (1-\alpha)^{1-\theta}\hat{q}^{\theta})^{\frac{1}{\theta}-1}\alpha^{1-\theta}\theta q^{\theta-1} - \lambda_1 P = 0$$

$$\frac{\partial L}{\partial \hat{q}} = \frac{1}{\theta}(\alpha^{1-\theta}q^{\theta} + (1-\alpha)^{1-\theta}\hat{q}^{\theta})^{\frac{1}{\theta}-1}(1-\alpha)^{1-\theta}\theta \hat{q}^{\theta-1} - \lambda_1 \hat{P} = 0$$

$$\frac{\alpha^{1-\theta}\theta q^{\theta-1}}{(1-\alpha)^{1-\theta}\theta\hat{q}^{\theta-1}} = \frac{\lambda_1 P}{\lambda_1 \hat{P}}$$

simplifying

$$q^{\theta-1} = (\frac{1-\alpha}{\alpha})^{1-\theta}\frac{P}{\hat{P}}\hat{q}^{\theta-1}$$

Then take the $\theta - 1$ root.

Example Stage 1

$$q = ((\frac{1-\alpha}{\alpha})^{1-\theta}\frac{P}{\hat{P}}\hat{q}^{\theta-1})^{\frac{1}{\theta-1}}$$

substitute this into the budget constraint

$$I - qP - \hat{q}\hat{P} = 0$$

To get

$$I - ((\frac{1-\alpha}{\alpha})^{1-\theta}\frac{P}{\hat{P}}\hat{q}^{\theta-1})^{\frac{1}{\theta-1}}P - \hat{q}\hat{P} = 0$$

This can be rewritten as

$$I - ((\frac{1-\alpha}{\alpha})^{1-\theta}\frac{P}{\hat{P}})^{\frac{1}{\theta-1}}\hat{q}P - \hat{q}\hat{P} = 0$$

Solve for $\hat{q}$ to get

$$\hat{q} = \frac{I}{((\frac{1-\alpha}{\alpha})^{1-\theta}\frac{P}{\hat{P}})^{\frac{1}{\theta-1}}P + \hat{P}}$$

This can be simplified still further!
Can you see how?

Expendíture functions

Eventually with further simplification we obtain the expenditure functions:

$$e(P, \hat{P}, I) = \frac{\alpha P^{\frac{\theta}{\theta-1}}}{\alpha P^{\frac{\theta}{\theta-1}} + (1-\alpha)\hat{P}^{\frac{\theta}{\theta-1}}} I$$

$$\hat{e}(P, \hat{P}, I) = \frac{(1-\alpha)\hat{P}^{\frac{\theta}{\theta-1}}}{\alpha P^{\frac{\theta}{\theta-1}} + (1-\alpha)\hat{P}^{\frac{\theta}{\theta-1}}} I$$

Stage 2 CES Utility

$$u = \left(\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho} \right)^{\frac{1}{\rho}}$$

$$\hat{u} = \left(\sum_{j=1}^{\hat{n}} \hat{a}_j^{1-\hat{\rho}} \hat{q}_j^{\hat{\rho}} \right)^{\frac{1}{\hat{\rho}}}$$

Derivation of stage 2 consumer demand

For processed foods (repeat again for the standard basket):

$$L_2 = \left(\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho}\right)^{\frac{1}{\rho}} + \lambda_2(e(P, \hat{P}, I) - \sum_{j=1}^{n} p_j q_j)$$

In the second stage it is assume that the expenditure function that enters the budget constraint is fixed and constant.

The first-order conditions for this are:

$$\frac{\partial L_2}{\partial q_l} = \frac{1}{\rho}\left(\sum_{j=1}^{n} a_j^{1-\rho} q_j^{\rho}\right)^{\frac{1}{\rho}-1} a_l^{1-\rho} \rho q_l^{\rho-1} - \lambda_2 p_l = 0, l = 1, \ldots, n$$

$$\frac{\partial L_2}{\partial \lambda_2} = e(P, \hat{P}, I) - \sum_{j=1}^{n} p_j q_j = 0$$

Result

$$q_i(p_1, \ldots, p_n, e) = \frac{a_i p_i^{\frac{1}{\rho - 1}}}{\sum a_j p_j^{\frac{\rho}{\rho - 1}}} e(P, \hat{P}, I)$$

$$\hat{q}_i(\hat{p}_1, \ldots, \hat{p}_n, \hat{e}) = \frac{\hat{a}_i \hat{p}_i^{\frac{1}{\hat{\rho} - 1}}}{\sum \hat{a}_j \hat{p}_j^{\frac{\hat{\rho}}{\hat{\rho} - 1}}} \hat{e}(P, \hat{P}, I)$$

Deriving expressions for P, $\hat{P}$

To do this consider the indirect utility functions (i.e. substitute the demand curves back into the utility functions):

$$v(p_1, \ldots, p_n) = \left(\sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}} \right)^{\frac{1-\rho}{\rho}} e(P, \hat{P}, I)$$

Observe that this can be rearranged to get

$$e(P, \hat{P}, I) = v(p_1, \ldots, p_n) \left(\sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}}$$

The term on the far right is clearly a price index and can set it equal to P the indirect utility expression can be interpreted as the number of units of processed food basket that are purchased.
We can proceed similarly for the standard food basket.

