Agricultural Marketing and Price Analysis
Lecture 8
Price linkages

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December 4, 2016

Introduction

- Horizontal price integration measures extent of price changes for a commodity spilling over on price changes for another commodity
- reason for this could be common demand shocks or common production shocks
- Cross-price substitution may smooths the the impacts of price changes by spreading them across multiple markets
- Horizontal price integration is exoected to be strong for storable commodities, e.g. corn, wheat, rice, etc.

Example

Release of an Australian crop report that reveals an unexpected decrease in forecasted wheat production is likely to cause the price of European corn to immediately increase. this increase occurs because traders will anticipate a higher wheat price, and will also anticipate that the higher wheat price will induce European farmers to shift acreage out of corn and into wheat. Traders will therefore expect a higher price of corn in the future due to the anticipated increase in demand and decrease in supply. An increase in the expected future corn price implies that more corn will be stored by European traders. The higher level of storage will reduce the current supply of corn , and this reduction will have an immediate and positive impact on the European price of corn. Substitution effects, combined with the capacity of firms to adjust inventories when relative prices change, implies that news about a future supply shock in the australian wheat market will have an immediate impact on the European price of corn.

Some measures of integration

- Correlation
- Co-integration (time series analysis)
- Copulas

Correlation table

	Corn	Wheat	Hogs	Cattle
Corn	1			
Wheat	0.493	1		
Hogs	0.082	0.068	1	
Cattle	0.043	0.076	0.160	1

Reasons for integration (or lack of)

- limited storability results in lack of integration
- weak substitution results in lack of integration
- supply response lags

Food for fuel debate

- Rising demand for ethanol and biodiesel has raised the demand for corn and soybeans
- The increase in the price of corn and soybeans has induced farmers to substitute away from other crops and toward corn and soybeans on the supply side.
- the price increase has also induced feedlots to to substitute away from corn and soybeans and toward feedgrains on the demand side.
- The combination of reduced supply and higher demand for crops other than corn and soybeans has placed upward pressure on prices for both non-processed and processed food products.

The Model

- We will assume two goods: Corn and the Other composite crop (OCC).
- C is the quantity of corn produced by the farmer.
- C_L qunatity of corn used by the feedlot sector
- C_B quantity of corn used by the biofuels sector.
- The quantity of OCC produced by the farmer is X

Adding up conditions

$$C = C_L + C_B$$

$$X = X_L + X_H$$

Farm production

The following is the production technology for the farm in implicit form:

$$f(C, X) = K$$

How much C and X can be produced with K units of capital?
Diminishing marginal productivity of farm capital implies the PPF is concave to the origin.

Feedlot production

$$g(C_L, X_L) = Q$$

Q units of livestock.
diminishing returns of livestock production to increased feed.

Aggregate willingness to pay for Corn

$M_B(C_B)$ is the aggregate willingness to pay for corn in the biofuel processing sector.
Increasing marginal cost of converting corn to ethanol implies $MB^{'}(C_B) > 0$ and $M^{''}(C_B) < 0$ concavity implies that $P_B(C_B) \equiv M_B^{'}(C_B)$ is downward sloping function.

Aggregate Willingness to pay for OCC

$M_H(X_H)$ is the aggregate willingness to pay for OCC by food processors. Concavity implies $P_H(X_H) = M'_H(X_H)$ is downward sloping.

Value of livestock

$M_L(Q)$ is society's valuation of Q units of livestock.
Livestock supply is assumed to be fixed and exogenous.

Social planner's problem

$$\max_{C_L, C_B, X_H, X_L} W(C_B, X_H) = M_B(C_B) + M_H(X_H) + M_L(Q) - F_K - F_L$$

subject to

$$f(C_L + C_B, X_L + X_H) = K$$

and

$$g(C_L, X_L) = Q$$

Lagrangian

$$L(C_L, C_B, X_H, X_L) = M_B(C_B) + M_H(X_H) + M_L(Q) - F_K - F_L +$$

$$\lambda_1[K - f(C_L + C_B, X_L + X_H)] + \lambda_2[Q - g(C_L, X_L)]$$

Recall,

$$P_B(C_B) = M_B^{'}(C_B)$$

$$P_H(X_H) = M_H^{'}(X_H)$$

First-order conditions

$$\frac{\partial L}{\partial C_L} = -\lambda_1 \frac{\partial f}{\partial C} - \lambda_2 \frac{\partial g}{\partial C_L} = 0$$

