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Title

- Text

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Exercise 1: Setup & Linear Least Squares

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Outline

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4. Exercises
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6. `git` workflow example

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Information

General

- Lecture material & problem sets available here
- Tutorial material available here

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Goals

Goals of Today

- Understand principle of linear least squares
- Derive closed-form solution for linear least squares
- Know different cases of data dimensionality & number of data points
- Be able to set up `git` and python properly
- Roughly understand the `git` workflow

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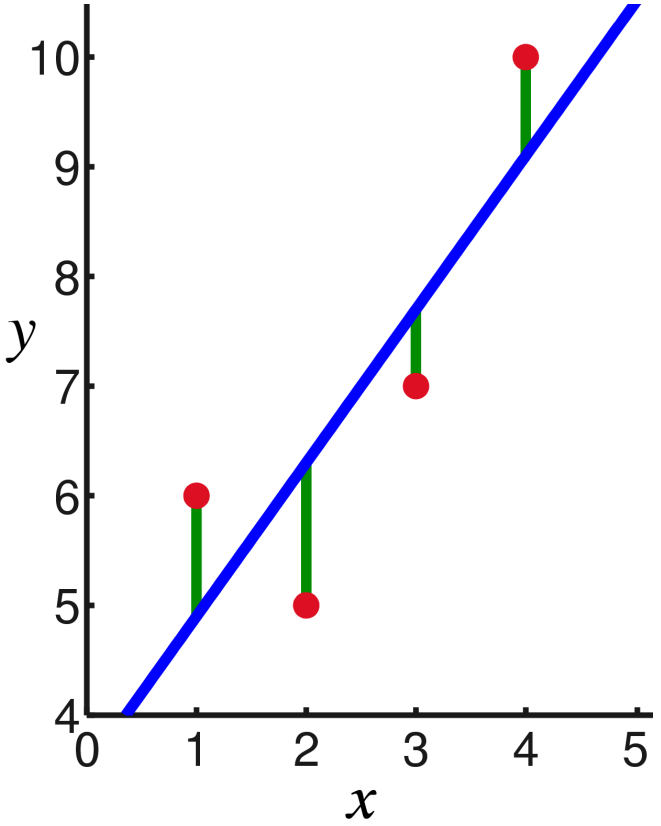
Theory / Recap

Linear Least Squares

- $D = \{(\mathrm{x}_1, y_1), \ldots, (\mathrm{x}_n, y_n)\}$ where $\mathrm{x}_i \in \mathbb{R}^d$
- Introduce $\bar{\mathrm{X}} \in \mathbb{R}^{n \times d}$ where x_i are rows
- Introduce $\bar{\mathrm{y}} \in \mathbb{R}^n$ where y_i are the scalar entries

- We want $\bar{\mathrm{X}}\,\mathrm{w} \approx \bar{\mathrm{y}}$ **how?**
- Minimize the square error $\|\bar{\mathrm{X}}\mathrm{w} - \bar{\mathrm{y}}\|_2^2$ **how?**



Example 1: Derive closed-form solution of LSQ

- Find an optimal w^* which minimizes $\|\bar{\mathrm{X}}\mathrm{w} - \bar{\mathrm{y}}\|_2^2$

- Tipps:
  - $\|\mathrm{a}\|_2^2 = \mathrm{a}^\mathrm{T}\mathrm{a}$
  - $(\mathrm{A} + \mathrm{B})^\mathrm{T} = \mathrm{A}^\mathrm{T} + \mathrm{B}^\mathrm{T}$
  - $(\mathrm{AB})^\mathrm{T} = \mathrm{B}^\mathrm{T}\mathrm{A}^\mathrm{T}$
  - $\frac{d}{d\mathrm{x}}\mathrm{Ax} = \mathrm{A}^\mathrm{T}$
  - $\frac{d}{d\mathrm{x}}\mathrm{x}^\mathrm{T}\mathrm{A} = \mathrm{A}$
  - $\frac{d}{d\mathrm{x}}\mathrm{x}^\mathrm{T}\mathrm{x} = 2\mathrm{x}$
  - $\frac{d}{d\mathrm{x}}\mathrm{x}^\mathrm{T}\mathrm{Ax} = \mathrm{Ax} + \mathrm{A}^\mathrm{T}\mathrm{x}$

Example 2: Write down the matrices

- $D = \{([x_{11}, x_{12}],\ y_1), ([x_{21}, x_{22}],\ y_2), ([x_{31}, x_{32}],\ y_3)\}$
- Proposed function design: $w_0 + w_1 x_1 + w_2 x_2 = y$

- Write down $\overline{\mathrm{X}}, \overline{\mathrm{y}}$ and w (don't compute anything!)

Example 3: Compute an example with LSQ

- $D = \{([1,1],\ 1), ([1,2],\ 3), ([2,4],\ 5)\}$

- Compute w^*
- Predict y for $\mathrm{x} = [5, 10]$

- Tipps:
  - If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

2 cases for LSQ

- $d > n$: Too many unknown parameters to fit
- $d \leq n$: If $\mathrm{X}^{\mathrm{T}}\mathrm{X}$ is invertible the closed form solution exist

- Alternatives exist which always yield a closed form solution (“Ridge Regression”)

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Exercises

Q1

- Practical example of using LSQ on 2D data

Q2

- Use LSQ on 3D data

Q3

- Advanced question – not mandatory: Give it a try anyways!

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Setup instructions

Setup python

- Download and install python distribution from here
- Download and install an editor such as Pycharm from here

Setup `git`

- Download and install `git` from here

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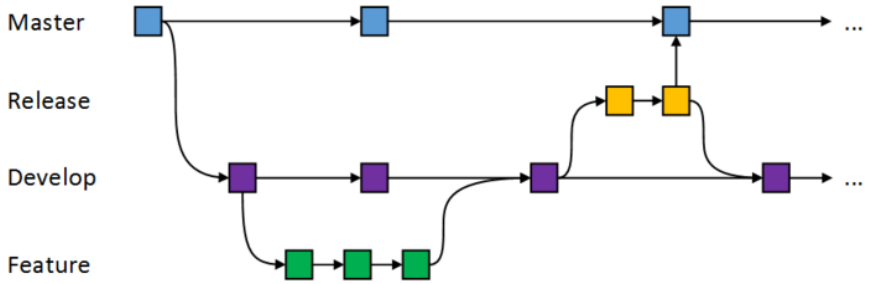
`git` workflow

Example 4: Setting up `git` project

1. Configure your `git`
   - `git config --global user.name "bacdavid”`
   - `git config --global user.email bacdavid@student.ethz.ch`
2. Create a project on `gitlab.ethz.ch` and locally (with Pycharm for instance) with a file `hello.py`
3. Navigate to your local project folder (use `cd` and `ls`)
   - `cd bacdavid/PycharmProjects/hallo_world`
4. Initialize `git` and push
   - `git init`
   - `git remote add origin https://gitlab.ethz.ch/bacdavid/hello_world.git`
   - `git add .`
   - `git commit –m “Initial commit”`
   - `git push –u origin master`

Example 5: Using `git` workflow



1. Create a new branch and switch to it
   - `git branch “goodbye”`
   - `git checkout goodbye`
   - `git branch # shows the branch`
2. Create a file `goodbye.py` next to `hello.py`
3. Commit and push
   - `git add .`
   - `git commit –m “added goodbye file”`
   - `git push origin goodbye # on gitlab you should have two branches now`
4. Switch to master branch and merge
   - `git checkout master`
   - `git merge goodbye`
   - `git push origin master`

Questions?