

Title

- Text

Exercise 5: B-Splines & NURBS

MAD

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Outline

1. Information
2. Goals
3. Theory/ Recap
4. Exercises

Information

General

- Lecture material & problem sets available [here](#)
- Tutorial material available [here](#)

Goals

Goals of Today

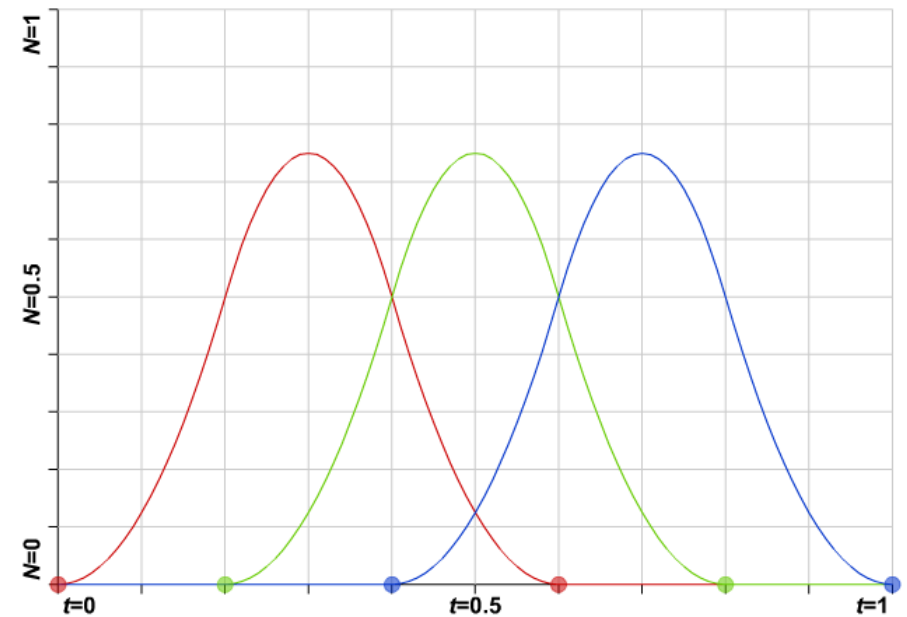
- Know how to construct B-Splines
- Know some properties of B-Splines
- Know what NURBS are and how to construct them
- Know how B-Splines are used in NURBS

Theory / Recap

B-Splines

- Basis function to define any other spline
- We are free to design them:
 - How many basis functions do we want? M
 - Of what degree should they be? d
 - From where to where they should run? t
- What's M, d, t ?

$$B_{i,d,t}(x)$$



Construction of B-Splines

- Recursive definition:

$$B_{i,0,\mathbf{t}} = \begin{cases} 1 & \text{if } t_i \leq x < t_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$B_{i,d,\mathbf{t}}(x) = \frac{x-t_i}{t_{i+d}-t_i} B_{i,d-1,\mathbf{t}}(x) + \frac{t_{i+d+1}-x}{t_{i+d+1}-t_{i+1}} B_{i+1,d-1,\mathbf{t}}(x)$$

- With:

d : degree of that B-spline

i : iterator 1, ..., M

\mathbf{t} : knot vector; in order; with $M + d + 1$ entries

ex: $\mathbf{t} = (0, 0.2, 0.4, 0.6, 0.8, 1)$

Example 1: Construction of B-Splines

- $M = 1$
- $d = 1$
- \mathbf{t} is equally spaced between 0 and 1
- Write down the function of the B-spline $B_{1,1,\mathbf{t}}(x)$
- Plot that spline
- Assume $M = 2$ and \mathbf{t} is equally spaced between 0 and 2, plot that as well

A few things on B-Splines

- Only non-zero between $t_i \leq x \leq t_{i+d+1}$
- Continuous derivatives up to degree $d - 1$, if not overlapping knots
- If overlapping, derivative continuity drops by 1
- Use $0/0 = 0$ if knot values are repeated

Non-Uniform Rational B-Splines (NURBS)

