A Mixed-Integer Convex Optimization Framework for Robust Multilegged Robot Locomotion Planning over Challenging Terrain

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***Abstract*— This paper introduces an optimization-based framework for robust multilegged walking motion planning. Many previous approaches use fixed gait sequences, and rely on Zero Moment Point (ZMP) to guarantee dynamic stability. While this combination works well on flat ground, it does not generalize to locomotion on uneven ground requiring aggressive gait or gait transition. To overcome such difficulty, in this paper, we present an optimization framework, that can plan both the contact location and gait sequence through a mixed-integer convex optimization program, so as to incorporate gait transition. Moreover, we rely on the Contact Wrench Cone (CWC) stability criterion, which generalizes the Zero Moment Point criterion to uneven terrain with friction cone constraints, and we plan the center of mass motion, together with the angular momentum through a convex optimization program. Our approach is successfully tested on a LittleDog quadruped over simulated scenarios, including where a ZMP planner would fail, always resulting on robust walking motions.**

***Index Terms*— Motion and Path Planning, Multilegged Robots, Robust Optimization.**

I. INTRODUCTION

Multilegged robots are powerful platforms when traversing rough terrains [1], [2], [3]. There are two major challenges for this motion planning task 1) The number of possible gait combinations grows exponentially with the number of footsteps; 2) It is non-trivial to guarantee that the planned motion is dynamically feasible, that the robot will not fall over. Previously researchers have used Central Pattern Generators (CPG) to plan the gait sequence [2], [4], [5], [6], and relied on Zero Moment Point (ZMP) for dynamic stability [7]. While CPG and ZMP work well for flat ground locomotion, neither of them extend naturally to uneven terrain. On challenging terrain, the robot needs to adjust its gait frequently to adapt to the terrain change, and CPG does not easily account for the transition gait. Moreover, while well defined on flat ground, the Zero Moment Point is not applicable to uneven ground with friction cone constraints. To this end, instead of using CPG, we will plan the gait sequence simultaneously to the contact plan by using a Mixed-Integer Convex Optimization, and guarantee the dynamical stability of the motion by explicitly optimizing the Contact Wrench Cone (CWC) stability margin [8], [9], rather than ZMP, by extending the work of [10] for multilegged robots.

There has been a series of work on planning footstep locations for bipedal robot, using mixed-integer convex optimization [11], [12]. This approach allows the planner to efficiently handle cluttered environments, rotations, and uneven terrain. In [11], the authors first find a set of collision free contact regions, and then use integer variables to assign each footstep to one region. We extend this approach to multi-leg robot. Compared to bipedal robot which has a unique footstep sequence, that only alternates between left and right leg; the multi-leg robot has extra complexity that the footstep sequence has exponentially number of combinations. Furthermore, such formulations do not consider heuristics in the optimization, leading to a purely geometric planning problem. In this paper we will extend previous approaches and use Mixed-Integer Convex Programming (MICP) to efficiently search for the optimal contact and gait sequences simultaneously, along with the incorporation of terrain heuristics, in order to negotiate different terrain conditions. This allows the planner to reason simultaneously about *where and when* to make contact.

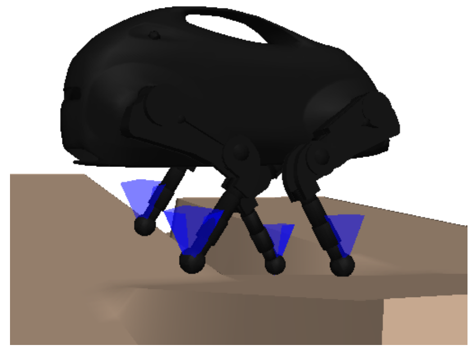

Fig. 1. LittleDog walking over rough terrain with non-coplanar contacts and linear friction cones (blue)

The main contribution of this work is a robust walking motion planning framework for multilegged robots, which can negotiate different gaits and contact paths, depending on the conditions of the terrain, and provide *formal robustness guarantees*, based on the CWC stability margin. We decouple the planning problem in two numerical optimization problems, leveraging MICP and Convex Optimization. Both stages are solved efficiently to their global optimum, using off-the-shelf optimization solvers.

