

The model

- We have n exclusive economic zones plus the high seas.
- We have a domestic fishing fleet in each zone that only fishes within the zone (This is a simplification but also true of many small island developing states)
- We have a single distant water fishing fleet (a simplification but generally there is no price discrimination in license fees, access fees are different and sometimes negotiated bilaterally)
- There is a single shared stock (This makes the problem game theoretic in nature)
- Fleets seek to maximize rents net of license fees
- Governments seek to maximize domestic rents plus license fee revenues (Sometimes there is interest in also adding on-shore benefits to national objectives, but I do not model that here)

Formalizing the problem

Single stock that is exploited by n domestic fleets but by a single distant water fleet spread across the n EEZ's and the high seas.

$$\frac{dX}{dt} = F(X) - \sum_{i=1}^n h_{i,D} - \sum_{i=1}^{n+1} h_{i,F}$$

with $h = qXE$ and $F(X) = rX(1 - \frac{X}{K})$.

Model is basically an extension of the static Gordon-Schaefer fisheries model.

The Agents' decision problems

Domestic fleets maximize the following static optimization problem

$$\Pi_{i,D} = pqX(E)e_{i,D} - c_De_{i,D} - f_{i,D}e_{i,D}, i = 1, \dots, n$$

where $E = \sum_{i=1}^n e_{i,D} + \sum_{i=1}^{n+1} e_{i,F}$

Foreign fleets maximize

$$\begin{aligned} \max_{\{e_{i,F}\}_{i=1}^{n+1}} \Pi_F &= \sum_{i=1}^{n+1} pqX e_{i,F} - c_F e_{i,F} - f_{i,F} e_{i,F} \\ &= \sum_{i=1}^{n+1} pqK \left(1 - \frac{q}{r}(E)\right) e_{i,F} \\ &\quad - c_F e_{i,F} - f_{i,F} e_{i,F} \end{aligned}$$

where $f_{n+1,F} = 0$.

First-order conditons: Domestic

$$\begin{aligned}\frac{\partial \Pi}{\partial e_{i,D}} &= pq \frac{\partial X}{\partial E} \frac{\partial E}{\partial e_{i,D}} e_{i,D} + pqX(E) - c_D - f_{i,D} = 0, i = 1, \dots, n \\ &= pqK \frac{-q}{r}(1)e_{i,D} + pqX(E) - c_D - f_{i,D} = 0 \\ &\quad -pK \frac{q^2}{r} e_{i,D} + pqX(E) - c_D - f_{i,D} = 0 \\ e_{i,D} &= \frac{pqX(E) - c_D - f_{i,D}}{pK \frac{q^2}{r}}\end{aligned}$$

So

$$s_{i,D} = \frac{e_{i,D}}{E} = \frac{pqX(E) - c_D - f_{i,D}}{pKE \frac{q^2}{r}}$$

First-order conditions: Foreign

$$\begin{aligned}\frac{\partial \Pi}{\partial e_{i,F}} &= pq \frac{\partial X}{\partial E} \frac{\partial E}{\partial e_{i,F}} e_{i,F} + pqX(E) - c_F - f_{i,F} = 0, i = 1, \dots, n \\ &= pqK \frac{-q}{r}(1)e_{i,F} + pqX(E) - c_F - f_{i,F} = 0 \\ &\quad -pK \frac{q^2}{r} e_{i,F} + pqX(E) - c_F - f_{i,F} = 0 \\ e_{i,D} &= \frac{pqX(E) - c_F - f_{i,F}}{pK \frac{q^2}{r}}\end{aligned}$$

So

$$s_{i,F} = \frac{e_{i,F}}{E} = \frac{pqX(E) - c_F - f_{i,F}}{pKE \frac{q^2}{r}}$$

