Commodities Futures Market - Lecture 7
Exotic Options II

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Readings

Chapter 5Section 5.4.3-5 and Ch. 6 Geman Agricultural Finance.

Monte-Carlo Pricing of Asian Options

Price dynamics are:

$$\frac{dS}{S} = (r - y)dt + \sigma d\hat{W}$$

Integrate this between 0 and t_1 using coefficient matching method:

$$S(t) = s(0)\exp\left\{(r - y - \frac{\sigma^2}{2})t_1 + \sigma \hat{W}(t_t)\right\}$$

Note we can integrate between any two points.

Paste solutions together

$$S(t_2) = \hat{S}(t_1) \exp\left\{ (r - y - \frac{\sigma^2}{2})(t_2 - t_1) + \sigma \hat{W}(t_2 - t_1) \right\}$$

Now take random draws of $\hat{W}(t_2 - t_1$ for equally spaced points in time. to calculate $\hat{S}(t_2)$.

Computation

Piecewise solutions allow us to calculate

$$a = \frac{\hat{S}_1(t_1}{+} \ldots + \hat{S}_2(t_i)n$$

Calculate $b_i = \max(0, a - k)$.
Repeat N times and calculate

$$C(0) = e^{-rT} \frac{b_1 + b_2 + \ldots + b_N}{N}$$

Calculating Δ

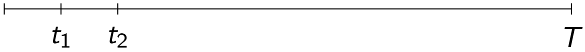
- Monte-Carlo gives us a good approximation of the call price
- However if you want to Δ hedge you can't just differentiate C to botain delta as you don't know the formula.
- Solution: Use a finite difference method

Finite difference approach to calculating delta

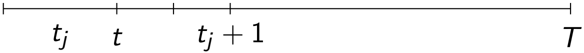
$$\Delta^{As} \approx \frac{C^{As}(s_0 + \frac{h}{2}) - C^{As}(S_0 - \frac{h}{2}}{h}$$

Exact results

Geman and Yor (1993) use the Laplace Transform method with reespect to maturity. Eydeland and Geman (1995) (how to invert the Laplace transform).
Valuation of Asian options depends on date.



At date 0, S at $t > 0$ is not known.
Assume date t lies between t_j and $t_j + 1$:



Then $S(t_1), \ldots, S(t_j)$ are full observed at date t. and

$$A(T) = \frac{S(t_1) + \ldots + S(t_j) + S(t_j \tilde{+} 1) + S(\tilde{t}_n)}{n}$$

Note: If prior spot prices are large then as one approaches maturity it is clear that the option is in the money because the last term is then relatively small as n becomes large.

Call formula

$$
\begin{aligned}
C(t) &= e^{-r(T-t)} E_Q \left[\max(0, A(T) - k) | F_t\right] \\
&= e^{-r(T-t)} E_Q \left[\max(A(T) - k) | F_t\right]
\end{aligned}
$$

Linearity of $E()$ lets us obtain:

$$
C^{As}(t) = S(t) \frac{1 - e^{-r(T-t)}}{rT} -
$$

$$
e^{-r(T-t)} \left[k - \frac{S(t_1) + \ldots + S(t_j)}{n} \right]
$$

Note the term in brackets is negative because the sum of the prices is large. Formula also useful for weather derivatives as these can be modelled as Asian options.

Comments

- volatility is reduced by averaging (volatility of an average is small)
- Risk return trade-off implies Asian option should be cheaper than a comparable plain vanilla option. However this is not true (Geman and Yor)

Example

Consider an Asian option with dates $t_1, t_2, t_3 = T$ on a dividend paying stock (comparable to commodity with convenience yield) with values $S(t_1) = 80$, $S(t_2) = 60$, $S(t_3) = 40$. $A(T) = 60$ and $S(T) = 40$ then for k between 40 and 60 the Asian option has positive payoff and the European call zero payoff. This means the price for the Asian option should be greater than the European call at date 0 despite lower volatility.
Implication for commodities: If the forward curve is backwardated (r-y ¡ 0) then the Asian call may be more expensive than a comparable Europena call.

Implications

Geman and Yor:

- If $r - y > 0$, forward curve is in contango and Asian option is cheaper.
- If $r - y < 0$, forward curve is backwardated and answer is unclear. Depends on relative volatility reduction due to averaging versus declining value of the the spot price.

Floating Strike asian Options

$$C_{fl}^{As}(T) = \max(0, S(T) - A(T))$$

Yes this is a call! Because call value rises with the strike price. Easily computed using Monte-Carlo. Alternatively

$$C_{fl}^{As}(T) = S(T)\max(0, 1 - \frac{A(T)}{S(T)})$$

using spot price as numeraire: