Commodities Futures Market - Lecture 7
Exotic Options I

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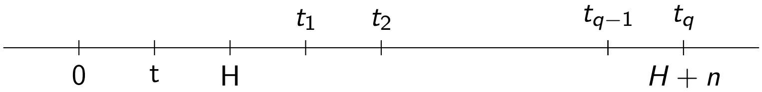
Readings

Chapter 4 Section 4.8 Geman Agricultural Finance.

Swaps and swaptions

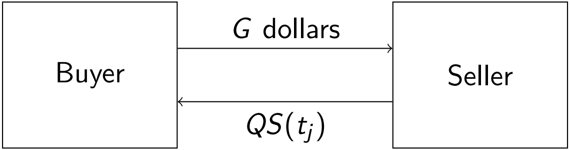
Defintion (Swap)

A portfolio of forward contracts entered at the same price



Buyer pays G dollars on each date t for Q units of the commodity.

Swap payments



where G is the fixed price of the swap and Q the quantity of the commodity and $S(t_j)$ the spot price taken from a public index at date t_j

Swaps

- Swaps become forward contracts if $q = 1$.
- Swaps are financially settled and do not involve physical delivery
- Over the counter (OTC) customized transactions for the purpose of hedging
- Existence of a reliable index necessary for a swap market to be viable

Main markets

- Gas
- Oil
- Electricity
- Agricultural swap markets are thin but becoming more liquid
- Example: Fertilizers

Market value and guaranteed price

At contract begin

$$V_P(t) = 0$$

market value of a long position paying fixed leg G. Fair or zero-sum game.

After contract begin

$$V_P(t') = \sum_{j=1}^{q} V_{P_j}(t')$$

Note from earlier in the course that forward contracts satisfy

$$V_{P_j}(t) = e^{-r(t_j - t} \left[F^{t_j}(t) - G\right]$$

So that the value of the swap is

$$V_{Swap} = \sum_{j=1}^{q} e^{-r(t_j - t} \left[F^{t_j}(t) - G\right]$$

Guaranteed Prce

Set $V_{Swap}(t) = 0$ to obtain

$$0 = \sum_{j=1}^{q} e^{-r(t_j - t)} \left[F^{t_j}(t) - G \right]$$

$$= \sum_{j=1}^{q} e^{-r(t_j - t)} F^{t_j}(t) - \sum_{j=1}^{q} e^{-r(t_j - t} G$$

$$= \sum_{j=1}^{q} e^{-r(t_j - t)} F^{t_j}(t) - G \sum_{j=1}^{q} e^{-r(t_j - t}$$

$$G = \frac{\sum_{j=1}^{q} e^{-r(t_j - t)} F^{t_j}(t)}{\sum_{j=1}^{q} e^{-r(t_j - t}}$$

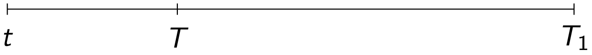
This is a weighted average

$$G = \sum_{j=1}^{q} w_j F^{t_j}(t)$$

with weights $w_i = \frac{e^{-r(t_j - t)}}{\sum_{j=1}^{q} e^{-r(t_j - t)}}$

Options on Swaps

Swaptions are options on swaps. Holder has the right to enter a swap at a future date T at a price G that is fixed at date t and prior to the maturity T_1.



Example

Buyer ays at date 0 underling price of swaption to seller. Maturity March 31, 2018. Payment dates of the swap are July, August September 2018 Payer swaption holder has right to enter contract as payer of fixed leg G and receiver of floating leg $S(j)$, where $j =$ June 30, July 31, $\ldots$

Call option

$$C_{swap}(T) = \max\left(0, V_{swap}(T)\right)$$

$$C(t) = e^{-r(T-t)} E_Q \left[\max\left(0, V_{swap}(T)\right) | F_t\right]$$

Example

An accumulator is a type of swap contract with a barrier.

- inception date t=0
- Maturity T, e.g. $T = 12$ months.
- Fixings, $N = 12$
- Fix price $k < f(t, T)$, e.g. $k = 80$ and $f(t, T) = 100$
- An upper barrier $U > f(0, T)$, e.g. $U = 115$
- at the end of each month the buyer of an accumulator is committed to buying 1 unit at a price k as long as $S < U$.
- If $S > U$ then the accumulator expires.

