

# Exercise 1: Linear Least Squares

MAD

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# Outline

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# Information

## General

- Slides by LAB are on the website
- These slides: <https://polybox.ethz.ch/index.php/s/9NFCvtriBbRBDnS> (pw: mad18)

# Programming

## Install Python & Packages

1. Install Pycharm (<https://www.jetbrains.com/pycharm/>)
2. Install Anaconda (<https://www.anaconda.com/download>)
3. Create environment `conda create --name mad`
4. Activate environment `source activate mad`
5. Install in that environment with pip `pip install tensorflow`
6. In Pycharm adjust the interpreter: Preferences/Project:NAME/Project Interpreter
7. Chose from the drop down or navigate to the env folder; select the python version in that folder

# Numpy

- Should be preinstalled, else use pip
- Similar to matlab, cheat sheet  
([https://sebastianraschka.com/blog/2014/matrix\\_cheatsheet\\_table.html](https://sebastianraschka.com/blog/2014/matrix_cheatsheet_table.html))
- Examples:  

```
A = np.zeros(10), A = np.zeros((10,2))  
np.dot(A,b), np.transpose(A), np.linalg.inv(A)  
random.uniform(0,1), random.seed(1234)
```
- Reference i.e. google “numpy transpose”  
(<https://docs.scipy.org/doc/numpy/reference/generated/numpy.transpose.html>)

## Plotting data

- Use matlab or matplotlib (python library)
- Matlab:
  1. Create a file in python `file = open('mad_ex1', 'w')`
  2. Write to file `file.write(str(x) + ',' + str(y) + str(\n))`
  3. Close file `file.close()`
  4. Import to matlab
- Matplotlib: examples on internet



## Questions regarding programming

- Think, search, think, question
- Every question has been answered:



# Exercises

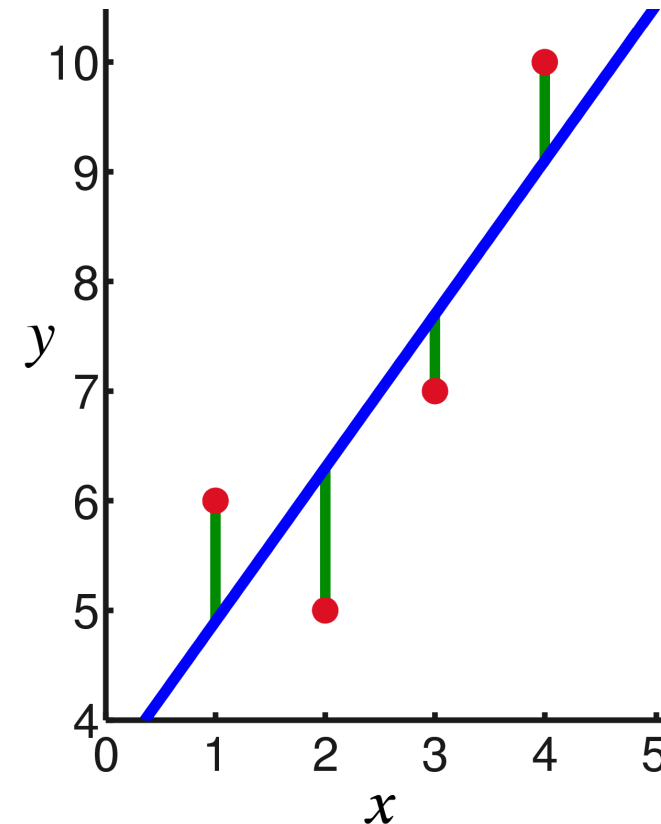
## Exercise 1: Linear Least Squares in 2d Theory

- $D = \{x_i, y_i\}$
- $f: x \rightarrow y$

1. Design a function, i.e.  $y = px$
2. Introduce data,  $\underbrace{\begin{pmatrix} 1 \\ 5 \end{pmatrix}}_{\bar{y}} = p \cdot \underbrace{\begin{pmatrix} 2 \\ 11 \end{pmatrix}}_a$
3. Cannot solve for  $p$ : Minimize  $E$   

$$E = (2p - 1)^2 + (11p - 5)^2 \Rightarrow \frac{dE}{dp} = 0$$
4. Solve for  $p$ :  $\bar{p} = \frac{a^T \bar{y}}{a^T a}$
5. Return to design:  $y = \bar{p}x$

- Why squared error?



## **Exercise 1: Linear Least Squares in 2d**

### **Questions**

- Write the code
- Find out how outliers & noise influence the fit “how robust is the method”

## Exercise 2: Linear Least Squares in 3d Theory

- $D = \{x_i, y_i, z_i\}$
- $f: x, y \rightarrow z$

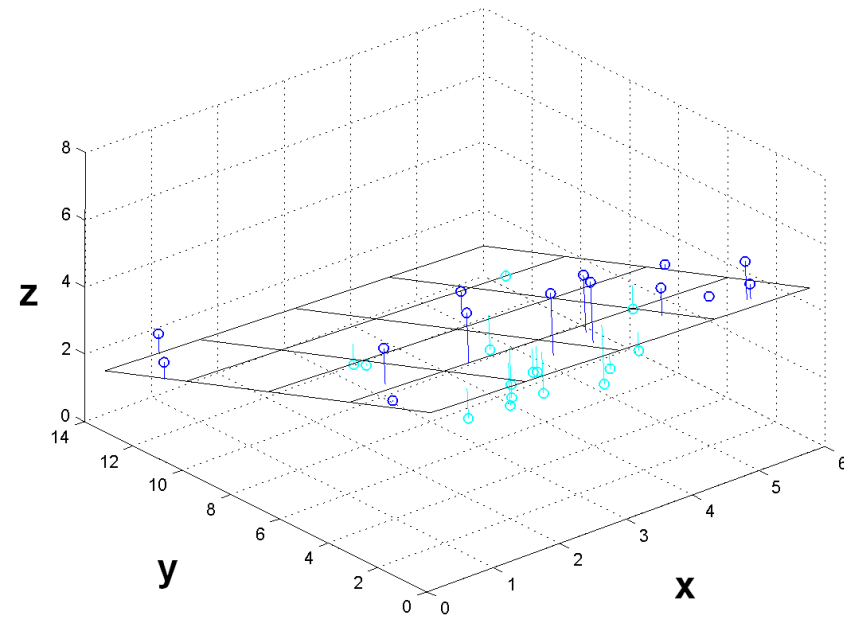
1. Function Design:  $z = a + bx + cy$

2. Rewrite:

$$\underbrace{\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \vdots & \vdots & \vdots \\ 1 & x_N & y_N \end{bmatrix}}_A \cdot \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{\bar{p}} = \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{bmatrix}}_{\bar{z}}$$

3. Minimize error:  $A^T A \bar{p} = A^T \bar{z}$

4. Insert  $\bar{p}$  into design



## Exercise 2: Linear Least Squares in 3d

### Questions

- Write the code
- Check how  $N$  affects the estimate

## Exercise 3: LSQ fit and noise

- Advanced (no points assigned), check lab slides

# Questions?

