ML course
Lecture 08:
Neural Networks basics

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girafe
ai

Outline

1. Neural Networks in different areas. Historical overview.
2. Backpropagation.
3. More on backpropagation.
4. Activation functions.
5. Playground.

History of Deep Learning

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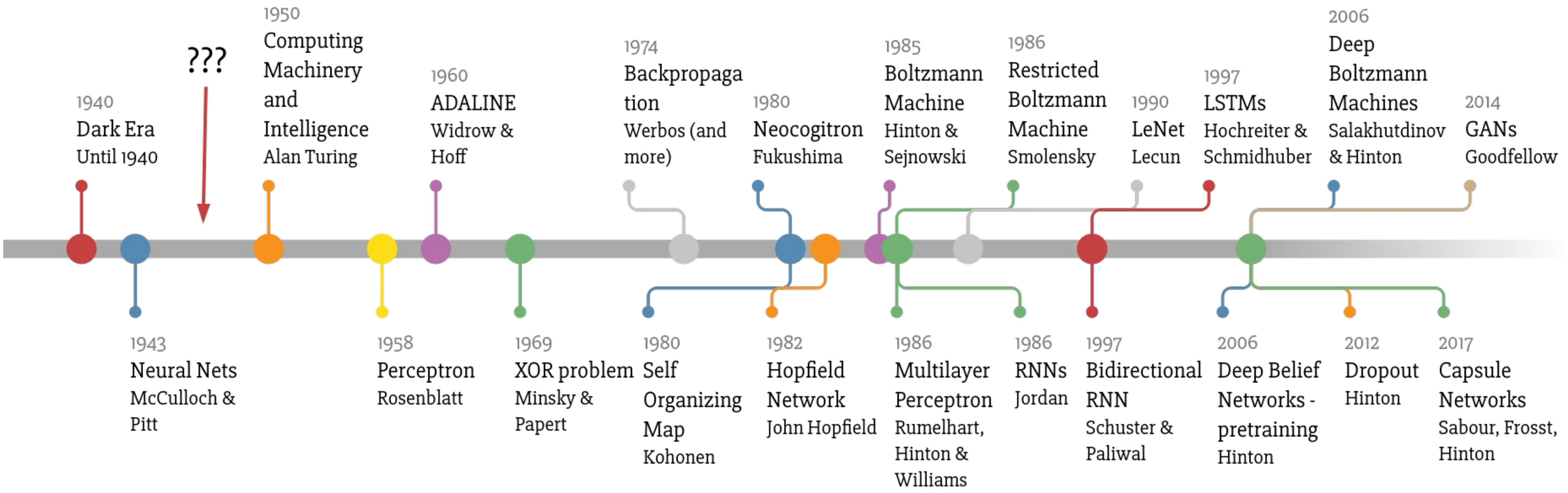
Deep Learning Timeline



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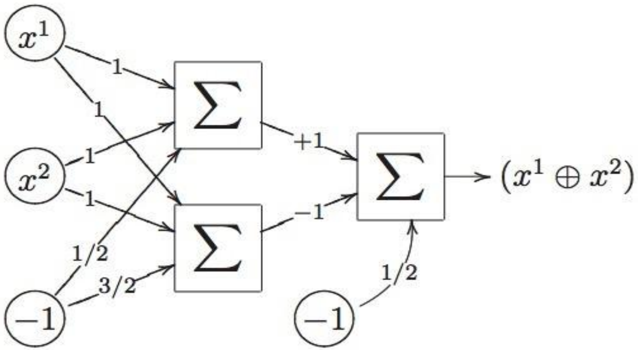
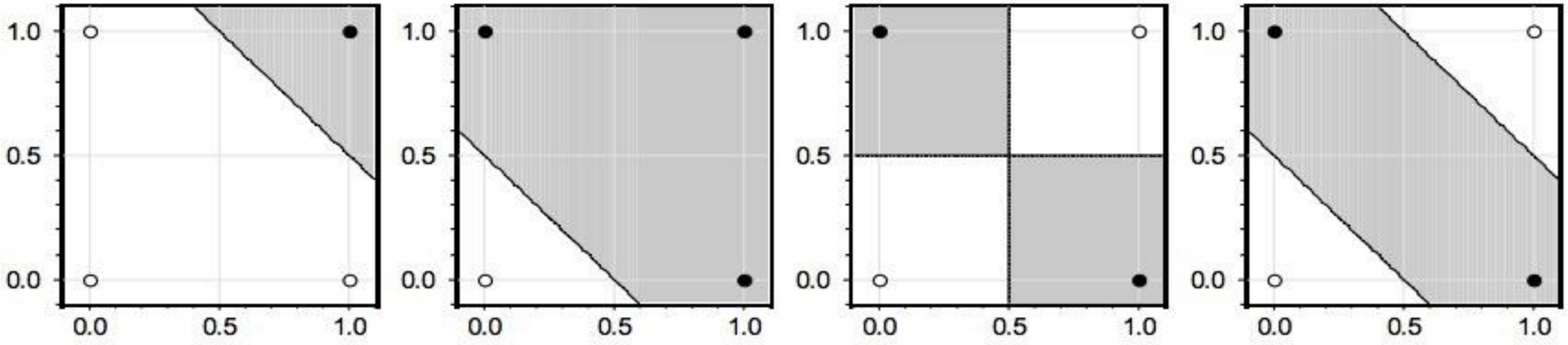
XOR problem



This 2-layer NN (on the left) implements XOR with only x1 and x2 features.

1-layer NN also can succeed, but only with extra feature x1*x2.



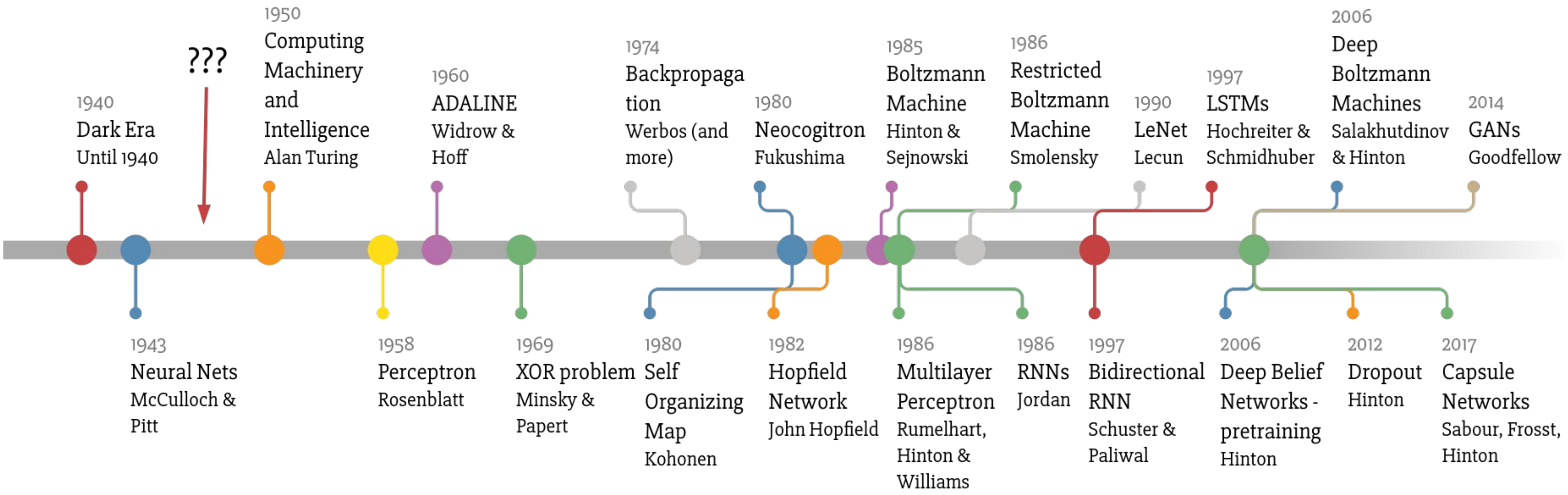
AND

OR

XOR(with x1*x2)

XOR

Deep Learning Timeline



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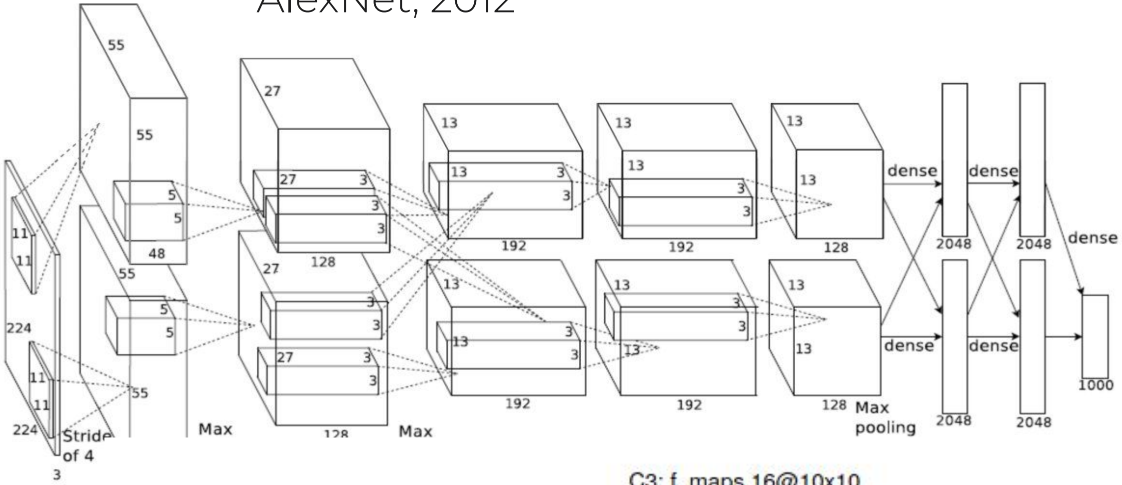
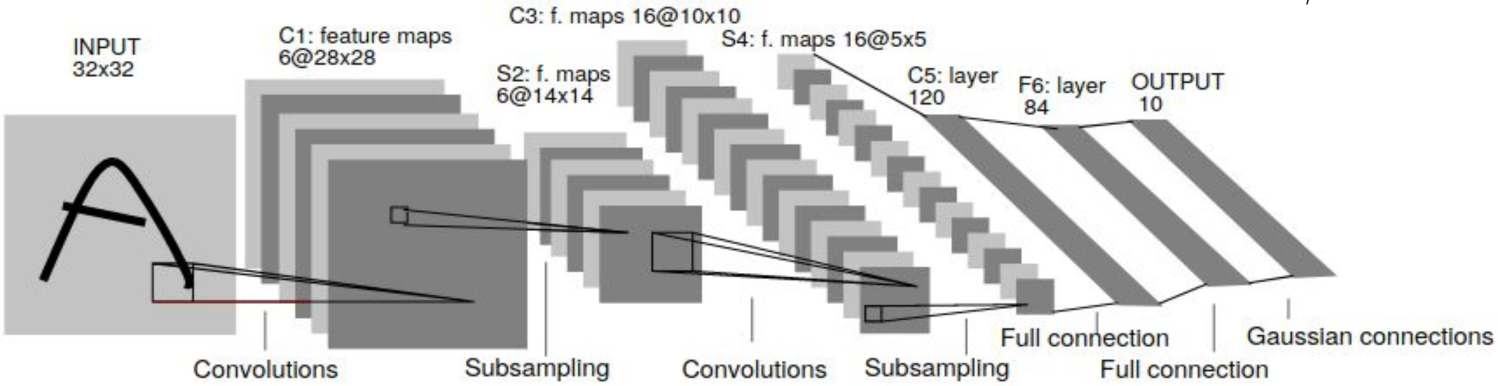
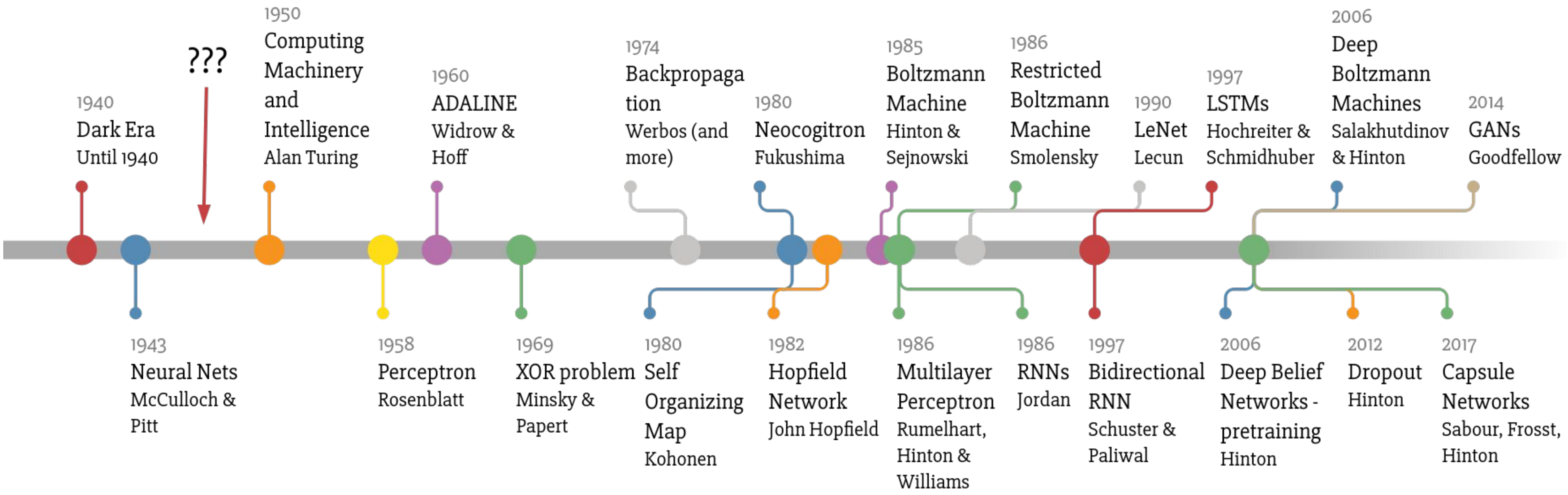
AlexNet, 2012



