A Mixed-Integer Convex Optimization Framework for Robust Multilegged Robot Locomotion Planning over Challenging Terrain

Bernardo Aceituno-Cabezas[1], *Student Member, IEEE*, Hongkai Dai[2], José Cappelletto[1], Juan C. Grieco[1], and Gerardo Fernández-López[1], *Member, IEEE*

***Abstract*— This paper introduces an optimization-based framework for robust multilegged walking motion planning. Previous approaches use fixed gait sequences, and rely on Zero Moment Point (ZMP) to guarantee dynamic stability. While this combination works well on flat ground, it does not generalize to uneven terrain requiring aggressive gait or gait transition. To overcome such difficulties, in this paper, we present an optimization framework, that can plan both the contact location and gait sequence simultaneously in a mixed-integer convex optimization program. Moreover, we rely on the Contact Wrench Cone (CWC) stability criterion, which generalizes the ZMP criterion to uneven terrain with friction cone constraints, and we plan the walking motion together with the angular momentum through a convex optimization program. Our approach is successfully tested on a LittleDog quadruped over simulated scenarios. We show that on the flat ground, our planner generates a periodic gait, same as Central Pattern Generator + ZMP planner; while on uneven terrain, our planner can successfully generate a motion containing different gaits, with a center-of-mass motion that respects the friction cone constraints, which are violated by ZMP planners. This improvement clearly demonstrates the advantage of our approach over traditional planning strategies.**

***Index Terms*— Motion and Path Planning, Multilegged Robots, Robust Optimization.**

I. INTRODUCTION

Multilegged robots are powerful platforms when traversing rough terrains [1], [2], [3]. There are two major challenges for this motion planning task: 1) The number of possible gait combinations grows exponentially with the number of footsteps; and 2) It is non-trivial to guarantee that the planned motion is dynamically feasible, so that the robot will not fall over. Previously, several researchers have used Central Pattern Generators (CPG) as an improved model to generate gait sequences [2], [4], [5], [6], and relied on Zero Moment Point (ZMP) for dynamic stability [7]. While CPG and ZMP work well locomotion over flat ground, neither of them extend naturally to uneven terrain. On challenging terrain, the robot needs to adjust its gait frequently to adapt to the terrain change [8], and CPG does not easily account for the transition gait [9]. Moreover, while well defined on flat ground, the Zero Moment Point is not applicable to uneven ground with friction cone constraints. To this end, instead of using CPG, we will plan the gait sequence simultaneously to the contact plan by using a Mixed-Integer Convex Optimization, and guarantee the dynamical stability of the motion by explicitly optimizing the Contact Wrench Cone (CWC) stability margin [10], [11], rather than ZMP, by extending the work of [12] for multilegged robots.

The authors are with (1) the Mechatronics Research Group, Simón Bolívar University {12-10764, cappelletto, jcgrieco, gfernandez}@usb.ve, and (2) is with the Toyota Research Institute hongkai.dai@tri.global

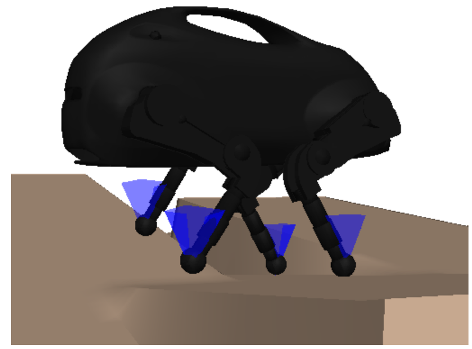

Fig. 1. LittleDog walking over rough terrain with non-coplanar contacts and linear friction cones (blue)

There has been a series of work on planning footstep locations for bipedal robot, using Mixed-Integer Convex Programming (MICP) [13], [14]. This approach allows the planner to efficiently handle cluttered environments, rotations, and uneven terrain. In [13], the authors first find a set of collision free contact regions, and then use integer variables to assign each footstep to one region. We extend this approach to multileg robot. Compared to bipedal robot which has a unique footstep sequence, where it only alternates between left and right leg; the multileg robot has extra complexity that the footstep sequence has exponentially number of combinations. Furthermore, such formulations do not consider heuristics in the optimization, leading to a purely geometric planning problem. In this paper we will use MICP to efficiently find the optimal contact and gait sequences, along with the incorporation of terrain heuristics. This allows the planner to reason simultaneously about *where and when* to make contact.

