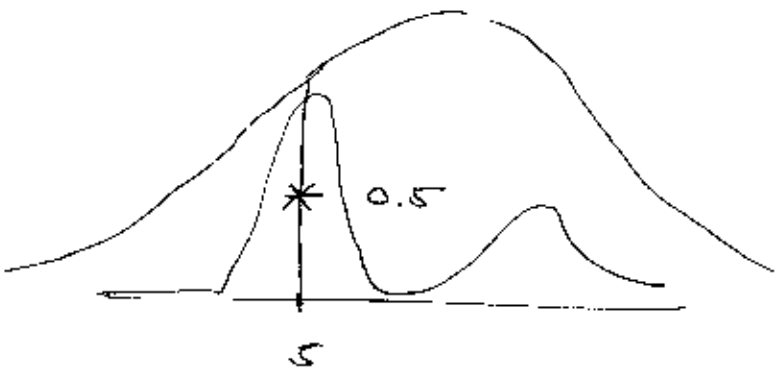
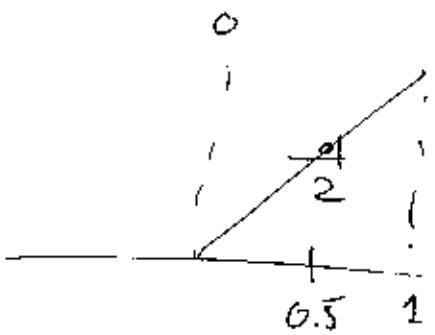
7- Probability Review

P 12



⇒ keep it is inside $p(x)$

P 13



- $p(y) \geq p(x_1)$ ✓ move (we always move if $p(y) \geq p(x_t)$)

  $B = \{0.5, 0.6\}$

- $0.9 \cdot p(x_1) > p(y)$ don't move

  $= 0.9 \quad = 0.8$

  $B = \{0.5, 0.5\}$

P 15

- $\int_{-1}^{1}\int_{-1}^{1} c\, dx\, dy \longrightarrow 4c = 1$ ← has to be due to constraint on prob. distribution

$$c = \frac{1}{4}$$

- $\int p(x,y)\, dy = p(x)$ ← marginalize

$$p(x) = \frac{1}{2}$$

$$P(x>0) = \int_0^b p(x)\, dx = \int_0^1 p(x)\, dx = \frac{1}{2}$$

tip

P 16

- $P(A|R) = \dfrac{P(R|A)\cdot P(A)}{P(R)}$ (1)

"given it's red what's the probability that it's from A"

marginalization

$$P(R) = \sum_x p(R,x) = \sum_x p(R|x)\cdot p(x)$$

both jars

plug in (1)

- $P(A|R) = \dfrac{P(R|A)\cdot P(A)}{\sum_x p(R|x)\cdot p(x)}$

since $p(x) = \frac{1}{2}$

$$= \frac{P(R|A)}{\sum_x p(R|x)} = \frac{7}{8}$$

count in jars:

$P(R|A) = \frac{7}{10}$

$P(R|B) = \frac{1}{10}$

p 17

- Center of mass → Expectation value
- Moment of inertia → Variance

p 18

$$p = \frac{1}{6}$$

$$\mathbb{E}[x] = \frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \frac{1}{6}\cdot 3 + \frac{1}{6}\cdot 4 + \frac{1}{6} 5 + \frac{1}{6}\cdot 6$$

$$= 3.5$$

p 19

- simulate a probabilistic model
- position ~~velocity~~ of rolling ball = constant velocity $\cdot \epsilon$ + noise

(noise: from random distribution sampled)

p 21

$$p(\text{spam} \mid \text{"gold"}) = \frac{p(\text{"gold"} \mid \text{spam})\, p(\text{spam})}{p(\text{"gold"})}$$

$$p(\text{"gold"}) = p(\text{"gold"} \mid \text{spam}) \cdot p(\text{spam}) + p(\text{"gold"} \mid \text{no spam}) \cdot p(\text{no spam})$$

$$\left.\begin{array}{l} p(\text{no spam}) + p(\text{spam}) = 1 \\ \end{array}\right\} \Rightarrow p(\text{no spam}) = 0.8$$