- Start of with data vectors $\mathbf{p}_i = (x_i, y_i)^T, i = 1, \dots, N$
- Set $M = N$, decide on d , and space \mathbf{t} (how many entries again?)
- Construct NURBS:

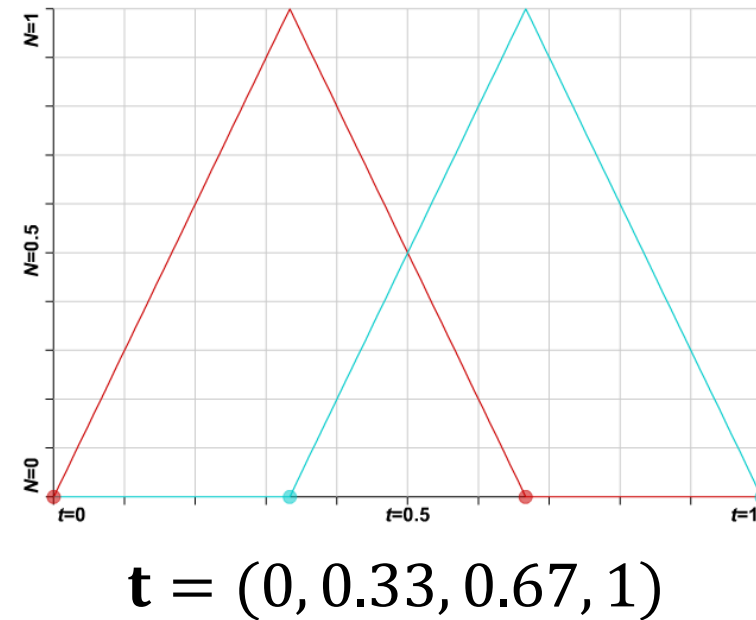
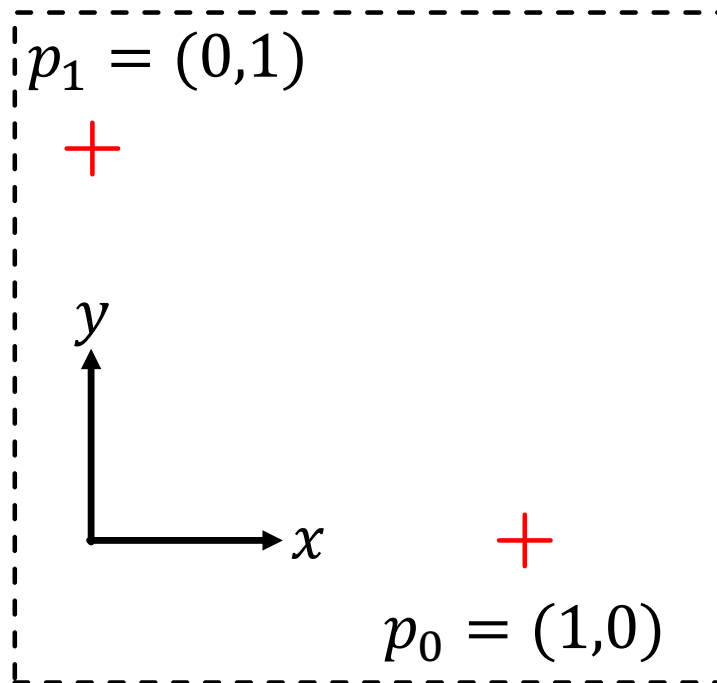
$$\mathbf{p}(s) = \sum_{i=1}^N R_{i,d,\mathbf{t}}(s) \mathbf{p}_i$$

- With $R_{i,d,\mathbf{t}} = \frac{B_{i,d,\mathbf{t}}(s)w_i}{\sum_{j=1}^N B_{j,d,\mathbf{t}}(s)w_j}$ in order to weight data

A few things on NURBS

- If non-weighted: $R_{i,d,t}(s) = B_{i,d,t}(s)$
- Due to the fact that $\sum_j B_{j,d,t}(s) = 1$