LeNet-5, 1998



Deep Learning Timeline



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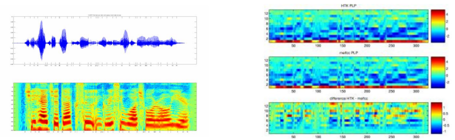
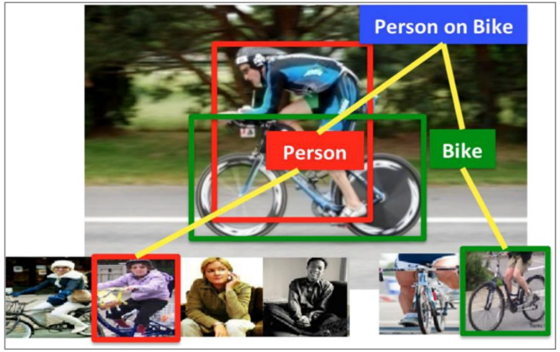
Real world applications

Audio Features



Spectrogram MFCC

- Object detection
- Action classification
- Image captioning
- …



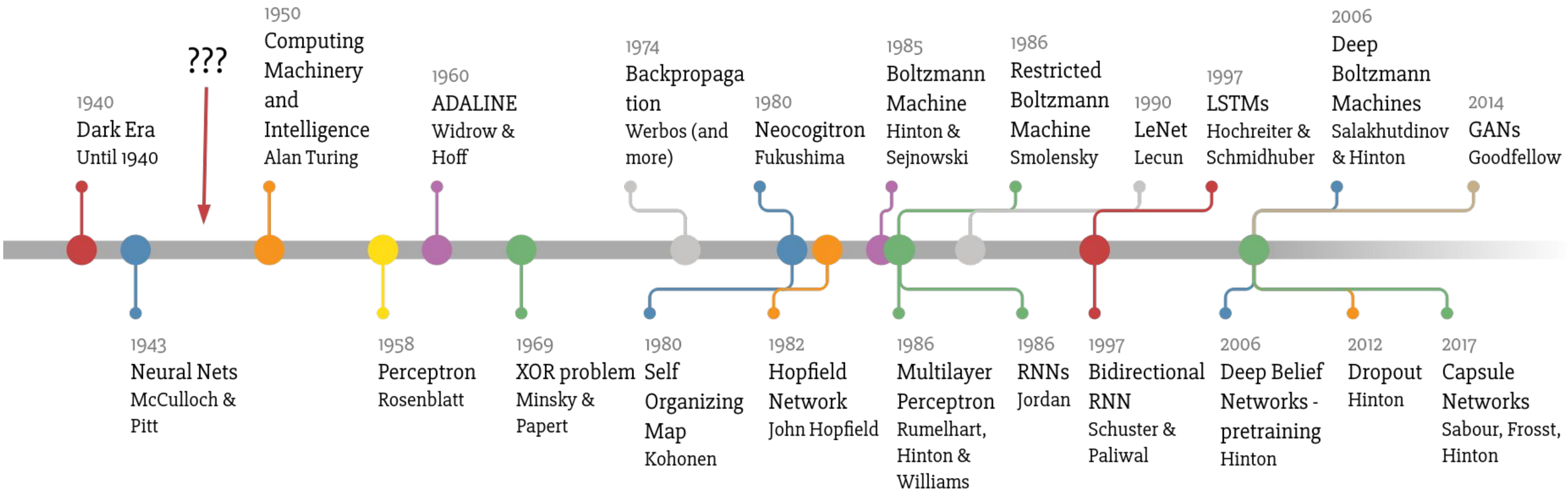
"man in black shirt is playing guitar."

GANs. 2014+

a) b) c) d)

https://thispersondoesnotexist.com/

Deep Learning Timeline



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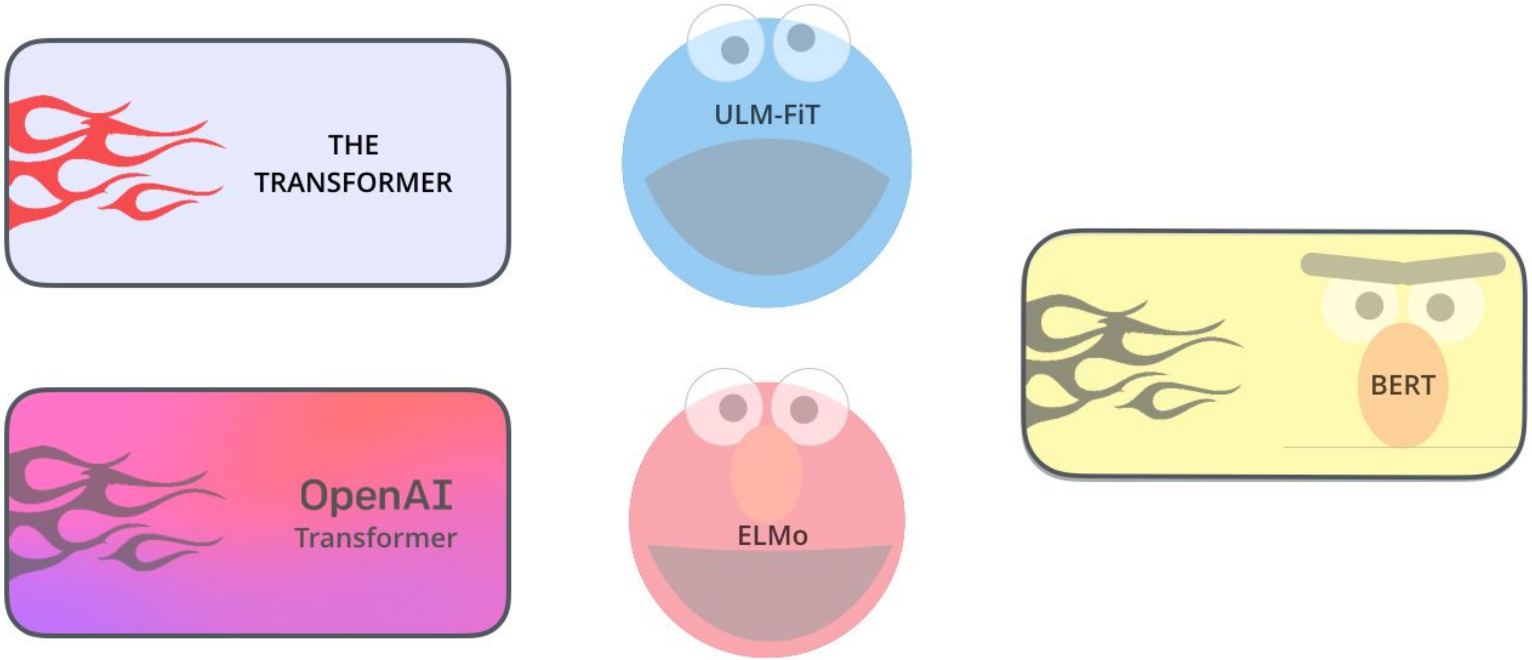
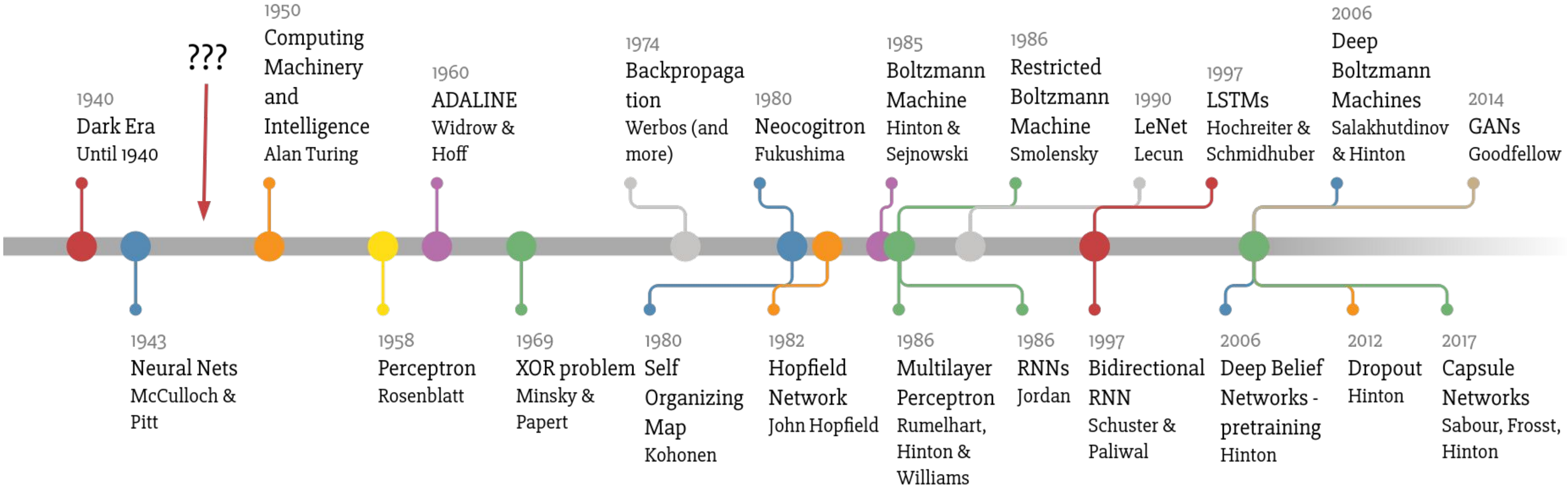
Transformer, BERT, GPT-2 and more, 2017+



Deep Learning Timeline



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Deep Learning: intuition

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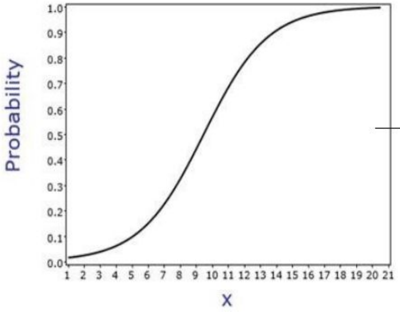
Logistic regression

X → Wx +b →



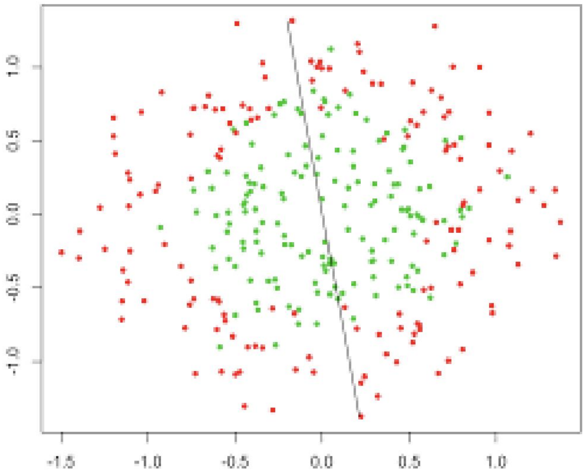
→ P(y)

$$P(y|x) = \sigma(w \cdot x + b)$$

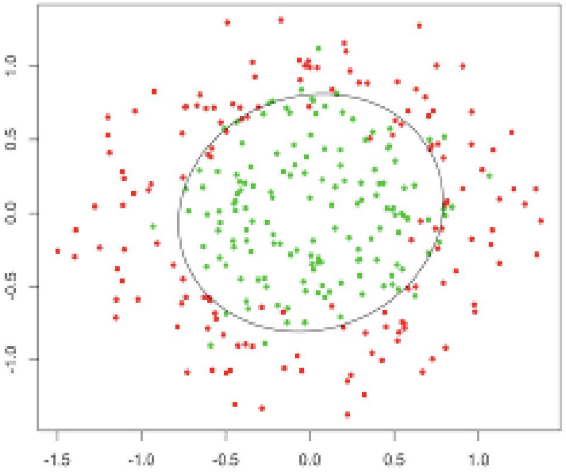
$$L = -\sum_{i} y_i \log P(y|x_i) + (1 - y_i) \log(1 - P(y|x_i))$$

Problem: nonlinear dependencies

Logistic regression (generally, linear model) need feature engineering to show good results.



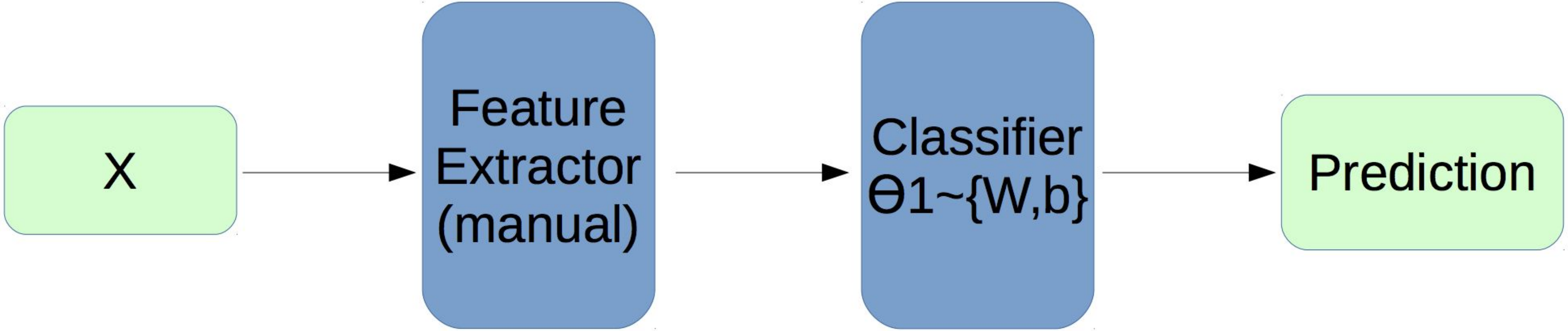
What we have



What we want

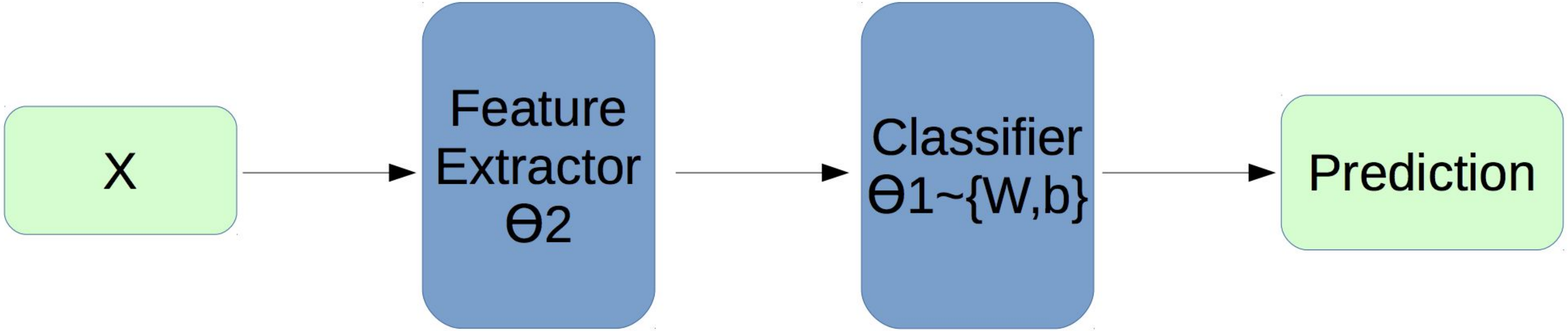
And feature engineering is an art.

Classic pipeline



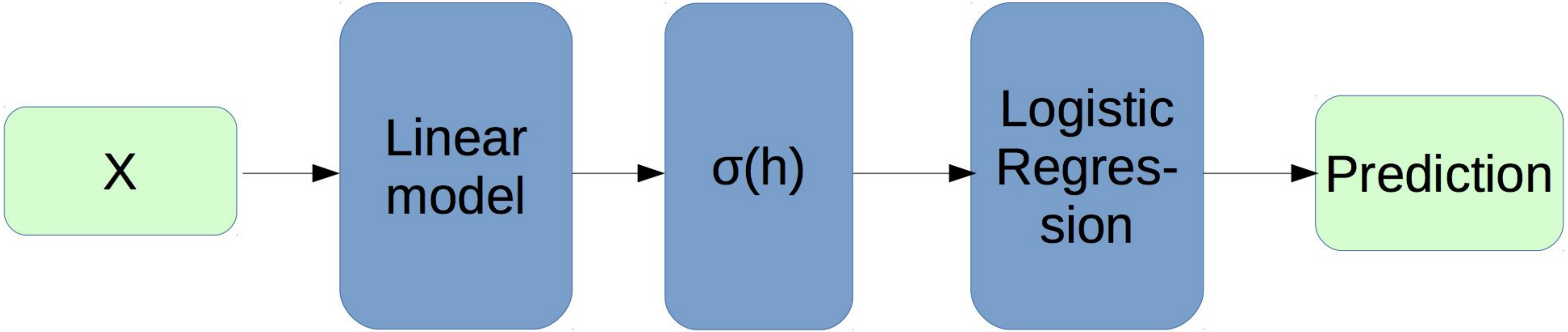
Handcrafted features, generated by experts.

NN pipeline



Automatically extracted features.

NN pipeline: example



E.g. two logistic regressions one after another.

Actually, it’s a neural network.

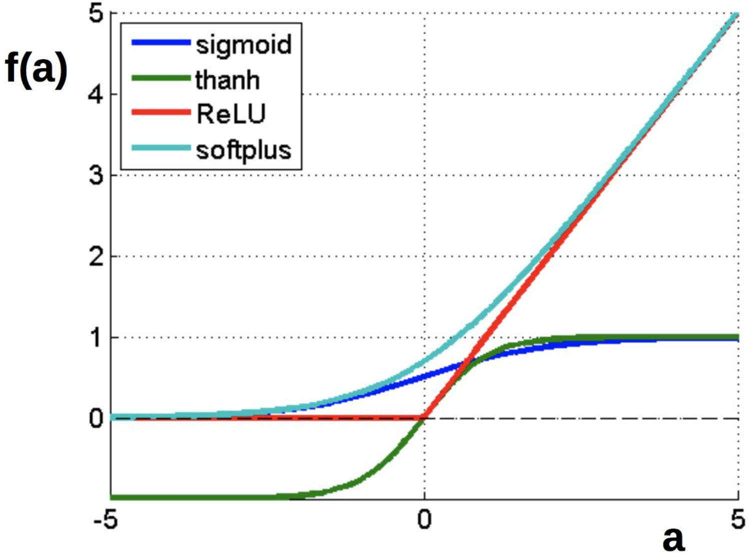
Activation functions: nonlinearities

$$f(a) = \frac{1}{1 + e^{-a}}$$

$$f(a) = \tanh(a)$$

$$f(a) = \max(0, a)$$

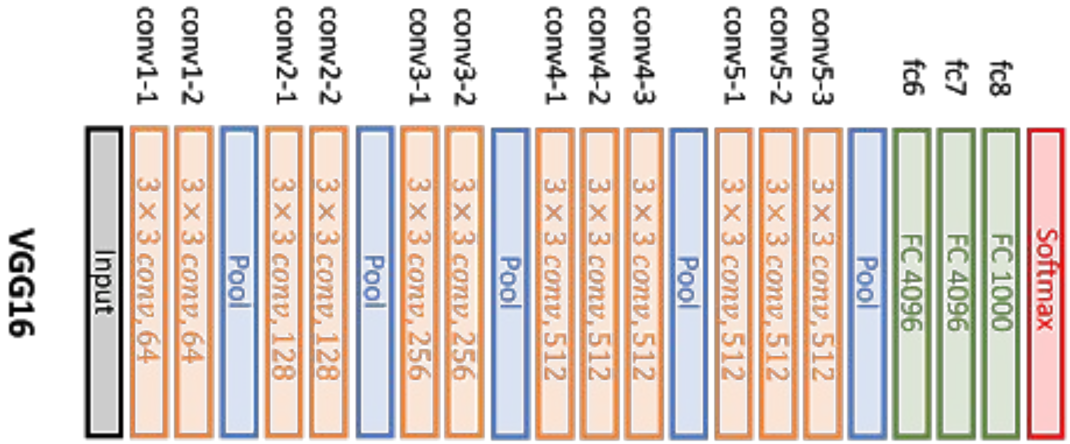
$$f(a) = \log(1 + e^{a})$$



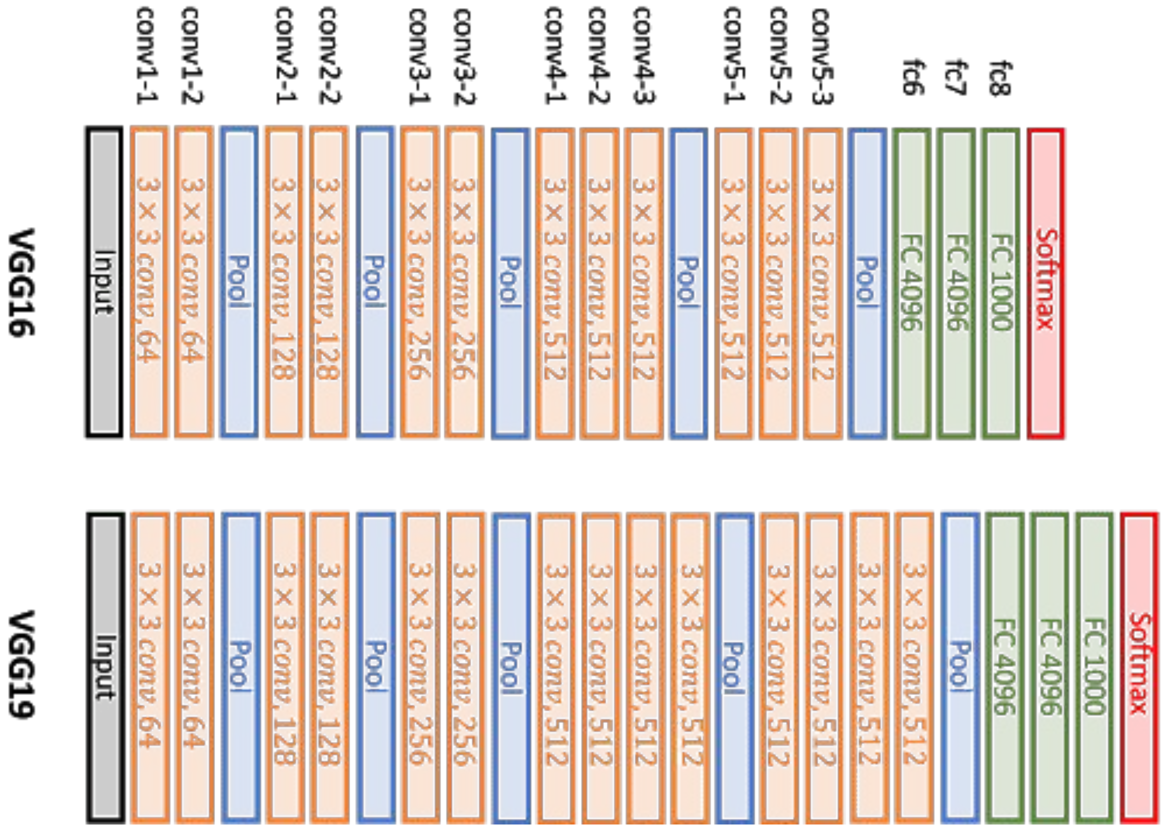
Some generally accepted terms

- Layer – a building block for NNs :
  - Dense/Linear/FC layer: $f(x) = Wx+b$
  - Nonlinearity layer: $f(x) = \sigma(x)$
  - Input layer, output layer
  - A few more we will cover later
- Activation function – function applied to layer output
  - Sigmoid
  - tanh
  - ReLU
  - Any other function to get nonlinear intermediate signal in NN
- Backpropagation – a fancy word for “chain rule”

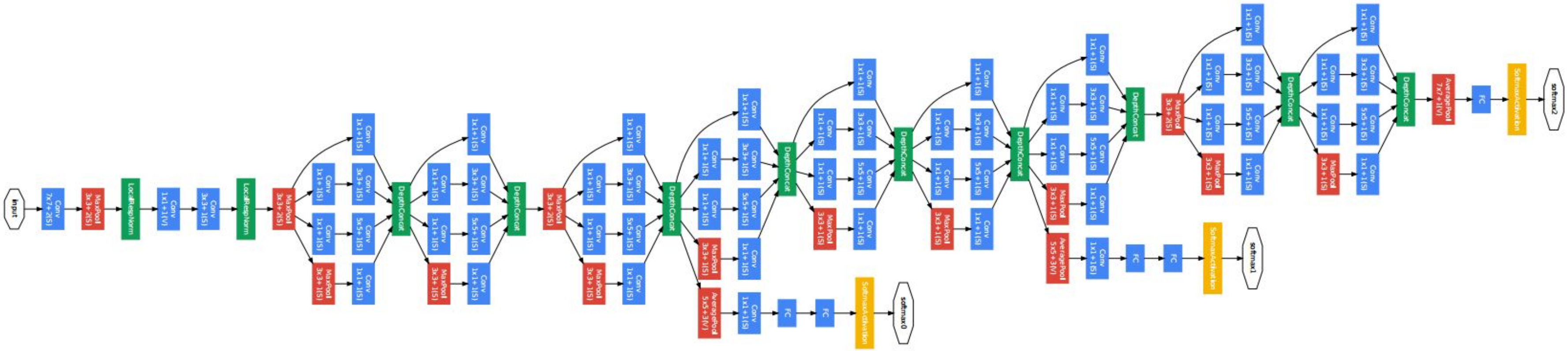
Actually, networks can be deep



And deeper...


VGG16
conv1-1
conv1-2
conv2-1
conv2-2
conv3-1
conv3-2
conv4-1
conv4-2
conv4-3
conv5-1
conv5-2
conv5-3
fc6
fc7
fc8
Input
3 × 3 conv, 64
3 × 3 conv, 64
Pool
3 × 3 conv, 128
3 × 3 conv, 128
Pool
3 × 3 conv, 256
3 × 3 conv, 256
Pool
3 × 3 conv, 512
3 × 3 conv, 512
3 × 3 conv, 512
Pool
3 × 3 conv, 512
3 × 3 conv, 512
3 × 3 conv, 512
Pool
FC 4096
FC 4096
FC 1000
Softmax
VGG19
Input
3 × 3 conv, 64
3 × 3 conv, 64
Pool
3 × 3 conv, 128
3 × 3 conv, 128
Pool
3 × 3 conv, 256
3 × 3 conv, 256
Pool
3 × 3 conv, 512
3 × 3 conv, 512
3 × 3 conv, 512
3 × 3 conv, 512
Pool
3 × 3 conv, 512
3 × 3 conv, 512
3 × 3 conv, 512
3 × 3 conv, 512
Pool
FC 4096
FC 4096
FC 1000
Softmax

Much deeper...



How to train it?

Backpropagation

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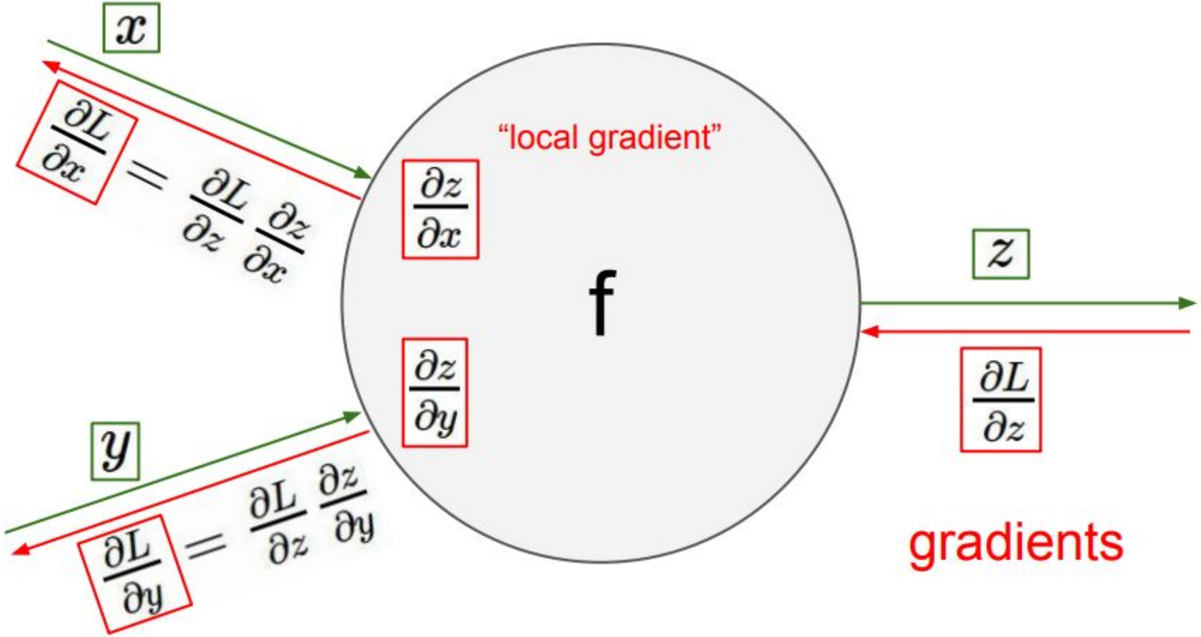
Backpropagation and chain rule

Chain rule is just simple math:

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$$

Backprop is just way to use it in NN training.



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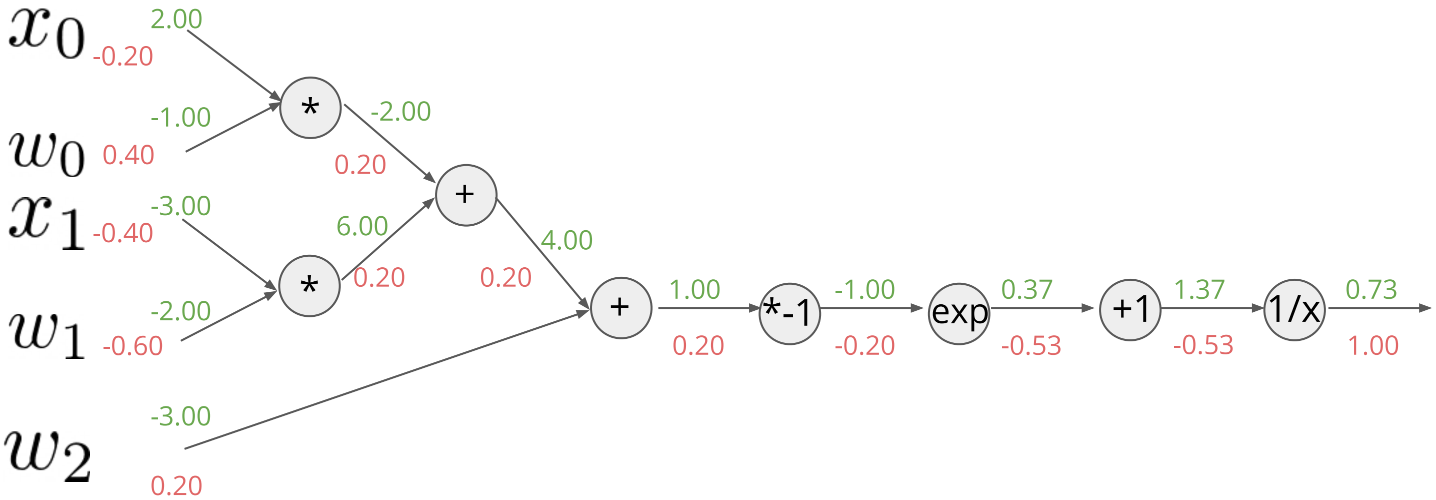
Backpropagation example

$$L(w, x) = \frac{1}{1 + \exp(-(x_0 w_0 + x_1 w_1 + w_2))}$$



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Backpropagation: matrix form

$$
\begin{aligned}
y_1 &= f_1(\mathbf{x}) = x_1 \\
y_2 &= f_2(\mathbf{x}) = x_2 \\
&\vdots \\
y_n &= f_n(\mathbf{x}) = x_n
\end{aligned}
$$

$$
\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \nabla f_1(\mathbf{x}) \\ \nabla f_2(\mathbf{x}) \\ \dots \\ \nabla f_m(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial \mathbf{x}} f_1(\mathbf{x}) \\ \frac{\partial}{\partial \mathbf{x}} f_2(\mathbf{x}) \\ \dots \\ \frac{\partial}{\partial \mathbf{x}} f_m(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x_1} f_1(\mathbf{x}) & \frac{\partial}{\partial x_2} f_1(\mathbf{x}) & \dots & \frac{\partial}{\partial x_n} f_1(\mathbf{x}) \\ \frac{\partial}{\partial x_1} f_2(\mathbf{x}) & \frac{\partial}{\partial x_2} f_2(\mathbf{x}) & \dots & \frac{\partial}{\partial x_n} f_2(\mathbf{x}) \\ & & \dots & \\ \frac{\partial}{\partial x_1} f_m(\mathbf{x}) & \frac{\partial}{\partial x_2} f_m(\mathbf{x}) & \dots & \frac{\partial}{\partial x_n} f_m(\mathbf{x}) \end{bmatrix}
$$

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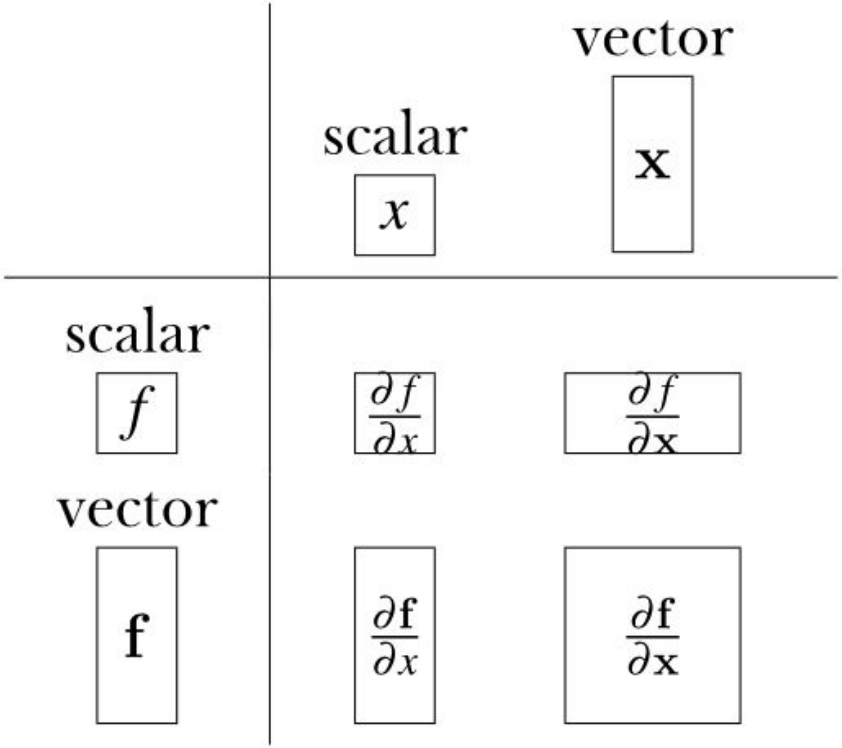
Backpropagation: matrix form



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Backpropagation: matrix form

$$
\begin{aligned}
\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial}{\partial \mathbf{x}} f_1(\mathbf{x}) \\ \frac{\partial}{\partial \mathbf{x}} f_2(\mathbf{x}) \\ \ldots \\ \frac{\partial}{\partial \mathbf{x}} f_m(\mathbf{x}) \end{bmatrix}
&= \begin{bmatrix} \frac{\partial}{\partial x_1} f_1(\mathbf{x}) & \frac{\partial}{\partial x_2} f_1(\mathbf{x}) & \ldots & \frac{\partial}{\partial x_n} f_1(\mathbf{x}) \\ \frac{\partial}{\partial x_1} f_2(\mathbf{x}) & \frac{\partial}{\partial x_2} f_2(\mathbf{x}) & \ldots & \frac{\partial}{\partial x_n} f_2(\mathbf{x}) \\ & \ldots & & \\ \frac{\partial}{\partial x_1} f_m(\mathbf{x}) & \frac{\partial}{\partial x_2} f_m(\mathbf{x}) & \ldots & \frac{\partial}{\partial x_n} f_m(\mathbf{x}) \end{bmatrix} \\
&= \begin{bmatrix} \frac{\partial}{\partial x_1} x_1 & \frac{\partial}{\partial x_2} x_1 & \ldots & \frac{\partial}{\partial x_n} x_1 \\ \frac{\partial}{\partial x_1} x_2 & \frac{\partial}{\partial x_2} x_2 & \ldots & \frac{\partial}{\partial x_n} x_2 \\ & \ldots & & \\ \frac{\partial}{\partial x_1} x_n & \frac{\partial}{\partial x_2} x_n & \ldots & \frac{\partial}{\partial x_n} x_n \end{bmatrix} \\
& \text{(and since } \frac{\partial}{\partial x_j} x_i = 0 \text{ for } j \neq i\text{)} \\
&= \begin{bmatrix} \frac{\partial}{\partial x_1} x_1 & 0 & \ldots & 0 \\ 0 & \frac{\partial}{\partial x_2} x_2 & \ldots & 0 \\ & & \ddots & \\ 0 & 0 & \ldots & \frac{\partial}{\partial x_n} x_n \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ & & \ddots & \\ 0 & 0 & \ldots & 1 \end{bmatrix} \\
&= I \quad \text{(}I\text{ is the identity matrix with ones down the diagonal)}
\end{aligned}
$$

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Activation functions

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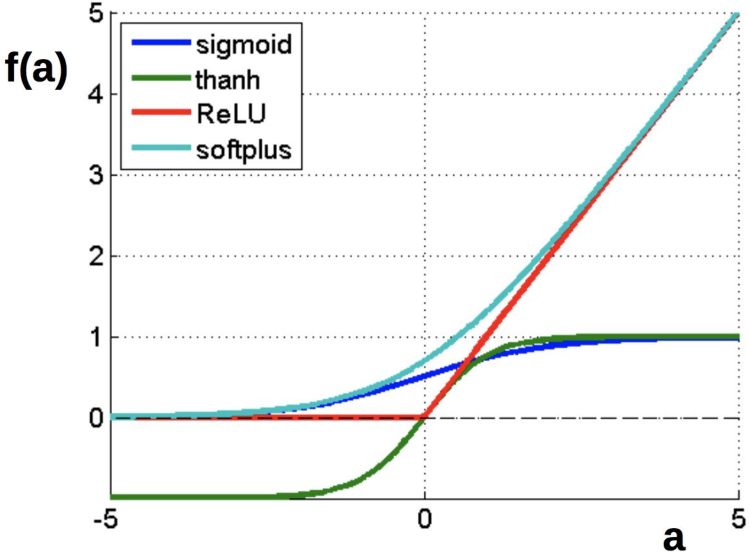
Once more: nonlinearities

$$f(a) = \frac{1}{1 + e^{-a}}$$

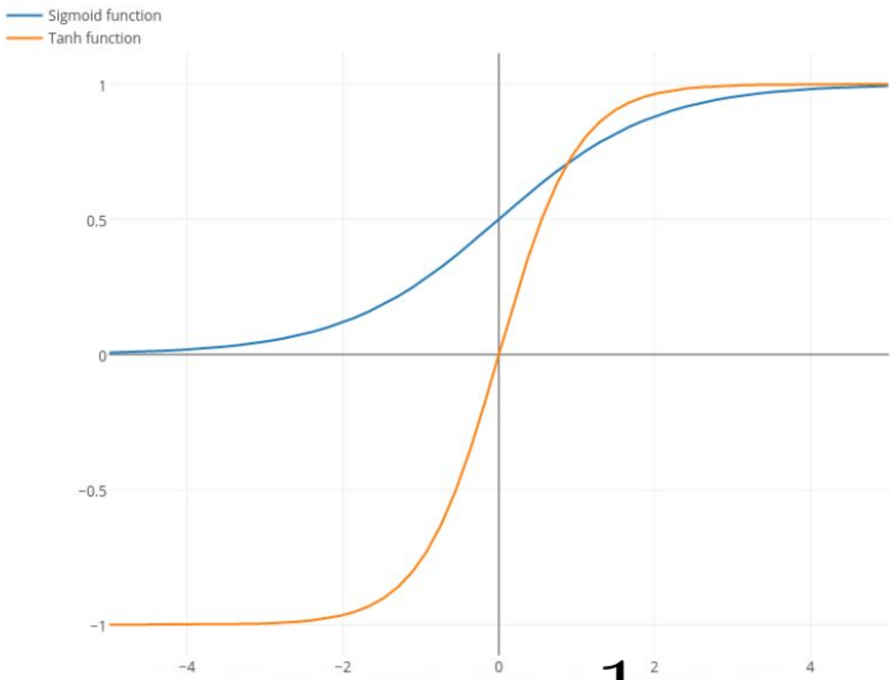
$$f(a) = \tanh(a)$$

$$f(a) = \max(0, a)$$

$$f(a) = \log(1 + e^{a})$$



Activation functions: Sigmoid



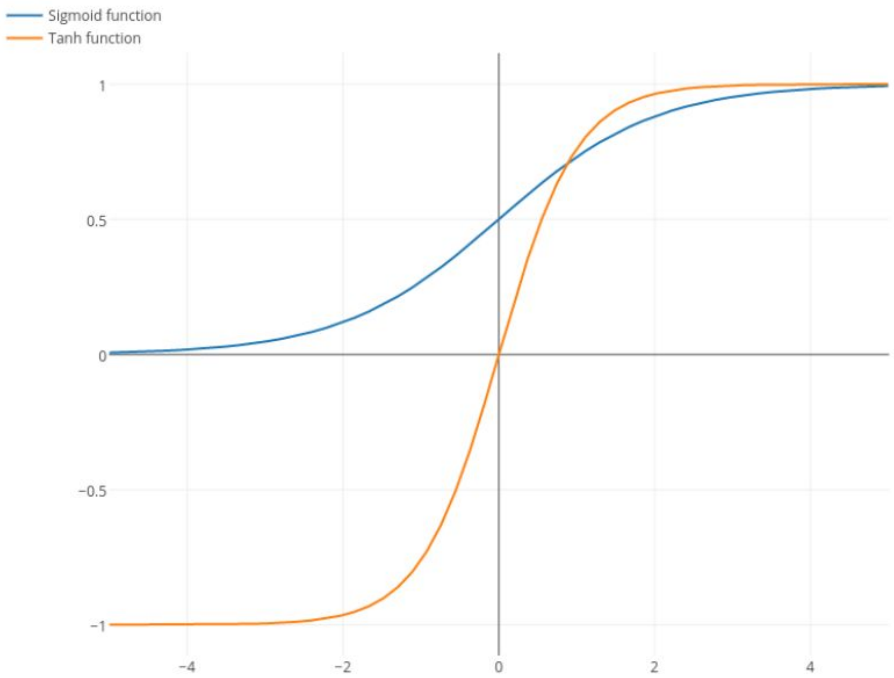
$$f(a) = \frac{1}{1 + e^{-a}}$$

- Maps R to (0,1)
- Historically popular, one of the first approximations of neuron activation

Problems:

- Almost zero gradients on the both sides (saturation)
- Shifted (not zero-centered) output
- Expensive computation of the exponent

Activation functions: tanh



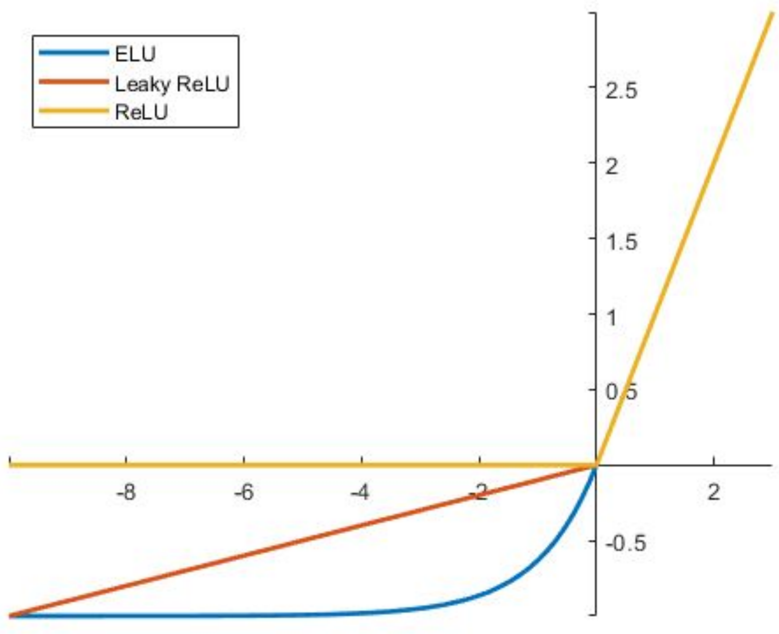
$$f(a) = \tanh(a)$$

- Maps R to (-1,1)
- Similar to the Sigmoid in other ways

Problems:

- Almost zero gradients on the both sides (saturation)
- ~~Shifted (not zero-centered) output~~
- Expensive computation of the exponent

Activation functions: ReLU



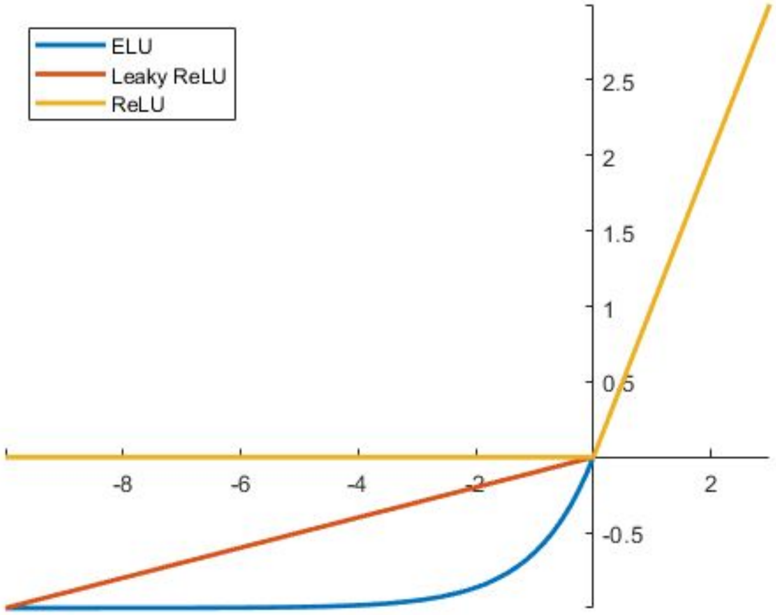
$$f(a) = \max(0, a)$$

- Very simple to compute (both forward and backward)
  - Up to 6 times faster than Sigmoid
- Does not saturate when x > 0
  - So the gradients are not 0

Problems:

- Zero gradients when x < 0
- Shifted (not zero-centered) output

Activation functions: LeakyReLU



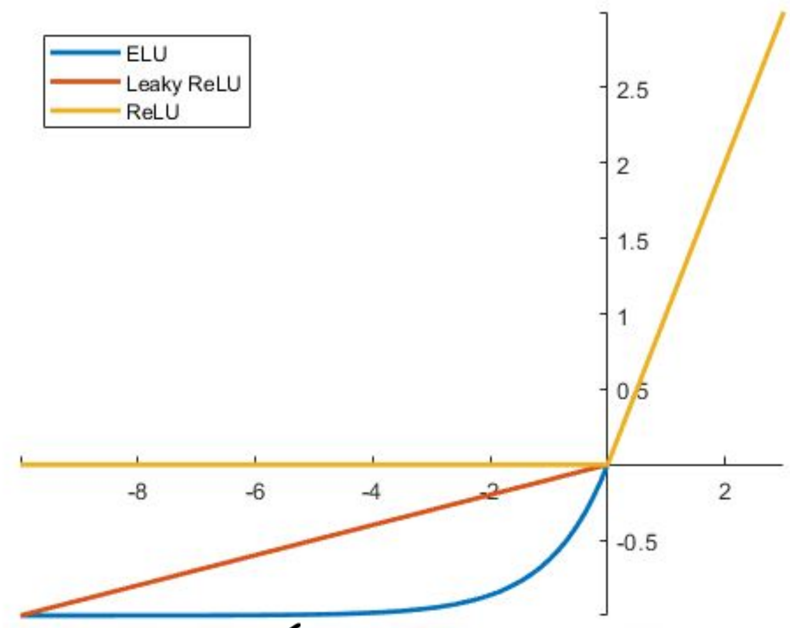
$$f(a) = \max(0.01a, a)$$

- Very simple to compute (both forward and backward)
  - Up to 6 times faster than Sigmoid
- Does not saturate when

Problems:

- Shifted, but not so much output

Activation functions: ELU



- Similar to ReLU
- Does not saturate
- Close to zero mean outputs

Problems:

- Requires exponent computation

$$f(a) = \begin{cases} a, & a > 0 \\ \alpha(\exp(a) - 1), & a \leq 0 \end{cases}$$

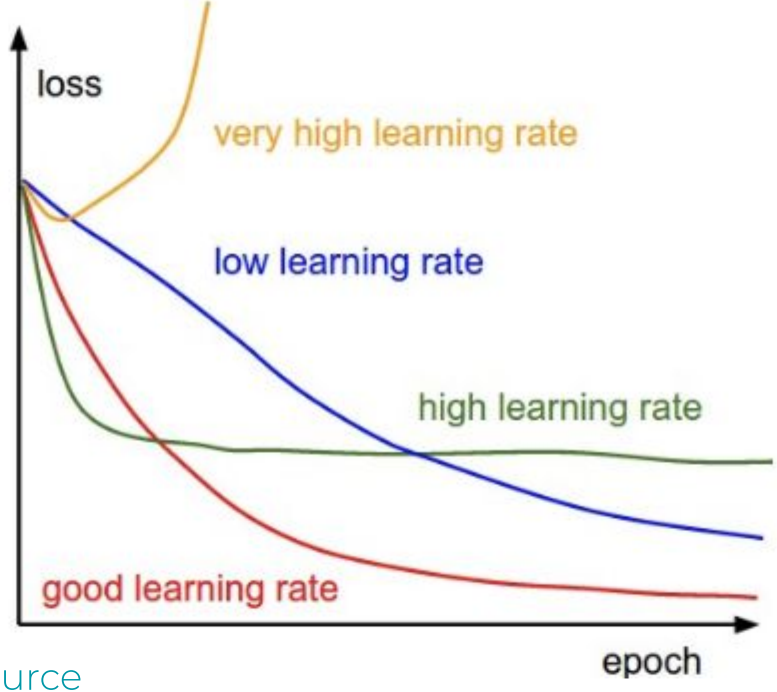
Activation functions: sum up

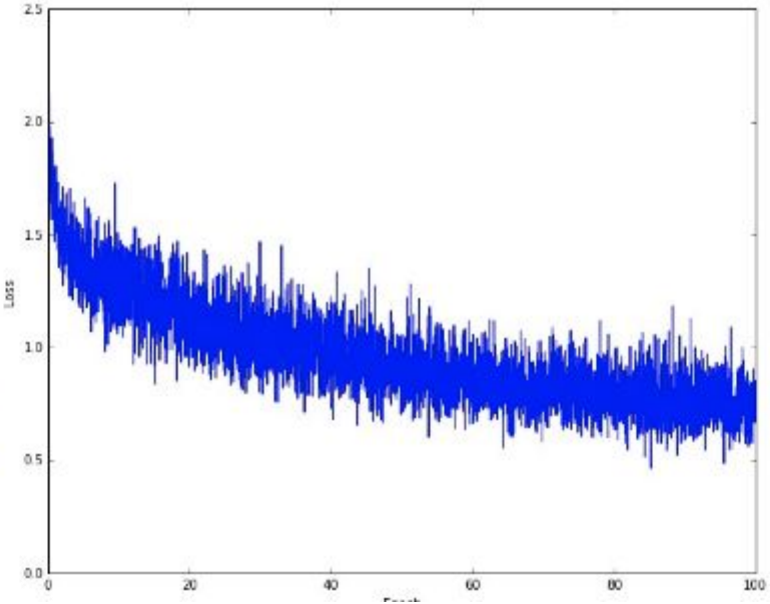
- Use ReLU as baseline approach
- Be careful with the learning rates
- Try out Leaky ReLU or ELU
- Try out tanh but do not expect much from it
- Do not use Sigmoid