The main contribution of this work is a robust walking motion planning framework for multilegged robots, which can negotiate different gaits and contact paths, depending on the conditions of the terrain, and provide *formal robustness guarantees*, based on the CWC stability margin. We decouple the planning problem in two numerical optimization problems, leveraging MICP and Convex Optimization. Both stages are solved efficiently to their global optimum, using off-the-shelf optimization solvers.

The remainder of this paper is organized as follows: Section II introduces the main concepts referred throughout the paper, Section III presents the formulation of our proposed framework, Section IV presents the results of our framework applied to a *LittleDog* quadruped, and Section V discusses and concludes on the contributions of this work.

II. BACKGROUND

In order for a foot to remain in contact, the Coulomb friction model requires that the contact wrench (concatenation of force/torque) stays within a wrench cone [10]. A common simplification to verify this condition is to rely on a n_e edged linear approximation of the cone at each contact [12], where the the CWC is defined as the admissible set of the total contact wrench. Such set can be computed by performing the *Minkowski* sum of the individual wrench polyhedrons at each support, as shown in Fig. 1, *always resulting in a convex set*. This polyhedron can be described by its facets, as:

$$CWC = \{w \in \mathbb{R}^6 \mid a_k^T w \leq 0, k = 1, \ldots, n_f\} \qquad (1)$$

Where n_f is the number of facets in the polyhedron and $a_k \in \mathbb{R}^6$ is the normal vector to each facet.

The dynamic nature of the CWC makes it a very useful space to determine the robustness of a motion [10]. To this end, the robot locomotion community introduced the *Contact Wrench Cone Margin*, defined in [12] as the smallest magnitude of wrench disturbance that the robot cannot resist, given the contact locations and friction cone constraints.

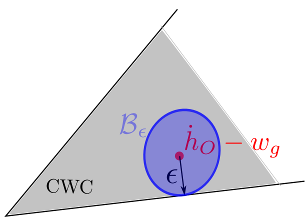

Fig. 2. Geometric interpretation of the CWC margin, where h_O is the centroidal momentum and w_g is the gravitational wrench

This formulation is equivalent to the maximum wrench disturbance magnitude ϵ, such that the contact wrench superimposed with the disturbance remains in the CWC. A representation of this notion in the plane is shown in Fig. 2. Algebraically:

$$\epsilon = \max \bar{\epsilon}$$

such that:

$$B_{\bar{\epsilon}} = \left\{ \dot{h}_O - w_g + T(p_d) w_d \;\middle|\; w_d^T Q_d w_d \leq \bar{\epsilon}^2 \right\} \subset CWC$$

where w_d is the disturbance wrench applied at $p_d \in \mathbb{R}^3$, $T(P_d) \in \mathbb{R}^{6\times 6}$ is the transform matrix that maps the disturbance as a wrench in the origin of the world frame and $Q_d \in \mathbb{R}^{6\times 6}$ is a symmetric matrix that encodes the norm in the wrench space. Using the facet description of the CWC, this representation can be rewritten as:

$$\epsilon = \min_{i=1,\ldots,n_f} -\bar{a}_i \begin{bmatrix} m\ddot{r} - m\mathbf{g} \\ \dot{k}_O - r \times m\mathbf{g} \end{bmatrix} \qquad (2)$$

where r is the robot's center of mass (CoM), m its mass, k_o its centroidal angular momentum, **g** the gravity acceleration, and $\bar{a}_i = (a_i^T T(p_d) Q_d^{-1} T(p_d)^T a_i)^{-1/2} a_i^T$. In the next section we will formulate an optimization problem to find the center of mass trajectory that maximizes this margin.

III. TECHNICAL APPROACH

We propose a robust motion planning framework to plan contact sequences and CoM trajectories. Our planning approach, thus, acts in two stages:

1) Contact and gait sequences that adapt to the conditions of the terrain.
2) CoM trajectory and angular momentum that maximize the robustness of the motion.

In this case, the gait sequence will be planned along the contact plan, within a single optimization problem.

A. Contact sequence optimization

In order the optimize contact sequences that adapt to the unevenness of the terrain, we propose a Mixed-Integer Programming formulation [15] which extends previous optimization-based footstep planners [13], [16] in order to account for gait sequence timing. Our goal is to compute the sequence of contacts, the order in which they will be followed, and assign supports according to the terrain, while minimizing the execution time.