The remainder of this paper is organized as follows: Section II introduces the main concepts referred throughout the paper, Section III presents the formulation of our proposed framework, Section IV presents the results of our framework applied to a *LittleDog* quadruped, and Section V discusses and concludes on the contributions of this work.

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II. BACKGROUND

In order for a foot to remain in contact, the Coulomb friction model requires that the contact wrench (concatenation of force/torque) stays within a wrench cone [8]. A common simplification to verify this condition is to rely on a n_e edged linear approximation of the cone at each contact [10], where the the CWC is defined as the admissible set of the total contact wrench. Such set can be computed by performing the *Minkowski* sum of the individual wrench polyhedrons at each support, as shown in Fig. 1, *always resulting in a convex set*. This polyhedron can be described by its facets, as:

$$CWC = \{w \in \mathbb{R}^6 \mid a_k^T w \leq 0, k = 1, \ldots, n_f\} \tag{1}$$

Where n_f is the number of facets in the polyhedron and $a_k \in \mathbb{R}^6$ is the normal vector to each facet.

The dynamic nature of the CWC makes it a very useful space to determine the robustness of a motion [8]. To this end, the robot locomotion community introduced the *Contact Wrench Cone Margin*, defined in [10] as the smallest magnitude of wrench disturbance that the robot cannot resist, given the contact locations and friction cone constraints.

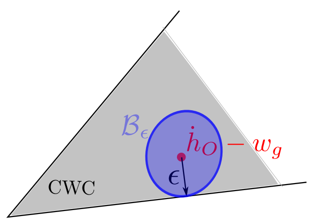

Fig. 2. Geometric interpretation of the CWC margin, where h_O is the centroidal momentum and w_g is the gravitational wrench

This formulation is equivalent to the maximum wrench disturbance magnitude ϵ, such that the contact wrench superimposed with the disturbance remains in the CWC. A representation of this notion in the plane is shown in Fig. 2. Algebraically:

$$\epsilon = \max \bar{\epsilon}$$

such that:

$$B_{\bar{\epsilon}} = \left\{ \dot{h}_O - w_g + T(p_d) w_d \;\middle|\; w_d^T Q_d w_d \leq \bar{\epsilon}^2 \right\} \subset CWC$$

where w_d is the disturbance wrench applied at $p_d \in \mathbb{R}^3$, $T(P_d) \in \mathbb{R}^{6\times 6}$ is the transform matrix that maps the disturbance as a wrench in the origin of the world frame and $Q_d \in \mathbb{R}^{6\times 6}$ is a symmetric matrix that encodes the norm in the wrench space. Using the facet description of the CWC, this representation can be rewritten as:

$$\epsilon = \min_{i=1,\ldots,n_f} -\bar{a}_i \begin{bmatrix} m\ddot{r} - m\mathbf{g} \\ \dot{k}_O - r \times m\mathbf{g} \end{bmatrix} \tag{2}$$

where r is the robot's center of mass (CoM), m its mass, k_o its centroidal angular momentum, $\mathbf{g}$ the gravity acceleration, and $\bar{a}_i = (a_i^T T(p_d) Q_d^{-1} T(p_d)^T a_i)^{-1/2} a_i^T$. In the next section we will formulate an optimization problem to find the center of mass trajectory that maximizes this margin.

III. TECHNICAL APPROACH

We propose a robust motion planning framework to plan contact sequences and CoM trajectories. Our planning approach, thus, acts in two stages:

1) Contact and gait sequences that adapt to the conditions of the terrain.
2) CoM trajectory and angular momentum that maximize the robustness of the motion.

In this case, the gait sequence will be planned along the contact plan, within a single optimization problem.

A. Contact sequence optimization

In order the optimize contact sequences that adapt to the unevenness of the terrain, we propose a Mixed-Integer Programming formulation [13] which extends previous optimization-based footstep planners [11], [14] in order to account for gait sequence timing. Our goal is to compute the sequence of contacts, the order in which they will be followed, and assign supports according to an unevenness metric of the terrain, while minimizing the execution time.

1) *Contact formulation*: In order to plan contact locations for n_l foots, also referred as footsteps, we will extend the work of [11] in order to account for the geometry of multilegged platforms. For this, we will represent the contact locations as an array of $N_f + n_l$ vectors in $\mathbb{R}^4$ of the form:

$$f = (x, y, z, \theta)$$

representing the position of each contact and the yaw orientation of the trunk when transitioning to the contact. This array is ordered by leg number, such that f_i represents a single foot contact position and f_{i+n_l} its next.

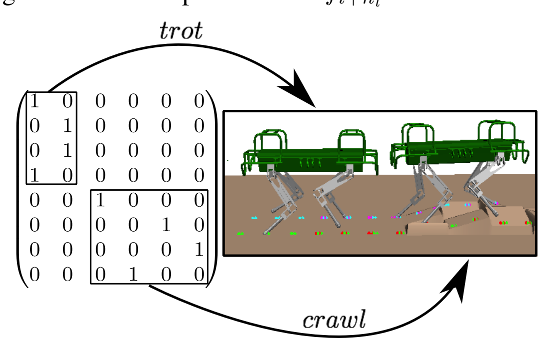

Fig. 3. Example of gait Transition matrix T

2) *Gait Formulation*: Gait sequences are often represented by phase diagrams [15], which indicate the n_l legged supports for a given support phase (horizontal axis). Inspired by this representation, we will describe the support phase assigned to each leg transfer cycle within the contact plan. To do this, we introduce a binary *transition matrix* $T \in \{0,1\}^{N_f \times N_f}$, where $T_{ij} = 1$ means that the robot will move to the i_{th} contact at the j_{th} support phase. Since each contact in the plan is reached once, we enforce this constraint as:

$$\sum_{j=1}^{N_f} T(i,j) = 1 \ , \ \forall i = 1, .., N_f \tag{3}$$

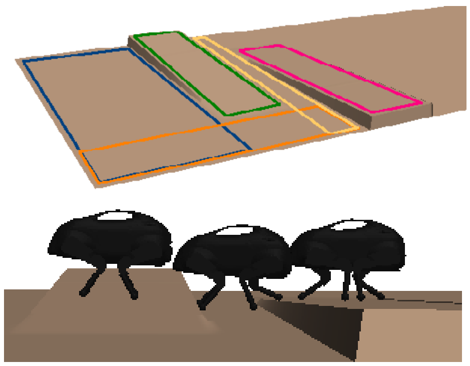

Fig. 4. Top: Example of segmented uneven terrain, with allowable safe-convex terrain regions, given as inputs to the contact sequence planner. Bottom: Example of an optimized gait sequence: the robot crawls when the climbing to higher platforms and trots when the terrain is flat enough.

This representation is equivalent to indicating the supports at each phase, since the contact plan is ordered by leg. Furthermore, we enforce that each cycle of n_l contacts must be reached before the next transfer cycle starts. In order to introduce this constraint, we define a vector $t \in \mathbb{Z}^{N_f}$ to compute the phase assigned to each movement, obtained as:

$$t = \begin{pmatrix} 1 & \dots & N_f \end{pmatrix} T^T$$

and then, since the gait follows a sequentially ordered contact plan, we enforce the following constraint:

$$t_j > t_{j-n_l} \ , \ \forall j = n_l + 1, \dots, N_f \tag{4}$$

Additional constraints can also be added depending on the geometry of the robot, since some support configurations might not be achievable (i.e. standing in only one support) or since one may want the sequence to follow an specific order.

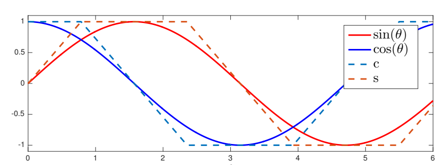

Fig. 5. Piecewise linear approximation of sin-cos functions, shown in [11]

