5- Numerical Integration

P10

example

1. $\int_{-1}^{1} x^2 dx = \frac{1}{3}[x^3]_{-1}^{1} = \frac{2}{3}$

2. ~~rectangular~~

$x^2|_0 \cdot 2 = 0$

3. $\frac{x^2|_{-1} + x^2|_1}{2} \cdot 2 = 2$

4. $\frac{x^2|_{-1} + 4 \cdot x^2|_0 + x^2|_1}{6} \cdot 2 = \frac{2}{3}$

perfect, since simpsons integrates quadratic polynomials perfectly

P12

$$I \approx \underbrace{(x_2 - x_0)}_{\Delta} \cdot \left(C_0^2 f(x_0) + C_1^2 f(x_1) + C_2^2 f(x_2) \right)$$

$$\sum C_i = 1 \Rightarrow C_0^2 + C_1^2 + C_2^2 = 1$$

$$C_i = C_{n-i} \Rightarrow C_0^2 = C_2^2 =: a$$

$$2a + \frac{2}{3} = 1 \Rightarrow a = \frac{1}{6}$$

$$\rightarrow \Delta \cdot \frac{f(x_0) + 4 f(x_1) + f(x_2)}{6} \Rightarrow \text{Simpson's Rule}$$

P 15

- „exact": $2n-1=3$

$n=2$ parameters required

- transform

$$z = x-1 \quad \rightarrow \quad z+1 = x \qquad dz = dx \qquad \Bigg| \quad I = \int_{-1}^{1} \underbrace{(z+1)^3}_{f(z)} dz$$

- in the table:

$$x_1 = -\sqrt{\tfrac{1}{3}}\,; \quad x_2 = \sqrt{\tfrac{1}{3}}\,; \quad \alpha_1 = \alpha_2 = 1$$

$$I = \int_{-1}^{1} (z+1)^3 dz = \left(-\sqrt{\tfrac{1}{3}}+1\right)^3 + \left(\sqrt{\tfrac{1}{3}}+1\right)^3 = \underline{\underline{4}}$$

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$f(x)=1$: $\displaystyle\int_{-1}^{1} 1\,dx = 2 = a\cdot 2 + b\cdot 0$

$\Rightarrow a = 1$

$f(x)=x^2$: $\displaystyle\int_{-1}^{1} x^2 dx = \frac{2}{3} = a(1+1) + b(-2-2)$

$= 2-4b$

$\rightarrow b = \underline{\underline{\frac{1}{3}}}$

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in script

P21

$$\int_{-1}^{1} a_0 + \cancel{a_1 x} + a_2 x^2 + \cancel{a_3 x^3} + a_4 x^4 + \cancel{a_5 x^5} \, dx$$

definition

$$= 2a_0 + \frac{2a_2}{3} + \frac{2a_4}{5} \stackrel{!}{=} 2a_0(a+b) + 2a_2(a+b\alpha^2) + 2a_4(a+b\alpha^4)$$

thus:

$$a+b = 1$$

$$a + b\alpha^2 = \frac{1}{3}$$

$$a + b\alpha^4 = \frac{1}{5}$$

solve: $a = \frac{1}{6}, \ b = \frac{5}{6}, \ \alpha = \frac{1}{\sqrt{5}}$