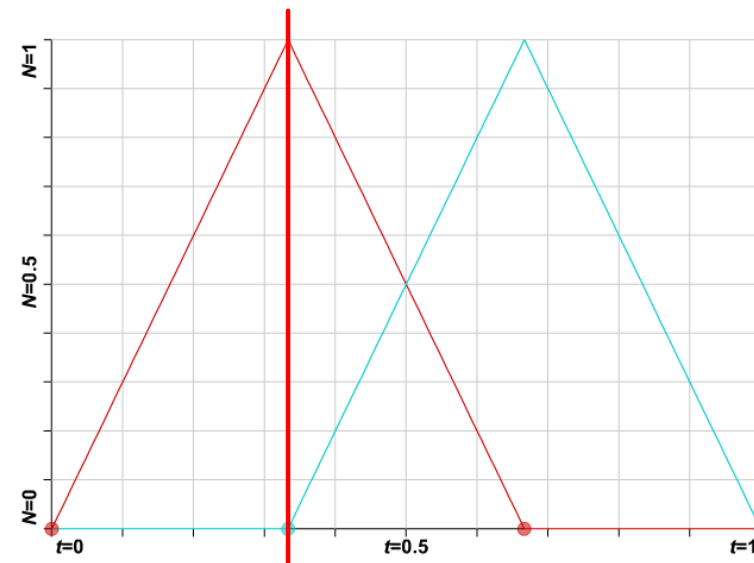
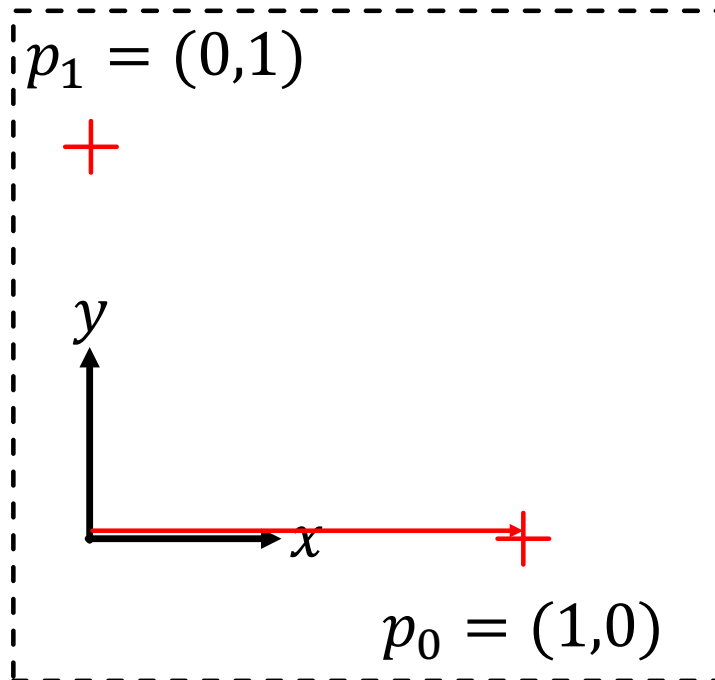
Example 2: Construction of NURBS



