Agricultural Marketing and Price Analysis
Lecture 5
Prices over time (commodity futures)

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Background

- Euronext
- CME Group
- Intercontinental exchange (ICE)
- Zhengzhou commodity exchange

- European commodities markets
- Formerly Chicago Board of Trade
- NYSE Commodities
- Chinese Agricultural Commodities Exchange

Euronext latest contracts

	DELIVERY	LAST	$+/-$	VOL	HIGH	LOW	SETTL
CORN FUTURES	Nov-16	159.25	0.5	1,492	159.25	158.50	159.25
MILLING WHEAT FUTURES	Dec-16	162.75	0.25	11,295	163.75	162.50	163.00
RAPESEED FUTURES	Nov-16	393.50	0.75	3,919	395.25	392.00	394.00

Commodities futures exchanges

- Centralized formal market
- Efficient price discovery
- Price risk transfer
- Low transaction costs

ICE

ICE became the center of global trading in soft commodities with its acquisition of the New York Board of Trade in 2007. Now known as ICE Futures U.S., the exchange offers futures and options on futures on soft commodities including coffee, cocoa, sugar, cotton and frozen concentrated orange juice. Sugar No. 11 is the benchmark contract for the global sugar market which is one of the world's ten largest agricultural futures markets. ICE Futures Europe lists London softs markets including cocoa, Robusta coffee and white sugar providing a range of global soft commodity products on the ICE platform. ICE Futures Canada lists the leading canola futures contract, an increasingly popular oilseed.

https://www.theice.com/agriculture

Backwardation and Contango

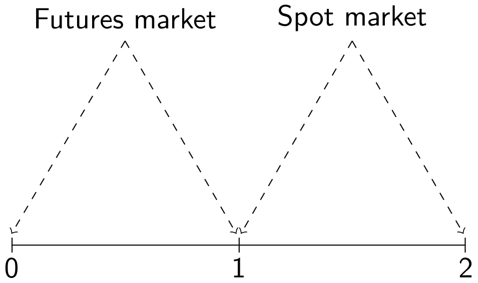
Contango

Futures price of long maturity future contract $>$ Future price of short maturity future contract

Backwardation

Futures price of long maturity future contract $<$ Future price of short maturity future contract

A model of commodity futures



Timing of decisions

Futures contracts at date 0

A date 1 contract
Commodity settlement
at date 1

A date 2 contract
Commodity settlement
at date 2

Futures contracts at date 1

A date 2 contract
Commodity settlement at date 2.
(Date 1 contracts have expired)

Spot and futures markets

Speculators

Speculator trade in the futures market

Producers and consumers

Trade in the spot market

At date 0 commodity inventories are S_0	
At date 1	Commodity $\tilde{Q}_1$ produced stochastically
At date 2	Commodity $\tilde{Q}_2$ produced stochastically

Production values are independent across time.

Demand

$$P = a - bQ$$

Spot prices at date 1

$$\tilde{P}_1 = a - b(S_0 + \tilde{Q}_1 - S)$$

Spot prices at date 2

$$\tilde{P}_2 = a - b(\tilde{Q}_2 + S)$$

S is the amount of the commodity stored from period 1 to 2.

Notation

Description	Symbol	Formula
Date 0		
Price of date 1 contract	f_0^1	$f_0^1 = E_0(\tilde{P}_1)$
Price of date 2 contract	f_0^2	$f_0^2 = E_0(\tilde{P}_2)$
Date 1		
Price of expiring date 1 contract	f_1^1	$f_1^1 = P_1$
Price of date 2 contract	f_0^2	$f_1^2 = E_1(\tilde{P}_2)$
Date 2		
Price of expiring date 2 contract	f_2^2	P_2

Optimal storage at date 1

Harvest in period 1 has just been completed Arbitrage leads to

$$E_1(\tilde{P}_2) = P_1 + m, \;\; S > 0$$

$$E_1(\tilde{P}_2) \leq P_1 + m, \;\; S = 0$$

we assume discounting is priced into m.