1) *Contact formulation*: In order to plan contact locations for n_l foots, also referred as footsteps, we will extend the work of [13] in order to account for the geometry of multilegged platforms. For this, we will represent the contact locations as an array of $N_f + n_l$ vectors in $\mathbb{R}^4$ of the form:

$$f = (x, y, z, \theta)$$

representing the position of each contact and the yaw orientation of the trunk when transitioning to the contact. This array is ordered by leg number, such that f_i represents a single foot contact position and f_{i+n_l} its next.

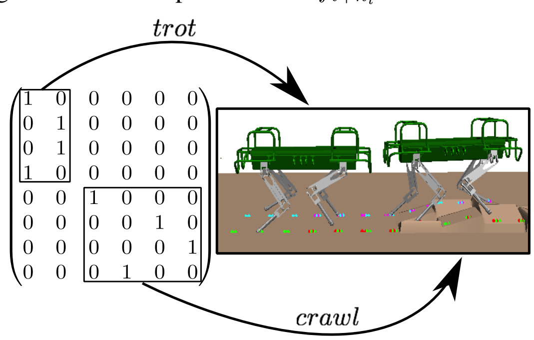

Fig. 3. Example of gait transition matrix T, with $N_f = 8$ and $N_t = 6$

2) *Gait Formulation*: Gait sequences are often represented by phase diagrams [8], which indicate the n_l legged supports for a given support phase (horizontal axis). Inspired by this representation, we will describe the support phase assigned to each leg transfer cycle within the contact plan. To do this, we introduce a binary *transition matrix* $T \in \{0, 1\}^{N_f \times N_t}$, where $T_{ij} = 1$ means that the robot will move to the i_{th} contact at the j_{th} support phase. Since each contact in the plan is reached once, we enforce this constraint as:

$$\sum_{j=1}^{N_t} T_{ij} = 1 \;,\; \forall i = 1, .., N_f \qquad (3)$$

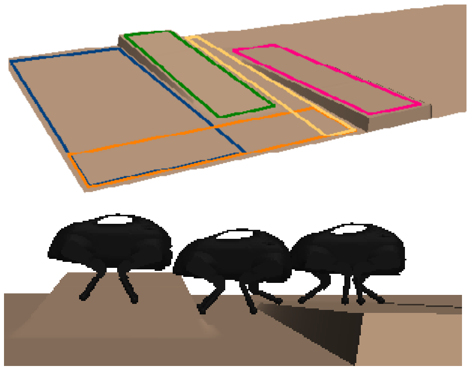

Fig. 4. Top: Example of segmented uneven terrain, with allowable safe-convex terrain regions, given as inputs to the contact sequence planner. Bottom: Example of an optimized gait sequence: the robot crawls when the climbing to higher platforms and trots when the terrain is flat enough.

This representation is equivalent to indicating the supports at each phase, since the contact plan is ordered by leg. Furthermore, we enforce that each cycle of n_l contacts must be reached before the next transfer cycle starts. In order to introduce this constraint, we define a vector $t \in \mathbb{Z}^{N_f \times 1}$ to compute the phase assigned to each movement, obtained as:

$$t = T \begin{pmatrix} 1 & \dots & N_t \end{pmatrix}^T$$

and then, since the gait follows a sequentially ordered contact plan, we enforce the following constraint:

$$t_j > t_{j-n_l} \ , \ \forall j = n_l + 1, \dots, N_t \tag{4}$$

Additional constraints can also be added depending on the geometry of the robot, since some support configurations might not be achievable (i.e. standing in only one support) or since one may want the sequence to follow an specific order.

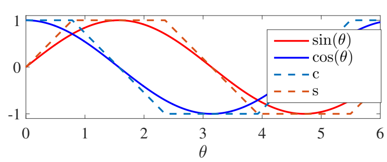

Fig. 5. Piecewise linear approximation of sin-cos functions, shown in [13]