3) ***Trunk Orientation*****:** To keep the problem convex, we will replace the trigonometric functions of the trunk orientation with piecewise linear approximations s and c, shown in Fig. 5. As in [11], we'll define binary matrices S and C in $\{0,1\}^{N_f \times N_s}$ to assign linear segments, where N_s is the number of segments. This is done by introducing the following constraints:

$$S_{ik} \Rightarrow \begin{cases} \psi_{k-1} \leq \theta_i \leq \psi_k \\ s_i = m_k \theta_i + n_k \end{cases} \quad C_{ik} \Rightarrow \begin{cases} \gamma_{k-1} \leq \theta_i \leq \gamma_k \\ c_i = m_k \theta_i + n_k \end{cases} \tag{5}$$

where the $\Rightarrow$ operator is represented with big-M formulation, ψ and γ represent the boundaries between each linear segment, and m and n represent its slope and intersection. Then, we enforce that every approximation lies within a single line segment, therefore for each i_{th} contact:

$$\sum_{s=1}^{N_s} S_{is} = 1 \qquad \sum_{s=1}^{N_s} C_{is} = 1 \tag{6}$$

4) ***Kinematic Constraints*****:** In order to ensure kinematic reachability, we must account for the workspace of each independent leg. To do this, we will approximate the position of the robot trunk when reaching the i_{th} contact as $p_i \in \mathbb{R}^3$. This is computed, for each contact, as:

$$p_i = \frac{\sum_{j \in C(f_i,T)} f_j}{n_l} \tag{7}$$

Where $C(f_i, T)$ is the set of contacts in support when transitioning to the i_{th} contact, defined by the transition matrix. Then, we can constrain that each contact is contained within the biggest square inscribed in the leg workspace, with side d_{lim}, as shown in Fig. 6. Algebraically:

$$\left| f_{i+n_l} - \left[p_i + L_i \begin{pmatrix} cos(\theta_i + \phi_i) \\ sin(\theta_i + \phi_i) \end{pmatrix} \right] \right| \leq d_{lim} \tag{8}$$

Where L_i is the approximate distance from the trunk to the leg, and ϕ_i is a known offset for each foot. Here, the trigonometric relations are decomposed in terms of c and s.

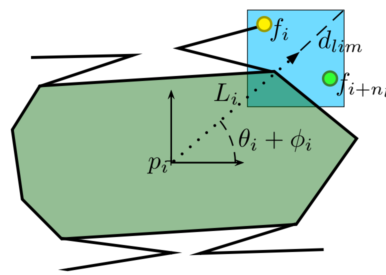

Fig. 6. Kinematic reachability constraint on a quadruped

5) ***Terrain heuristics*****:** In order to adapt the gait to the conditions of the terrain we will require a terrain heuristic. Here, we will use the height deviation between contact i and the previous cycle as a metric, defined as:

$$\Delta z_i = |f_{zi} - p_{z,i-n_l}| \ , \ \forall i = n_l + 1, \dots, N_f$$

We want to penalize the height deviation Δz_i if it is above a given threshold γ_z, otherwise we put zero cost on the deviation. To do so, we introduce two binary variables $\delta^+_{z,i}, \delta^-_{z,i}$, with the constraint:

$$\delta^+_{z,i} = 1 \Rightarrow f_i - p_{i-n_l} \geq \gamma_z \tag{9}$$
$$\delta^-_{z,i} = 1 \Rightarrow f_i - p_{i-n_l} \leq -\gamma_z \tag{10}$$
$$\delta_{z,i} = (\delta^+_{z,i} + \delta^-_{z,i})\gamma_z \tag{11}$$

The variable $\delta_{z,i}$ can only take two values, 0 or γ_z. We will impose a cost on this variable δ_{z_i} to penalize the height change from step to step during aggressive motions. This approach can also be applied to other terrain metrics, such as friction or slope.

6) ***Obstacle avoidance*****:** To avoid obstacles, we will constrain the contacts to lie within a set of N_r *obstacle-free convex regions*, as show in Fig. 4. Each safe region R is then represented as a convex hull $R = \{x \in \mathbb{R}^3 \mid A_r x \leq b_r\}$. The assignment of contacts to these regions will be done through a binary matrix $\mathcal{H} \in \{0,1\}^{N_f \times N_r}$. Then, each contact is assigned to a single safe region with the following linear constraint:

$$\sum_{r=1}^{N_r} \mathcal{H}_{\imath r} = 1 \tag{12}$$

Then, each contact is constrained to a safe region with the following Mixed-Integer constraint:

$$\mathcal{H}_{\imath r} \Rightarrow A_r f_\imath \leq b_r \tag{13}$$

7) *Objectives*: As explained above, this formulation has three main objectives:

1) Minimize the distance to the goal.
2) Minimize the duration of the plan.
3) Maximize the number of supports (minimize the legs in transfer) according to the conditions of the terrain.