$$p(\text{"gold"}) = 0.6 \cdot 0.2 + 0.01 \cdot 0.8 = 0.128$$

$$p(\text{spam} \mid \text{"gold"}) = \frac{0.6 \cdot 0.2}{0.128} = 0.9375$$

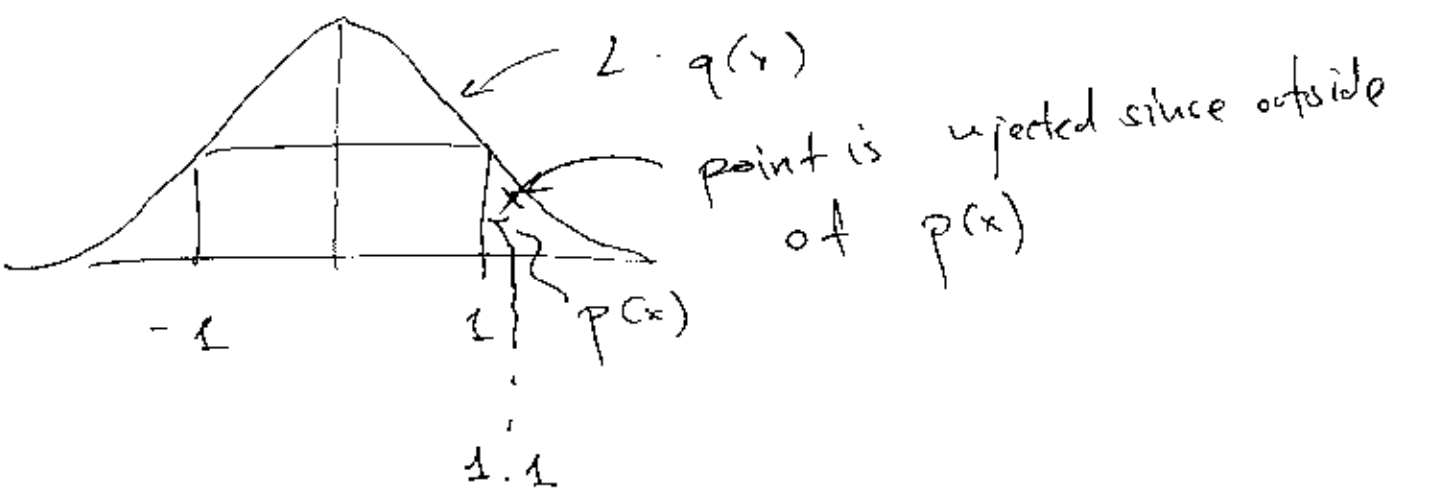
p 22

$$L = \sup_x \left(\frac{p(x)}{q(x)} \right)$$

$$= \sup_x \left(\frac{\sqrt{\pi}}{2} \cdot e^{x^2} \right) = \frac{\sqrt{\pi}}{2} \sup_x \left(e^{x^2} \right)$$

$$= \frac{\sqrt{\pi}}{2} e^{1^2}$$

only no-zero on $[-1, 1]$; the function e^{x^2} is the largest thus at -1 or 1.



p 23

$y_1 = x_1 + s_1 = 0.6 \Rightarrow B = \{0.6\}$

y_1 from tips

0.6 accepted since "up hill"

$y_2 = 0.6 - 0.2$

$= 0.4$

$0.3 \cdot p(0.6) \overset{?}{\leq} p(0.4)$

$0.36 \overset{\checkmark}{\leq} 0.8 \Rightarrow B = \{0.6, 0.4\}$

P29

$$p \approx \frac{n_{\text{hit the line}}}{n_{\text{total}}}$$

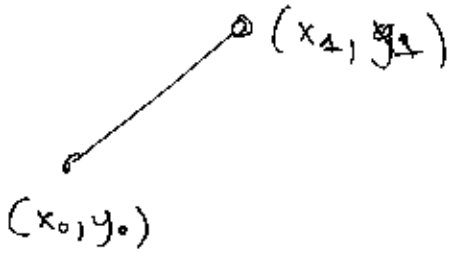
1) Simulate needles:

a) Pick random point (x_0, y_0)

b) Pick random angle

c) $x_1 = x_0 + \cos\theta$

$y_1 = y_0 + \sin\theta$



2) check if $y_{\text{line}1} \in [y_0, y_1)$

$\vdots$

$y_{\text{line}3} \in [y_0, y_1]$

3) update counters.