3) ***Trunk Orientation*****:** To keep the problem convex, we will replace the trigonometric functions of the trunk orientation with piecewise linear approximations s and c, shown in Fig. 5. As in [13], we'll define binary matrices S and C in $\{0, 1\}^{N_f \times N_s}$ to assign linear segments, where N_s is the number of segments. This is done by introducing the following constraints:

$$S_{ik} \Rightarrow \begin{cases} \psi_{k-1} \leq \theta_i \leq \psi_k \\ s_i = m_k \theta_i + n_k \end{cases} \quad C_{ik} \Rightarrow \begin{cases} \gamma_{k-1} \leq \theta_i \leq \gamma_k \\ c_i = m_k \theta_i + n_k \end{cases} \tag{5}$$

where the $\Rightarrow$ operator is represented with big-M formulation, ψ and γ represent the boundaries between each linear segment, and m and n represent its slope and intersection. Then, we enforce that every approximation lies within a single line segment, therefore for each i_{th} contact:

$$\sum_{s=1}^{N_s} S_{is} = 1 \qquad \sum_{s=1}^{N_s} C_{is} = 1 \tag{6}$$

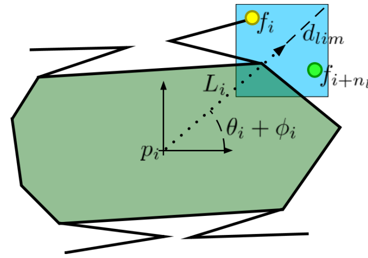

Fig. 6. Kinematic reachability constraint on a quadruped

4) ***Kinematic Constraints*****:** In order to ensure kinematic reachability, we must account for the workspace of each independent leg. To do this, we will approximate the position of the robot trunk when reaching the i_{th} contact as $p_i \in \mathbb{R}^3$. This is computed, for each contact, as:

$$p_i = \frac{\sum_{j \in C(f_i, T)} f_j}{n_l} \tag{7}$$

Where $C(f_i, T)$ is the set of contacts in support when transitioning to the i_{th} contact, defined by the transition matrix. Then, we can constrain that each contact is contained within the biggest square inscribed in the leg workspace, with side d_{lim}, as shown in Fig. 6. Algebraically:

$$\left| f_{i+n_l} - \left[p_i + L_i \begin{pmatrix} \cos(\theta_i + \phi_i) \\ \sin(\theta_i + \phi_i) \end{pmatrix} \right] \right| \leq d_{lim} \tag{8}$$

Where L_i is the approximate distance from the trunk to the leg, and ϕ_i is a known offset for each foot. Here, the trigonometric relations are decomposed in terms of c and s.

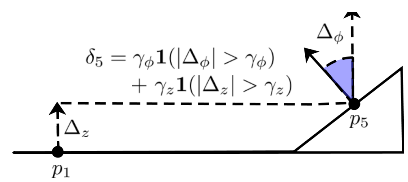

Fig. 7. Example of terrain heuristics, where $\mathbf{1}()$ is a binary return function.

5) ***Terrain heuristics*****:** In order to adapt the gait to the conditions of the terrain we will require a terrain heuristic. Here, we will present a general approach for discrete heuristics, this can be applied for terrain metrics such as height deviation, friction or slope, as show in Fig. 7. Here, we will use the height deviation between contact i and the previous cycle as a heuristic, defined as:

$$\Delta z_i = |f_{z,i} - p_{z,i-n_l}| \ , \ \forall i = n_l + 1, \dots, N_f$$

We want to penalize the height deviation Δz_i if it is above a given threshold γ_z, otherwise we put zero cost on the deviation. To do so, we introduce two binary variables $\delta^+_{z,i}, \delta^-_{z,i}$, with the constraint:

$$\delta^+_{z,i} \Rightarrow f_{z,i} - p_{z,i-n_l} \geq \gamma_z \tag{9a}$$

$$\delta^-_{z,i} \Rightarrow f_{z,i} - p_{z,i-n_l} \leq -\gamma_z \tag{9b}$$

$$\delta_{z,i} = (\delta^+_{z,i} + \delta^-_{z,i}) \gamma_z \tag{9c}$$

The variable $\delta_{z,i}$ can only take two values, 0 or γ_z. We will impose a cost on this variable δ_{z_i} to penalize the height change from step to step during aggressive motions. This can also be applied to continuous heuristics, by replacing the binary variables with a quadratic cost.