For the first objective, we minimize quadratic distance between the final steps f_g and the goal g. Algebraically:

$$(f_g - g)^T Q_g (f_g - g) \tag{14}$$

Where Q_g is a positive-semidefinite weight matrix. For the second objective, one can simply minimize the sum of all the assigned phases, equivalent to reducing the number phases used in the plan. Algebraically:

$$\sum_{j=1}^{N} c_t t_j \tag{15}$$

where c_t is a a positive weight. For the second objective we require to relate the transition matrix T with the terrain heuristic δ_z. This can be easily done by performing the product between both. Thus we define a vector u such that:

$$u = \delta_z^T T$$

This product effectively tells whether an specific phase would yield an undesired configuration on the transfer legs, and therefore we aim to minimize it. However, each j_{th} element of the vector u represents a bilinear relation between δ_{zj} and T_{ji}. Nevertheless, since the matrix T is binary, we can simply replace the product with an intermediate variable β, such that $u_j = \sum_{i=1}^{N_f} \beta_{ji}$, computed as:

$$\beta_{ji} = \delta_{zj} T_{ji} = \begin{cases} \delta_{zj} & \text{if } T_{ji} = 1 \\ 0 & \text{if } T_{ji} = 0 \end{cases}$$

which can be easily represented with mixed-integer constraints by using big-M formulation. We then add a quadratic cost on each of elements of u:

$$\sum_{j=1}^{N_f} u_j^T Q_u u_j \tag{16}$$

where Q_u is a positive-semidefinite weight matrix. This cost will ensure that the planner assigns more support to a transfer phase if the terrain is significantly rough, while seeking the fastest gait when the terrain is relatively flat. Moreover, if weight on this cost is too high, then the planner will be biased towards more conservative walking on flat regions.

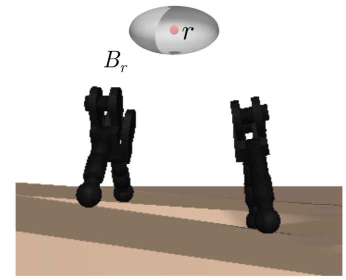

Fig. 7. Ellipsoidal uncertainty set B_r (gray) of the CoM position r (red)

B. Robust motion planning

This section presents a convex optimization formulation to generate CoM and angular momentum plans, first proposed in [10]. Here we will summarize the formulation of [10], which maximizes the CWC margin, and minimizes the centroidal angular momentum of the motion. In this paper, We extend this formulation to multilegged robots, while [10] is limited for bipedal robots.

1) *Contact Wrench Cone Margin:* In order to maximize the CWC margin we must compute the variable as a convex constraint. Using the representation shown in (2) we can compute it at each knot-point with the following constraint:

$$\min_{\epsilon} -\epsilon \quad s.t. \quad \epsilon_L \leq \epsilon \leq -\bar{a}_i \begin{bmatrix} m\ddot{r} - m\mathbf{g} \\ \dot{k}_O - r \times m\mathbf{g} \end{bmatrix} \tag{17}$$

where ϵ_L is a margin lower bound. Using this representation we effectively maximize the CWC margin at each knot-point.

2) *Centroidal Angular Momentum:* Generally, natural walking motions tend to regulate the centroidal angular momentum k_G (angular momentum respect to the CoM). In order to make the body motions more natural via linear constraints, we will minimize the L_1 norm of this variable $|k_G|_1$, which can be computed, at each knot-point, as:

$$k_G = k_O - mr \times \dot{r} = k_O + m\lfloor \dot{r} \rfloor_\times r \tag{18}$$

and $|k_G|_1 = \max_j \omega_j k_G$, where ω_j is a weight vector for the absolute value, such that: $\omega_j = [\pm 1, \pm 1, \pm 1]$. Unfortunately, the cross product relation in (18) is a bilinear relation between r and $\dot{r}$, making it a non-convex constraint. To avoid this, we will treat r as an *uncertain variable* within an admissible set, defined by the contacts, and approach the problem with robust optimization [16]. We will assume that the uncertainty set of the CoM is an ellipsoid B_r, with radial matrix Q_r and centered around r^*, as shown in Fig. 7.

$$|k_G|_1 \leq \omega_j k_O + m\omega_j \lfloor \dot{r} \rfloor_\times r^* + m\sqrt{\omega_j \lfloor \dot{r} \rfloor_\times Q_r^{-1} \lfloor \dot{r} \rfloor_\times^T \omega_j^T} \leq s$$

By adding this constraint we can minimize over an upper bound s of the non-convex representation (18). In this case, the bilinear relations are relaxed and the cross product can be represented as *second-order cone constraint*, therefore, making the problem convex and, thus, efficiently solvable.

3) *Objectives:* Following the example of [10], this formulation introduces three goals for each knot-point:

1) Minimize the upper bound of $|k_G|_1$.
2) Minimize the CoM acceleration.
3) Maximize the CWC margin.

therefore, the cost function for the planner will be:

$$\min_{r,k_O} \; J = \sum_{i=1}^{N} c_k s^T s + c_a \ddot{r}^T \ddot{r} - c_\epsilon \epsilon \qquad (19)$$

Where c_k, c_a and c_ϵ are positive weights on each cost. This cost function, along with the constraints imposed above, make the formulation a convex optimization problem.

IV. RESULTS

The capabilities of our approach are tested on a *BostonDynamics* © LittleDog quadruped by performing simulations in MATLAB 2015b, under the Drake Toolbox for planning and control [17]. For the optimization sections we rely on Gurobi [18] as a solver for the contact planner and Mosek 7 [19] for the robust motion planner. All tests are performed on a *Intel Core2Quad* processor clocked at 2.4 Ghz.

A. Flat Terrain

To test a basic scenario, we present a comparison between a forward trajectory over the x axis with a classic ZMP planner and with the proposed convex formulation on a flat terrain, with a friction coefficient of $\mu_s = 1.0$. The robust planner is configured with an admissible ellipsoidal set of dimensions $1.0cm \times 1.0cm \times 0.5cm$, and the contact planner is configured with 16 footsteps. The obtained gait sequence and the body motions are shown in Fig. 8.

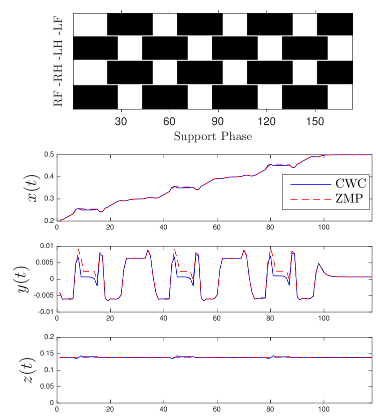

Fig. 8. Phase diagram of gait sequence and CoM trajectory for flat terrain

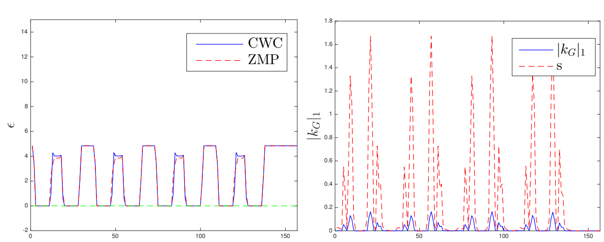

Fig. 9. Left: CWC Margin comparison using ZMP planning and the proposed approach. Right: $|k_G|_1$ for flat terrain

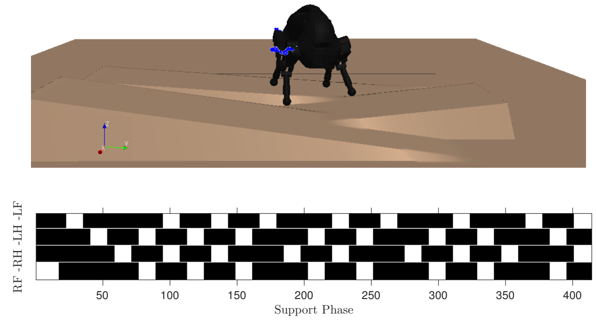

Fig. 10. Top: LittleDog following a robust plan obtained with our approach on a rough terrain with $\mu_s = 0.5$. Bottom: Generated gait sequence for the rough terrain.

