1 Least Squares

p7

example

$$\|\vec{x}w - \vec{y}\|_2^2 = (\vec{x}w - \vec{y})^T(\vec{x}w - \vec{y})$$

$$= w^2 \vec{x}^T\vec{x} - 2w\vec{x}^T\vec{y} + \vec{y}^T\vec{y}$$

derivative $\frac{\partial}{\partial w}() = 2w\vec{x}^T\vec{x} - 2\vec{x}^T\vec{y} \stackrel{!}{=} 0$

$$\rightarrow w^* = \underline{\underline{\frac{\vec{x}^T\vec{y}}{\vec{x}^T\vec{x}}}}$$

p8

example

$$\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} w = \begin{bmatrix} 0 \\ 1.5 \\ 2.2 \end{bmatrix}$$

$$\frac{d}{dw}\left[(w - 1.5)^2 + (2w - 2.2)^2\right] = 0$$

$$w^* = \underline{\underline{1.18}}$$

thus:

$$y = 1.18 \cdot 10 = \underline{\underline{11.8}}$$

T 11

example

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \quad \hat{y} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\vec{w}^* = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 5 & 25 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = 9$$

p 16

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 4 \\ 1 & 2 & 16 \end{bmatrix} \quad \hat{y} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\vec{w}^* = \begin{bmatrix} 6.33 \\ -6.0 \\ 0.66 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 10 & 100 \end{bmatrix} \begin{bmatrix} 6.33 \\ -6. \\ 0.66 \end{bmatrix} = 13$$

p 17

$$y = \omega_1 \exp(\omega_2 x + \omega_3 x^2) \quad | \ln \circ$$

$$\ln y = \ln(\omega_1) + \ln \exp(\omega_2 x + \omega_3 x^2)$$

$$\ln y = \ln \omega_1 + \omega_2 x + \omega_3 x^2$$

$$\Rightarrow \tilde{y} = \tilde{\omega}_1 + \omega_2 x + \omega_3 x^2$$

p 18

$$X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$\Rightarrow$ if we can compute the eigenvalues ~~sig~~ singular vals
they are the same as the

$\lambda_1 = 0.5, \ \lambda_2 = 1, \ \lambda_3 = 2$

$K = \frac{2}{0.5} = 4$ „well conditioned" p 20

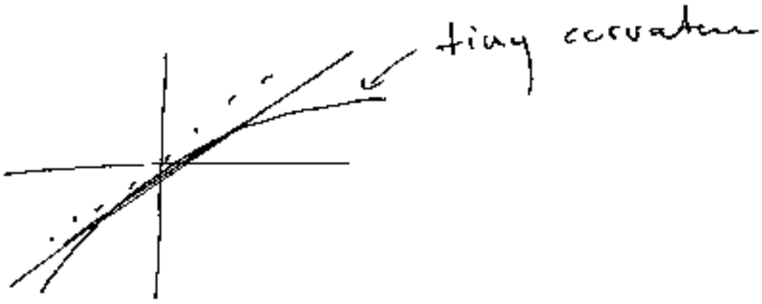
$$\| X\vec{\omega} - \vec{y} \|_2^2 = (X\vec{\omega} - \vec{y})^T (X\vec{\omega} - \vec{y})$$
$$= \vec{y}^T\vec{y} - \vec{y}^T X \vec{\omega} - \vec{\omega}^T X^T \vec{y} + \vec{\omega}^T X^T X \vec{\omega}$$

derivative $\frac{\partial}{\partial \vec{\omega}}(\cdot) = -2X^T\vec{y} + 2X^T X\vec{\omega} = 0$

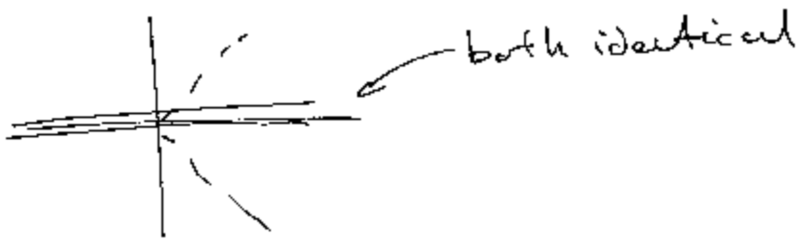
$$\Rightarrow \omega^* = \underline{(X^TX)^{-1}} X \vec{y}$$

p 21

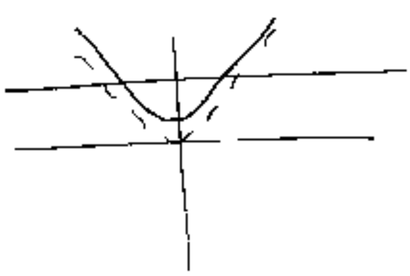
a)



b)



c)



⑤

p22

$$\omega_1^* = \arg\min \sum_{i\neq j} (\cancel{x_i} + \omega_1 - \cancel{x_i} - \tilde{\omega}_1)^2$$

$$+ (\cancel{x_j} + \omega_1 - \cancel{x_j} - \tilde{\omega}_1 - \varepsilon_j)^2$$

$$\frac{d}{d\omega_1}(.) = \cancel{2}\sum_{i\neq j}(\omega_1 - \tilde{\omega}_1) + \cancel{2}(\omega_1 - \tilde{\omega}_1 - \varepsilon_j) \stackrel{!}{=} 0$$

$$= (N-1)(\omega_1 - \tilde{\omega}_1) + \omega_1 - \tilde{\omega}_1 - \varepsilon_j = 0$$

$$\longrightarrow \omega_1 = \underline{\underline{\frac{\tilde{\omega}_1 N + \varepsilon_j}{N}}}$$

p23

$X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$ $\rightsquigarrow$ rank = 2 $\Rightarrow$ not invertible

decomposition (from the tips)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

hence:

$$X^{-1} \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}}}$$

then:

$$X\vec{\omega} = \vec{y} \Rightarrow \vec{\omega}^* = X^{-1}\vec{y} = \underline{\underline{\begin{bmatrix} 2 \\ 10 \\ 0 \end{bmatrix}}}$$