6) ***Obstacle avoidance*:** To avoid obstacles, we will constrain the contacts to lie within a set of N_r *obstacle-free convex regions*, as show in Fig. 4. Each safe region $\mathcal{R}$ is then represented as a convex hull $\mathcal{R} = \{x \in \mathbb{R}^3 \mid A_r x \leq b_r\}$. The assignment of contacts to these regions will be done through a binary matrix $H \in \{0,1\}^{N_f \times N_r}$. Then, each contact is assigned to a single safe region with the linear constraint:

$$\sum_{r=1}^{N_r} H_{\imath r} = 1 \tag{10}$$

Then, each contact is constrained to a safe region with the following Mixed-Integer constraint:

$$H_{\imath r} \Rightarrow A_r f_\imath \leq b_r \tag{11}$$

7) ***Objectives*:** As explained above, this formulation has three main objectives:

1) Minimize the distance to the goal.
2) Minimize the duration of the plan.
3) Maximize the number of supports (minimize the legs in transfer) according to the conditions of the terrain.

For the first objective, we minimize quadratic distance between the final steps f_g and the goal g. Algebraically:

$$(f_g - g)^T Q_g (f_g - g), \tag{12}$$

where Q_g is a positive-semidefinite weight matrix.

For the second objective, one can simply minimize the sum of all the assigned phases, equivalent to reducing the number phases used in the plan. Algebraically:

$$\sum_{j=1}^{N} c_t t_j \tag{13}$$

where c_t is a a positive weight.

To achieve the third objective, we introduce a cost, based on the heuristics, at each transition (Eq (14))

$$\sum_{i=1}^{N_t} \left(\sum_{j=1}^{N_f} \beta_{ij} \right)^2 \tag{14}$$

$$\text{where } \beta_{ij} = \begin{cases} \delta_{z,i} & \text{if } T_{ij} = 1 \\ 0 & \text{if } T_{ij} = 0 \end{cases} \tag{15}$$

where β_{ij} represents the cost on the terrain change, if the j'th transfer occurs at step i. It can be easily represented with mixed-integer constraints by using big-M formulation. Cost (14) would be smaller, if two transfers in rough terrain occur at different steps, rather than in the same step, due to the inequality $(a+b)^2 \geq a^2 + b^2$, $\forall a, b \geq 0$. This cost will ensure that the planner assigns more support to a transfer phase if the terrain is significantly rough, while seeking the fastest gait when the terrain is relatively flat. Moreover, if weight on this cost is too high, then the planner will be biased towards more conservative walking on flat regions.

B. Robust motion planning

This section presents a convex optimization formulation to generate CoM and angular momentum plans, first proposed in [12]. Here we will summarize the formulation of [12], which maximizes the CWC margin, and minimizes the centroidal angular momentum of the motion. In this paper, We extend this formulation to multilegged robots, while [12] is limited for bipedal robots.

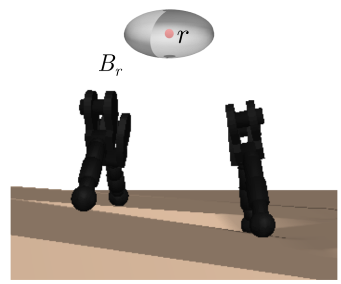

Fig. 8. Ellipsoidal uncertainty set B_r (gray) of the CoM position r (red)

In order to represent centroidal angular momentum k_G, keeping the motion natural [8], we will treat r as an *uncertain variable* within an admissible set, defined by the contact locations, and approach the problem with robust optimization [17]. We will assume that the uncertainty set of the CoM is an ellipsoid B_r, as shown in Fig. 8.

1) ***Objectives:*** Following the example of [12], this formulation introduces three goals:

1) Minimize a convex upper bound of $|k_G|_1$.
2) Minimize the CoM acceleration.
3) Maximize the CWC margin.

For a detailed explanation of this formulation, and its implementation on humanoids, the reader can refer to [12].

IV. RESULTS

The capabilities of our approach are tested on a *BostonDynamics* © LittleDog quadruped by performing simulations in MATLAB 2015b, under the Drake Toolbox for planning and control [18]. For the optimization sections we rely on Gurobi [19] as a solver for the contact planner and Mosek 7 [20] for the robust motion planner. All tests are performed on a *Intel Core2Quad* processor clocked at 2.4 Ghz. Hereby, all distances will be measured in meters, and forces in Newtons.