Profit maximizing food companies

Profit is given by

$$\Pi = (p_i - w_i - c)q_i$$

where

- p_i is price of the processed food product
- w_i is the raw commodity price
- c is the unit cost of processing

analysis is short-run with no entry or exit so we ignore fixed costs.

Assumptions

- Case 1: Firm is a monopsonist (sets price equal to supply schedule
- Case 2: Firm is a competitive price taker

Condition for a profit maximum is

Marginal Revenue = Marginal Outlay + Cost of food processing

$$p_i + q_i \frac{dp_i}{dq_i} = w_i^s(q_i) + \delta_i \frac{dw_i^s(q_i)}{dq_i} q_i + c$$

δ_i is an indicator function taking on the value 1 if the firm is a monopsonist and 0 if the firm is a price taker on the raw commodity market.

Marginal Revenue and Marginal Outlay

Recall:

$$P = \left(\sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}}$$

and

$$q_i(p_1, \ldots, p_n, e) = \frac{a_i p_i^{\frac{1}{\rho-1}}}{\sum a_j p_j^{\frac{\rho}{\rho-1}}} e(P, \hat{P}, I)$$

Inverting the first of these gives:

$$P^{\frac{\rho}{\rho-1}} = \sum_{j=1}^{n} a_j p_j^{\frac{\rho}{\rho-1}}$$

which is just the denominator in the demand equation.

$$q_i(p_1, \ldots, p_n, e) = \frac{a_i p_i^{\frac{1}{\rho-1}}}{P^{\frac{\rho}{\rho-1}}} e(P, \hat{P}, I)$$

Raising both sides to the power of $\rho - 1$:

$$q_i(p_1, \ldots, p_n, e)^{\rho-1} = \frac{a_i^{\rho-1} p_i}{P^{\rho}} e(P, \hat{P}, I)^{\rho-1}$$

This can be rearranged to obtain:

$$p_i = (a_i e(P, \hat{P}, I))^{1-\rho} P^{\rho} q_i^{\rho-1}$$

We can now use this to calculate the marginal revenue

Marginal Revenue

Marginal revenue:

$$MR = p_i + \frac{dp_i}{dq_i} q_i$$

Differentiating p_i we get

$$\frac{dp_i}{dq_i} = (a_i e(P, \hat{P}, I))^{1-\rho} P^{\rho} (\rho - 1) q_i^{\rho-2}$$

substituting we get

$$MR = (a_i e(P, \hat{P}, I))^{1-\rho} P^{\rho} q_i^{\rho-1} +$$

$$(a_i e(P, \hat{P}, I))^{1-\rho} P^{\rho} (\rho - 1) q_i^{\rho-2} q_i$$

$$= \rho (a_i e(P, \hat{P}, I))^{1-\rho} P^{\rho} q_i^{\rho-1}$$

$$= \rho p_i$$

Intuitive interpretation

- $0 < \rho < 1$ firm prices where $MR > 0$ or on the elastic part of the demand curve.
- When $\rho = 1 MR = p_i$ and demand is infinitely elastic
- When $\rho = 0$ demand is Cobb-Douglas and demand has an elasticity of -1, $MR = 0$
- In general decreasing *rho* makes demand more inelastic and increases the firms market power

Market outlay

Start with the market supply :

$$w_i^S(q_i) = \beta_0 + \beta_1 q_i$$

Marginal outlay is

$$w_i 6S(q_i) + \delta_i q_i \frac{dw_i^S}{dq_i}$$

$$= \beta_0 + \beta_1 q_i + \delta_i q_i \beta_1$$

$$= \beta_0 + (1 + \delta_i)\beta_1 q_i$$

Market equilibrium

In equilibrium marginal revenue equals marginal outlay plus marginal conversion costs.

$$\rho(a_i e(P, \hat{P}, I))^{1-\rho} P^{\rho} q_i^{\rho-1} - (\beta_0 + (1+\delta_i)\beta_1 q_i) - c = 0, i = 1, \ldots, n$$

and the same again for the standard good. Giving $n + \hat{n}$ equations that need to be solved for $(q_i, \hat{q}_i)$.

Conclusion

- In a competitive market LOP ensures price at each stage of the chain reflects added costs of transporting processing and warehousing
- Competitive market does not fit food supply chains because firms are small in number and their are large barriers to entry
- Prices determined by supply chain costs and market power.
- Large marketing margins
- Relationship between pricing and outside demand is no longer easily predictable due to non-linearities.
- Large empirical literature on estimating marketing margins in supply-chains (not cost data is in general not avaiable and must be inferred indirectly).