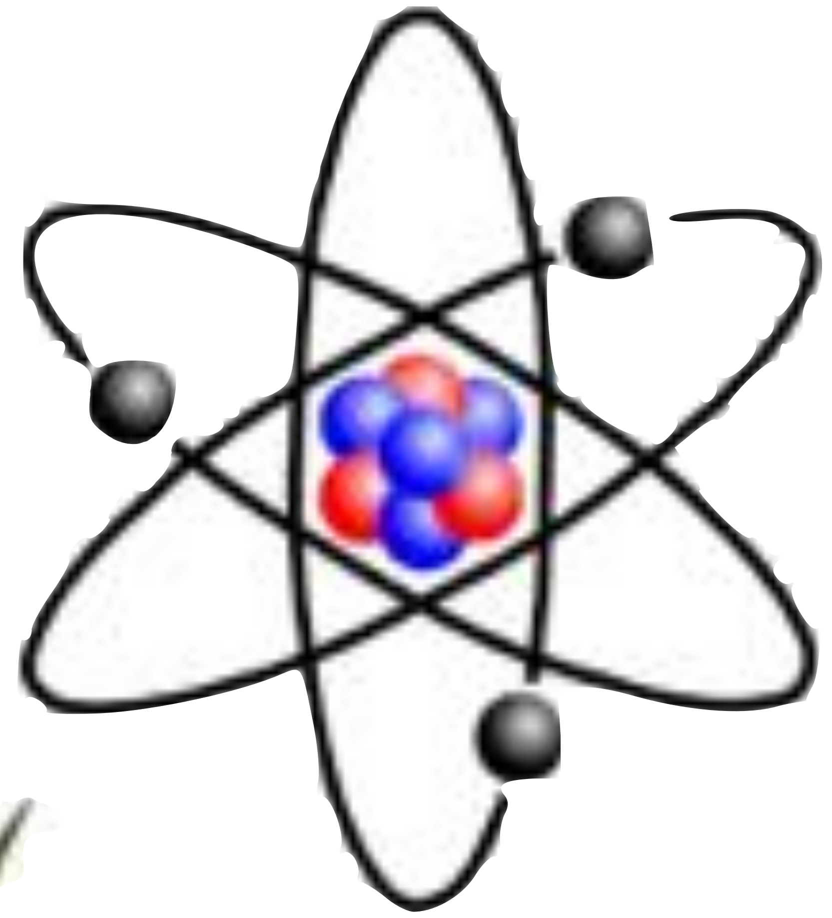
set Notation

Elements



Lithium

Foundations, building blocks of sets

Can be anything

Messi

Google

Aspirin

Structured

Numbers

→ 0 1 2 3 4

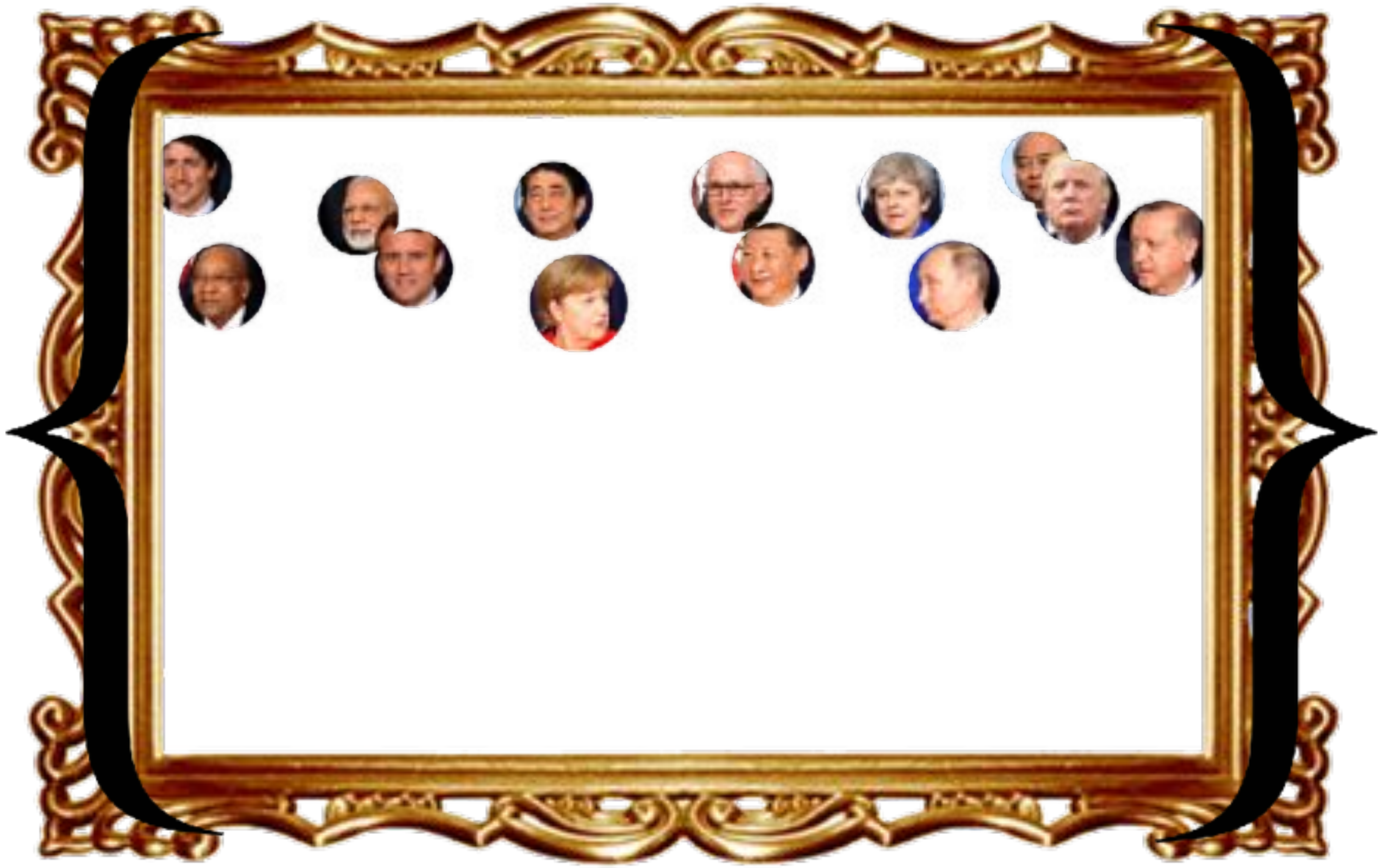
Elements to Sets

Beyond individual elements

"Bigger picture"

Set Collection of elements

Define specify elements



Specification

Explicit	Coin	{ heads, tails }
	Bits	{ 0, 1 }
	Die	{ 1, 2, 3, 4, 5, 6 }
Implicit	Digits	{ 0, 1, …, 9 }
	Letters	{ a, b, …, z }
	Days	{ Monday, …., Sunday}
Descriptive	{four-letter words} = {love, like, dear ,…}	