A. Flat Terrain

To test a basic scenario, we test a forward trajectory (along the $+x$ axis of the world coordinate frame) on a flat terrain, with a high friction coefficient of $\mu_s = 1.0$. The robust planner is configured with an admissible ellipsoidal set of dimensions $1.0cm \times 1.0cm \times 0.5cm$, and the contact planner is configured with 20 footsteps. The obtained gait sequence and the body motions are shown in Fig. 9. For the sake of comparison, we implement a traditional ZMP planner, transcribed as a Quadratic Program, in order to find dynamically stable CoM trajectories with zero angular momentum.

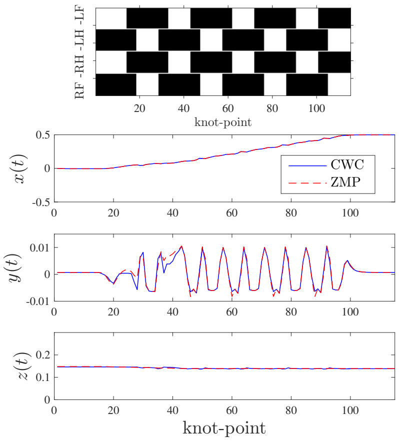

Fig. 9. Top: Phase diagram of the resulting trotting gait sequence, where black represents contact. Bottom: CoM trajectory for flat terrain

In this case, the cost on plan duration and the fact that the terrain is flat, allow the planner to converge to the fastest gait, a trot. Similar to CPG, the resulting plan converges to a periodic oscillatory motion in the CoM position, which resembles a ZMP trajectory. Here, the results are consistent with [12], as the flat terrain with high friction results in a trajectory similar to a ZMP planner. Small differences between ZMP and CWC trajectories are likely caused by the optimization set-up and the variations in angular momentum.

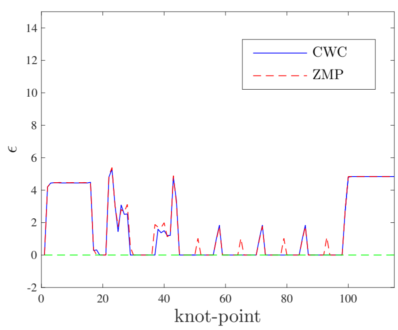

Fig. 10. CWC margin comparison using ZMP planning and the proposed approach, for flat terrain

On the other hand, the robustness metrics for this motion are presented in Fig. 10. As expected, the motion obtained with our scheme results in a trajectory similar to the ZMP planner's [2], since the friction coefficient is high, and the gait sequence remains constant as the terrain doesn't change. Our implementation of the contact planner runs at a approximately 0.3 secs and the robust motion is solved to optimality in 0.2 secs. An interesting remark is regards the computation time. Here, we use long sequences of 20 contacts, resulting in a slower performance due to the number of integer variables. In the case where the planner is configured for 8 contacts, this computation is significantly reduced to under 20 ms.

B. Rough Terrain

Our second test is performed on a rough terrain mock-up with a low-friction, in this case we choose a friction coefficient of $\mu_s = 0.5$. The robust planner is configured with the same parameters, and the contact planner is set to

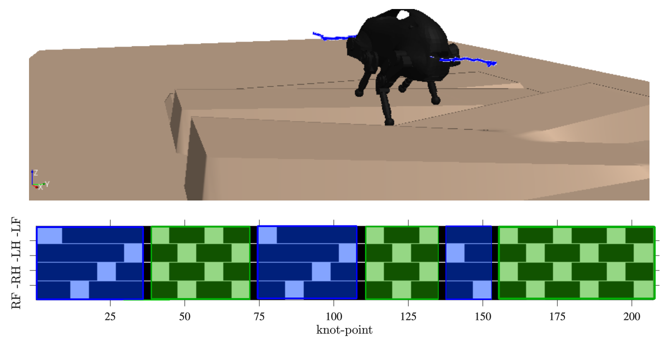

Fig. 11. Top: LittleDog following a robust CoM trajectory (blue) obtained with our approach on a rough terrain with $\mu_s = 0.5$. Bottom: Generated gait sequence for the rough terrain, green: trot gait, blue: crawl gait.

concatenate four successive plans of 16 footsteps. In this case, the obtained plan and the optimized gait sequence for the entire plan are shown in Figs. 11 and 12.

