Lecture 12: Language and Sequence Modeling

Iurii Efimov





Outline

- 1. RNN intuitions
- 2. Language models
- 3. Memory concept: LSTM
- 4. RNN as encoder for sequential data
- 5. Vanishing gradient



RNNs generating...



Shakespeare

Algebraic Geometry (Latex)

Linux kernel (source code)

PANDARUS:

Alas, I think he shall be come approached and the day When little srain would be attain'd into being never fed, And who is but a chain and subjects of his death. I should not sleep.

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Clown:

Come, sir, I will make did behold your worship.

VIOLA:

I'll drink it.

```
Proof. Omitted.
Lemma 0.1. Let C be a set of the construction.
  Let C be a gerber covering. Let F be a quasi-coherent sheaves of O-modules. We
have to show that
                                    \mathcal{O}_{\mathcal{O}_{x}} = \mathcal{O}_{X}(\mathcal{L})
Proof. This is an algebraic space with the composition of sheaves \mathcal{F} on X_{tasts} we
have
                          \mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}
where G defines an isomorphism F \to F of O-modules.
Lemma 0.2. This is an integer Z is injective.
Proof. See Spaces, Lemma ??.
Lemma 0.3. Let S be a scheme. Let X be a scheme and X is an affine open
covering. Let U \subset X be a canonical and locally of finite type. Let X be a scheme.
Let X be a scheme which is equal to the formal complex.
The following to the construction of the lemma follows.
Let X be a scheme. Let X be a scheme covering. Let
                      b: X \to Y' \to Y \to Y \to Y' \times_X Y \to X.
be a morphism of algebraic spaces over S and Y.
Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let F be a
```