- Draw the NURB

Example 2: Construction of NURBS

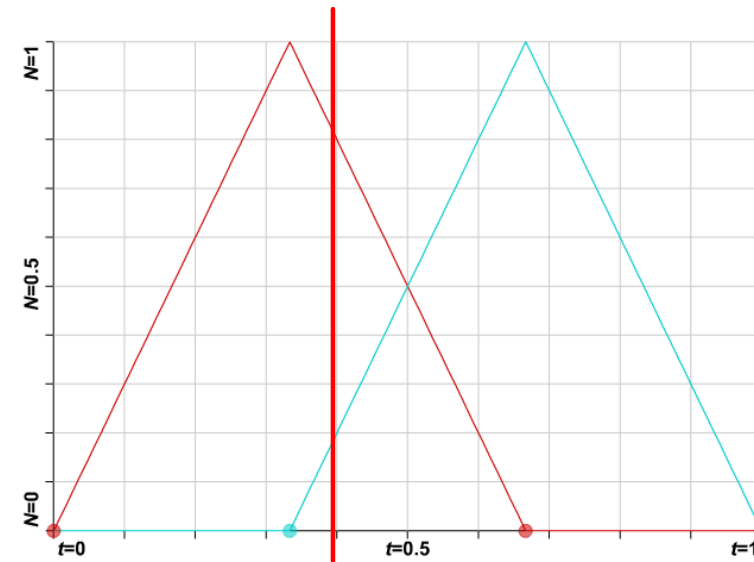
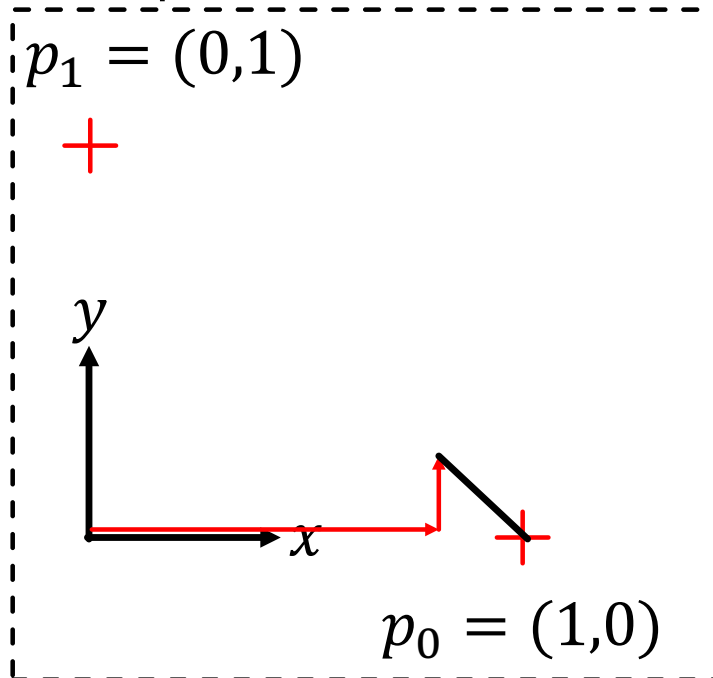
- Inspect a few points



David Bachmann

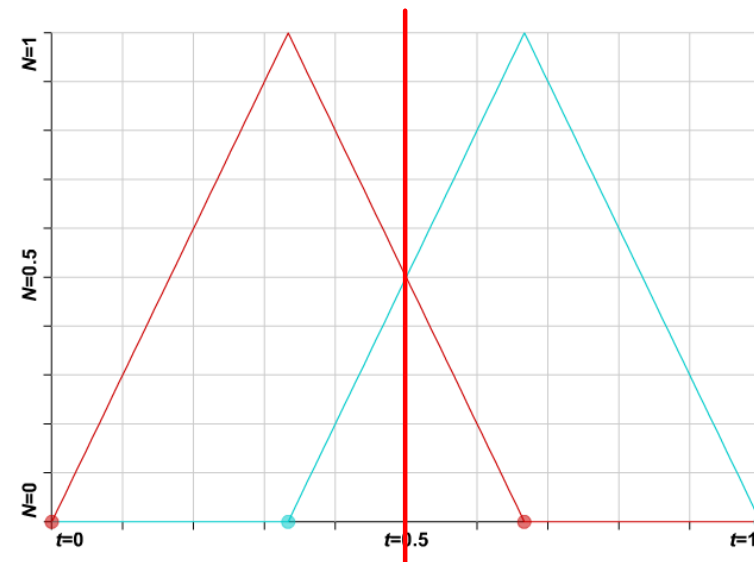
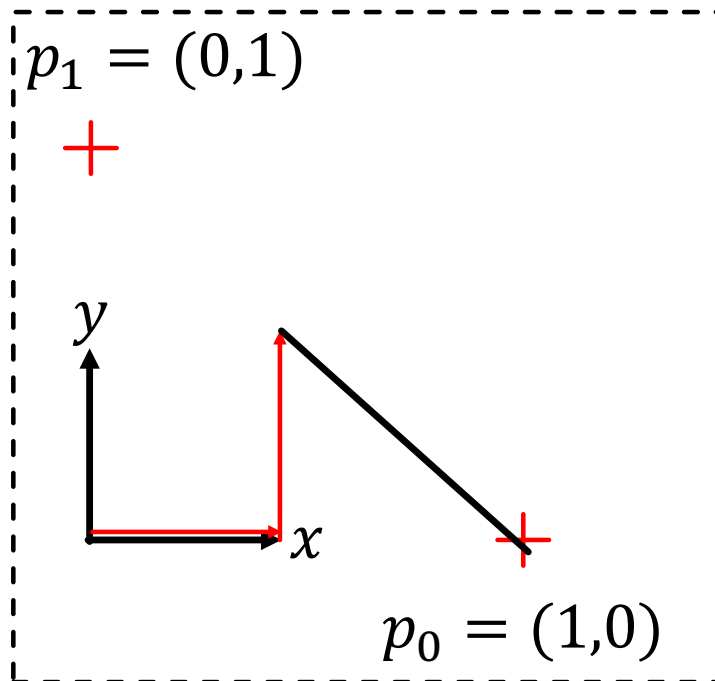
Example 2: Construction of NURBS

