2 Non Linear Systems

example ①

p 9

$$x^3 = 10 \longrightarrow x^3 - 10 = 0$$

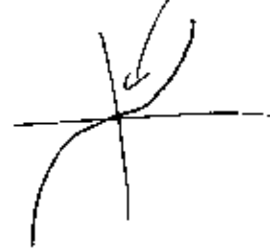
$x^0 = 2$ since $2^3 = 8 \approx 10$

$$x^1 = x^0 - \frac{(x^0)^3 - 10}{3(x^0)^2} = 2 - \frac{8 - 10}{12} = \underline{\underline{2.17}}$$

exact $\sqrt[3]{10} = 2.15$ ← not bad!

chose $x^0 = 0$

$$x^1 = x^0 - \frac{(x^0)^3 - 10}{3(x_i^0)^2}$$



flat gradient
no update
possible !
-10

p 12

example

$$x_1 + x_2 - x_1 x_2 + 2 = 0$$

$$x_1 \cdot \exp(-x_2) - 1 = 0$$

Jacobian: $$\bar{J}(\bar{x}) = \begin{bmatrix} 1 - x_2 & 1 - x_1 \\ \exp(-x_2) & -x_1 \exp(-x_2) \end{bmatrix}$$

$$\bar{J}(x^0) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

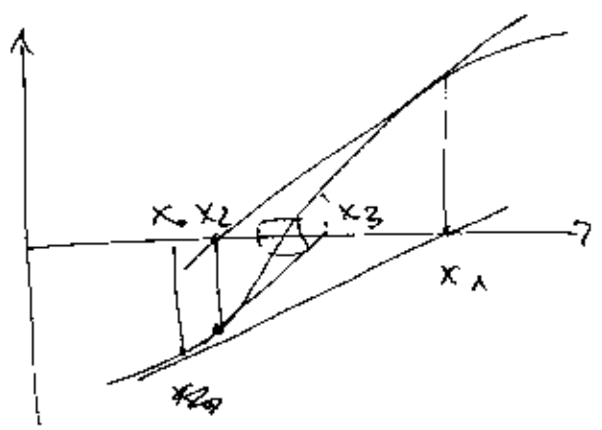
... p19

$$J^0 y^0 = -F^0$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_1^0 \\ y_2^0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \longrightarrow y_1^0 = 1, \; y_2^0 = -3$$

$$x^1 = y^0 + x^0 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

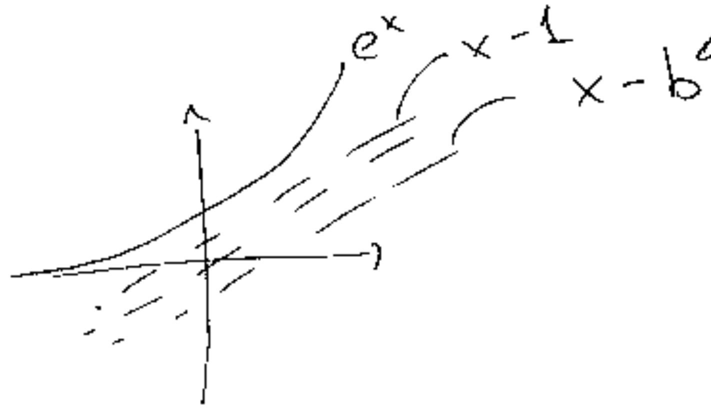
p 16



→ image probably too inaccurate print & redo.

⇝ Secant Method cannot be used since we used two initial points in order to estimate the gradient.

p 17



also show the moving of the line we move until we touch e^x

$x = 1$

p 17

$$f(x) = e^x - x - 1$$

$$f'(x) = e^x - 1$$

$$x^0 = 1 \longrightarrow x^{(1)} = x^0 - \frac{f(x^0)}{f'(x^0)} = 1.164$$

$$x^2 = 1.1461$$
$$x^3 = 1.14617$$
$$\vdots$$

p 19

```
while (b-a) > tol do
    m <- (a+b)/2
    if sign f(a) = sign f(m) then
        a <- m
    else
        b <- m
    end if
end while
```

p 20

$$\frac{d}{dx} x^2 = 0 \qquad x^1 = x_0 - \frac{2x_0}{2} = \underline{\underline{0}}$$

$$2x = 0$$

$$f(x) = 2x$$
$$f'(x) = 2$$