quasi-coherent sheaf of O_X -modules. The following are equivalent

 F is an algebraic space over S. (2) If X is an affine open covering.

```
* If this error is set, we will need anything right after that BSD.
                                                                                static void action new function(struct s stat info *wb)
                                                                                  unsigned long flags:
                                                                                 int lel idx bit = e->edd, *sys & -((unsigned long) *FIRST COMPAT);
                                                                                 buf[0] = 0xFFFFFFFF & (bit << 4);
                                                                                 min(inc, slist->bytes);
                                                                                 printk(KERN WARNING "Memory allocated %02x/%02x, "
                                                                                   "original MLL instead\n"),
                                                                                   min(min(multi run - s->len, max) * num data in),
                                                                                   frame pos, sz + first seg);
                                                                                 div u64 w(val, inb p);
                                                                                  spin unlock(&disk->queue lock);
                                                                                 mutex unlock(&s->sock->mutex);
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finite type.



Proof. Omitted.

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$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

Proof. This is an algebraic space with the composition of sheaves \mathcal{F} on $X_{\acute{e}tale}$ we have

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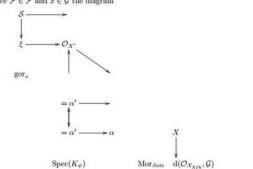
be a morphism of algebraic spaces over S and Y.

Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let \mathcal{F} be a quasi-coherent sheaf of \mathcal{O}_X -modules. The following are equivalent

- F is an algebraic space over S.
- (2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type.

This since $\mathcal{F} \in \mathcal{F}$ and $x \in \mathcal{G}$ the diagram



is a limit. Then $\mathcal G$ is a finite type and assume S is a flat and $\mathcal F$ and $\mathcal G$ is a finite type f_* . This is of finite type diagrams, and

- the composition of G is a regular sequence,
- O_{X'} is a sheaf of rings.

Proof. We have see that $X = \operatorname{Spec}(R)$ and $\mathcal F$ is a finite type representable by algebraic space. The property $\mathcal F$ is a finite morphism of algebraic stacks. Then the cohomology of X is an open neighbourhood of U.

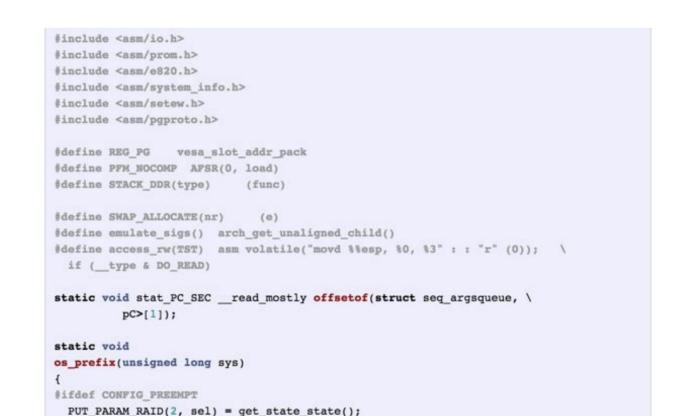
Proof. This is clear that G is a finite presentation, see Lemmas ??. A reduced above we conclude that U is an open covering of C. The functor F is a "field

$$\mathcal{O}_{X,x} \longrightarrow \mathcal{F}_{\overline{x}} -1(\mathcal{O}_{X_{\ell tale}}) \longrightarrow \mathcal{O}_{X_{\ell}}^{-1}\mathcal{O}_{X_{\lambda}}(\mathcal{O}_{X_{\eta}}^{\overline{v}})$$

is an isomorphism of covering of \mathcal{O}_{X_i} . If \mathcal{F} is the unique element of \mathcal{F} such that X is an isomorphism.

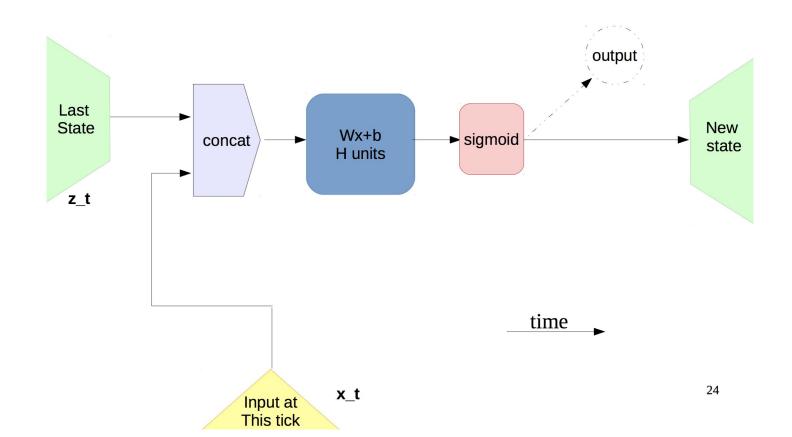
The property \mathcal{F} is a disjoint union of Proposition ?? and we can filtered set of presentations of a scheme \mathcal{O}_{X} -algebra with \mathcal{F} are opens of finite type over S. If \mathcal{F} is a scheme theoretic image points.

If \mathcal{F} is a finite direct sum $\mathcal{O}_{X_{\lambda}}$ is a closed immersion, see Lemma ??. This is a sequence of \mathcal{F} is a similar morphism.

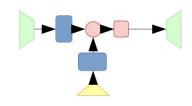




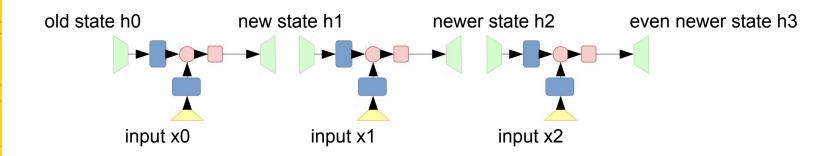






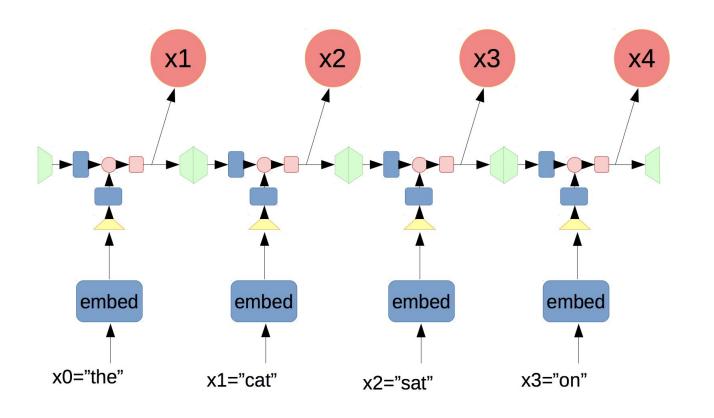






We use same weight matrices for all steps





Recurrent neural network: with formulas



$$h_0 = \bar{0}$$

$$h_1 = \sigma(\langle W_{\text{hid}}[h_0, x_0] \rangle + b)$$

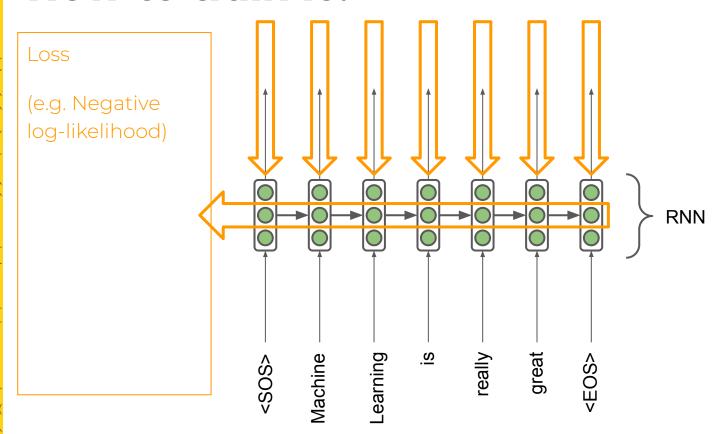
$$h_2 = \sigma(\langle W_{\text{hid}}[h_1, x_1] \rangle + b) = \sigma(\langle W_{\text{hid}}[\sigma(\langle W_{\text{hid}}[h_0, x_0] \rangle + b, x_1] \rangle + b)$$

$$h_{i+1} = \sigma(\langle W_{\text{hid}}[h_i, x_i] \rangle + b)$$

$$P(x_{i+1}) = \operatorname{softmax}(\langle W_{\text{out}}, h_i \rangle + b_{\text{out}})$$

How to train it?





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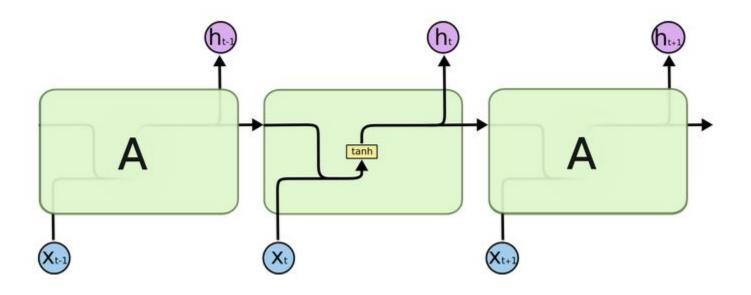
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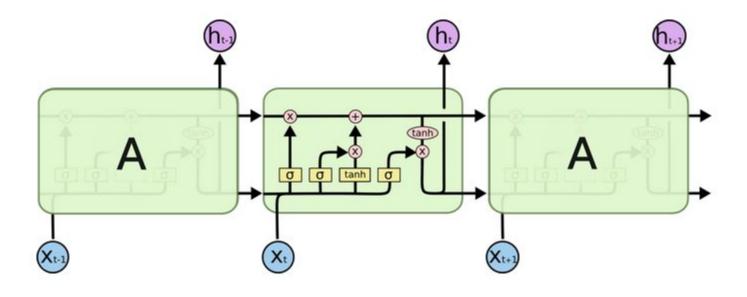
Vanilla RNN



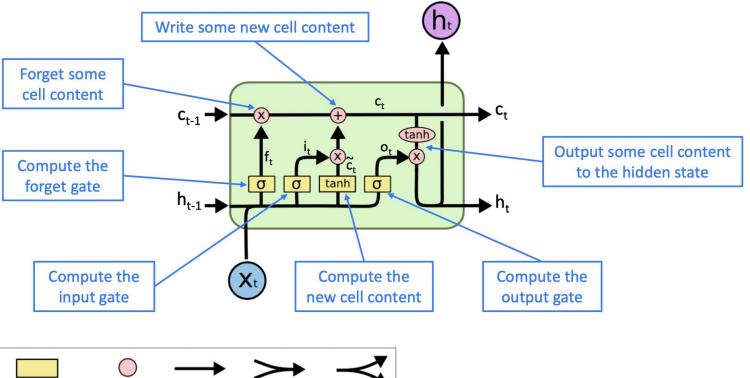


LSTM











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Forget gate: controls what is kept vs forgotten, from previous cell state

Input gate: controls what parts of the new cell content are written to cell

Output gate: controls what parts of cell are output to hidden state

New cell content: this is the new content to be written to the cell

Cell state: erase ("forget") some content from last cell state, and write ("input") some new cell content

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Sigmoid function: all gate values are between 0 and 1

$$egin{aligned} oldsymbol{f}^{(t)} &= \sigma \left(oldsymbol{W}_f oldsymbol{h}^{(t-1)} + oldsymbol{U}_f oldsymbol{x}^{(t)} + oldsymbol{b}_f
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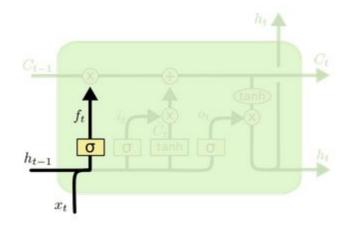
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$$arrow oldsymbol{h}^{(t)} = oldsymbol{o}^{(t)} \circ anh oldsymbol{c}^{(t)}$$

Gates are applied using element-wise product

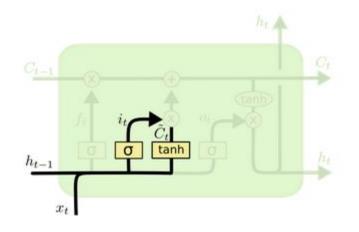
All these are vectors of same length *n*





$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] + b_f\right)$$

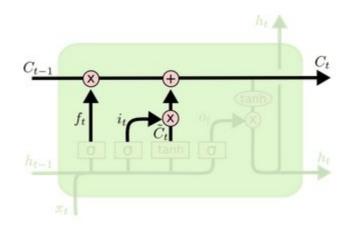




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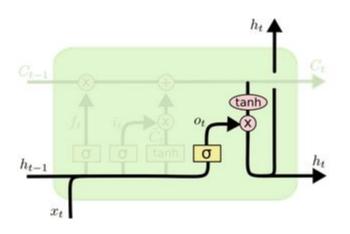
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$





$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$





$$o_t = \sigma (W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh (C_t)$$

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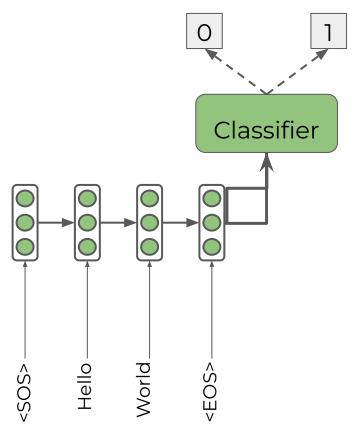
$$m{h}^{(t)} = m{o}^{(t)} \circ anh m{c}^{(t)}$$

Gates are applied using element-wise product

All these are vectors of same length *n*

RNN as encoder for sequential data

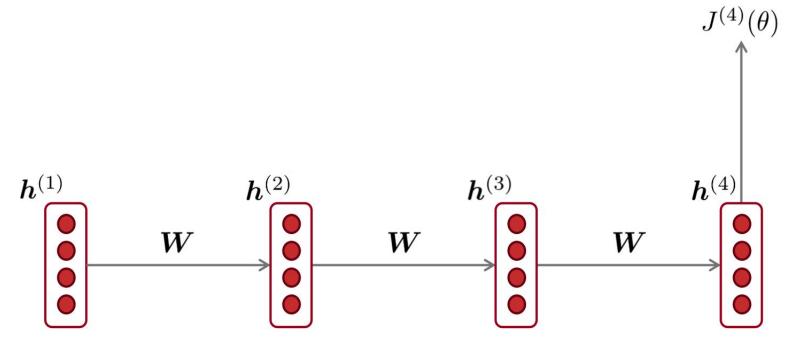




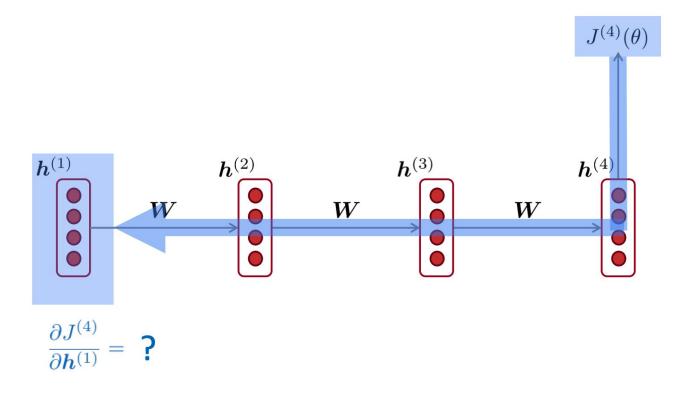
RNNs can be used to encode an input sequence in a fixed size vector.

This vector can be treated as a representation of input sequence.

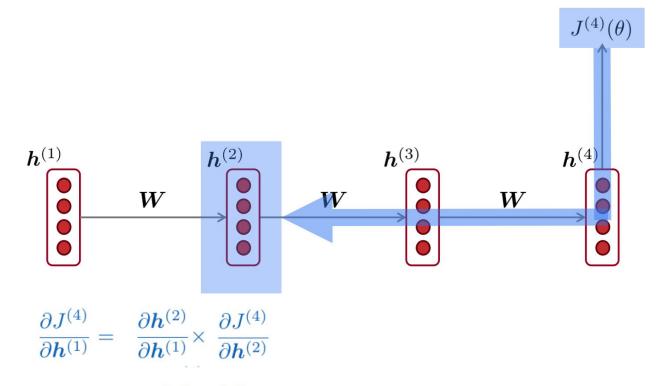






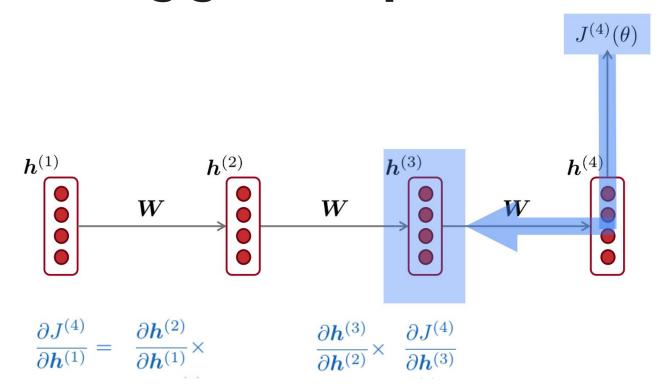






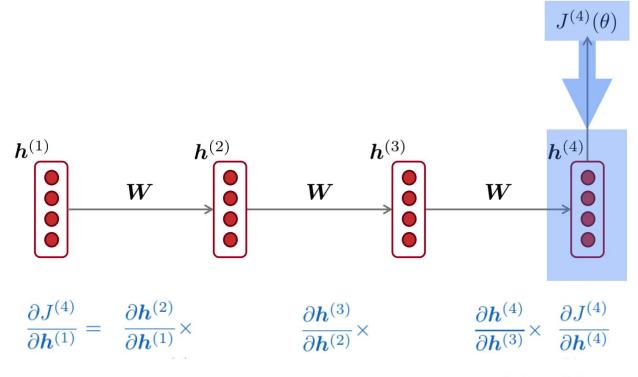
chain rule!





chain rule!



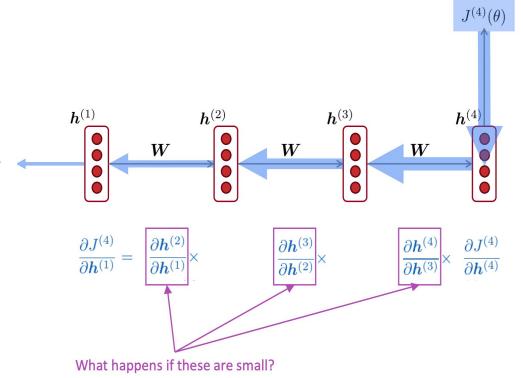


chain rule!



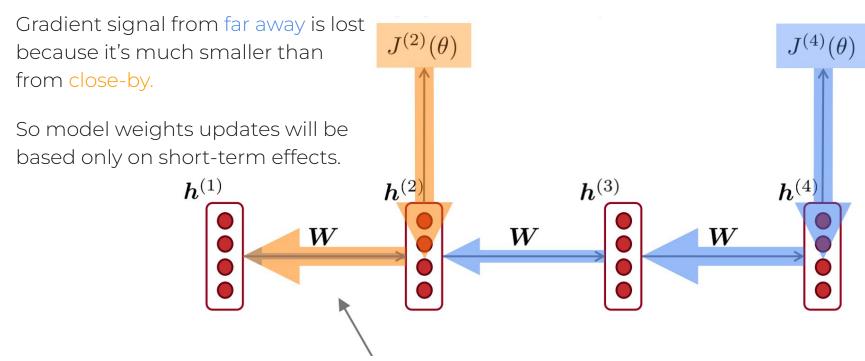
Vanishing gradient problem:

When the derivatives are small, the gradient signal gets smaller and smaller as it backpropagates further



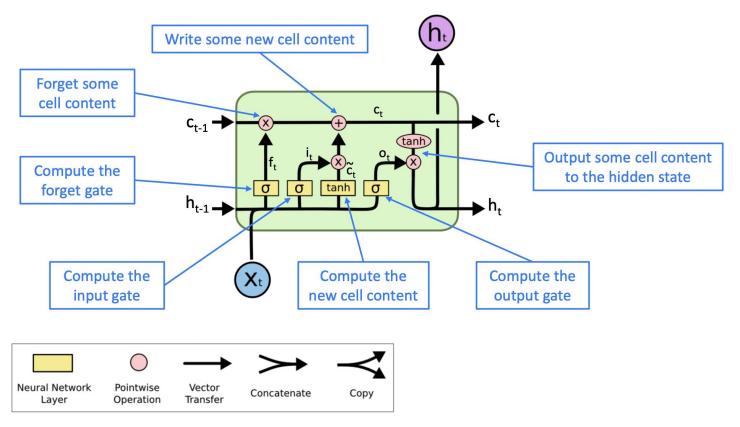
More info: "On the difficulty of training recurrent neural networks", Pascanu et al, 2013 http://proceedings.mlr.press/v28/pascanul3.pdf





Vanishing gradient: LSTM







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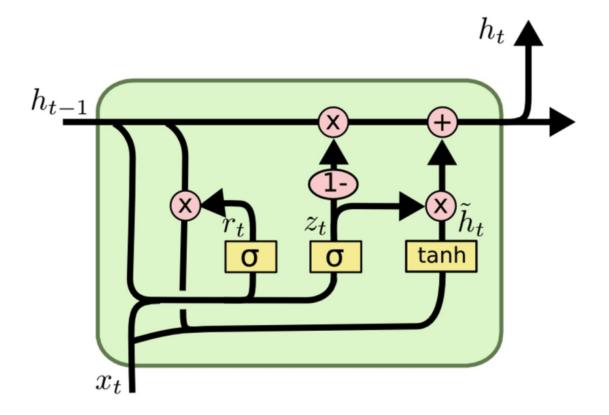
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All these are vectors of same length *n*

Vanishina aradient: GDII





Vanishing gradient: GRU



<u>Update gate:</u> controls what parts of hidden state are updated vs preserved

Reset gate: controls what parts of previous hidden state are used to compute new content

New hidden state content: reset gate selects useful parts of prev hidden state. Use this and current input to compute new hidden content.

Hidden state: update gate simultaneously controls what is kept from previous hidden state, and what is updated to new hidden state content

$$oxed{u^{(t)}} = \sigma \left(oldsymbol{W}_u oldsymbol{h}^{(t-1)} + oldsymbol{U}_u oldsymbol{x}^{(t)} + oldsymbol{b}_u
ight)$$
 $oxed{r^{(t)}} = \sigma \left(oldsymbol{W}_r oldsymbol{h}^{(t-1)} + oldsymbol{U}_r oldsymbol{x}^{(t)} + oldsymbol{b}_r
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$$oldsymbol{ ilde{h}}^{(t)} = anh\left(oldsymbol{W}_h(oldsymbol{r}^{(t)} \circ oldsymbol{h}^{(t-1)}) + oldsymbol{U}_h oldsymbol{x}^{(t)} + oldsymbol{b}_h
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How does this solve vanishing gradient?
Like LSTM, GRU makes it easier to retain info long-term (e.g. by setting update gate to 0)

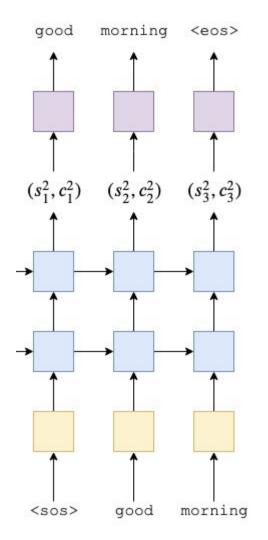
Vanishing gradient: LSTM vs GRU



- LSTM and GRU are both great
 - o GRU is quicker to compute and has fewer parameters than LSTM
 - There is no conclusive evidence that one consistently performs better than the other
 - LSTM is a good default choice (especially if your data has particularly long dependencies, or you have lots of training data)

Outro

- RNN is a great choice for data with sequential structure
- Multi-layer RNN can also be of great use
- Rule of thumb: start with LSTM, but switch to GRU if you want something more efficient





Q & A

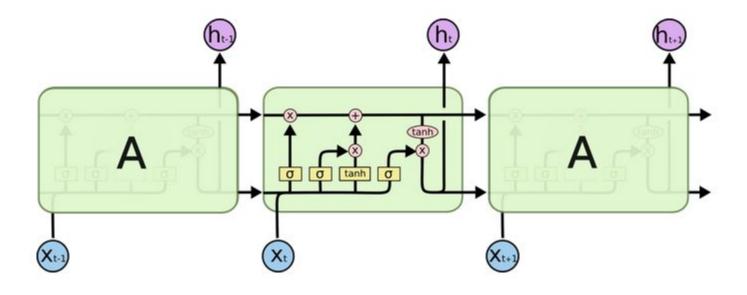


That's all. Feel free to ask any questions.