Gradient optimization

Stochastic gradient descent (and variations) is used to optimize NN parameters.

$$x_{t+1} = x_t - \text{learning rate} \cdot dx$$





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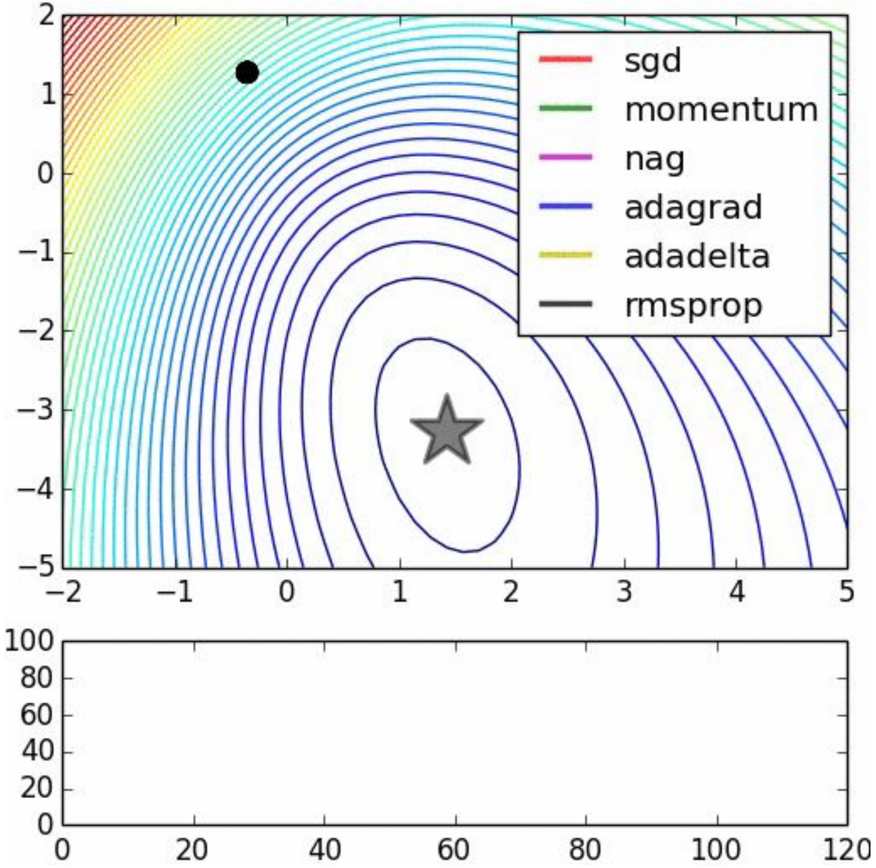
Optimizers

There are much more optimizers:

- Momentum
- Adagrad
- Adadelta
- RMSprop
- Adam
- ...
- even other NNs

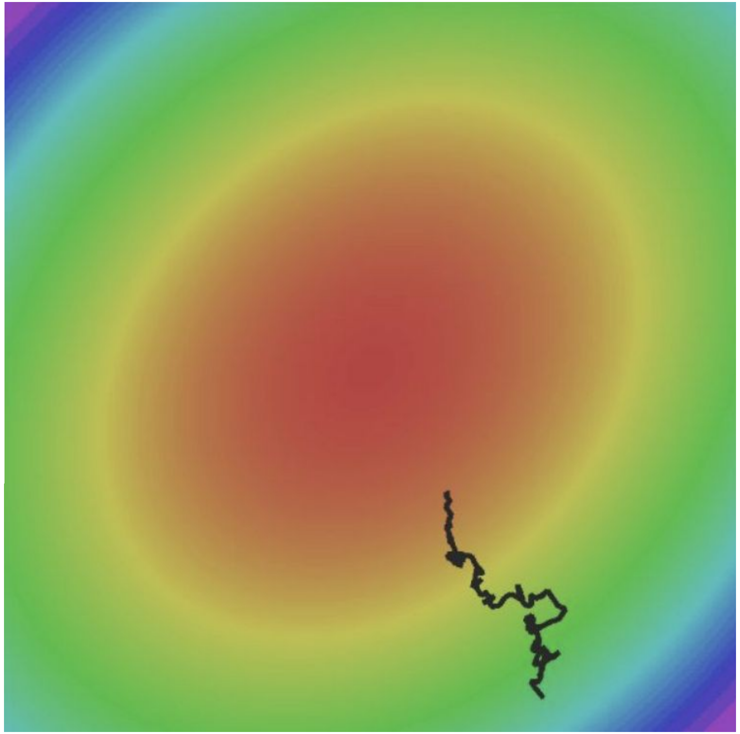


Optimization: SGD

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(x_i, y_i, W)$$

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W)$$

Averaging over minibatches ->noisy gradient



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First idea: momentum

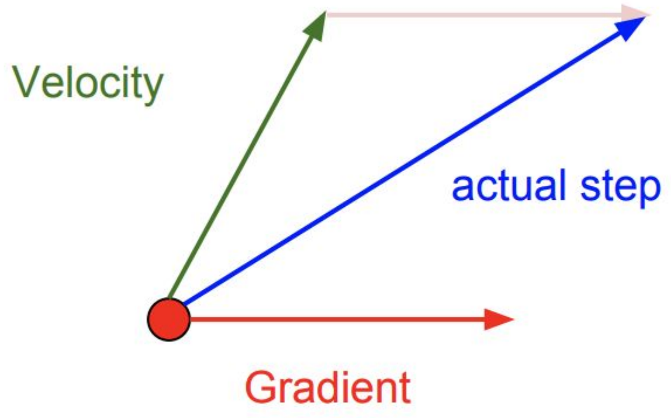
Simple SGD

$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$

SGD with momentum

$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

Momentum update:



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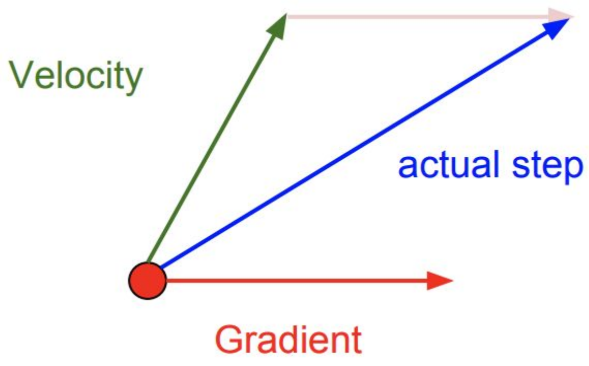
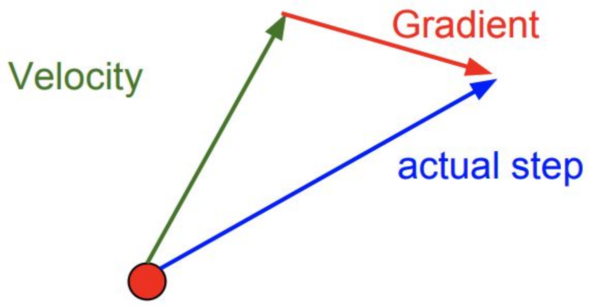
Nesterov momentum

Momentum update:



$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

Nesterov Momentum



$$v_{t+1} = \rho v_t - \alpha \nabla f(\boxed{x_t + \rho v_t})$$
$$x_{t+1} = x_t + v_{t+1}$$

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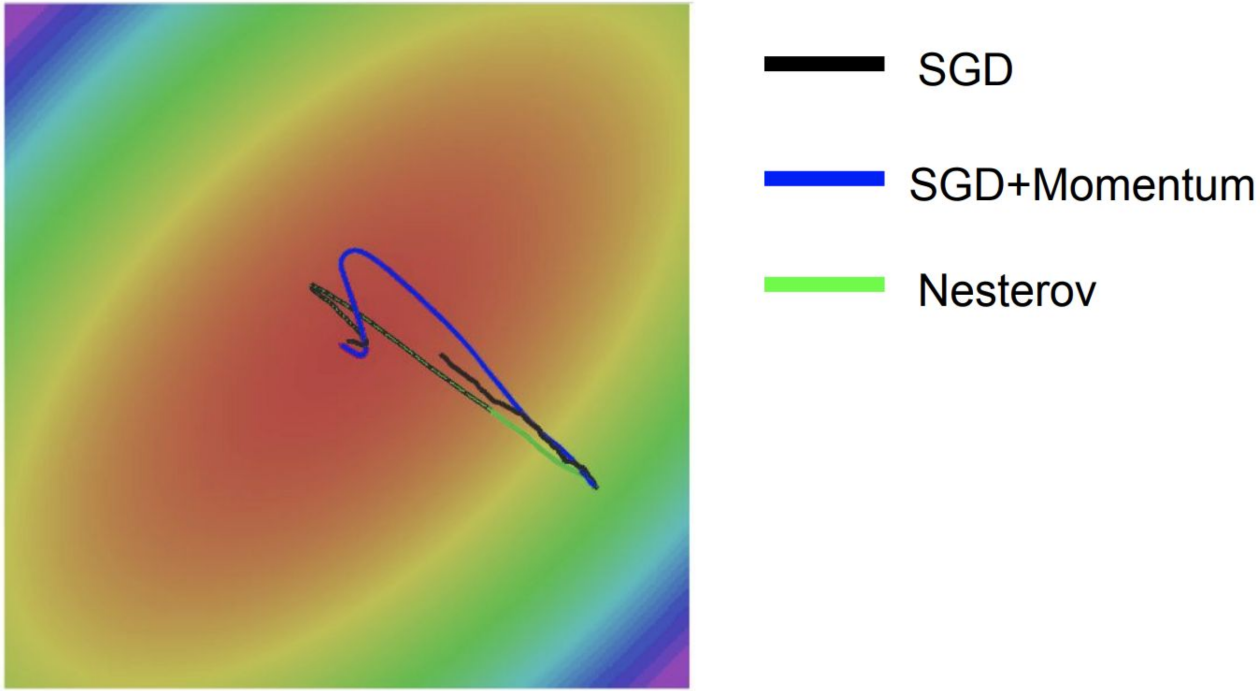
Comparing momentums



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Second idea: different dimensions are different

Adagrad: SGD with cache

$$\text{cache}_{t+1} = \text{cache}_t + (\nabla f(x_t))^2$$

$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{\text{cache}_{t+1} + \varepsilon}$$

Second idea: different dimensions are different

Adagrad: SGD with cache

$$\text{cache}_{t+1} = \text{cache}_t + (\nabla f(x_t))^2$$

$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{\text{cache}_{t+1} + \varepsilon}$$