First-order conditions: High Seas

$$\begin{aligned}\frac{\partial \Pi}{\partial e_{n+1,F}} &= pq \frac{\partial X}{\partial E} \frac{\partial E}{\partial e_{n+1,F}} e_{n+1,F} + pqX(E) - c_F - f_{n+1,F} = 0 \\ &= pqK \frac{-q}{r}(1)e_{n+1,F} + pqX(E) - c_F - f_{n+1,F} = 0 \\ &\quad - pK \frac{q^2}{r} e_{n+1,F} + pqX(E) - c_F - f_{n+1,F} = 0 \\ e_{n+1,D} &= \frac{pqX(E) - c_F - f_{n+1,F}}{pK \frac{q^2}{r}}\end{aligned}$$

So

$$s_{n+1,F} = \frac{e_{n+1,F}}{E} = \frac{pqX(E) - c_F - f_{n+1,F}}{pKE \frac{q^2}{r}}$$

Solution method

The game is an aggregative game, so one could try to solve it via the replacement function method, in practice using share functions is sometimes simpler. I use share functions and solve for the aggregate level of fishing effort and then solve for distant water fleets optimal effort level. See Cornes, R., & Hartley, R. (2007). Aggregative public good games. *Journal of Public Economic Theory*, 9(2), 201-219 and related literature on aggregative games. Also my paper on oligopoly Beard, R. (2015). N-Firm Oligopoly with General Iso-Elastic Demand. *Bulletin of Economic Research*, 67(4), 336-345 uses a similar approach.

Share functions

$$\sum_{i=1}^n s_{i,D} + \sum_{i=1}^n s_{i,F} + s_{n+1,F} = 1$$

so

$$\begin{aligned} \sum_{i=1}^n \frac{pqX(E) - c_D - f_{i,D}}{pKE \frac{q^2}{r}} + \sum_{i=1}^n \frac{pqX(E) - c_F - f_{i,F}}{pKE \frac{q^2}{r}} \\ + \frac{pqX(E) - c_F}{pKE \frac{q^2}{r}} = 1 \end{aligned}$$

Solving for E^*

$$\sum_{i=1}^n \frac{pqX(E) - c_D - f_{i,D}}{pKE \frac{q^2}{r}} + \sum_{i=1}^n \frac{pqX(E) - c_F - f_{i,F}}{pKE \frac{q^2}{r}} + \frac{pqX(E) - c_F}{pKE \frac{q^2}{r}} = 1$$

$$npqX(E) - nc_D - \sum f_{i,D} + npqX(E) - nc_F - \sum f_{i,F} + pqX(E) - c_F = pK \frac{q^2}{r} E$$

Solving for E^* continued

$$\begin{aligned} npqK(1 - \frac{q}{r}E) - nc_D - \sum f_{i,D} + npqK(1 - \frac{q}{r}E) - nc_F - \sum f_{i,F} \\ + pqK(1 - \frac{q}{r}E) - c_F = pK\frac{q^2}{r}E \end{aligned}$$

\implies

$$(2n+1)pqK - c_F - nc_D - \sum f_{i,D} - nc_F - \sum f_{i,F} = 2(n+1)pK\frac{q^2}{r}E$$

\implies

$$\frac{r((2n+1)pqK - nc_D - (n+1)c_F - \sum(f_{i,D} + f_{i,F}))}{2(n+1)pKq^2} = E^*$$

Nash equilibrium

$$e_{i,D}^* = \frac{r(pqK(1 - \frac{q}{r}E^*) - c_D - f_{i,D})}{pKq^2}, i = 1, \dots, n$$

$$e_{i,F}^* = \frac{r(pqK[1 - \frac{q}{r}E^*] - c_F - f_{i,F})}{pKq^2}, i = 1, \dots, n$$

As the optimal in-zone effort. This can be interpreted as the demand for fishing licenses.

and

$$e_{n+1,F}^* = \frac{r(pqK[1 - \frac{q}{r}E^*] - c_F)}{pq^2K}$$

is the high seas effort of distant water fishing nations.

With

$$E^* = \frac{r((2n+1)pqK - nc_D - (n+1)c_F - \sum(f_{i,D} + f_{i,F}))}{2(n+1)pq^2K}$$