The model

- We have *n* exclusive economic zones plus the high seas.
- We have a domestic fishing fleet in each zone that only fishes within the zone (This is a simplification but also true of many small island developing states)
- We have a single distant water fishing fleet (a simplification but generally there is no price discrimination in license fees, access fees are different and sometimes negotiated bilaterally
- There is a single shared stock (This makes the problem game theoretic in nature)
- Fleets seek to maximize rents net of license fees
- Governments seek to maximize domestic rents plus license fee revenues (Sometimes there is interest in also adding on-shore benefits to national objectives, but I do not model that here)

Formalizing the problem

Single stock that is exploited by n domestic fleets but by a single distant water fleet spread across the n EEZ's and the high seas.

$$\frac{dX}{dt} = F(X) - \sum_{i=1}^{n} h_{i,D} - \sum_{i=1}^{n+1} h_{i,F}$$

with $h = qXE$ and $F(X) = rX(1 - \frac{X}{K})$.
Model is basically an extension of the static Gordon-Schaefer fisheries model.

The Agents' decision problems

Domestic fleets maximize the following static optimization problem

$$\Pi_{i,D} = pqX(E)e_{i,D} - c_D e_{i,D} - f_{i,D} e_{i,D}, i = 1, \dots, n$$

where $E = \sum_{i=1}^{n} e_{i,D} + \sum_{i=1}^{n+1} e_{i,F}$

Foreign fleets maximize

$$\begin{aligned}\max_{\{e_{i,F}\}_{i=1}^{n+1}} \Pi_F &= \sum_{i=1}^{n+1} pqXe_{i,F} - c_F e_{i,F} - f_{i,F} e_{i,F} \\ &= \sum_{i=1}^{n+1} pqK(1 - \frac{q}{r}(E))e_{i,F} \\ &\quad - c_F e_{i,F} - f_{i,F} e_{i,F}\end{aligned}$$

where $f_{n+1,F} = 0$.

First-order conditons: Domestic

$$
\begin{aligned}
\frac{\partial \Pi}{\partial e_{i,D}} &= pq\frac{\partial X}{\partial E}\frac{\partial E}{\partial e_{i,D}}e_{i,D} + pqX(E) - c_D - f_{i,D} = 0, i = 1, \dots, n \\
&= pqK\frac{-q}{r}(1)e_{i,D} + pqX(E) - c_D - f_{i,D} = 0 \\
&-pK\frac{q^2}{r}e_{i,D} + pqX(E) - c_D - f_{i,D} = 0 \\
&e_{i,D} = \frac{pqX(E) - c_D - f_{i,D}}{pK\frac{q^2}{r}}
\end{aligned}
$$

So

$$
s_{i,D} = \frac{e_{i,D}}{E} = \frac{pqX(E) - c_D - f_{i,D}}{pKE\frac{q^2}{r}}
$$

First-order conditions; Foreign

$$\frac{\partial \Pi}{\partial e_{i,F}} = pq\frac{\partial X}{\partial E}\frac{\partial E}{\partial e_{i,F}}e_{i,F} + pqX(E) - c_F - f_{i,F} = 0, i = 1, \ldots, n$$

$$= pqK\frac{-q}{r}(1)e_{i,F} + pqX(E) - c_F - f_{i,F} = 0$$

$$-pK\frac{q^2}{r}e_{i,F} + pqX(E) - c_F - f_{i,F} = 0$$

$$e_{i,D} = \frac{pqX(E) - c_F - f_{i,F}}{pK\frac{q^2}{r}}$$

So

$$s_{i,F} = \frac{e_{i,F}}{E} = \frac{pqX(E) - c_F - f_{i,F}}{pKE\frac{q^2}{r}}$$