Critical value of Q

For $Q_1^* \geq Q_1$ a stockout will occur ($S^* = 0$) and for $Q_1 > Q_1^*$, $S^* > 0$ use this and the LOP condition (previous slide) to derive Q^*.
Set $E_1(\tilde{P}_2) = P_1 + m$ and substitute for P_1 and $\tilde{P}_2$ to get (using the inverse demand expressions):

$$E_1(\tilde{P}_2) = P_1 + m$$

$$a - b(E_1(\tilde{Q}_2) + S_1(Q_1^*)) = a - b(S_0 + Q_1 - S_1(Q_1^*)) + m$$

For the stockout case set $S_1^* = 0$ to get:

$$a - b(E_1(\tilde{Q}_2)) = a - b(S_0 + Q_1) + m$$

$$bQ_1 = bE_1(\tilde{Q}_2) - bS_0 + m$$

$$Q_1^* = E_1(\tilde{Q}_2) - S_0 + \frac{m}{b} = \mu_2 - S_0 + \frac{m}{b}$$

Critical value of Q, no stockout

With no stockout $Q_1 > Q_1^*$, in this case solve for $S^*(Q_1)$:

$$E_1(\tilde{P}_2) = P_1 + m$$

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Critical value of Q, no stockout

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$$a - b(E_1(\tilde{Q}_2) + S_1(Q_1^*)) = a - b(S_0 + Q_1 - S_1(Q_1^*)) + m$$

$$0 = b(E_1(\tilde{Q}_2) + S_1(Q_1^*)) - b(S_0 + Q_1 - S_1(Q_1^*)) + m$$

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$$b(E_1(\tilde{Q}_2) + S_1(Q_1^*) - S_0 - Q_1 + S_1(Q_1^*)) + m = 0$$

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$$b(E_1(\tilde{Q}_2) + 2S_1(Q_1^*) - S_0 - Q_1) + m = 0$$

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$$b(E_1(\tilde{Q}_2) + 2S_1(Q_1^*) - S_0 - Q_1) + m = 0$$

$$2bS_1(Q_1^*) - b(S_0 + Q_1 - E_1(\tilde{Q}_2)) = -m$$

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$$2bS_1(Q_1^*) - b(S_0 + Q_1 - E_1(\tilde{Q}_2)) = -m$$

$$2bS_1(Q_1^*) = b(S_0 + Q_1 - E_1(\tilde{Q}_2)) - m$$

$$S_1(Q_1^*) = \frac{b(S_0 + Q_1 - E_1(\tilde{Q}_2)) - m}{2b} = \frac{b(S_0 + Q_1 - \mu_2 - m)}{2b}$$

Spot prices - date 1

Recall

Spot prices at date 1

$$\tilde{P}_1 = a - b(S_0 + \tilde{Q}_1 - S_1(\tilde{Q}_1))$$

$$P_1(Q_1) = \begin{cases} a - b(S_0 + Q_1) & \text{for} 0 \leq Q_1 \leq Q_1^* \\ a - b(S_0 + Q_1 - S^*(Q_1)) & \text{for } Q_1 > Q_1^* \end{cases}$$

Spot prices - date 2

Spot prices at date 2

$$\tilde{P}_2 = a - b(\tilde{Q}_2 + S_1(\tilde{Q}_1))$$

$$P_2(Q_1, \tilde{Q}_2) = \begin{cases} a - b(Q_2) & \text{for } 0 \leq Q_1 \leq Q_1^* \\ a - b(S^*(Q_1) + \tilde{Q}_2) & \text{for } Q_1 > Q_1^* \end{cases}$$

Expected prices

$$E_0\left\{P_1(\tilde{Q}_1)\right\} = a - b(S_0 + \mu_1) + b\int_{Q_1^*}^{\infty} S^*(Q_1)df(Q_1)dQ_1$$

and

$$E_0(P_2(\tilde{Q}_1, \tilde{Q}_2)) = a - b\mu_2 - b\int_{Q_1^*}^{\infty} S^*(Q_1)f(Q_1)dQ_1$$

$$E_1\left\{P_2(\tilde{Q}_2|Q_1)\right\} = \begin{cases} a - b(\mu_2) & \text{for } 0 < Q_1 < Q_1^* \\ a - b(\mu_2 + S^*(Q_1) & \text{for } Q_1 \geq Q_1^* \end{cases}$$

Summary

- We have looked at commodities exchanges
- Futures markets with standardized contracts
- Price formation in futures markets and storage
- Expected prices
- Next week: Market equilibrium and an application
- Also impact of production uncertainty and convenience yield