Commodity Futures Markets Options

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March 20, 2017

Introduction

Puts and Calls

Trading strategies

Trading strategies

Readings

- H. Geman, Ch.4. Agricultural Commodity Spot Markets, in: *Agricultural Finance*, John Wiley & Sons, 2015.

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Trading strategies

Introduction

- underlying asset: stock, stock index, currency, bond, commodity S.
- Option types: Call C or Put P.
- Buyer (long) or seller (short)
- Strike price: guaranteed purchase or selling price k
- Maturity date T, for European options this is the exercise date.
- Option price or premium $C(0)$ or $P(0)$.

Continuous compounding

Recall

$$FV(m,r,t) = P_0(1+\frac{r}{m})^{mt}$$

we want to show that

$$\lim_{m\to\infty} FV(m,r,t) = P_0 e^{rt}$$

Proof

Define

$$L = \lim_{m\to\infty} P_0(1+\frac{r}{m})^{mt}$$

$$ln(L) = \ln(\lim_{m\to\infty} P_0(1+\frac{r}{m})^{mt})$$

$$= \lim_{m\to\infty} \ln(P_0(1+\frac{r}{m})^{mt})$$

$$= \lim_{m\to\infty} \ln(P_0) + \ln((1+\frac{r}{m})^{mt})$$

$$= \lim_{m\to\infty} \ln(P_0) + mt\ln(1+\frac{r}{m})$$

Proof continued

Use the approximation $lm(1+h) \sim h$ because $\lim_{m\to\infty} \frac{r}{m} = 0$

$$= \lim_{m\to\infty} (\ln(P_0) + mt\frac{r}{m})$$

$$= \lim_{m\to\infty} (\ln(P_0) + rt)$$

$$ln(L) = \ln(P_0) + rt$$

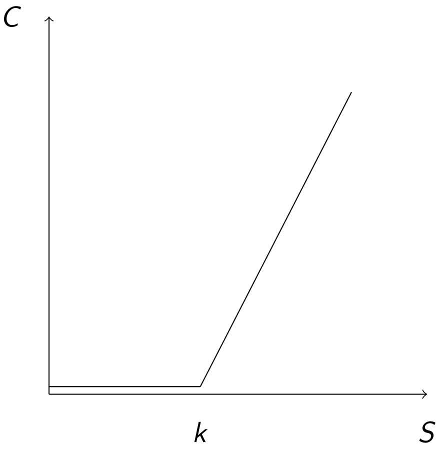
Take anti-logs

$$L = P_0 e^{rt}$$

or

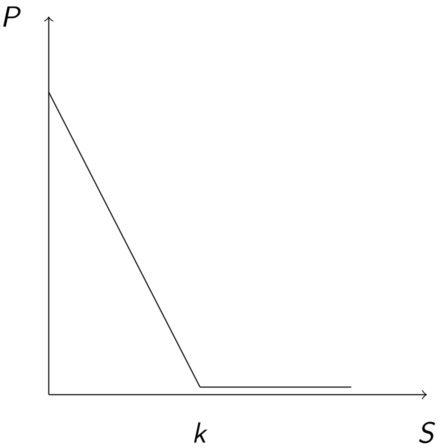
$$\lim_{m\to\infty} FV(r, m, t) = P_0 e^{rt}$$

