ETH zürich

Title

- Text

Exercise 4: Lagrange Polynomials & Cubic Splines

MAD
bacdavid@student.ethz.ch

Outline

1. Information
2. Goals
3. Theory/ Recap
4. Exercises

Information

ETH zürich

General

- Lecture material & problem sets available here
- Tutorial material available here

Goals

Goals of Today

- Understand Lagrange polynomials
- Understand Lagrange interpolation
- Understand connection between polynomial degree & datapoints
- Understand derivation of cubic splines

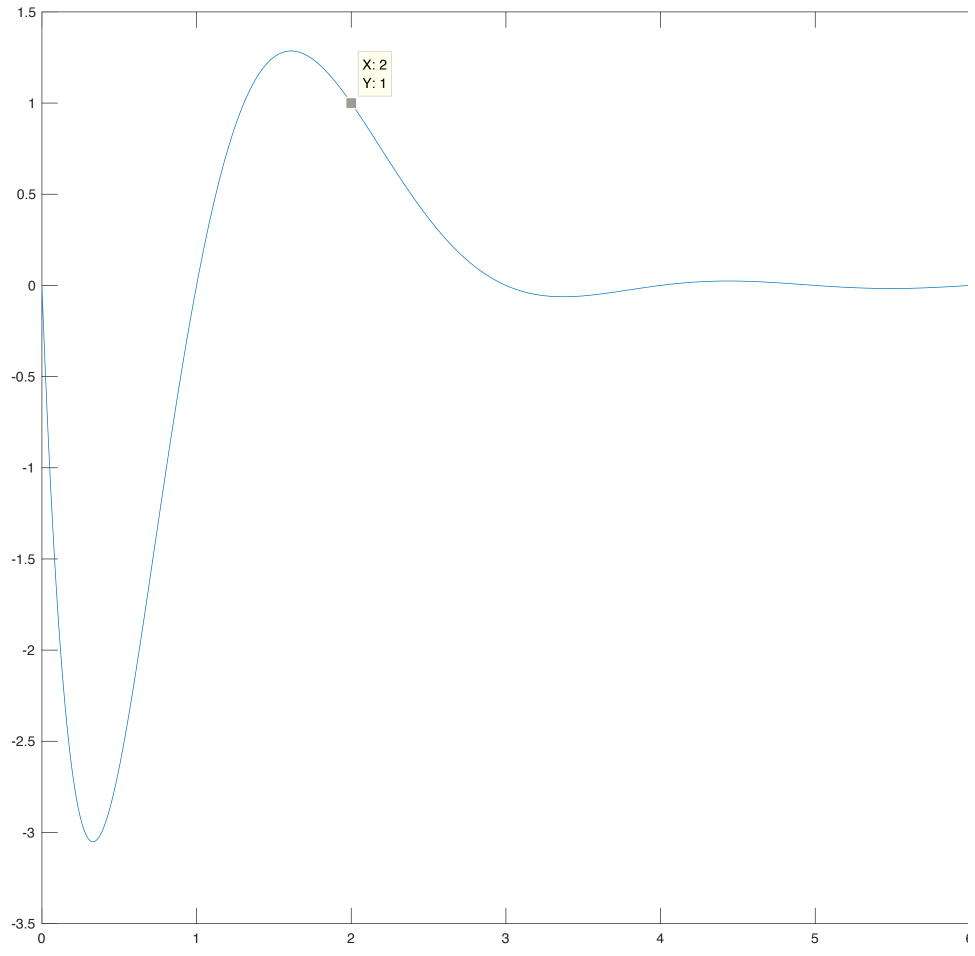
Theory / Recap

ETH zürich

Lagrange Polynomial

- Is zeros everywhere, except at the position x_i it is equal to 1:

$$l_i(x) = \prod_{1 \le m \le N, m \neq i} \frac{x - x_m}{x_i - x_m}$$

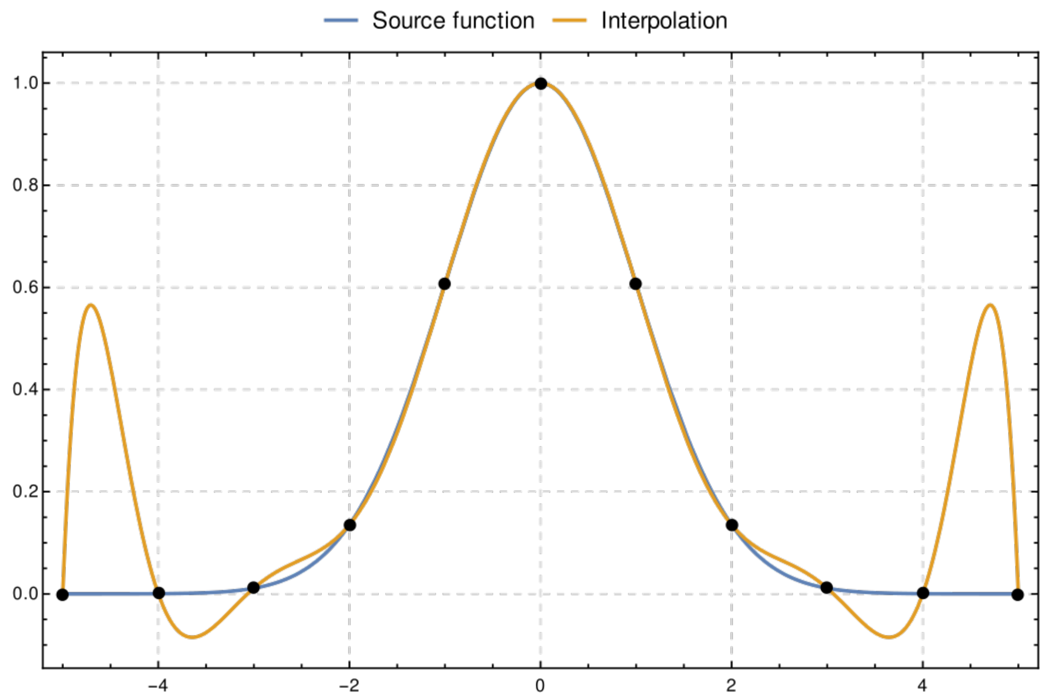


ETH zürich

Lagrange Interpolation

- Dataset $D = \{(x_1, y_1), \ldots, (x_N, y_N)\}$
- Want to create a function that goes through all points in D - how?
- Lagrange interpolation:

$$L(x) = \sum_D y_i \cdot l_i(x)$$



https://en.wikipedia.org/wiki/Lagrange_polynomial#/media/File:Runge%27s_phenomenon_in_Lagrange_polynomials.svg

ETH zürich

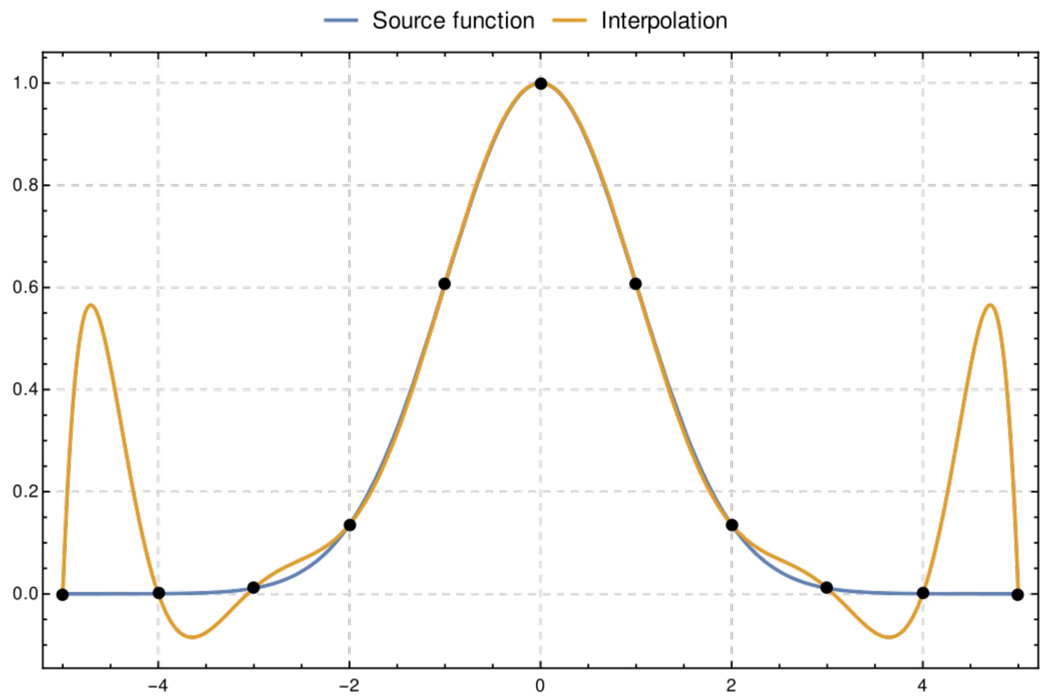
Example 1: Lagrange Polynomial

- $D = \{(1,1),(2,2)\}$

- How many degrees will the Lagrange polynomial have?
- Can you guess the function?
- Write down the Lagrange Polynomial

