8 - Numerical Integration of 3

p6

- composite Trapezoidal rule:

(write single trapezoidal rule for each interval and add) $= \frac{h}{2} f_1 + h f_2 + h f_3 + \frac{h}{2} f_4$

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f_{11}	f_{12}	f_{13}
f_{21}	f_{22}	f_{23}
f_{31}	f_{32}	f_{23}

$$A_1 = \frac{h}{2} f_{11} + f_{21} + \frac{h}{2} f_{31}$$

$$A_2 = \dots$$

$$A_3 = \dots$$

$$V = \frac{h}{2} A_1 + A_2 + \frac{h}{2} A_3$$

$$= \frac{h}{4} f_{11} + \frac{h}{2} f_{21} + \frac{h}{4} f_{31} + \frac{h}{2} f_{12} + f_{22} + \frac{h}{2} f_{32}$$

$$+ \frac{h}{4} f_{13} + \frac{h}{2} f_{23} + \frac{h}{4} f_{33}$$

$\underline{P\ 4}$ ②

a) $\int_{-3}^{3}\int_{-3}^{3} (0.1x^2 + 0.1y^3 + 4.)\, dx\, dy$

$= \underline{154.8}$

b) $h = 2$

$A_1 = 2\cdot\left(0.1(-2)^2 + 0.1(-2)^3 + 4\right) + 2\left(0.1(0)^2 + 0.1(-2)^3 + 4\right)$

$+ 2\left(0.1(2)^2 + 0.1(-2)^3 + 4\right)$

$= 20.8$

$A_2 = 25.6$

$A_3 = 30.4$

$V = 2A_1 + 2\cdot A_2 + 2\cdot A_3$

$= \underline{153.6}$

$e = \underline{1.2}$

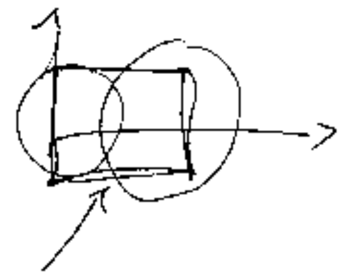
c) Rectangular rule for 2D $O(h^2)$ for 3D $O(h)$

so linear : Decrease by 4.

a) Envelope the area of interest:

$x = \text{rand} \cdot 12$

$y = $ ~~rand · 10 - 5~~ rand*10 - 5



repeat
1. Draw coordinates
2. Check if in both circles:

$x^2 + y^2 < 25$ and $(x-12)^2 + y^2 < 100$

3. If inside $n_{inside} \leftarrow n_{inside} + 1;$
else $n_{outside} \leftarrow n_{outside} + 1;$

4. $A \approx \frac{n_{inside}}{n_{inside} + n_{outside}} \cdot (12 \cdot 5)$

Area of sampling space

b) Compute

$$\varepsilon_N \approx 2 \sqrt{\frac{1}{N-1} \left(\underline{E\{f^2\}} - \underline{E^2\{f\}} \right)}$$

total number: $n_{inside} + n_{outside}$

$\frac{1}{n_{inside} + n_{outside}} \cdot n_{inside}$

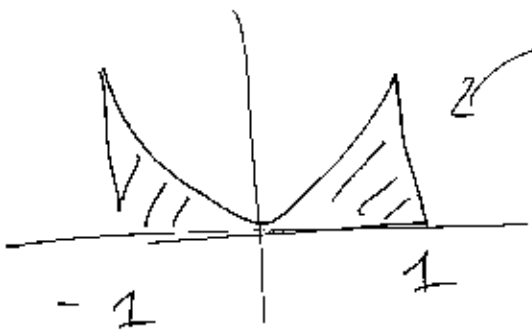
$\left(\frac{1}{n_{inside} + n_{outside}} \cdot n_{inside} \right)^2$

c) $\varepsilon \sim \frac{1}{\sqrt{M}} = \frac{1}{\sqrt{10}}$

d) $\varepsilon = \sqrt{\frac{\mathrm{Var}(f)}{M}}$

variance of uniform distribution scales $\mathcal{O}(\Delta^2)$;
hence doubling $\mathrm{Var} \sim 4$;
$\sqrt{4} = 2 \leftarrow$ doubles error as well

P12



$$\varphi(x,y) = \begin{cases} 1 & \text{if } |x| \leq 1 \text{ and } 0 \leq y \leq x^2 \\ 0 & \text{else} \end{cases}$$

$$\mathbb{E}[q(x,y)] = \frac{1}{4} \int_{-1}^{1} \int_{-1}^{1} \varphi(x,y)\,dx\,dy = \underline{\frac{1}{4} \cdot \frac{2}{3}}$$

$$\rightsquigarrow I = |\Omega| \cdot \mathbb{E}[q(x,y)] = \underline{\underline{\frac{2}{3}}}$$

usually we cannot compute this:
we approximate by Monte Carlo!

P 13

a)
$$P = \frac{V^2}{R}$$

b)
$$\bar{P} = \mathbb{E}[P] = \mathbb{E}_{p(R)}\left[\frac{V^2}{R}\right] = \int_{10}^{20} \frac{V^2}{R}\, p(R)\, dR$$

c)
$$\bar{P} \approx \frac{1}{4}\left(\frac{10^2}{12} + \frac{10^2}{17} + \frac{10^2}{15} + \frac{10^2}{16}\right) = 6.78 \text{ Watts}$$

$\mathbb{E}[x] \approx$ Average $\frac{1}{N}\sum x$