More

Compact ✅

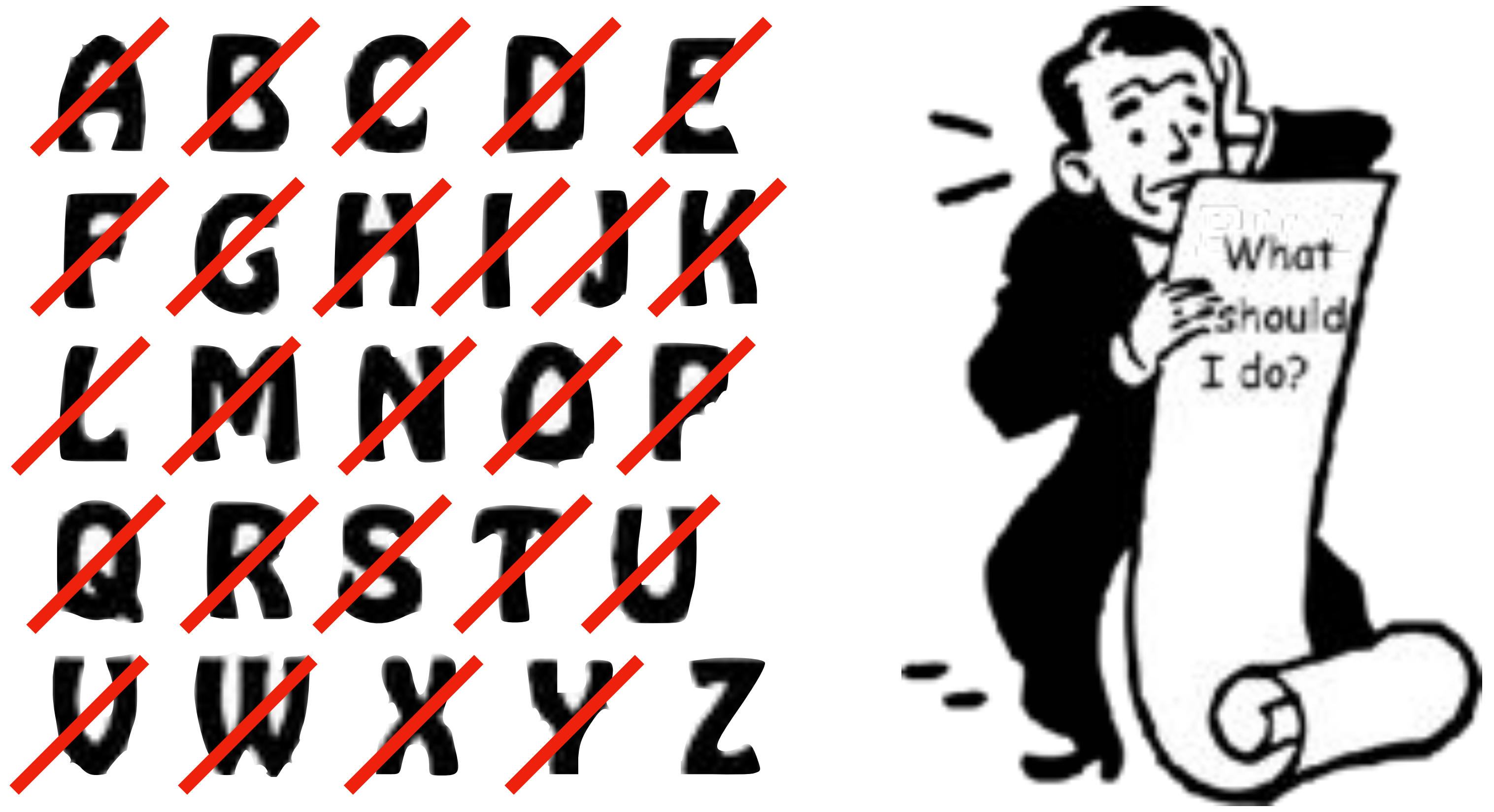
Expressive ✅

Ambiguous ⚠️

Common Sets

Integers	{ …, -2, -1, 0, 1, 2, … }	$\mathbb{Z}$
Naturals	{ 0, 1, 2, … }	$\mathbb{N}$
Positives	{1, 2, 3,… }	$\mathbb{P}$
Rationals	{ integer ratios m/n, $n \neq 0$ }	$\mathbb{Q}$
Reals	{ … Google … }	$\mathbb{R}$

Mnemonic



Zahl - number

CONVEN IENT
CONVEN TION

Sets	UPPER CASE	A
Elements	lower case	a

Membership

If element x is in a set A, it is a member of, or belongs to A, denoted $x \in A$

$0 \in \{0,1\}$ $1 \in \{0,1\}$ $\pi \in \mathbb{R}$

Equivalently, A contains x, written $A \ni x$

$\{0,1\} \ni 0$ $\{0,1\} \ni 1$ $\mathbb{R} \ni \pi$

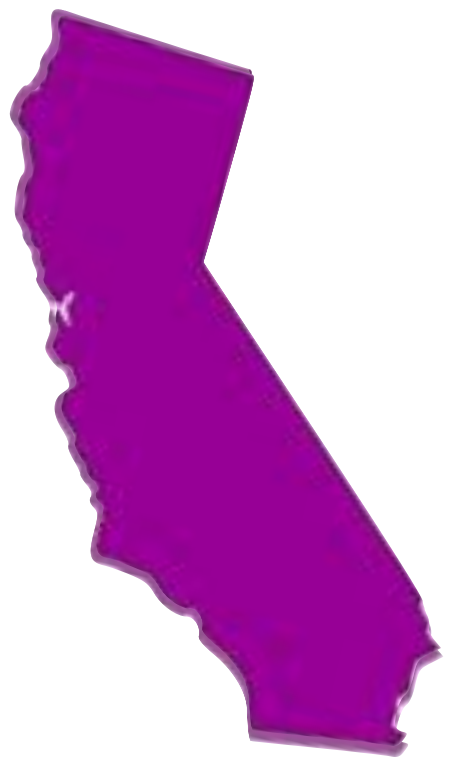
If x is not in A, then x is not a member, or does not belong to A, denoted $x \notin A$