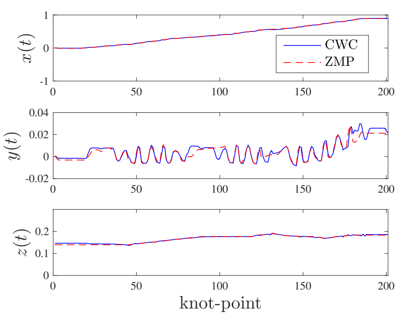

Fig. 12. CoM trajectory for uneven terrain with $\mu_s = 0.5$

Its important to remark how the gait sequence transition emerges when switching between very uneven platforms, specifically around the timesteps 30, 70, 110, 130, and 155. In this case unlike a CPG, the optimization problem won't converge to a pattern (such as trot or crawl); instead it generates a transition gait, in order to favor stability between stepping platforms. Furthermore, the stability metrics for the optimized walking motion are shown in Fig. 13. In this case, the ZMP planner motion has segments with a negative CWC margin, specially when switching between gaits, causing the robot to fall. On the other hand, the proposed approach results in a robust plan across the entire terrain (with a positive margin at all time), thus the robot can maintain stability even during gait transitions. The contact plan is solved in 7.3 secs and the robust motion is solved in under 2.3 secs. Similar to the flat case, when the number of contacts in the plan is set to 8, the computation time is 40 ms.

C. Discussion

The obtained results lead to several interesting remarks. In particular, it's important to notice the trade-offs existing at the gait planning stage. Depending on the terrain heuristics used, the planner can distribute the contacts at each leg and seek for smooth or aggressive transitions between gaits. This is particularly noticeable in very rough terrain mock-ups, where the changes in height and slope lead the planner to direct changes between different gaits. Furthermore, the cost presented in eq. 13 allows the planner to seek faster gaits, like trot, when the terrain is evenly distributed.

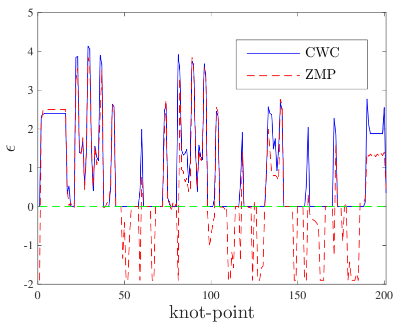

Fig. 13. CWC margin comparison using ZMP planning and the proposed approach, for uneven terrain with $\mu_s = 0.5$

V. CONCLUSION

We have presented a robust multilegged locomotion planning framework, which can negotiate different paths and gaits that adapt to the condition of the terrain, based on unevenness metrics applied as heuristics. Furthermore, we haven extend an algorithm to generate CoM trajectories that maximize the CWC margin and minimize the Centroidal Angular Momentum of the motion. Our approach has been successfully tested on a LittleDog quadruped; it generates transitional gaits on uneven terrain, and has proven to provide robust motions in situations where a classic Zero-Moment Point planner would fail.

A. Future Work

We are highly interested in providing experimental validation of this approach on modern multilegged platforms [3], [1]. Additionally, the proposed formulation is not restricted to quadrupeds, hence, we are also interested on testing our approach in robots with different geometries and possible gaits. We also are interested in the extending our approximate kinematic model, in order to account for collisions with the environment, similar to [13], [21].

In this paper we present a two stage planner to compute the footstep location and the center of mass motion separately. A good extension is to consider using the approaches described in [14], [3], to plan them simultaneously in a single mixed-integer convex optimization problem. This would allow for automatic gait discovery and a richer analysis on the dynamic conditions of the environment.

For the specific case of the gait planning stage, we are considering the addition of further terrain unevenness metrics, such as the convex region orientation or the support area described by the feet. Furthermore, our motion planning approach does not consider joint limits, therefore, some motions might not be reachable on robot's without powerful enough motors. Adding this consideration would increase the complexity of the formulation, for this reason, we are also interested in simplifying the formulation of the CWC planner in order to account for further constraints.

B. Source Code

The authors of this work have made the entire source code of this implementation publicly available on GitHub[1].

[1] https://github.com/baceituno

REFERENCES

[1] M. Hutter, C. Gehring, D. Jud, A. Lauber, C. D. Bellicoso, V. Tsounis, J. Hwangbo, K. Bodie, P. Fankhauser, M. Bloesch, *et al.*, "Anymal-a highly mobile and dynamic quadrupedal robot," in *Intelligent Robots and Systems (IROS), 2016 IEEE/RSJ International Conference on*. IEEE, 2016, pp. 38–44.