RNNs, we are coming. Time to generate some names!

Recap: LSTM





Exploding gradient problem



- If the gradient becomes too big, then the SGD update step becomes too big: $\theta^{new}=\theta^{old}-\alpha\nabla_{\theta}J(\theta)$
- This can cause bad updates: we take too large a step and reach a bad parameter configuration (with large loss)
- In the worst case, this will result in Inf or NaN in your network (then you have to restart training from an earlier checkpoint)

Exploding gradient solution



 Gradient clipping: if the norm of the gradient is greater than some threshold, scale it down before applying SGD update

Algorithm 1 Pseudo-code for norm clipping

$$\hat{\mathbf{g}} \leftarrow \frac{\partial \mathcal{E}}{\partial \theta}$$
 $\mathbf{if} \ \|\hat{\mathbf{g}}\| \geq threshold \ \mathbf{then}$
 $\hat{\mathbf{g}} \leftarrow \frac{threshold}{\|\hat{\mathbf{g}}\|} \hat{\mathbf{g}}$
end \mathbf{if}

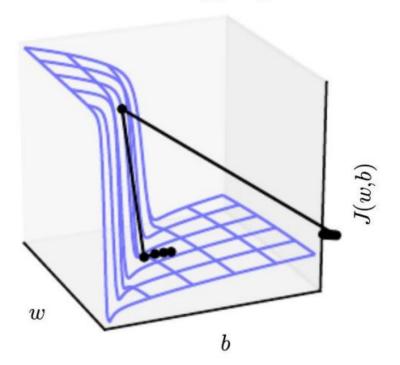
Intuition: take a step in the same direction, but a smaller step

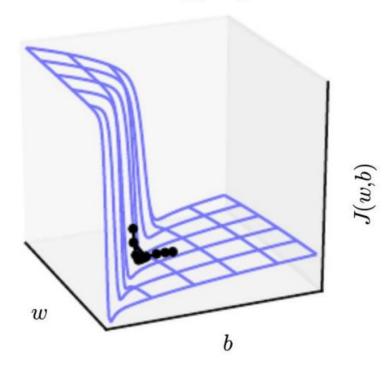
Exploding gradient solution



Without clipping

With clipping





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Vanishing gradient in non-RNN



Vanishing gradient is present in all deep neural network architectures.

- Due to chain rule / choice of nonlinearity function, gradient can become vanishingly small during backpropagation
- Lower levels are hard to train and are trained slower
- Potential solution(but not actually for that problem): dense connections (just like in DenseNet)

Conclusion:

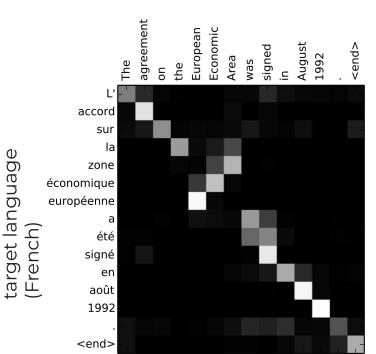
Though vanishing/exploding gradients are a general problem, RNNs are particularly unstable due to the repeated multiplication by the same weight matrix [Bengio et al, 1994]. Gradients magnitude drops exponentially with connection length.

