

Advanced Agribusiness Management

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SEQUENCING AND SCHEDULING: ALGORITHMS AND
COMPLEXITY E.L. Lawler J.K. Lenstra A.H.G. Rinnooy
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Pinedo, Scheduling Theory

Sequencing and scheduling is concerned with the optimal allocation of scarce resources to activities over time.(Lawler et al.)

- ▶ Crop planning (crop rotation)
- ▶ Harvest scheduling
- ▶ Irrigation scheduling

Deterministic (Machine) Scheduling

- ▶ A *machine* is a resource that can perform at most one activity at any time.
- ▶ Example 1: A tractor cannot be used to plow two fields at the same time.
- ▶ Example 2: The same area of land cannot simultaneously grow corn and soybeans.
- ▶ Activities are referred to as *jobs*.
- ▶ Plowing, cultivating, harvesting, irrigating (look in Farm logs to see what your activity choices are).

- ▶ Suppose m machines have to process n jobs. Then a schedule is an *allocation* of one or more time intervals on one or more machines to one or more jobs.
- ▶ Example: A farmer has four fields, a schedule might be plow the North Field Monday, plant the south field Tuesday and Wednesday and Irrigate the crop on the West field on Thursday spray the East Field on Friday.

A schedule is feasible if

- ▶ no two time intervals allocated to the same machine overlap (You can't plan and harvest the same field at the same time).
- ▶ no two time intervals allocated to the same job overlap (You can't plow the top field and the bottom field at the same time).

- ▶ Not all schedules are optimal
- ▶ We can define criteria that when minimized mean our schedule is optimal according to the criteria

- ▶ Scheduling theory uses a very specific notation known as Graham notation to denote different scheduling problems named after Ronald Graham, mathematician and trampolinist).
- ▶ Assume we have m machines M_i where $i = 1, \dots, m$ and we have n jobs J_j , where $j = 1, \dots, n$. Schedules allocate time intervals on machines to jobs.

- ▶ Graham notation specifies three fields $\alpha|\beta|\gamma$.
- ▶ The α field refers to the machine environment.
- ▶ So for example the number of machines can be constant or variable and there may be a single machine, a number of identical parallel machines, uniform parallel machines in which jobs are processed at constant speed on different machines, or unrelated parallel machines with job dependent processing speeds.

A farm may have a tractor and a plow but harvesting might be contracted out, so that the harvester is not always available. This would involve a variable number of machines.

The β field refers to the job characteristics. So for example whether jobs can be preempted (interrupted and resumed later), precedence relations (whether some jobs have to be processed before others), release dates and processing requirements or times.

γ

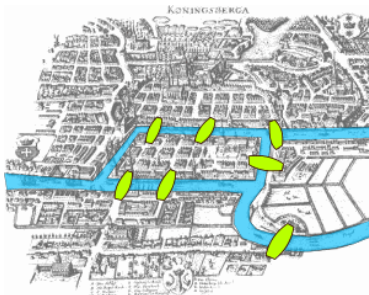
The γ field represents an optimality criteria that defines the particular objective function to be minimized.

- ▶ Graham notation allows us to systematically classify scheduling problems and formulate them in a compact way.
- ▶ If we consider the α field, $\alpha = \alpha_1\alpha_2$ where $\alpha_1 \in \{., P, Q, R\}$ where P represents identical parallel machines, Q represents uniform parallel machines and R unrelated parallel machines. α_2 can be a positive integer m , the number of machines or it can be . which means that the number of machines is variable.
- ▶ $\beta \in \{\beta_1, \dots, \beta_n\}$ where β_1 can take on the values pmin which allows preemption or . which indicates no preemption. Preemption means a job can be interrupted and restarted later.
- ▶ $\beta_2 \in \{prec, tree, .\}$. Where prec is a precedence relation. What is a precedence relation?

The Königsberg Bridge Problem

To understand precedence relations we need to introduce some graph theory. A graph G is an object made up of vertices (v_1, \dots, v_n) and edges (e_1, \dots, e_n) that link the vertices. Graph theory has its beginning with the Königsberg bridge problem. Königsberg was the capital of the former German province of East Prussia and is now known as Kaliningrad and is part of the Russian Baltic exclave. A stylized map of Königsberg is depicted below.

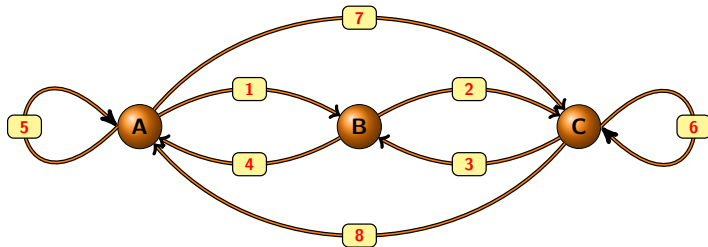
The Königsberg Bridges



The river Pregel is depicted in blue and the seven bridges are in green. The K bridge problem formulated by Euler was to ask whether it is possible to walk through the city by crossing each bridge only once? This problem gave birth to graph theory.

It is easy to see that the problem is a scheduling problem concerned with the sequence in which the bridges should be crossed.

Graphs can be used to define precedence relations.



$$\gamma \in \left\{ f_{\max}, \sum f_j \right\}$$

- ▶ the *completion time* of the j -th job C_j
- ▶ the *Lateness* $L_j = C_j - d_j$
- ▶ *Tardiness* $T_j = \max\{0, C_j - d_j\}$
- ▶ unit penalty $U_j = 0$, if $C_j \leq d_j$, $U_j = 1$ otherwise.

Example of common optimality criteria:

Minimize

$$f_{\max} \in \{C_{\max}, L_{\max}\}$$

For example we may want to minimize the completion time of the harvest. Or we may have a delivery deadline and want to minimize how late our delivery is. Scheduling theory lets us formulate all these problems formally.

$$1|prec|L_{\max}$$

Describes the problem of minimizing maximum lateness on a single machine with precedence constraints (easy problem-polynomial time solution).

- ▶ Single machine 1
- ▶ Identical parallel machines P_m
- ▶ Parallel machines with different speeds Q_m
- ▶ Unrelated parallel machines R_m
- ▶ Flow shop, machines in series F_m
- ▶ Flexible Flow shop (FFc)
- ▶ Job shop shop each job has a predetermined route to follow e.g. Harvest the first field, then the second field then the third. Plow the fourth, fifth and sixth fields.

Depending on factory layout different machine environments may exist, think of fruit canning factories or different types of food processing facilities and how they might be organized.

These depict time allocations to jobs and resources. Used in project management but also in operations management for scheduling.

HENRY LAURENCE GANTT (1861? 1919) Henry Laurence Gantt was an industrial engineer and a disciple of Frederick W.Taylor. He developed his now famous charts during World War I to compare production schedules with their realisations. Gantt discussed the underlying principles in his paper "Efficiency and Democracy, " which he presented at the annual meeting of the American Society of Mechanical Engineers in 1918. The Gantt charts currently in use are typically a simplification of the originals both in purpose and in design.

Gantt charts