Degree of Lagrange Polynomial

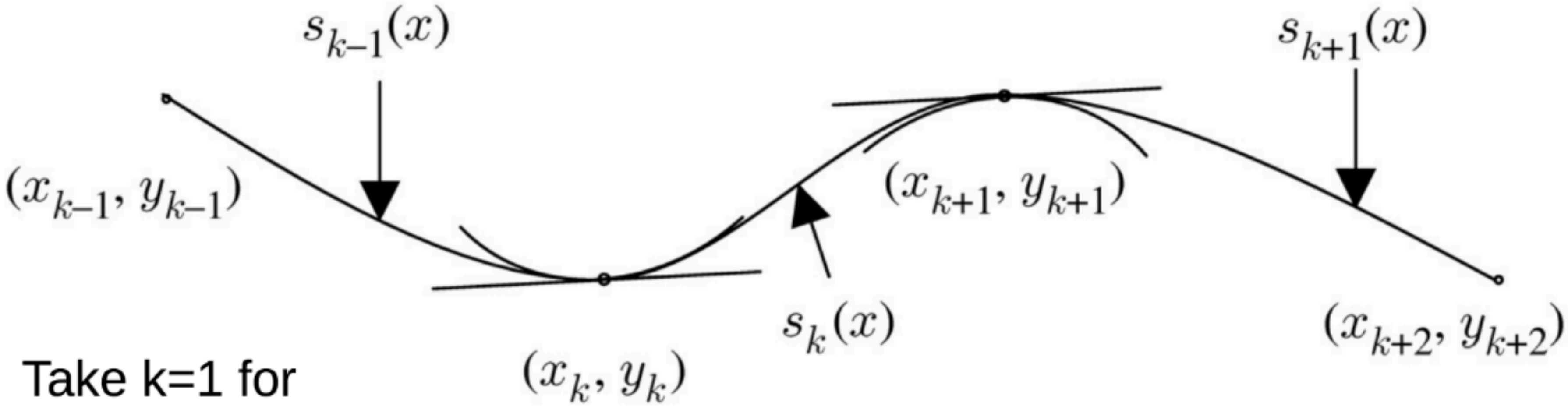
- Lagrange Polynomial has a degree of $N - 1$
- High degree polynomial tend to oscillate: BAD!
- This is called **Runge's** phenomenon



https://en.wikipedia.org/wiki/Lagrange_polynomial#/media/File:Runge%27s_phenomenon_in_Lagrange_polynomials.svg

Solution to Runge's: Cubic Splines

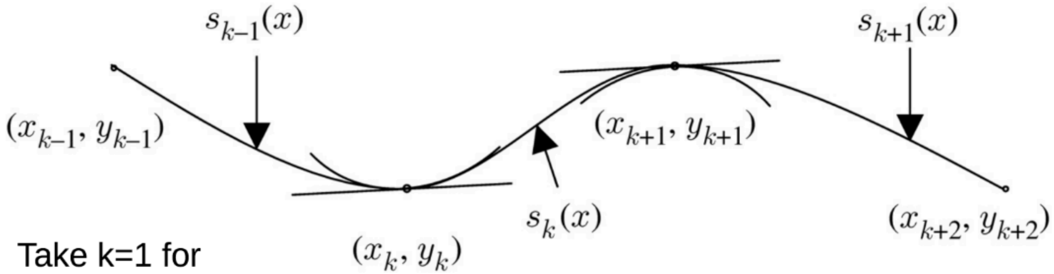
- Set of piecewise third-order polynomials
- Force them to be C^2 continuous – what does this mean?



Take k=1 for this example

ETH zürich

Example 2: Cubic Spline



Take k=1 for this example

- 4 data-points $D = \{(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)\}$

- How many piecewise functions required?
- Write them down with parameters (a – for first spline, b – for second spline, etc.)
- Write down the constraints (C^2 cont.)
- How many unknows? How many constraints?
- What additional constraints can you come up with?

Example 3: Quadratic spline with 3 datapoints

- 3 data-points $D = \{(0,0),(1,1),(2,0)\}$
- Use quadratic functions, eg.: $S_1(x) = a_1 + a_2x + a_3x^2$
- Force C^2 cont. at $(1,1)$

- Write down the 2 functions.
- Write down the constraints.
- Write in matrix form: $\mathrm{A} \cdot [a_1, a_2, a_3, b_1, b_2, b_3]^{\mathrm{T}} = \mathrm{b}$ Don’t solve!
- Look at the problem again – solve for the parameters: It should be very simple!

ETH zürich

Exercises

Q1

- Implement Lagrange Interpolation
- Revisit Runge’s phenomenon

Q2

- Derive Cubic Splines

ETH zürich

Q3

- Implement Cubic Splines

Questions?