Problem: gradient fades with time

Second idea: different dimensions are different

Adagrad: SGD with cache

$$\text{cache}_{t+1} = \text{cache}_t + (\nabla f(x_t))^2$$

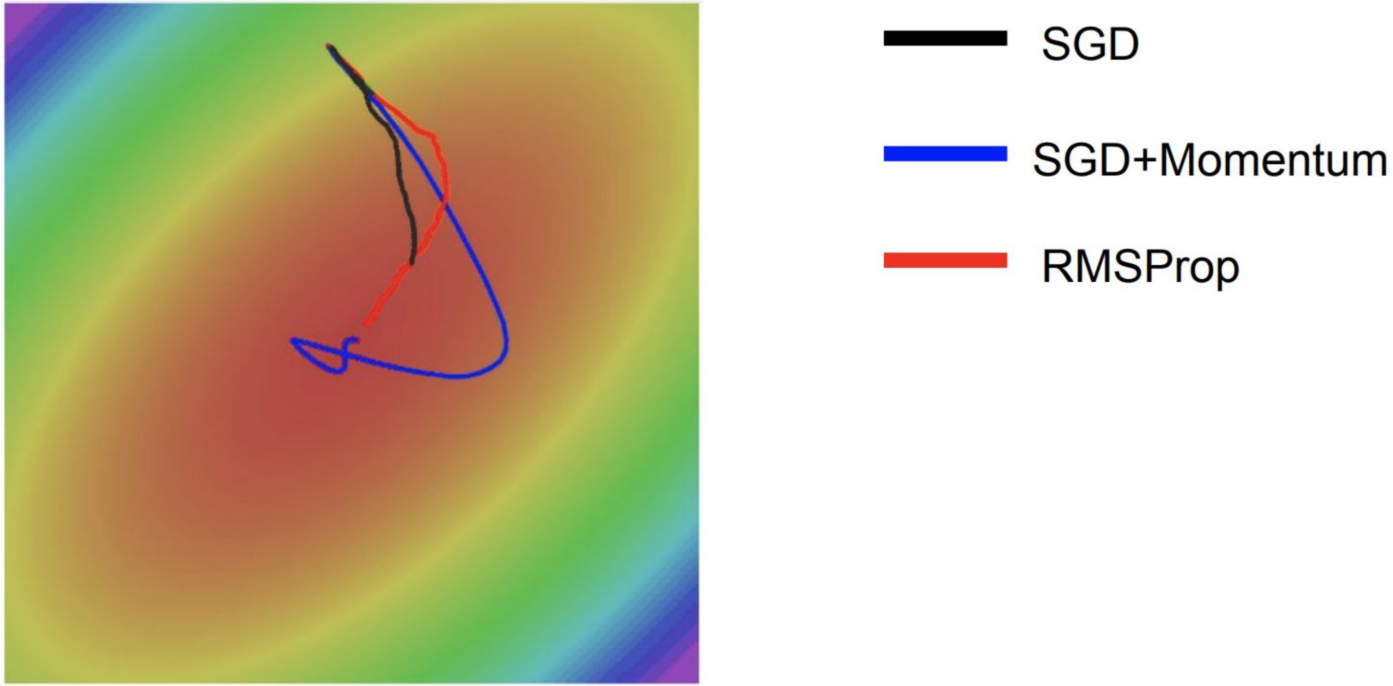
$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{\text{cache}_{t+1} + \varepsilon}$$

RMSProp: SGD with cache with exp. Smoothing

$$\text{cache}_{t+1} = \beta \text{cache}_t + (1 - \beta)(\nabla f(x_t))^2$$

$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{\text{cache}_{t+1} + \varepsilon}$$

Slide 29 Lecture 6 of Geoff Hinton's Coursera class



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Adam

Let’s combine the momentum idea and RMSProp normalization:

$$v_{t+1} = \gamma v_t + (1-\gamma)\nabla f(x_t)$$
$$\text{cache}_{t+1} = \beta \text{cache}_t + (1-\beta)(\nabla f(x_t))^2$$
$$x_{t+1} = x_t - \alpha \frac{v_{t+1}}{\text{cache}_{t+1} + \varepsilon}$$

Adam full form involves bias correction term. See link for more info.

Adam

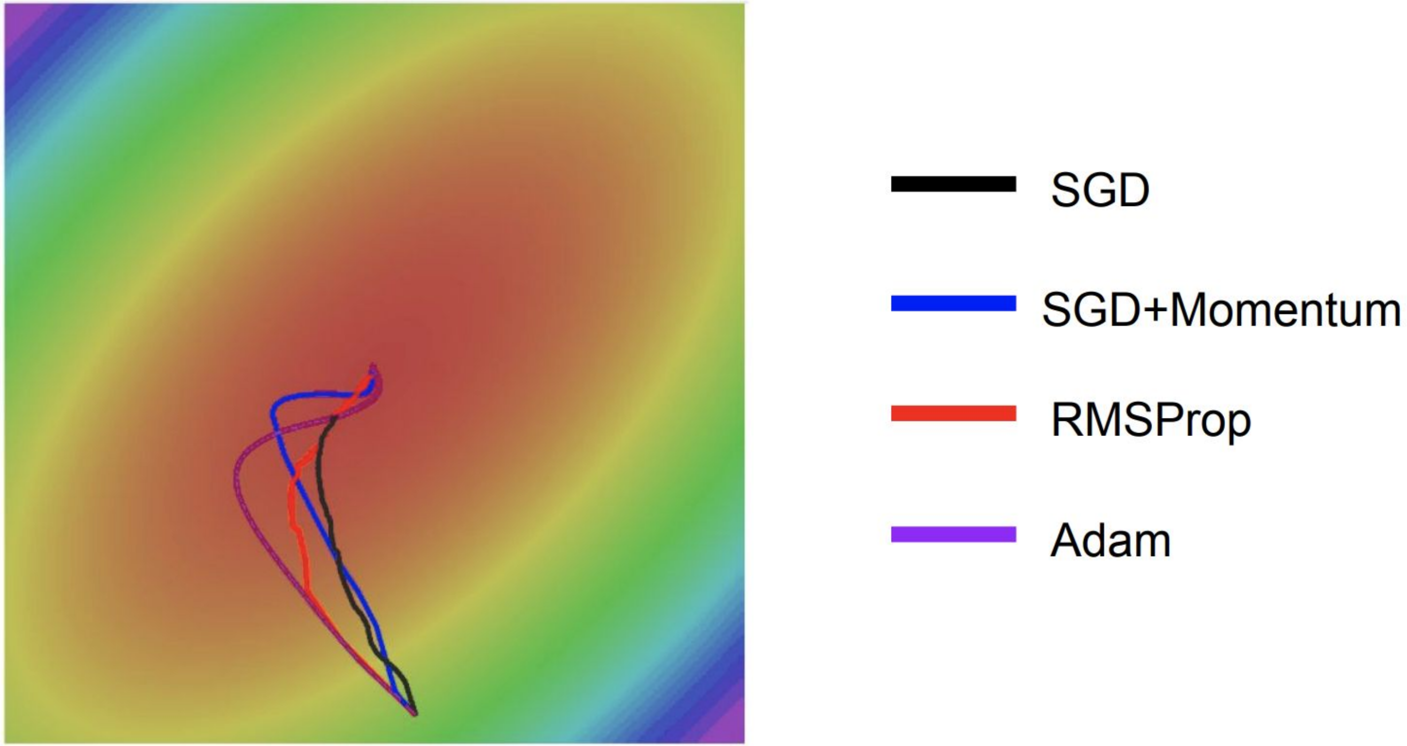
Let's combine the momentum idea and RMSProp normalization:

$$v_{t+1} = \gamma v_t + (1-\gamma)\nabla f(x_t)$$
$$\text{cache}_{t+1} = \beta \text{cache}_t + (1-\beta)(\nabla f(x_t))^2$$
$$x_{t+1} = x_t - \alpha \frac{v_{t+1}}{\text{cache}_{t+1} + \varepsilon}$$

Actually, that's not quite Adam.

Adam full form involves bias correction term. See link for more info.

Comparing optimizers



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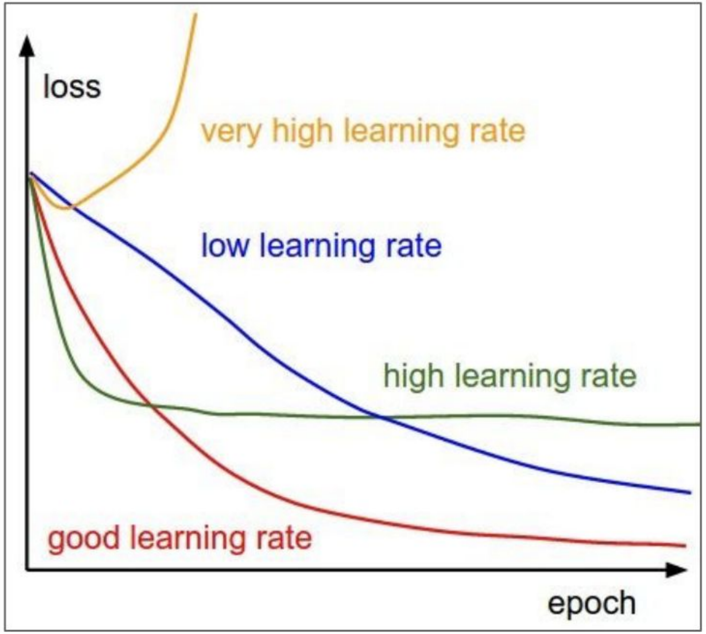
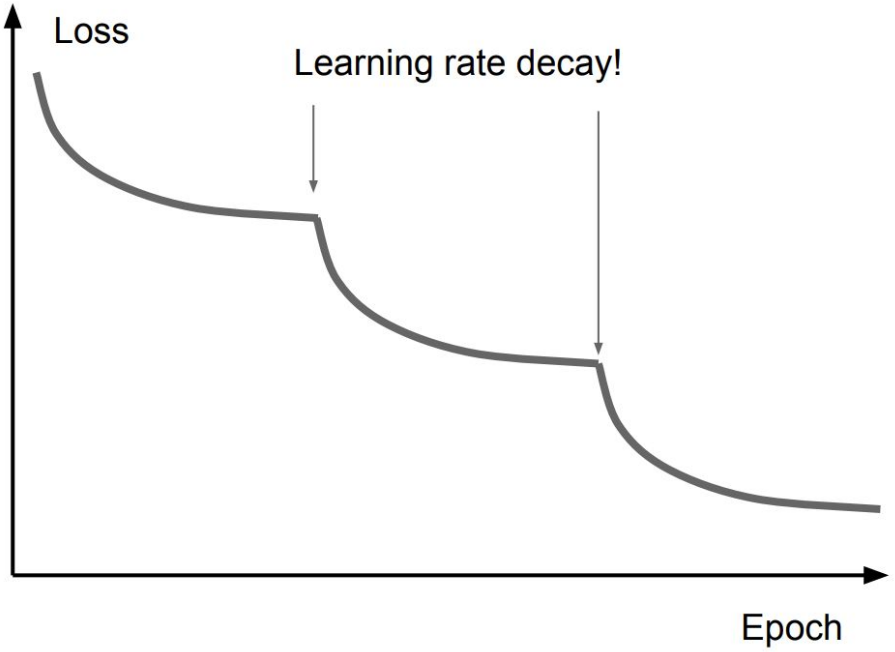
Once more: learning rate





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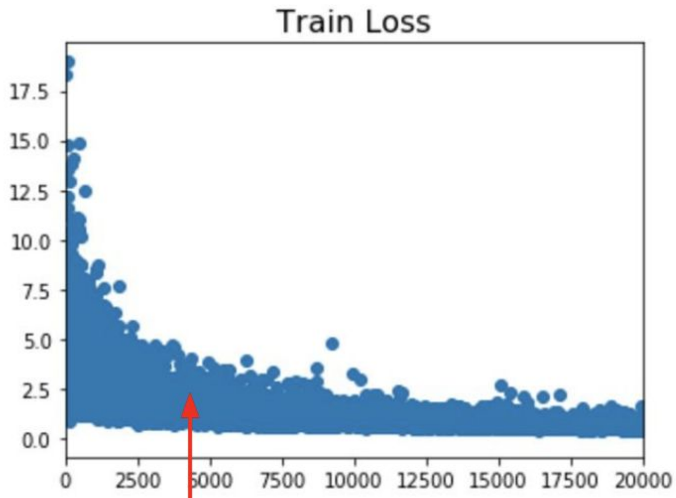
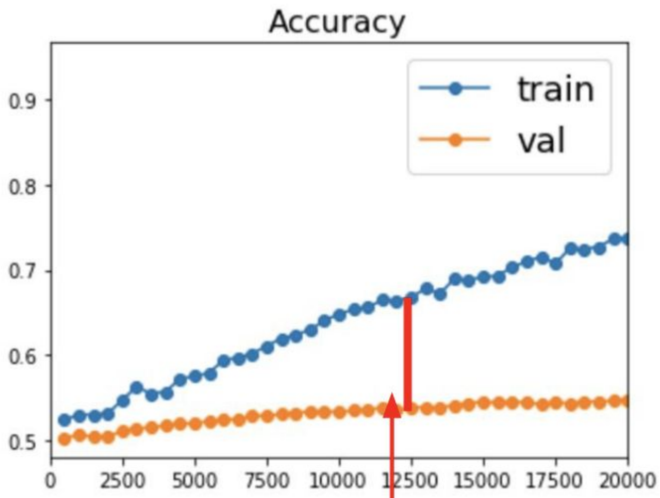
Sum up: optimization

- Adam is great basic choice
- Even for Adam/RMSProp learning rate matters
- Use learning rate decay
- Monitor your model quality





Better optimization algorithms help reduce training loss

But we really care about error on new data - how to reduce the gap?

source

Fancy neural networks

girafe
ai

RNN

Shakespeare

```
PANDARUS:
Alas, I think he shall be come approached and the day
When little srain would be attain'd into being never fed,
And who is but a chain and subjects of his death,
I should not sleep.

Second Senator:
They are away this miseries, produced upon my soul,
Breaking and strongly should be buried, when I perish
The earth and thoughts of many states.

DUKE VINCENTIO:
Well, your wit is in the care of side and that.

Second Lord:
They would be ruled after this chamber, and
my fair nues begun out of the fact, to be conveyed,
Whose noble souls I'll have the heart of the wars.

Clown:
Come, sir, I will make did behold your worship.

VIOLA:
I'll drink it.
```

Algebraic Geometry (Latex)

Proof. Omitted. $\square$

Lemma 0.1. *Let $\mathcal{C}$ be a set of the construction.*

Let $\mathcal{C}$ be a gerber covering. Let $\mathcal{F}$ be a quasi-coherent sheaves of $\mathcal{O}$-modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

.