$2 \notin \{0,1\}$ $\pi \notin \mathbb{Q}$

Equivalently, A does not contain x, $A \not\ni x$

$\{0,1\} \not\ni 2$ $\mathbb{Q} \not\ni \pi$

Set of States
United



$\in$

$\notin$

$\in$

$\notin$

$\in$

$\notin$

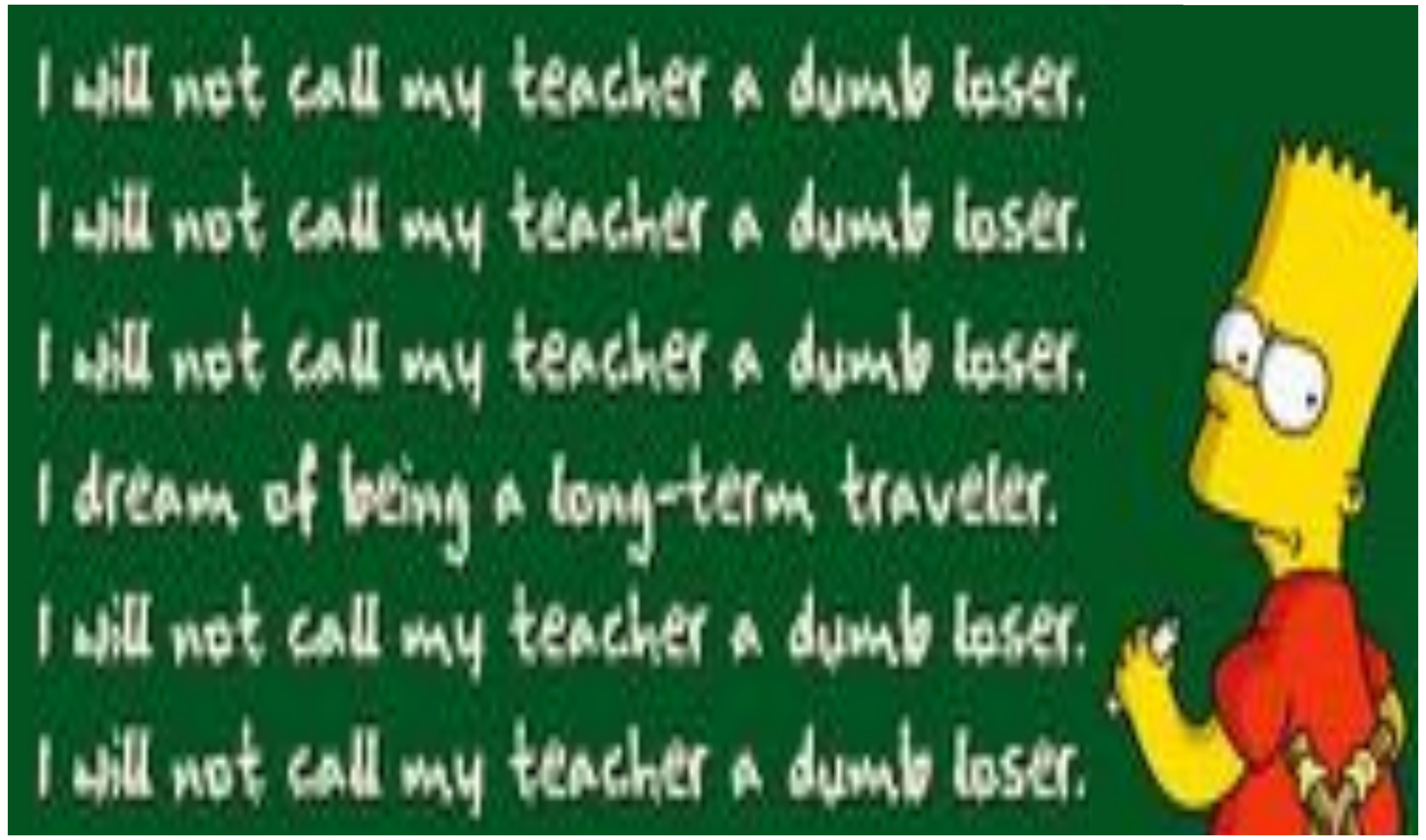
$\in$

$\notin$

Doesn't Matter

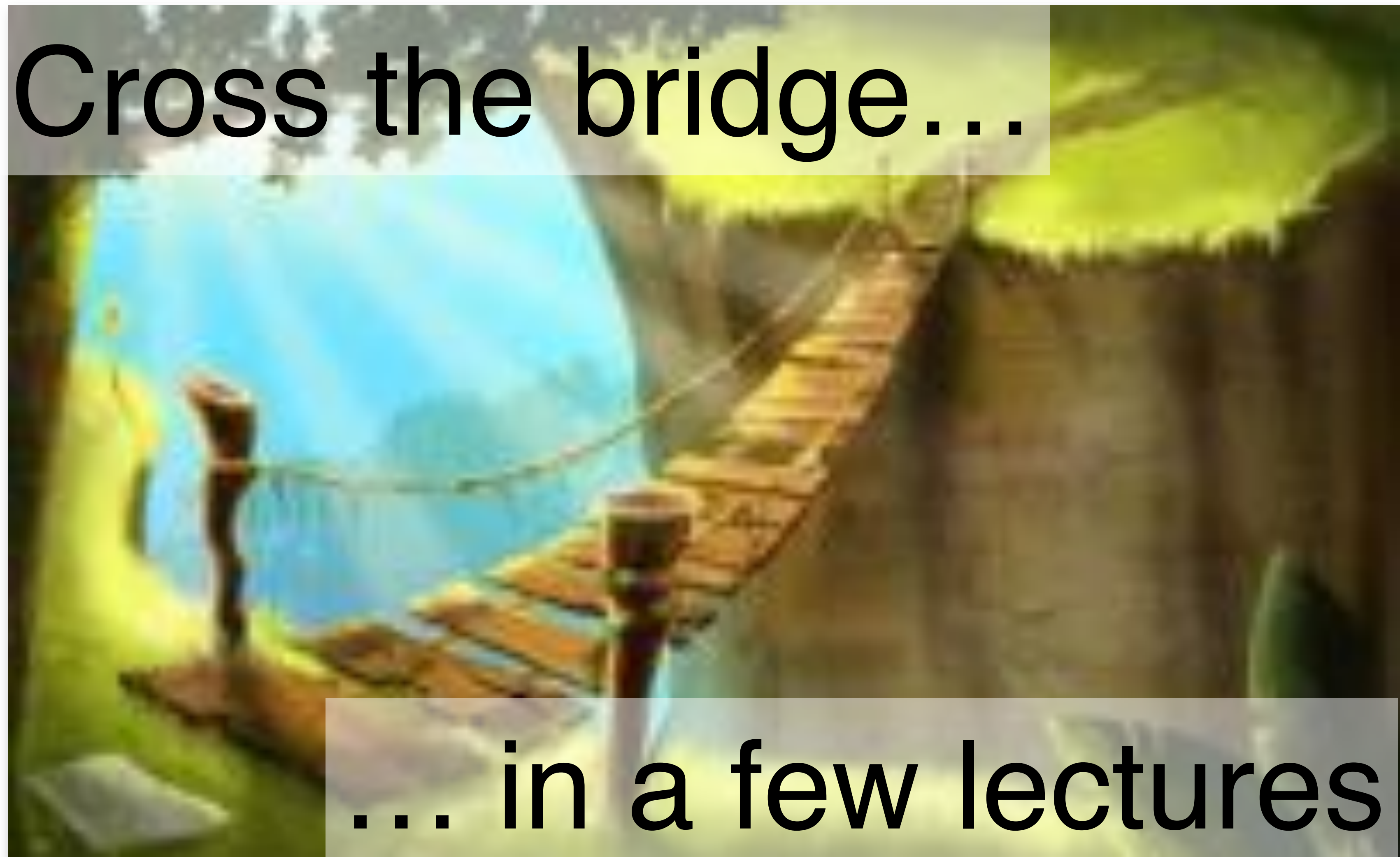
Order $\{0, 1\} = \{1, 0\}$

Repetition $\{0,1\} = \{0,1,1,1\}$



Order matters: use ordered tuples $(0,1) \neq (1,0)$

Repetition matters: use multisets, or bags



Special Sets

Empty set | contains no elements | $\varnothing$ or $\{\,\}$ | $\forall x,\ x \notin \varnothing$ | $\forall$ - All, every

Universal set | all possible elements | Ω | $\forall x,\ x \in \Omega$

Ω lets us consider only relevant elements

$\Omega = \mathbb{Z}$ integers

"prime"

Means 2, 3, 5,…

Not

amazon Prime

Ω depends on application

temperature $\Omega = \mathbb{R}$

text $\Omega = \{\text{words}\}$

Only one $\varnothing$

set with no elements

Sets

Set Definition

{…} or **`set({…})`**

```
Set1 = {1,2}
print(Set1)
{1,2}
```

```
Set2 = set({2,3})
print(Set2)
{2,3}
```

For empty set use only **`set()`** or **`set({})`**

```
Empty1 = set()
type(Empty1)
set
print(Empty1)
set{}
```

```
Empty2 = set({})
type(Empty2)
set
print(Empty2)
set{}
```

```
NotASet = {}
type(NotASet)
dict
```

{} is not an empty set

Membership

$\in$ - **in**

```
Furniture = {'desk','chair'}
'desk' in Furniture
True
'bed' in Furniture
False
```

$\notin$ - **not in**

```
Furniture = {'desk','chair'}
'desk' not in Furniture
False
'bed' not in Furniture
True
```

