Advanced Agribusiness Management

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Readings

Agricultural decision analysis and decision theory

- Farmers, Agribusinesses and Policymakers all need to make decisions in an environment of considerable uncertainty.
- Agriculture as a business activity is exposed to uncertain weather,
- uncertain prices,
- threats in the form of diseases and natural disasters
- and on occasion an uncertain policy environment.

Pascal's Wager

Decision theory traces its origins to Pascal's famous wager in a rational being is asked to gamble on by choosing beliefs on whether god exists or not. Pascal points out that beliefs must be chosen and that reason *rationality* fails to guide us as to what beliefs we should have, rather a decision must be made it is necessary to choose and exercise free will. Pascal discusses the decision in terms of choices and consequences, and in terms of probabilities.

The St. Petersburg Paradox

The St Petersburg game involves flipping a coin until it comes up tails

n	$P(n)$	Prize	Expected payoff
1	1/2	\$2	\$ 1
2	1/4	\$4	\$1
3	1/8	\$8	\$1
4	1/16	\$16	\$1
5	1/32	\$32	\$1
6	1/64	\$64	\$1
7	1/128	\$128	\$1
8	1/256	\$256	\$1
9	1/512	\$512	\$1
10	1/1024	\$1024	\$1

Payoff's sum to infinity if one continues to play the game but few people would be prepared to pay to play this this game.

Bernoulli solution: utility

using $u = log(prize)$

n	$P(n)$	Prize	Utility	Expected Utility
1	1/2	$2	0.301	0.1505
2	1/4	$4	0.602	0.1505
3	1/8	$8	0.903	0.1129
4	1/16	$16	1.204	0.0753
5	1/32	$32	1.505	0.0470
6	1/64	$64	1.806	0.0282
7	1/128	$128	2.107	0.0165
8	1/256	$256	2.408	0.0094
9	1/512	$512	2.709	0.0053
10	1/1024	$1024	3.010	0.0029

Expected utility now converges to a constant as n becomes large. Is the paradox solved?

Expected Utility and Expected Monetary Value

$$E(X) = \sum_{k=1}^{K} p_k(X = x)X_k$$

$$E(U(X)) = \sum_{k=1}^{K} p_k(X = x)U(X_k)$$

Savage Utility

$$U(x) = \sum_{k} P(s_k)u(x(s_k))$$

The latter is just a slightly more detailed version of expected utility.

Savage's Theorem

Let $\succeq$ be a weak preference relation on X If $\succeq$ satisfies Savage's axioms, then the following holds:

- The agent's confidence in the actuality of the states in S can be represented by a unique (and finitely additive) probability function, P;
- the strength of her desires for the ultimate outcomes in O can be represented by a utility function, u, that is unique up to positive linear transformation;
- and the pair (P, u)gives rise to an expected utility function, U, that represents her preferences for the alternatives in X; i.e., for any $x, y \in X$

$$x \succeq y, \iff U(x) \geq U(y)$$

Savage decision table

	no rain	rain
stroll without umbrella	very comfortable stroll	miserable wet stroll
stroll with umbrella	comfortable stroll	comfortable stroll
constant act	miserable wet stroll	miserable wet stroll

Comparative Beliefs

Suppose E and F are two events (i.e., subsets of S). Suppose A and$B$ are two outcomes and x and y two acts, with the following properties:

- $x(s_i) = A$ for all $s_i \in E$,
- $x(s_j) = B$ for all $s_j \notin E$,
- $y(s_i) = A$ for all $s_i \in F$,
- $y(s_j) = B$for all $s_j \notin F$,
- $A \succeq B$.

Then $E \succeq F \iff x \succeq y$.

Example

Consider the following situation a farmer intends to plant a high yield but not drought tolerant crop but is unsure whether it will rain or not, his choices are to plant the crop x not to plant the crop but plant a lower yield drought tolerant crop y, the outcomes are good harvest (A), bad harvest (B) . If it rains (E) planting the crop (x) leads to a good harvest, but if it doesn't rain ($s_j \notin E$) then the result is a bad harvest (B). If he plants the drought tolerant crop (y) then he will achieve a good harvest if it doesn't rain but a bad harvest if it does rain. A good harvest is obviously preferred to a bad harvest ($A \succeq B$). From this comparative beliefs let's us conclude that she would rather it rain than not rain if and only if she prefers to plant the high yield crop rather than the lower yielding drought tolerant crop.

Notes on the example

Distinguishing between states and events gives us more flexibility. Events can be compound combinations of states, the power subjective probability lies in its ability to compare things that are difficult to compare from a frequentist (classical) perspective, e.g. statements like it is more likely to rain than for the price of wheat to go up would be incommensurable in classical probability and decision theory but not in Savage's framework.

Ordering

The relation $\succeq$ is complete and transitive. (Ordering)

Sure Thing Principle

If x, y and $x^{'}, y^{'}$ are such that:
x agrees with y and $x^{'}$ agrees with $y^{'}$ in event E^c, x agrees with$x^?$and y agrees with$y^?$ in event E, and $x \succeq y$, then $x^{'} \succeq y^{'}$

Null events

An Event E is null just in case for any alternatives $x, y \in F$, $x \sim y$ given E.

State Neutrality

If f$x(s_i) = A$ and $y(s_i) = B$ whenever $s_i \in E$ and $E$is not null, then$x \succeq y$ given E just in case $A \succeq B$
This means preferences are not affected by whether one know the state or not.

P4

Consider the following acts:

	E	E^c
x	A	$A\prime$
y	Y	$Y\prime$
	F	F^c
$x\prime$	A	$A\prime$
$y\prime$	B	$B\prime$

Now suppose:

$$A \succeq A\prime, B \succeq B\prime, x \succeq x\prime$$

Then

$$y \succeq y\prime$$

In words preferring to bet A when taking action$x$ means that you consider E more probable than F

This will lead us to subjective probability.

P5

There are some $x, y \in F$ such that $x \succ y$.

Subjective probability

$\succeq$ can now be interpreted as a probability relation (ranking probabilities) and can now be represented by a probability function analogous to the way utility functions represent preferences in economics.
A last axiom is needed (analogous to local non-satiation, that is non-atomicity.

Anscombe-Aumann

Review of Kolmogorov Axioms

Cox's Theorem

Reggazzini