- Inspect a few points



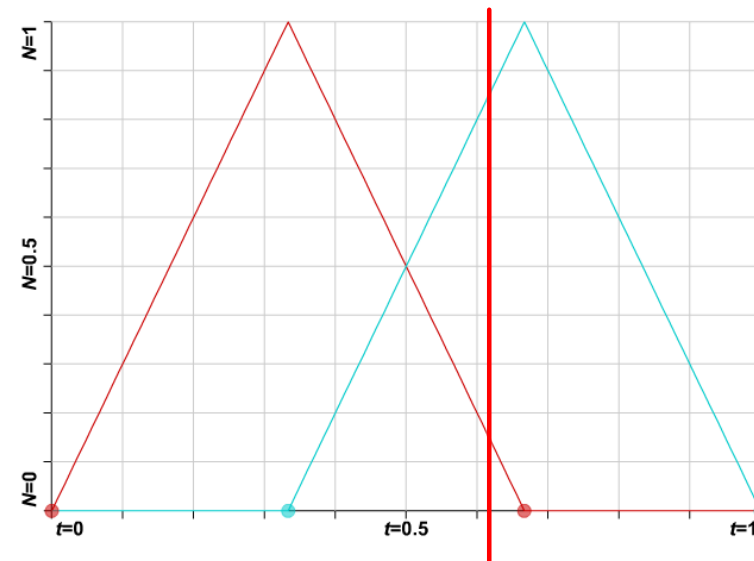
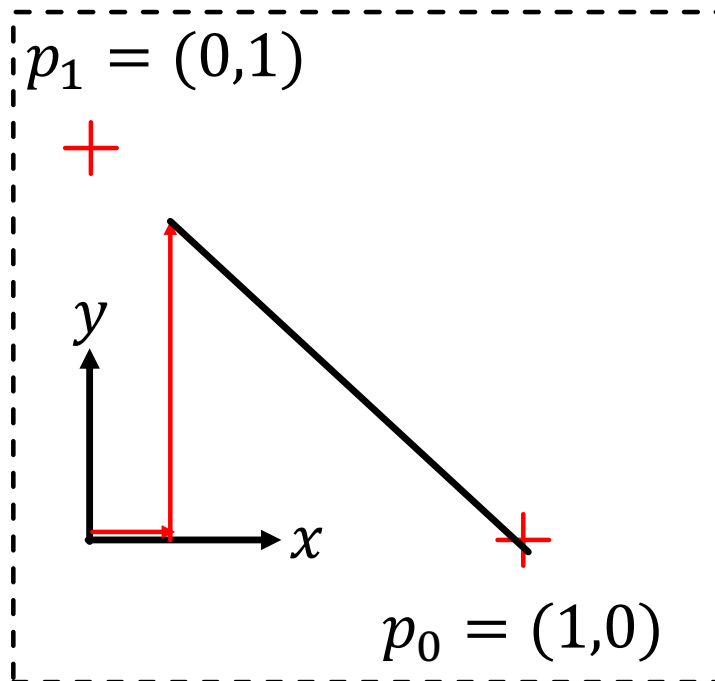
Example 2: Construction of NURBS