Testing if Empty, Size

Test empty

`not`

```
S = {1,2}
not S
False
```

```
T = set()
not T
True
```

Size

`len()`

```
print(len(S))
2
```

```
print(len(T))
0
```

Check if size is 0

`len()==0`

```
print(len(S)==0)
False
```

```
print(len(T)==0)
True
```


gram
set Visualization

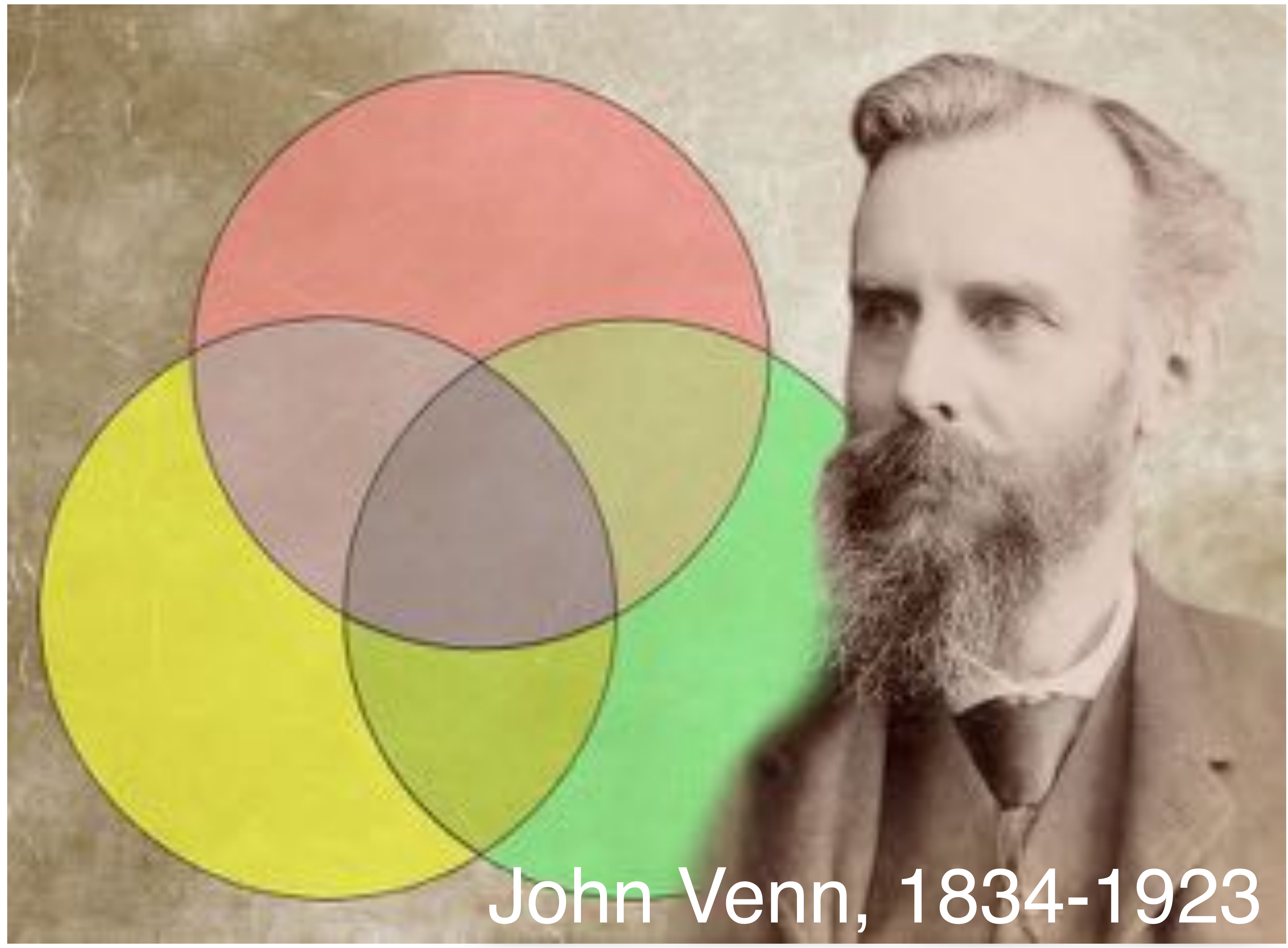
Venn Diagram

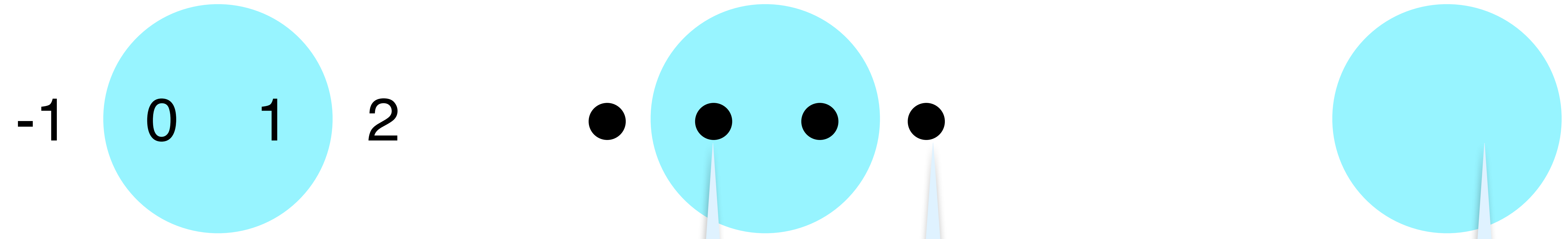
Instagram

Visualize

Sets — Regions

Elements — Points





{ 0, 1 }

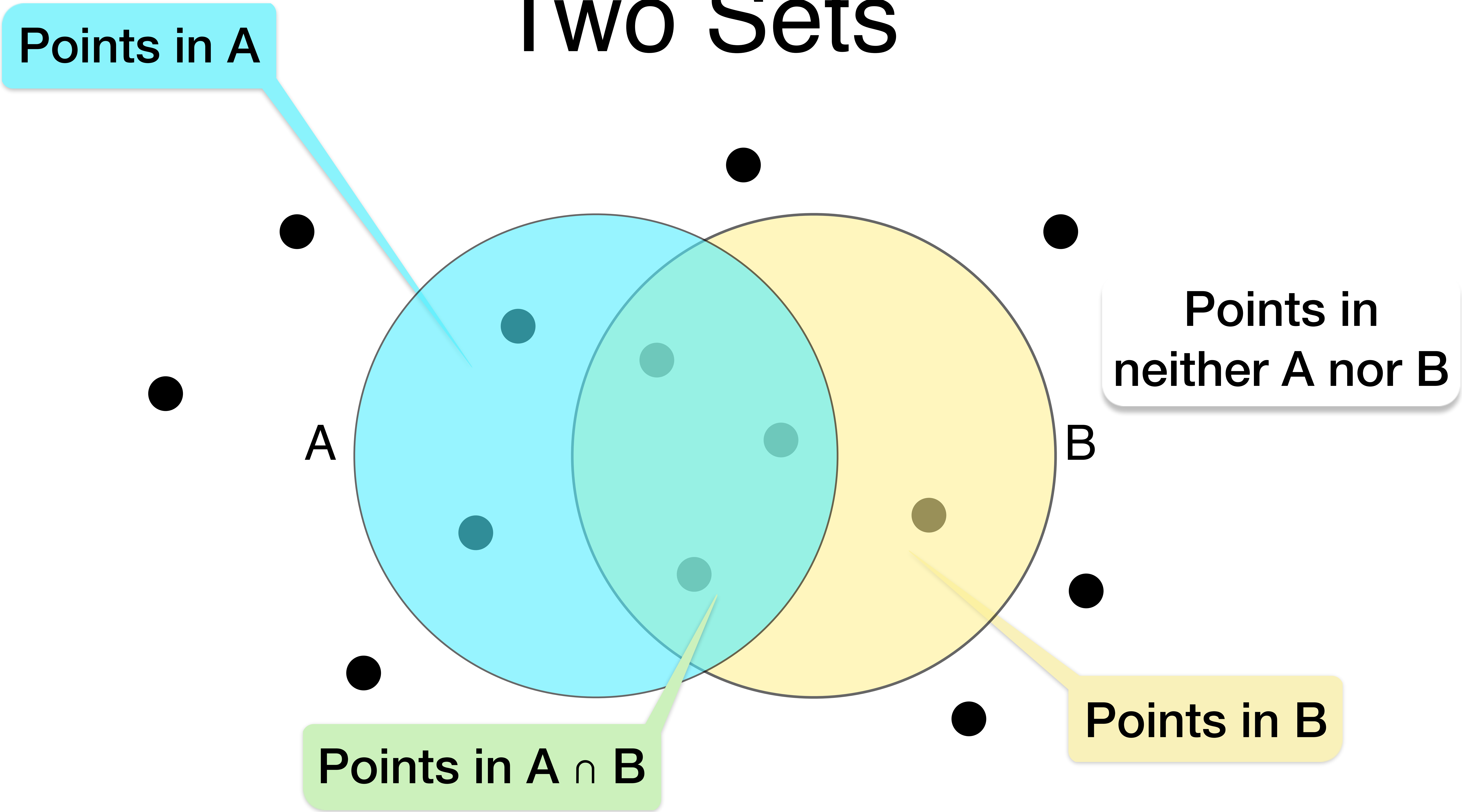
Elements in set

Elements not in set

Elements in set

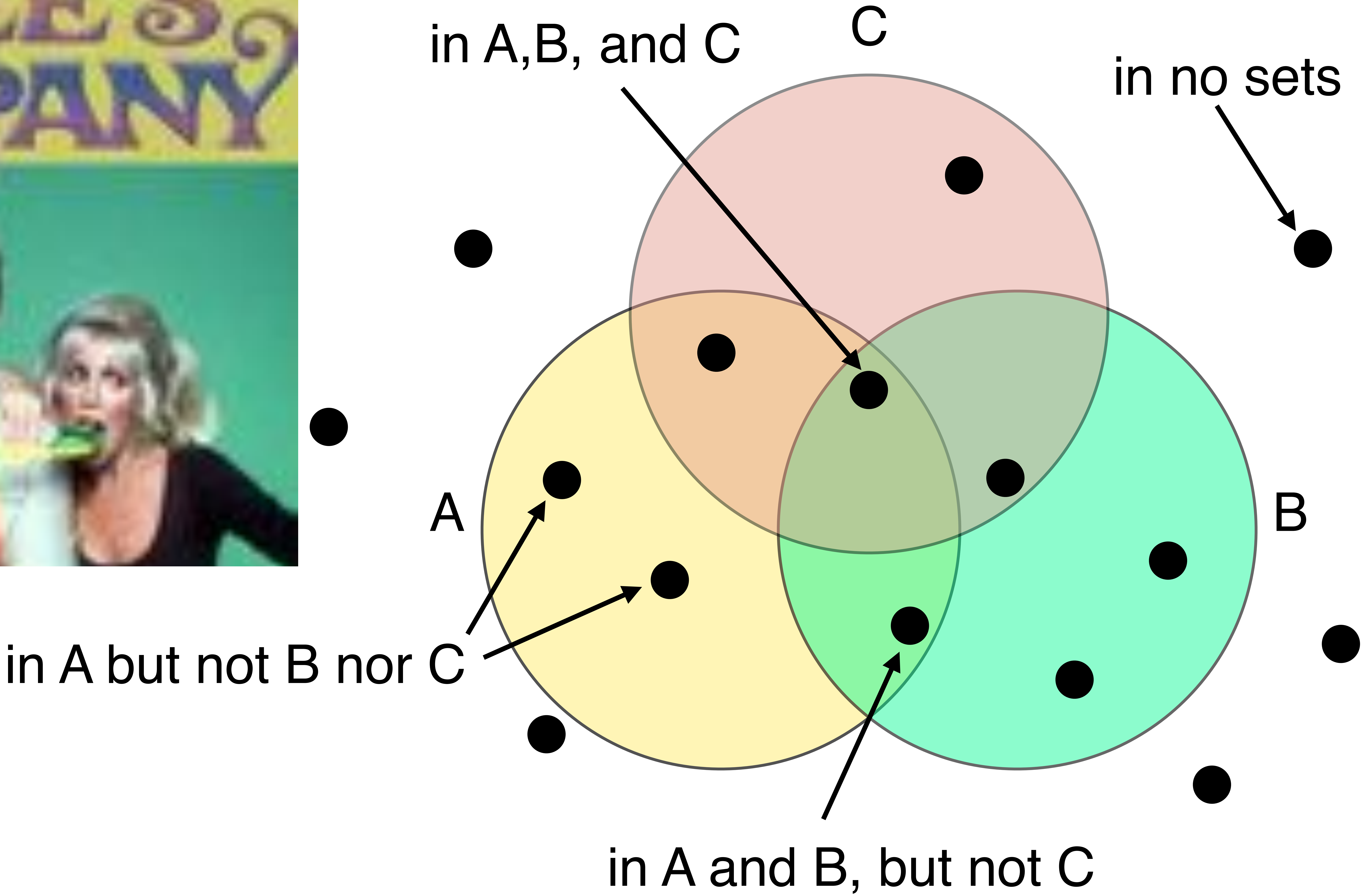
