# EXAMPLE: TWO COUNTRY LICENSING MODEL

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## 1. Introduction

Consider a world with two coastal nations and a distant water fishing fleet that is split between the EEZ's and the high seas.

Our problem is to maximize the rent for the domestic fleets plus the lciense fee revenue they receive from the distant water fleet. The distant water fleets problem is to distribute it's effort between the two EEZ's and the high seas while competing for fish with the two doemstic fleets, in such a way as to maximize it's rent.

## 2. Simplified model

$$\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - qx(t)E(t) \tag{1}$$

Typically one assumes that the biological model consists of a simple mass-balance relationship in which

$$\text{Stock change} = \text{Growth} - \text{Catch} \tag{2}$$

If growth exceed the catch, then the stock will grow, if the catch exceeds growth the stock will decline.

$$\text{Growth} > \text{Catch} \implies \text{stock increases} \tag{3}$$

In this chapter we will assume that the growth of the stock and catch balance exactly, so that the stock remains in equilibrium (steady-state).

$$\text{Growth} = \text{Catch} \implies \text{steady-state} \tag{4}$$

Assume that growth $F(x) = rx(t)(1 - \frac{x(t)}{K})$ and that catch $h(x) = qx(t)E(t)$ then in steady-state we have

$$F(x) = h(x) \implies rx(t)(1 - \frac{x(t)}{K}) = qx(t)E(t) \tag{5}$$

Cancelling the $x(t)$ on both sides we get

$$r(1 - \frac{x(t)}{K}) = qE(t) \tag{6}$$

and rearranging we obtain

$$x(t) = K(1 - \frac{q}{r}E) \tag{7}$$

This is the steady state level of the stock as a function of fishing effort. We can suppress the dependency on $t$ because in steady-state time no longer influences the solution, the solution is "timeless". From this we can see that as fishing effort increases the stock goes down. We can do this using SymPy in Python

**Example 2.1.**

```
import math as math
from sympy import *
import numpy as np

P, p, q, c_D, c_F, r, K, f_D, f_F, e_D, e_F, X
= symbols('P p q c_D c_F  r K f_D f_F e_D e_F X')

eqn = r*X*(1 - X/K) - q*X*e_D - q*X*e_F
simplify(eqn)

ss = solve(eqn,X)

print("Steady-state value of X:", ss[1])
```

☞**Click to run the code**

The rent for the two fleets is given by

$$\Pi_D = ph_D(e_D, e_f) - c_D e_D - f_D e_D$$

$$\Pi_F = ph_F(e_D, e_f) - c_F e_F - f_F e_F$$

where $h_D(e_D, e_F) = qXe_D$ and $h_F(e_D, e_F) = qXe_F$. In steady-state these become

$$h_D(e_D, e_F) = qXe_D = qK(1 - \frac{q}{r}(e_D + e_F))e_D$$

$$h_F(e_D, e_F) = qXe_F = qK(1 - \frac{q}{r}(e_D + e_F))e_F$$

substituting these into the rent functions gives

$$\Pi_D = pqK(1 - \frac{q}{r}(e_D + e_F))e_D) - c_D e_D - f_D e_D$$

$$\Pi_F = pqK(1 - \frac{q}{r}(e_D + e_F))e_F - c_F e_F - f_F e_F$$

This pair of equations defines a two player game. The Python code for this is given below

**Example 2.2.**

```
rent_F = p*q*X*e_F - c_F*e_F - f_F*e_F
print(rent_F)

rent_D = p*q*X*e_D - c_D*e_D - f_D*e_D
print(rent_D)
```

To solve this game. We maximize each of these functions by choosing the corresponding level of fishing effort.

The first -order conditions are

$$-\frac{Ke_F pq^2}{r} + \frac{-Kpq(e_D q + e_F q - r) + r(-c_F - f_F)}{r} = 0$$
$$-\frac{Ke_D pq^2}{r} + \frac{-Kpq(e_D q + e_F q - r) + r(-c_D - f_D)}{r} = 0$$

Using Python we can simplify the rent equations and then derive the fist-order conditions for a rent maximum as follows:

**Example 2.3.**

```
rent_Fss = simplify(rent_F.subs(X, ss[1]))
print(rent_Fss)

rent_Dss = simplify(rent_D.subs(X, ss[1]))
print(rent_Dss)


foc1 = diff(rent_Fss, e_F)
print(foc1)

foc2 = diff(rent_Dss, e_D)
print(foc2)
```

Solving these for $e_D$ and $e_F$ we get

$$e_D^* = \frac{Kpqr - 2c_D r + c_F r - 2f_D r + f_F r}{3Kpq^2}$$
$$e_F^* = \frac{Kpqr + c_D r - 2c_F r + f_D r - 2f_F r}{3Kpq^2}$$

**Example 2.4.**

```
sol = solve([foc1, foc2], (e_F, e_D), dict=True)

print("Foreign demand:", sol[0][e_F])
```

```
print("Domestic demand:", sol[0][e_D])
```

These are the demand functions for domestic and foreign licenses respectively. Notice that the sign on the domestic license fee is negative implying a downward sloping demand curve for domestic licenses also the demand function is linear in the license fee. However, the demand function also captures a substitution effect. An increase in the foreign license fee will increase the demand for domestic licenses and vice versa. So if one attempts to increase revenue by increasing the license fee you are likely to see some substitution towards domestic vessels in the absence of outside options(other EEZ's and the high seas). To assess leakage to other EEZs and the high seas we would need a regional model.

