Mixed-Integer Convex Optimization of Non-Gaited Multi-Legged Walking Sequences

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I. Motivation

There has been a series of work on planning optimal footstep plans for bipedal robots, using Mixed-Integer Convex Programming (MICP). In Deits' work [1], the authors planned the footsteps for bipedal robots. We extend this approach to multi-leg robots. Unlike the bipedal robot case whose leg sequence is merely alternating the left and right leg, the multi-leg robot has extra complexity that the leg sequence has exponentially number of combinations.

Previously researchers have used Central Pattern Generators (CPG) to plan the gait sequence. While CPG works well on flat terrain, it doesn't extend naturally to uneven terrain. Therefore. most contact and walking motion planning approaches can't simultaneously reason about *where and when* to place the foots. Here, we will propose a MICP formulation to efficiently search for the foot locations and gait sequences simultaneously.

II. Proposed Approach

In order the optimize gait sequences that adapt to the terrain, we propose a MICP formulation that can work as a part of optimization-based footstep planners, such as proposed by Deits [1] or Aceituno [2]. Thus, this approach will optimize over the following decision variables:

1) Contact locations: an array of N_f vectors of the form $f = (x, y, z, \theta)$ representing the position of each contact the yaw orientation of the trunk when transitioning to the contact. This array is ordered by leg number, such that for n_l legs f_i represents a single leg position and f_{i+n_l} represents its next contact position.
2) Safe-region assignment: done with a binary matrix H, such that $H_{rj} \Rightarrow A_r f_j \leq b_r$, assigns contact location j to a convex region r, where $\sum_{r=1}^{N_r} H_{rj} = 1$.
3) Gait sequence: a binary matrix that assigns each transition of the leg from a contact to the next to a certain timestep. This also defines the workspace of the robot at each contact phase, ensuring dynamic reachability.

Gait sequences are often represented by phase diagrams, which indicate the n_l legged supports for a given support phase. Inspired by this representation, we will describe the support phase assigned to each leg transfer cycle within the contact plan. To do this, we introduce a binary matrix $T \in \{0, 1\}^{N_f \times N_f}$, where $T_{ij} = 1$ means that the i_{th} foot will

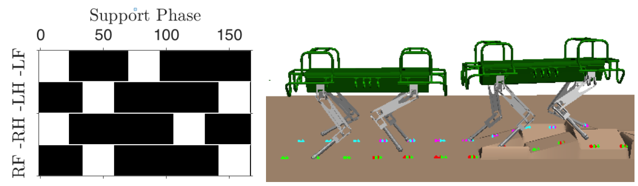

Fig. 1. Left: Phase diagram of an optimized gait sequence on rough terrain. Right: Execution of the optimized motion, HyQ trots when the terrain is flat and starts crawling when the terrain is rough enough.

transfer from foot location $i - n_l$ to i at the j_{th} support phase. Since each foot location can only be reached once in the plan, we constrain this as:

$$\sum_{j=1}^{N_f} T(i, j) = 1 \ , \ \forall i = 1, .., N_f \tag{1}$$

This representation is equivalent to indicating the supports at each phase, since we assume that the contact vector is ordered by leg number. Furthermore, we enforce that each cycle of n_l contacts must be reached before the next transfer cycle starts.

Finally, we aim to minimize a convex objective function, which includes the following costs:

- Aggresive movements with roughness between adjacent contact locations.
- Distance between final foot locations and the goal.
- Relative displacement between contacts.
- Plan execution time.

This cost function can ensure that the planner assigns more support to a transfer phase over rough terrain, while seeking the fastest gait when the terrain is relatively flat.

References

[1] R. Deits and R. Tedrake, "Footstep planning on uneven terrain with mixed-integer convex optimization," in *Humanoid Robots (Humanoids), 2014 14th IEEE-RAS International Conference on*. IEEE, 2014, pp. 279–286.

[2] B. Aceituno-Cabezas, J. Cappelletto, J. C. Grieco, and G. Fernandez-Lopez, "A Generalized Mixed-Integer Convex Program for Multilegged Footstep Planning on Uneven Terrain," *arXiv preprint arXiv:1612.02109*, 2017.