Proof. This is an algebraic space with the composition of sheaves $\mathcal{F}$ on $X_{étale}$ we have

$$\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where $\mathcal{G}$ defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of $\mathcal{O}$-modules. $\square$

Lemma 0.2. *This is an integer $\mathcal{Z}$ is injective.*

Proof. See Spaces, Lemma ??. $\square$

Lemma 0.3. *Let S be a scheme. Let X be a scheme and X is an affine open covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. Let X be a scheme which is equal to the formal complex.*

The following to the construction of the lemma follows.

Let X be a scheme. Let X be a scheme covering. Let

$$b : X \to Y' \to Y \to Y \to Y' \times_X Y \to X.$$

be a morphism of algebraic spaces over S and Y.

Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let $\mathcal{F}$ be a quasi-coherent sheaf of $\mathcal{O}_X$-modules. The following are equivalent

(1) $\mathcal{F}$ is an algebraic space over S.
(2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type. $\square$

Linux kernel (source code)

```c
/*
 * If this error is set, we will need anything right after that BSD.
 */
static void action_new_function(struct s_stat_info *wb)
{
  unsigned long flags;
  int lel_idx_bit = e->edd, *sys & ~((unsigned long) *FIRST_COMPAT);
  buf[0] = 0xFFFFFFFF & (bit << 4);
  min(inc, slist->bytes);
  printk(KERN_WARNING "Memory allocated %02x/%02x, "
    "original MLL instead\n"),
    min(min(multi_run - s->len, max) * num_data_in),
    frame_pos, sz + first_seg);
  div_u64_w(val, inb_p);
  spin_unlock(&disk->queue_lock);
  mutex_unlock(&s->sock->mutex);
  mutex_unlock(&func->mutex);
  return disassemble(info->pending_bh);
}
```

source

Proof. Omitted. □

Lemma 0.1. *Let $\mathcal{C}$ be a set of the construction.*

Let $\mathcal{C}$ be a gerber covering. Let $\mathcal{F}$ be a quasi-coherent sheaves of $\mathcal{O}$-modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

.

Proof. This is an algebraic space with the composition of sheaves $\mathcal{F}$ on $X_{\acute{e}tale}$ we have

$$\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where $\mathcal{G}$ defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of $\mathcal{O}$-modules. □

Lemma 0.2. *This is an integer $\mathcal{Z}$ is injective.*

Proof. See Spaces, Lemma ??. □

Lemma 0.3. *Let S be a scheme. Let X be a scheme and X is an affine open covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. Let X be a scheme which is equal to the formal complex.*

The following to the construction of the lemma follows.

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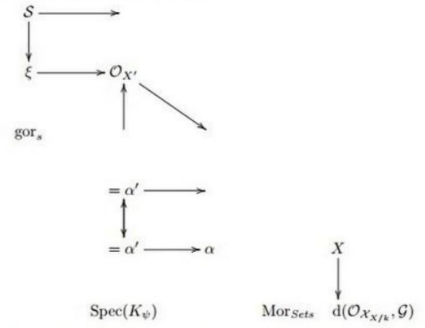
(1) $\mathcal{F}$ is an algebraic space over S.
(2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type. □

This since $\mathcal{F} \in \mathcal{F}$ and $x \in \mathcal{G}$ the diagram



is a limit. Then $\mathcal{G}$ is a finite type and assume S is a flat and $\mathcal{F}$ and $\mathcal{G}$ is a finite type f_*. This is of finite type diagrams, and

- the composition of $\mathcal{G}$ is a regular sequence,
- $\mathcal{O}_{X'}$ is a sheaf of rings.

□

Proof. We have see that $X = \mathrm{Spec}(R)$ and $\mathcal{F}$ is a finite type representable by algebraic space. The property $\mathcal{F}$ is a finite morphism of algebraic stacks. Then the cohomology of X is an open neighbourhood of U. □

Proof. This is clear that $\mathcal{G}$ is a finite presentation, see Lemmas ??.
A *reduced above* we conclude that U is an open covering of $\mathcal{C}$. The functor $\mathcal{F}$ is a "field

$$\mathcal{O}_{X,x} \longrightarrow \mathcal{F}_{\overline{x}} \ \text{-}1(\mathcal{O}_{X_{\acute{e}tale}}) \longrightarrow \mathcal{O}_{X_\ell}^{-1}\mathcal{O}_{X_\lambda}(\mathcal{O}_{X_\eta}^{\overline{v}})$$

is an isomorphism of covering of $\mathcal{O}_{X_i}$. If $\mathcal{F}$ is the unique element of $\mathcal{F}$ such that X is an isomorphism.
The property $\mathcal{F}$ is a disjoint union of Proposition ?? and we can filtered set of presentations of a scheme $\mathcal{O}_X$-algebra with $\mathcal{F}$ are opens of finite type over S.
If $\mathcal{F}$ is a scheme theoretic image points. □

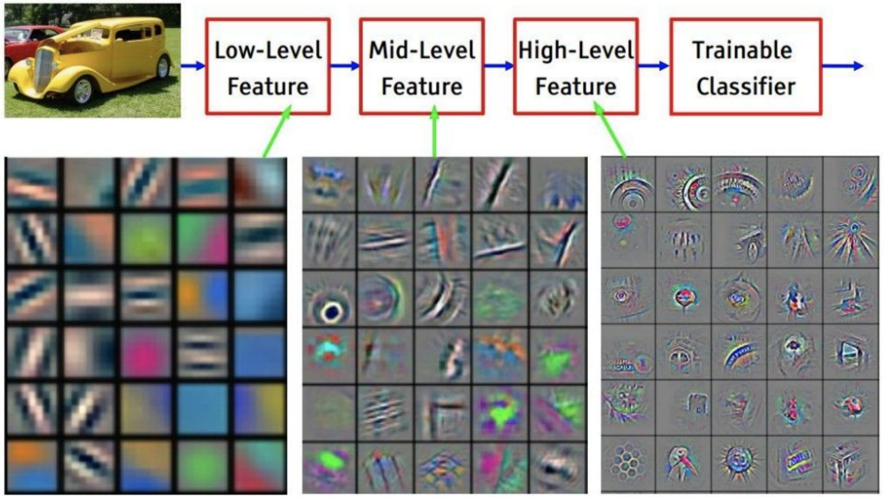
If $\mathcal{F}$ is a finite direct sum $\mathcal{O}_{X_\lambda}$ is a closed immersion, see Lemma ??. This is a sequence of $\mathcal{F}$ is a similar morphism.

source

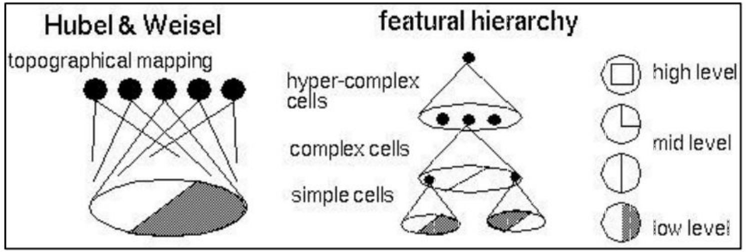
```c
#include <asm/io.h>
#include <asm/prom.h>
#include <asm/e820.h>
#include <asm/system_info.h>
#include <asm/setew.h>
#include <asm/pgproto.h>

#define REG_PG    vesa_slot_addr_pack
#define PFM_NOCOMP  AFSR(0, load)
#define STACK_DDR(type)     (func)

#define SWAP_ALLOCATE(nr)     (e)
#define emulate_sigs()  arch_get_unaligned_child()
#define access_rw(TST)  asm volatile("movd %%esp, %0, %3" : : "r" (0));   \
  if (__type & DO_READ)

static void stat_PC_SEC __read_mostly offsetof(struct seq_argsqueue, \
          pC>[1]);

static void
os_prefix(unsigned long sys)
{
#ifdef CONFIG_PREEMPT
  PUT_PARAM_RAID(2, sel) = get_state_state();
  set_pid_sum((unsigned long)state, current_state_str(),
           (unsigned long)-1->lr_full; low;
}
```

source

CNN:Convolutional layer and visual cortex



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

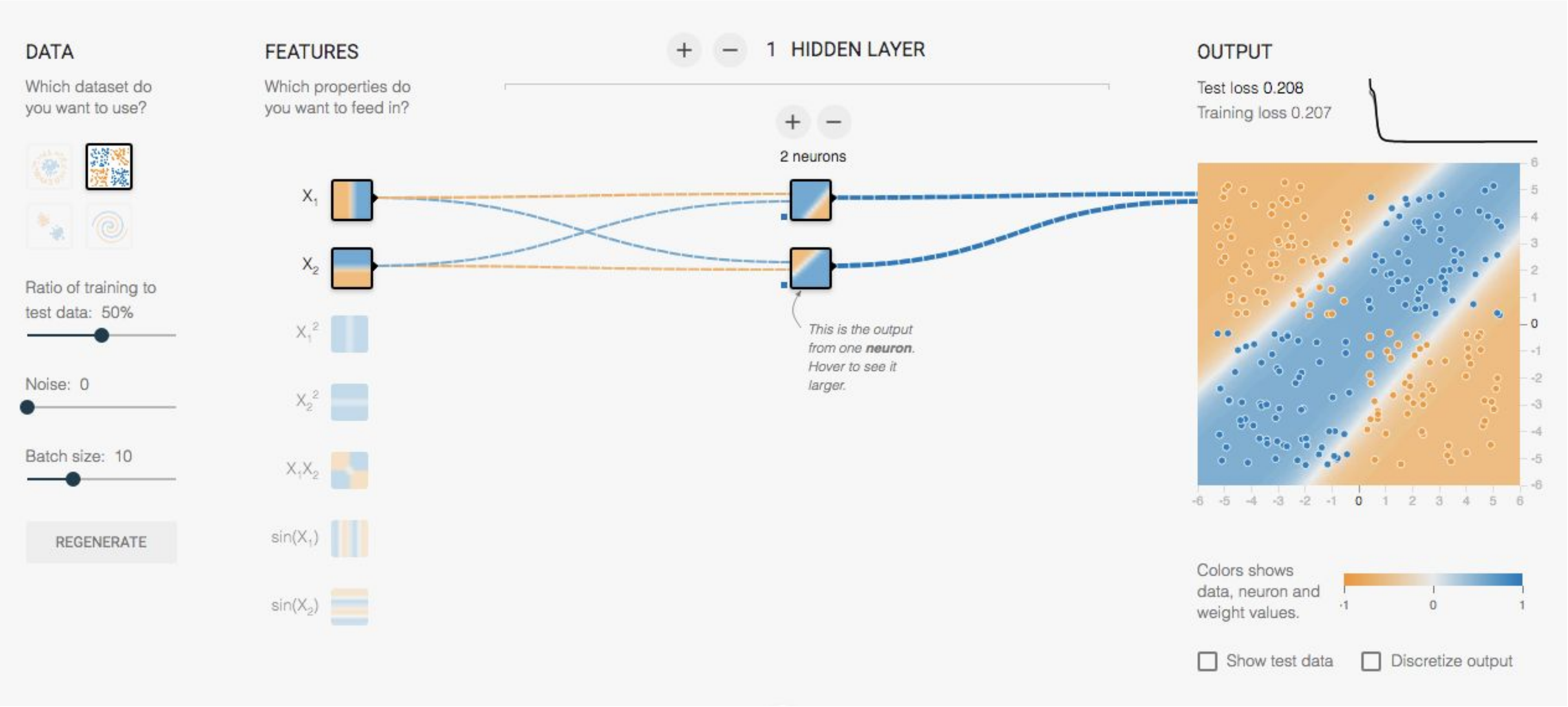


[From Yann LeCun slides]

CNN:Convolutional layer and visual cortex

source

Don’t miss the interactive playground



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WHO'S AWESOME?
You're Awesome

Outro

- Neural Networks are great
    - Especially for data with specific structure
- All operations should be differentiable to use backpropagation mechanics
    - And still it is just basic differentiation
- Many techniques in Deep Learning are inspired by nature
    - Or general sense
- Do not hesitate to ask questions (and answer them as well)

More materials for self-study: link

Revise

1. Neural Networks in different areas. Historical overview.
2. Backpropagation.
3. More on backpropagation.
4. Activation functions.
5. Playground.

Thanks for attention!

Questions?

girafe
ai