European Call



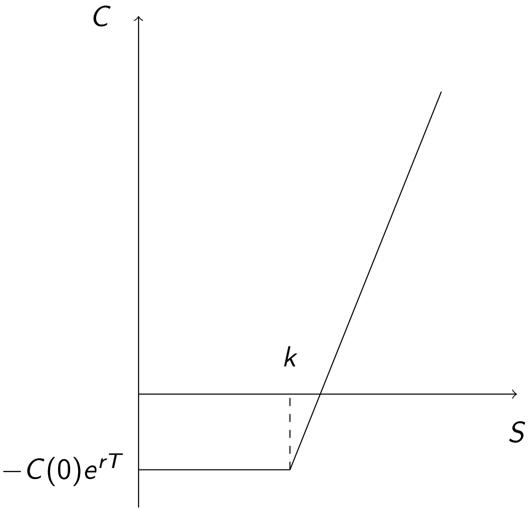
$$C = \max(0, S - k)$$

European Put



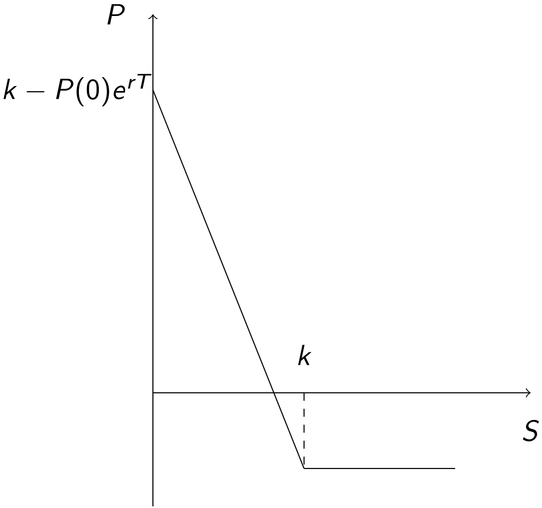
$$P = \max(0, k - S)$$

Long call

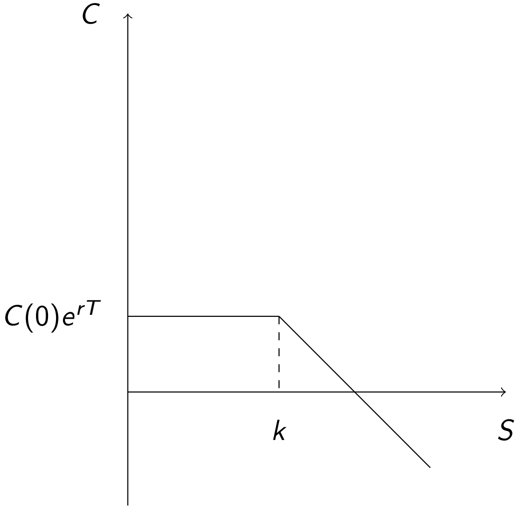


$$C = -C(0)e^{rT} + \max(0, S - k)$$

Long Put

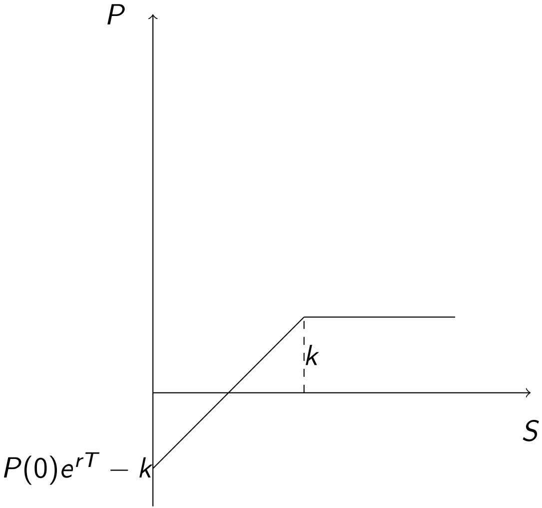


Short call



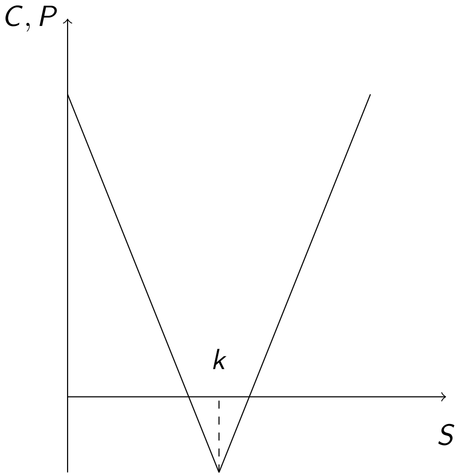
$$C = C(0)e^{rT} + \max(0, S - k)$$

Short Put

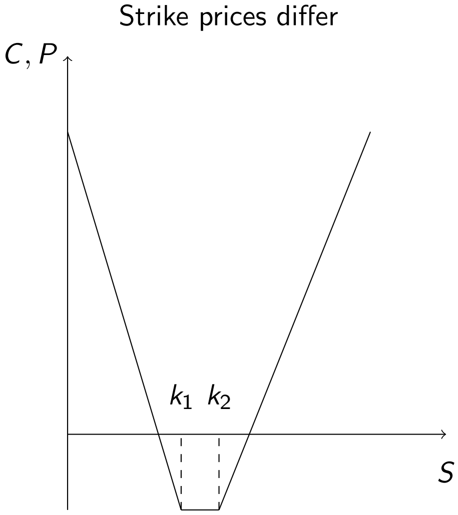


Long Straddle

Simultaneous purchase of a call and a put option



Strangle



Put call parity

$$P(t) + S(t) = C(t) + ke^{r(T-t)}$$

- No taxes, no transaction costs: "frictionless markets"
- the underlying stock pays no dividend over the lifetime of the option
- interest rates r are constant. r is the continuously compounded interest rate
- Zero arbitrage. With zero initial wealth and zero risk at date 0 the final wealth will be zero at date T.

Position over interval (t, T)

Proof of put-call parity:

	t	T	
		$S(T) < k$	$S(T) > k$
buy the stock	$-S(t)$	$S(T)$	$S(T)$
buy the put	$-P(t)$	$k - S(T)$	0
sell the call	$+C(t)$	0	$-(S(T) - k)$
Sum	$-ke^{r(T-t)}$	k	k

The End

Thanks for listening!