Example

An agri-food producer has bought a 12 month accumulator from a coffee producer for 1 tonne of coffee per month.

1. If coffee prices rise above k the agri-food company makes a profit as long as $S < U$, by buying at a lower prices than the spot.
2. If coffee prices fall below k the agri-food company.
3. If prices stay flat the company still wins because $k < f(0, T)$.

Calendar Spreads

Assume $t < T_1 < T$ traders can hedge a single commodity during the period (T_1, T).
Example a forward-start call option:

$$C^{fs}(T) = \max(0, S(T) - S(T_1))$$

The strike is therefore fixed as the spot price on a certain date.

Example

Buyer wishes to hedge against a rise in corn prices between June 1 and July 1, with a current date of March 1. By purchasing a calendar spread option to buy they are able to hedge against future potential prices increases for the fixed period (e.g. a harvest period for example).

Price dynamics under the risk-neutral measure

$$\frac{dS}{S} = (r - y)dt + \sigma d\tilde{W}$$

$$S(T) = S(T_1)\exp\{(r - y - \frac{\sigma^2}{2})(T - T_1) + \sigma\hat{W}(T - T_1)\}$$
$$= S(T_1)\exp\{U\}$$

where

$$U = (r - y - \frac{\sigma^2}{2})(T - T_1) + \sigma\hat{W}(T - T_1)$$

Payoff

$$C^{fs}(T) = S(T_1)\max(0, exp\{U\} - 1)$$

and

$$C^{fs}(t) = e^{rt} E_Q\left[S(T_1)(exp\{U\} - 1)\mathbb{1}e|F_t\right]$$

$$e = \{\text{state of nature where } S(T) > S(T_1)\}$$

Call option formula

$$C^{fs}(t) = S(t)e^{-r(T-t)}N(d_1) - Ft, T_1 e^{-r(T-t)}N(d_2)$$

$$d_1 = \frac{\ln \frac{S(t)e^{-y(T-t)}}{F(t,T_1)e^{-r(T-t)}} + \frac{1}{2}\sigma^2(T - T_1)}{\sigma\sqrt{T - T_1}}$$

$$d_2 = d_1 - \sigma\sqrt{T - T_1}$$

Asian options: Example 1

A treasurer of a multinational company located in Switzerland (coffeee trader) will receive daily or weekly cash flows (100) denominated in JPY (japanese Yen). He needs to hedge his exposure to the yen by buying put options on the JPY/CHF. He can do this with 100 an ordinary put options. Transactions costs make this a bad strategy. Instead he can buy a put option on the arithmetic average of the 100 exchange rates, this is an Asian put option. Similarly, an Asian call option can be used to hedge against the rising price of coffee imported from Brazil by writing a call option on the BRZ/CHF.

Price Averages

$$\frac{dS}{S} = (r - y)dt + \sigma d\hat{W}$$

$$S(t_i) = S(0)\exp\left\{(r - y - \frac{\sigma^2}{2})t_i + \sigma\hat{W}(t_i)\right\}, t_1, t_2, \ldots, t_n$$

Average is given by

$$A(T) = \frac{S(t_1}{+}S(t_2) + \ldots + S(t_n)n$$

This is a sum of exponentials which is too complicated! Can't obtain a Black-Scholes formula.

Geometric average as an approximation

$$A^{geom} = [S(t_1)S(t_2)\dots S(t_n)]^{\frac{1}{n}}$$

Asian option at date zero priced with Black-Scholes:

$$C(0) = e^{-\alpha T}N(d_1) - ke^{rT}N(d_2)$$

$$\alpha = \frac{1}{2}(y - r + \frac{\sigma^2}{6})$$

$$d_1 = \ln\frac{S(0)}{k} + \frac{1}{2}(r - y + \frac{\sigma^2}{6})\frac{T}{\sigma\sqrt{\frac{T}{3}}}$$

$$d_2 = d_1 - \sigma\sqrt{T}3$$