$$\frac{\partial L}{\partial C_B} = P_B(C_B) - \lambda_1 \frac{\partial f}{\partial C} = 0$$

$$\frac{\partial L}{\partial X_L} = -\lambda_1 \frac{\partial f}{\partial X} - \lambda_1 \frac{\partial g}{\partial X_L} = 0$$

$$\frac{\partial L}{\partial X_H} = P_H(X_H) - \lambda_1 \frac{\partial f}{\partial X_H} = 0$$

$$f(C_L + C_B, X_L + X_H) = K$$

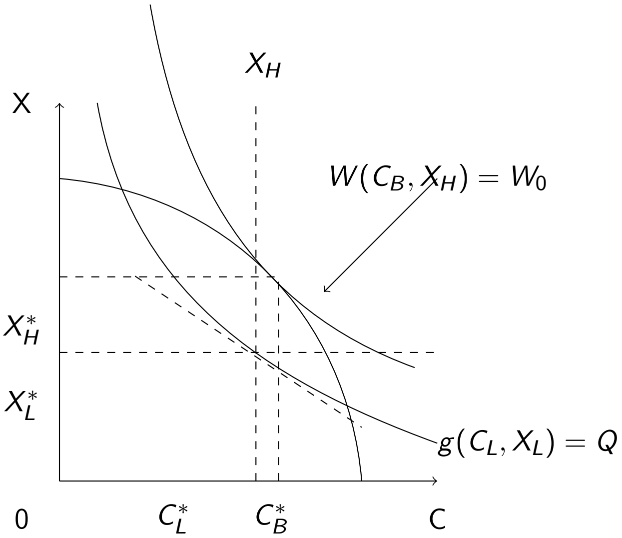
$$g(C_L, X_L) = 0$$

Equilibrium conditions

$$\frac{P_B(C_B)}{P_H(C_H)} = \frac{\frac{\partial f()C,X)}{\partial C}}{\frac{\partial f(C,X)}{\partial X}}$$

$$\frac{P_B(C_B)}{P_H(X_H)} = \frac{\frac{\partial g(C_L,X_L)}{\partial C_L}}{\frac{\partial g(C_L,X_L)}{\partial X_L}}$$

Graphical solution: Social planner's problem



Derivation of multi-market equilibrium

Farmer's profit maximization problem:
Maximize:

$$\Pi = P_c C + P_X X - F_K$$

subject to

$$f(C, X) = K$$

where

- P_C is the price of corn
- P_X is the price of the OCC
- F_K is a fixed capital cost

Lagrangian

$$L = P_c C + P_X X - F_K + \lambda(K - f(C, X))$$

The first-order conditions are:

$$\frac{\partial L}{\partial C} = P_C - \lambda \frac{\partial f}{\partial C} = 0$$

$$\frac{\partial L}{\partial X} = P_X - \lambda \frac{\partial f}{\partial X} = 0$$

$$\frac{\partial L}{\partial \lambda} = K - f(C, X) = 0$$

Dividing one equation by the other (after rearranging):

$$\frac{P_C}{P_X} = \frac{\lambda \frac{\partial f}{\partial C}}{\lambda \frac{\partial f}{\partial X}}$$

Lagrange variable cancels

$$\frac{P_C}{P_X} = \frac{\frac{\partial f}{\partial C}}{\frac{\partial f}{\partial X}}$$

The **farmers should allocate capital** such that the **opportunity cost of corn relative to the OCC (price ratio) equals the marginal rate of transformation** of the OCC into corn)

From this we would obtain the market supply schedules $C^S(P_C, P_X)$ and $X^S(P_C, P_X)$ with further manipulation (Technology constaint stuill hasn't been used).

The Derived Demand for Feed Grains

Minimize the costs of production of the feedlot subject to the feedlots production technology:

$$P_C C_L + P_X X_L$$

subject to

$$g(C_L, X_L) = Q$$

Lagrangian

$$L = P_C C_L + P_X X_L + \lambda(Q - g(C_L, X_L))$$

The first-order conditions are:

$$\frac{\partial L}{\partial C_L} = P_C - \lambda \frac{\partial g}{\partial C_L} = 0$$

$$\frac{\partial L}{\partial X_L} = P_X - \lambda \frac{\partial g}{\partial X_L} = 0$$

$$\frac{\partial L}{\partial \lambda} = Q - g(C_L, X_L) = 0$$

Proceeding as before we get

$$\frac{P_C}{P_X} = \frac{\frac{\partial g}{\partial C_L}}{\frac{\partial g}{\partial X_L}}$$

and

$$Q = g(C_L, X_L)$$

The Derived demand for Feed Grains

Derived demand is obtained by solving these to obtain

$$C_L^D(P_C, P_X)$$

and

$$X_L^D(P_C, P_X)$$

The Biofuels Sector

A biofuels processor chooses C_B (the quantity of corn to be used for biofuels production) to maximize surplus (willingness to pay minus cost of purchasing corn).