At this point if the question of interest is simply how many licenses to issue, then we are done. License fees are often set in legislation or by government regulations or other policy instruments. If there is no intention to review or change fees then substituting the existing license fees into the above formula will give us the demand for licenses, noting that this would be typically measured in something like vessel days, or possibly number of vessels. If effort is measured in hooks then the fee basis is not likely to be based on hooks so hooks needs to be converted to vessels, so we need to take the average number of hooks set per vessel and divide the effort by the average number of hooks per vessel. So if $e_D$ and $e_F$ are measured in hooks, we need to divide the fees $f_D$ and $f_F$ by the average number of hooks per vessel. This converts it to a fee per hook. Sometimes we do this in terms of hundred hooks.

If however we intend reviewing the license fees themselves, then the benchmark should be the optimal license fee. In this case we ned to define a social welfare function in terms of license fees and maximize it. Note that if the fee is known already then that predetermines the demand and the number of licenses then issued may not be optimal or efficient. Likewise if the demand is known and one wishes to set a fee, then one would have solved the game above for the fees rather than for effort and would just plug-in the demand to find the fee, simultaneously setting both a fee and the number of licenses to issue will not be efficient unless consideration is given to what the social objectives are and how these are to be captured in a social welfare function.

## 3. Social planner's problem

The social planner wishes to maximize social welfare (socio-economic benefits). We will consider two cases: i) assume social welfare is given by the sum of the domestic fleets rent net of fees and license fee revenue, ii) assume social welfare is given by domestic rent gross of fees plus license fee revenue.

### 3.1. **Case I.**

$$\max_{f_D, f_F} W = pqK(1 - \frac{q}{r}(e_D^* + e_F^*))e_D^* - c_D e_D^* - f_D e_D^* + e_D^* f_D + e_F^* f_F$$

Note that domestic license fee revenue nets out.

$$W = pqK(1 - \frac{q}{r}(e_D^* + e_F^*))e_D^* - c_D e_D^* + e_F^* f_F$$

$$= .pqK(e_D^* - pK\frac{q^2}{r}((e_D^*)^2 + e_F^* e_D^*)) - c_D e_D^* + e_F^* f_F$$

The first-order conditions are given by

$$\frac{\partial W}{\partial f_D} = \frac{r(-Kpq + 2c_D - c_F - 4f_D + 2f_F)}{9Kpq^2} = 0$$

$$\frac{\partial W}{\partial f_F} = \frac{r(5Kpq - c_D - 4c_F + 2f_D - 10f_F)}{9Kpq^2} = 0$$

Now solve these for $f_D, f_F$ to obtain

$$f_D^* = \frac{c_D - c_F}{2}$$

$$f_F^* = pq\frac{K}{2} - \frac{c_F}{2}$$

Recall from the notes on the Gordon-Schaefer model that $X_{MSY} = \frac{K}{2}$. So the fee for foreign vessels should be set to the value of CPUE at MSY minus half the cost of fishing.

In the absence of a domestic fleet the social planner maximizes license fee revenue. I will leave this as an exercise.

### 3.2. **Case II.**

$$\max_{f_D, f_F} W = pqK(1 - \frac{q}{r}(e_D^* + e_F^*))e_D^* - c_D e_D^* + e_D^* f_D + e_F^* f_F$$

the first-order conditions are given by

$$\frac{\partial W}{\partial f_D} = \frac{r(2Kpq - 4c_D + 2c_F - 16f_D + 5f_F)}{9Kpq^2} = 0$$

$$\frac{\partial W}{\partial f_F} = \frac{r(5Kpq - c_D - 4c_F + 5f_D - 10f_F)/}{9Kpq^2} = 0$$

Solving these for $F_D, f_F$ results in

$$f_D^* = pq\frac{K}{3} - \frac{c_D}{3}$$

$$f_F^* = pq\frac{2}{3}K - \frac{4}{15}c_D - \frac{2}{5}c_F$$

In the former case the fee is set relative to a reference point below MSY and in the latter case relative to a reference point above MSY. The result is a lower doemstic fee is set relative to case I and a higher foreign fee is set relative to case I.

It should be noted that this approach results in higher socio-economic benefits than both managing to MEY and to case I.

## References

[1] Andersen, P. (1983). On Rent of Fishing Grounds: a translation of Jens Warming's 1911 Article, with an introduction. *History of Political Economy*, 15(3), 391-396.

[2] Anderson, L.G.. *The Economics of Fisheries Management.* Revised and enlarged edition.the John Hopkins University Press, 2004.

[3] Conrad, J., *Resource Economics*, Cambridge University Press; 2 edition (June 14, 2010).

[4] Fisher, A.C.. *Lecture notes on resource and environmental economics*, Springer 1st Edition, 2020.

[5] Gordon, H. S. (1954). The Economic Theory of a Common-Property Resource: The Fishery. Journal of Political Economy, 62(2), 124-142.

[6] Hannesson,R. *The economics of fishing*, Agenda Publishing, 1st edition (January 21, 2021).

[7] Karp, L., *Natural Resources as Capital*, The MIT Press (November 17, 2017).

[8] Schaefer, Milner B. Some aspects of the dynamics of populations important to the management of the commercial marine fisheries. *Inter-American Tropical Tuna Commission Bulletin* 1.2 (1954): 23-56.

[9] Warming, J. 1911. Om Grundrente af Fiskgrunde.*Nationaløkonomisk Tidsskrift* 49:499–505.