On the other hand, the robustness metrics for this motion are presented in Fig. 9. As expected, the motion obtained with our scheme converges to the same trajectory as the ZMP planner [2], since the friction coefficient is high, and the gait sequence remains constant as the terrain doesn't change. Our implementation of the contact planner runs at 0.3 secs and the robust motion is solved to optimality in 0.7 secs.

B. Rough Terrain

Another test is performed on a very uneven terrain mock-up with a low-friction coefficient of $\mu_s = 0.5$. The robust planner is configured with the same parameters, and the contact planner is set to concatenate four successive plans of 16 footsteps. In this case, the obtained plan and the optimized gait sequence for the entire plan are shown in Figs. 11 and 10. Its important to remark how the gait sequence transition emerges when switching between very uneven platforms, specifically around the timestep intervals $(150, 200)$, $(240, 280)$ and $(330, 375)$. In this case unlike a CPG, the optimization problem won't converge to a pattern (such as trot or crawl); instead it generates a transition gait, in order to favor stability between stepping platforms.

Furthermore, the stability metrics for the optimized walking motion are shown in Fig. 12. In this case, the ZMP planner motion has segments with a negative CWC margin, specially when switching between gaits, causing the robot to fall. On the other hand, the proposed approach results in a robust plan across the entire terrain (with a positive margin at all time), thus the robot can maintain stability even during gait transitions.

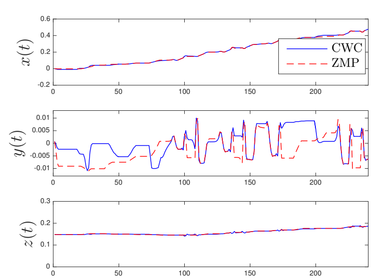

Fig. 11. CoM trajectory for uneven terrain with $\mu_s = 0.5$

The contact plan is solved in 7.3 secs and the robust motion is solved in under 2.3 secs.

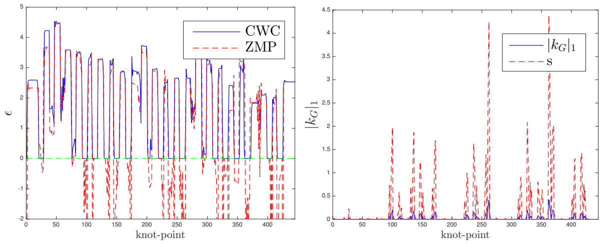

Fig. 12. Left: CWC Margin comparison using ZMP planning and the proposed approach. Right: $|k_G|_1$ for uneven terrain with $\mu_s = 0.5$

V. CONCLUSION

We have presented a robust multilegged locomotion planning framework, which produces contact sequences that adapt to the unevenness of the terrain, and CoM trajectories that maximize the CWC Margin and minimize the Centroidal Angular Momentum of the motion. Our approach has been successfully tested on a LittleDog quadruped; it generates transitional gaits on uneven terrain, and has proven to provide robust motions in situations where a classic Zero-Moment Point planner would fail.

A. Future Work

In the future, we are interested in providing experimental validation of this approach on modern multilegged platforms [3], [1]. Additionally, the proposed formulation is not restricted to quadrupeds, hence, we are also interested on testing our approach in robots with different geometries and locomotion modes.

In this paper we present a two stage planner to compute the footstep location and the center of mass motion separately. A good extension is to consider using the approaches described in [12], [3], to plan them simultaneously in a single mixed-integer convex optimization problem.

In the specific case of the gait planning stage, we are interested in considering additional metrics for the unevenness of the terrain, such as the convex region orientation or the support area described by the feet. Furthermore, our motion planning approach does not consider join limits, therefore, some motions might not be reachable on robot's without powerful enough motors. Adding this consideration would increase the complexity of the formulation, for this reason, we are also interested in simplifying the formulation of the CWC planner in order to account for further constraints.

B. Source Code

The authors of this work have made the entire source code of our implementation publicly available on GitHub[1].

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