A Mixed-Integer Convex Formulation for Simultaneous Contact, Gait and Motion Optimization on Multi-Legged Robots

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I. Motivation

Planning motions for multi-legged robots is a complex task, as it involves both the discrete choice on gait sequence, and continuous decisions on foot location and robot dynamics. For this reason, most approaches have decoupled the problem in two stages: searching contacts and planning motions. However, by ignoring the dynamics in the first stage, it might restrict the feasible motions on the second stage when considering the robot dynamics. Hence in this work we combine the two stages into one single problem using mixed-integer optimization.

Previous research has addressed this problem with non-convex optimization, which requires significant computation time and cannot guarantee optimality. Others have used Mixed-Integer Convex Programs (MICP) [1], [2], [3], using a fixed gait. Here, we propose a MICP formulation to simultaneously plan the gaits and dynamically feasible motions on rough terrain, by efficiently solving a MICP.

II. Proposed Formulation

In order to plan the robot's motion, we assume that it is divided in N_t time-slots over which we generate leg swings. Here, we rely on a centroidal dynamics model. Hence, our decision variables are the center of mass $\mathbf{r}$, end-effectors $\mathbf{p}_e$, and contact forces $\boldsymbol{\lambda}_e$ trajectories.

1) Contact Locations: we optimize over N_c contacts, which are represented by their position and the trunk orientation at their swing time.

2) Gait Sequence: we optimize the gait sequence by assigning the transition to each contact, given a determined time-slot N_t, in a gait transition matrix $\mathbf{T} \in \{0,1\}^{N_c \times N_t}$.

3) Safe-regions assignment: As in previous work [2], we avoid obstacles by assigning contacts to one of N_r convex safe regions, done with a binary matrix $\mathbf{H} \in \{0,1\}^{N_c \times N_r}$.

4) Kinematic reachability: we approximate the kinematic reachability of each end-effector by adding bounding box constraints on the contact locations. For this, we use the CoM location as a reference for the workspace of each leg.

5) Contact Force Constraints: we ensure stability by constraining each contact force to lie in its corresponding friction cone. We use a) the binary matrix $\mathbf{H}$ to determine each cone, and b) the gait transition matrix $\mathbf{T}$ to activate the contact forces.

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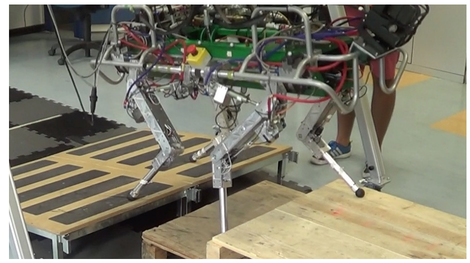

Fig. 1. Our approach generates robust plans on challenging terrains, simultaneously reasoning about contact locations and body motions. We validate on the HyQ robot by transversing different challenging terrains in which considering non-coplanar contacts dynamics and friction-cone constraints are important for the success of the task.

6) Approximate Torque Limits: To improve feasibility in the execution, we rely on the quasi-static assumption on each leg $\mathbf{J}_e^T \boldsymbol{\lambda}_e \leq \boldsymbol{\tau}_{max}$, constraining leg torques over the approximate bounds of the operational space Jacobian $\mathbf{J}_e$.

7) Angular dynamics: We exploit the relation presented in [1], in order to represent bilinear terms of the angular momentum rate as a difference of convex quadratic terms.

Given these constraints, we formulate a MICP and add costs on the acceleration, angular momentum rate, execution time, and negative friction cone stability margin.

III. Experimental Validation

We validate our approach on the HyQ robot traversing challenging terrain tracks (see Fig. 1), which would not be easily handled by non-convex trajectory optimization techniques or without friction cone constraints. Our approach generates robust trajectories in all scenarios, where each locomotion cycle (4 time-slots or one stride) is optimized in about 0.5 secs.

References

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