[2] K. Byl, "Metastable legged-robot locomotion," Ph.D. dissertation, Massachusetts Institute of Technology, 2008.

[3] C. Mastalli, M. Focchi, I. Havoutis, A. Radulescu, S. Calinon, J. Buchli, D. G. Caldwell, and C. Semini, "Trajectory and foothold optimization using low-dimensional models for rough terrain locomotion," in *International Conference on Robotics and Automation (ICRA)*, 2017.

[4] V. Barasuol, J. Buchli, C. Semini, M. Frigerio, E. R. D. Pieri, and D. G. Caldwell, "A reactive controller framework for quadrupedal locomotion on challenging terrain," in *2013 IEEE International Conference on Robotics and Automation*, May 2013.

[5] M. Oliveira, C. P. Santos, L. Costa, V. Matos, and M. Ferreira, "Multi-objective parameter cpg optimization for gait generation of a quadruped robot considering behavioral diversity," in *2011 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2011.

[6] D. Wettergreen and C. Thorpe, "Gait generation for legged robots," in *Intelligent Robots and Systems, 1992., Proceedings of the 1992 lEEE/RSJ International Conference on*. IEEE, 1992.

[7] M. Vukobratovic and D. Juricic, "Contribution to the synthesis of biped gait," *IEEE Transactions on Biomedical Engineering*, vol. BME-16, no. 1, pp. 1–6, Jan 1969.

[8] M. Hildebrand, "The quadrupedal gaits of vertebrates," 1989.

[9] A. J. Ijspeert, "Central pattern generators for locomotion control in animals and robots: a review," *Neural networks*, vol. 21, no. 4, pp. 642–653, 2008.

[10] P.-B. Wieber, "On the stability of walking systems," in *Proceedings of the international workshop on humanoid and human friendly robotics*, 2002.

[11] H. Hirukawa, S. Hattori, K. Harada, S. Kajita, K. Kaneko, F. Kanehiro, K. Fujiwara, and M. Morisawa, "A universal stability criterion of the foot contact of legged robots-adios zmp," in *Proceedings 2006 IEEE International Conference on Robotics and Automation, 2006. ICRA 2006*. IEEE, 2006.

[12] H. Dai and R. Tedrake, "Planning robust walking motion on uneven terrain via convex optimization," in *Humanoid Robots (Humanoids), 2016 IEEE-RAS 16th International Conference on*. IEEE, 2016, pp. 579–586.

[13] R. Deits and R. Tedrake, "Footstep planning on uneven terrain with mixed-integer convex optimization," in *Humanoid Robots (Humanoids), 2014 14th IEEE-RAS International Conference on*. IEEE, 2014.

[14] B. Ponton, A. Herzog, S. Schaal, and L. Righetti, "A convex model of humanoid momentum dynamics for multi-contact motion generation," in *Humanoid Robots (Humanoids), 2016 IEEE-RAS 16th International Conference on*. IEEE, 2016, pp. 842–849.

[15] A. Richards and J. How, "Mixed-integer programming for control," in *Proceedings of the 2005, American Control Conference, 2005*. IEEE, 2005, pp. 2676–2683.

[16] B. Aceituno-Cabezas, J. Cappelletto, J. C. Grieco, and G. Fernandez-Lopez, "A generalized mixed-integer convex program for multilegged footstep planning on uneven terrain," *arXiv preprint arXiv:1612.02109*, 2017.

[17] D. Bertsimas, D. B. Brown, and C. Caramanis, "Theory and applications of robust optimization," *SIAM review*, vol. 53, no. 3, pp. 464–501, 2011.

[18] R. Tedrake and the Drake Development Team, "Drake: A planning, control, and analysis toolbox for nonlinear dynamical systems," 2016. [Online]. Available: http://drake.mit.edu

[19] I. Gurobi Optimization, "Gurobi optimizer reference manual," 2015. [Online]. Available: http://www.gurobi.com

[20] M. ApS, *The MOSEK optimization toolbox for MATLAB manual. Version 7.1 (Revision 28).*, 2015. [Online]. Available: http://docs.mosek.com/7.1/toolbox/index.html

[21] S. Tonneau, A. Del Prete, J. Pettré, C. Park, D. Manocha, and N. Mansard, "An efficient acyclic contact planner for multiped robots," 2016.