Source: "Learning Long-Term Dependencies with Gradient Descent is Difficult", Bengio et al. 1994, http://ai.dinfo.unifi.it/paolo//ps/tnn-94-gradient.pdf

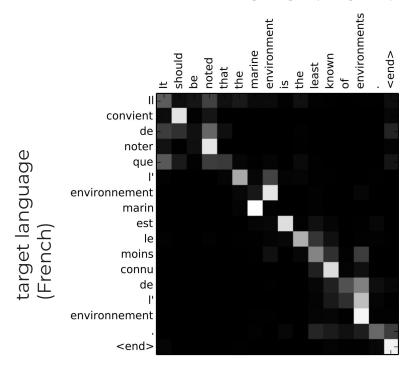
Attention maps in translation



source language (English)



source language (English)



Bahdanau et al. "Neural Machine Translation by Jointly Learning to Align and Translaters"

Vanishing gradient in non-RNN



Vanishing gradient is present in **all** deep neural network architectures.

- Due to chain rule / choice of nonlinearity function, gradient can become vanishingly small during backpropagation
- Lower levels are hard to train and are trained slower
- **Potential solution:** direct (or skip-) connections (just like in ResNet)

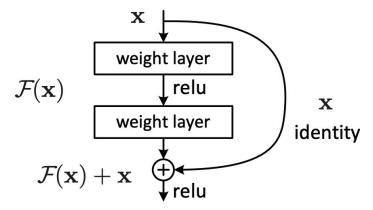


Figure 2. Residual learning: a building block.

Source: "Deep Residual Learning for Image Recognition", He et al, 2015.

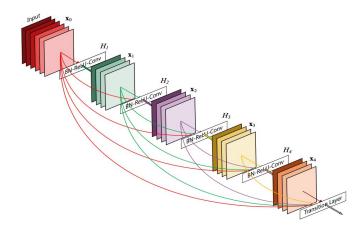
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Vanishing gradient in non-RNN



Vanishing gradient is present in **all** deep neural network architectures.

- Due to chain rule / choice of nonlinearity function, gradient can become vanishingly small during backpropagation
- Lower levels are hard to train and are trained slower
- Potential solution: dense connections (just like in DenseNet)



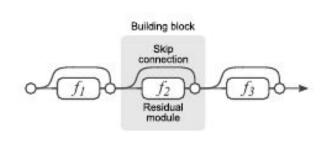
Source: "Densely Connected Convolutional Networks", Huang et al, 2017 https://arxiv.org/pdf/1608.06993.pdf



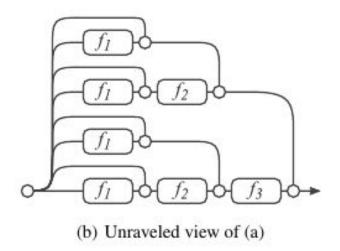
Another view on ResNets and vanishing gradient



"Residual Networks Behave Like Ensembles of Relatively Shallow Networks"



(a) Conventional 3-block residual network



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Revise

- 1. RNN intuitions
- 2. Language models
- 3. Memory concept: LSTM
- 4. RNN as encoder for sequential data
- 5. Vanishing gradient



Thanks for attention!

Questions?