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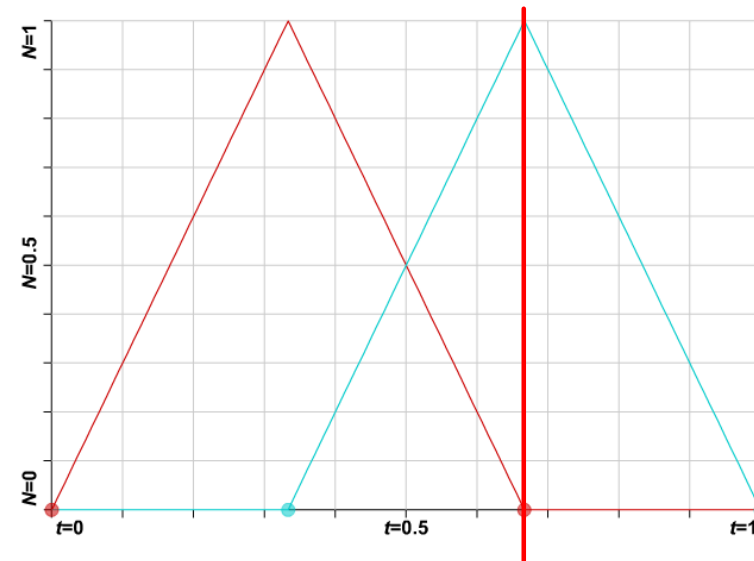
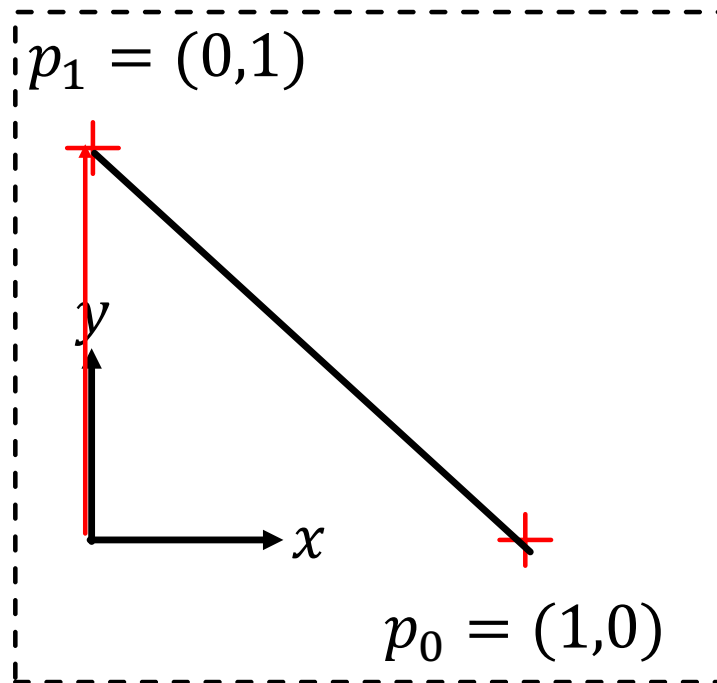
Example 2: Construction of NURBS

- Inspect a few points



Example 2: Construction of NURBS

- Inspect a few points



Example 3: Intuition for NURBS

- <http://geometrie.foretnik.net/files/NURBS-en.swf> (enable flash)
- Select «Non-Uniform B» from presets
- If we move p_4 which parts of the curve are unaffected?

Exercises

Q1

- Write down B-Splines

Q2

- Construct NURBS on a data-set

Questions?