$$M_B(C_B) - P_B C_B$$

the first-order condition for a maximum is

$$M_B'(C_B) = P_B$$

This is equivalent to

$$P_B(C_B) = P_C$$

Inverting this

$$C_B = P_B^{-1}(P_C)$$

gives us the derived demand for corn for the biofuels sector.

Food processing sector

A food processor choses X_H to maximize:

$$M_H(C_H) - P_X X_H$$

The first-order condition for this is

$$M_H^{'}(X_H) = P_X$$

This is equivalent to

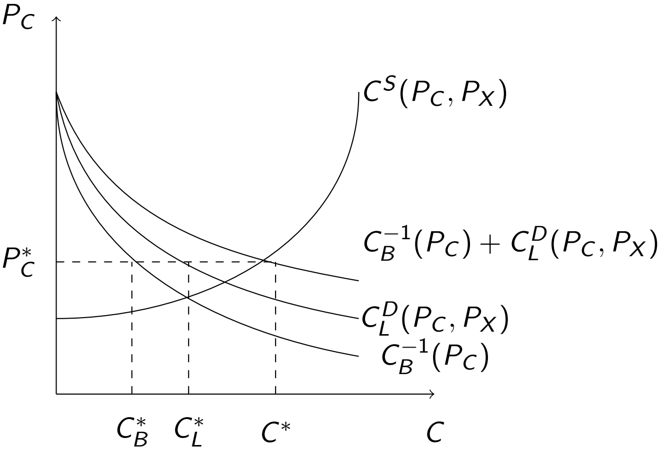
$$P_H(X_H) = P_X$$

Inversting this

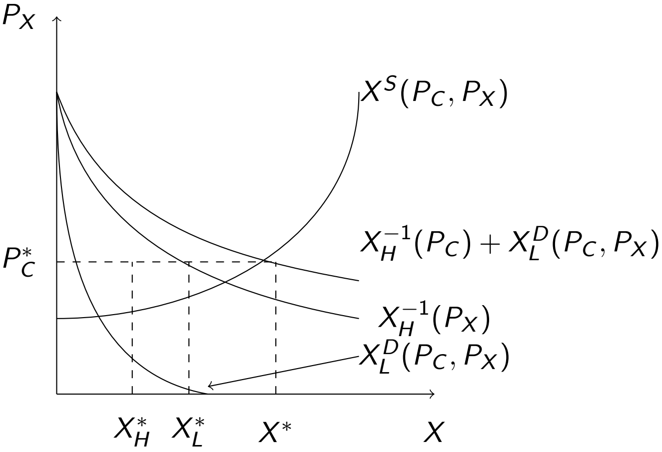
$$X_H = P_H^{-1}(P_X)$$

gives us the derided demand for X_H by the food processor.

Competitive equilibrium - Corn



Competitive equilibrium - OCC



Competitive Equilibrium = Social Planner's Outcome

- Substitute $P_B(C_B) = P_C$ and $P_H(X_H) = P_X$ into the competitive market FOC(first-order conditions) for farming and livestock
- $\frac{P_C}{P_X} = \frac{\frac{\partial f}{\partial C}}{\frac{\partial f}{\partial X}}$ and $\frac{P_C}{P_X} = \frac{\frac{\partial g}{\partial C_L}}{\frac{\partial g}{\partial X_L}}$
- Result is identical to FOC for social planner

Example - Farm Supply and Feedlot Demand Schedules

Consider a constant returns to scale CES function:

$$Z = H(a_C C^b + a_X X^b)^{\frac{1}{b}}$$

Z can be interpreted in different ways:

- Z is stock of farm capital and C and X are Corn and OCC.
- If Z is feedlot's level of livestock output then C and X are the input levels of C and OCC.
- H is a scale parameter
- a_C and a_X are share parameters and b is a substitution parameter.