First-order conditions; High Seas

$$\frac{\partial \Pi}{\partial e_{n+1,F}} = pq\frac{\partial X}{\partial E}\frac{\partial E}{\partial e_{n+1,F}}e_{n+1,F} + pqX(E) - c_F - f_{n+1,F} = 0$$

$$= pqK\frac{-q}{r}(1)e_{n+1,F} + pqX(E) - c_F - f_{n+1,F} = 0$$

$$-pK\frac{q^2}{r}e_{n+1,F} + pqX(E) - c_F - f_{n+1,F} = 0$$

$$e_{n+1,D} = \frac{pqX(E) - c_F - f_{n+1,F}}{pK\frac{q^2}{r}}$$

So

$$s_{n+1,F} = \frac{e_{n+1,F}}{E} = \frac{pqX(E) - c_F - f_{n+1,F}}{pKE\frac{q^2}{r}}$$

Solution method

The game is an aggregative game, so one could try to solve it via the replacement function method, in practice using share functions is sometimes simpler. I use share functions and solve for the aggregate level of fishing effort and then solve for distant water fleets optimal effort level. See Cornes, R., & Hartley, R. (2007). Aggregative public good games. *Journal of Public Economic Theory*, 9(2), 201-219 and related literature on aggregative games. Also my paper on oligopoly Beard, R. (2015). N-Firm Oligopoly with General Iso-Elastic Demand. *Bulletin of Economic Research*, 67(4), 336-345 uses a similar approach.

Share functions

$$\sum_{i=1}^{n} s_{i,D} + \sum_{i=1}^{n} s_{i,F} + s_{n+1,F} = 1$$

so

$$\sum_{i=1}^{n} \frac{pqX(E) - c_D - f_{i,D}}{pKE\frac{q^2}{r}} + \sum_{i=1}^{n} \frac{pqX(E) - c_F - f_{i,F}}{pKE\frac{q^2}{r}}$$
$$+ \frac{pqX(E) - c_F}{pKE\frac{q^2}{r}} = 1$$

Solving for E^*

$$\sum_{i=1}^{n} \frac{pqX(E) - c_D - f_{i,D}}{pKE\frac{q^2}{r}} + \sum_{i=1}^{n} \frac{pqX(E) - c_F - f_{i,F}}{pKE\frac{q^2}{r}}$$

$$+\frac{pqX(E) - c_F}{pKE\frac{q^2}{r}} = 1$$

$$npqX(E) - nc_D - \sum f_{i,D} + npqX(E) - nc_F - \sum f_{i,F}$$

$$+pqX(E) - c_F = pK\frac{q^2}{r}E$$

Solving for E^* continued

$$npqK(1-\frac{q}{r}E) - nc_D - \sum f_{i,D} + npqK(1-\frac{q}{r}E) - nc_F - \sum f_{i,F}$$
$$+pqK(1-\frac{q}{r}E) - c_F = pK\frac{q^2}{r}E$$

$$\Longrightarrow$$

$$(2n+1)pqK - c_F - nc_D - \sum f_{i,D} - nc_F - \sum f_{i,F} = 2(n+1)pK\frac{q^2}{r}E$$

$$\Longrightarrow$$

$$\frac{r((2n+1)pqK - nc_D - (n+1)c_F - \sum(f_{i,D}+f_{i,F}))}{2(n+1)pKq^2} = E^*$$

Nash equilibrium

$$e^*_{i,D} = \frac{r(pqK(1 - \frac{q}{r}E^*) - c_D - f_{i,D})}{pKq^2}, i = 1, \ldots, n$$

$$e^*_{i,F} = \frac{r(pqK[1 - \frac{q}{r}E^*] - c_F - f_{i,F})}{pKq^2}, i = 1, \ldots, n$$

As the optimal in-zone effort. This can be interpreted as the demand for fishing licenses.
and

$$e^*_{n+1,F} = \frac{r(pqK[1 - \frac{q}{r}E^*] - c_F)}{pq^2K}$$

is the high seas effort of distant water fishing nations.
With

$$E^* = \frac{r((2n+1)pqK - nc_D - (n+1)c_F - \sum(f_{i,D} + f_{i,F}))}{2(n+1)pq^2K}$$