Elasticity of substitution

Totally differentiate $Z = H(a_C C^b + a_X X^b)^{\frac{1}{b}}$

$$dZ = \frac{\partial Z}{\partial C} dC + \frac{\partial Z}{\partial X} dX = 0$$

$$\frac{1}{b} H(a_C C^b + a_X X^b)^{\frac{1}{b}-1} a_C b C^{b-1} dC +$$

$$\frac{1}{b} H(a_C C^b + a_X X^b)^{\frac{1}{b}-1} a_X b X^{b-1} dX$$

Simplifying

$$a_C C^{b-1} dC +$$

$$a_X b X^{b-1} dX = 0$$

Solve for the slope to get

$$\frac{dX}{dC} = -\frac{a_C}{a_X} \left(\frac{C}{X} \right)^{b-1}$$

Elasticity of substitution

- For supply this is the marginal rate of transformation (slope of PPF)
- for demand this is the marginal rate of technical substitution (MRTS) (Iso-quant $g(.)$)
- the elasticity of substitution of C for X ρ is the percentage change in $\frac{C}{X}$ due to a percentage change in $\frac{dX}{dC}$

Invert $\frac{dX}{dC} = -\frac{a_C}{a_X}\left(\frac{C}{X}\right)^{b-1}$

$$\frac{C}{X} = \left(-\frac{a_X}{a_C}\frac{dX}{dC}\right)^{\frac{1}{b-1}}$$

Now calculate

$$\frac{\partial \frac{C}{X}}{\partial \frac{dX}{dC}} \frac{\frac{dX}{dC}}{\frac{C}{X}} = \frac{1}{1-b} \equiv \rho$$

Deriving the farm supply and feedlot demand schedules

In equilibrium the slope of the PPF and the slope of the Iso-quant equal the price ratio $\frac{P_C}{P_X}$:
Set $\frac{P_C}{P_X} = -\frac{a_C}{a_X}\left(\frac{C}{X}\right)^{b-1}$ rearrange to obtain

$$C = \left(-\frac{P_C}{P_X}\frac{a_X}{a_C}\right)^{\frac{1}{b-1}} X$$

Together with

$$Z = K(a_C C^b + a_X X^b)^{\frac{1}{b}}$$

These can be be solved to obtain the feedlot demand schedules

Farm supply/feedlot demand schedules

$$C(P_C, P_X) = \left(\frac{P_C}{a_C}\right)^{\frac{1}{b-1}} \left(a_C(\frac{P_C}{a_C})^{\frac{b}{b-1}} + a_X(\frac{P_X}{a_X})^{\frac{b}{b-1}}\right)^{\frac{1}{b}} \frac{Z}{K}$$

$$X(P_C, P_X) = \left(\frac{P_X}{a_X}\right)^{\frac{1}{b-1}} \left(a_C(\frac{P_C}{a_C})^{\frac{b}{b-1}} + a_X(\frac{P_X}{a_X})^{\frac{b}{b-1}}\right)^{\frac{1}{b}} \frac{Z}{K}$$

The interpretation of these depends on the value of b.

- These are supply schedules for $b \geq 1$
- these are demand schedules for $b < 1$

ρ

$$\rho = \frac{1}{1-b}$$

- So $b = 1 - \frac{1}{\rho} = \frac{\rho-1}{\rho}$
- If $\rho < 0$ then $b > 1$ so we ahve a PPF
- $\rho > 0$ implies $b < 1$ so we have an iso-quant
- $\rho \to -\infty$ then $b \to 1$ then we have perfect substitutes (supply side)
- $\rho \to \infty$ then $b \to 1$ then we have perfect substitutes (demand side)
- $\rho \to 0$ fixed proportions eiher PPF or iso-quant depending on the direction of convergence
- $\rho \to 1$ implies $b \to 0$ implies Cobb-douglas

Food and biofuel demand

Recall $P_B(C_B) = M_B'(C_B)$ and $P_H(X_H) = M_H'(X_H)$ are marginal willingness to pay
Assume the following functional forms for the exercises:

$$P_C = h_C C_B^{-\eta_C}$$

$$P_X = h_X X_H^{-\eta_H}$$

Conclusion

- We had three aims with this topic: Show the equivalence of the social planner's problem and the market equilibrium
- Show how the CES function can be used to represent different technologies in multi-market models
- show linkages across markets using the farm production, feedlot and biofuels, consumer sectors