Elements not in set

Two Sets



Points in A
A
B
Points in neither A nor B
Points in A ∩ B
Points in B

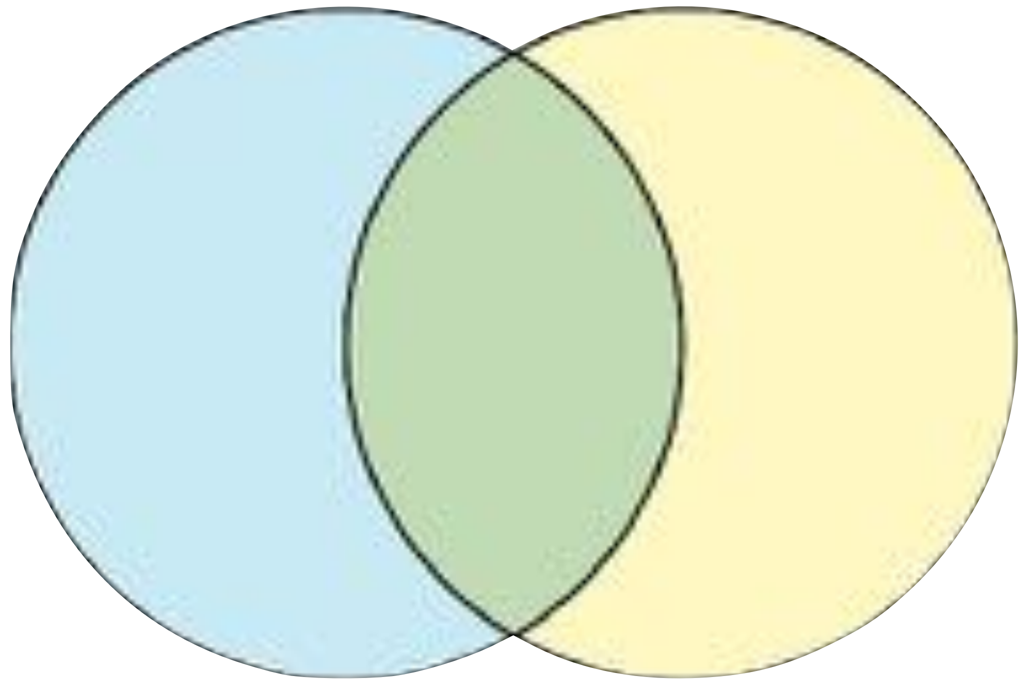

THREE'S COMPANY


in A,B, and C
C
in no sets
A
B
in A but not B nor C
in A and B, but not C

Why Venn

=

10^3



=

Visualize definitions & proofs

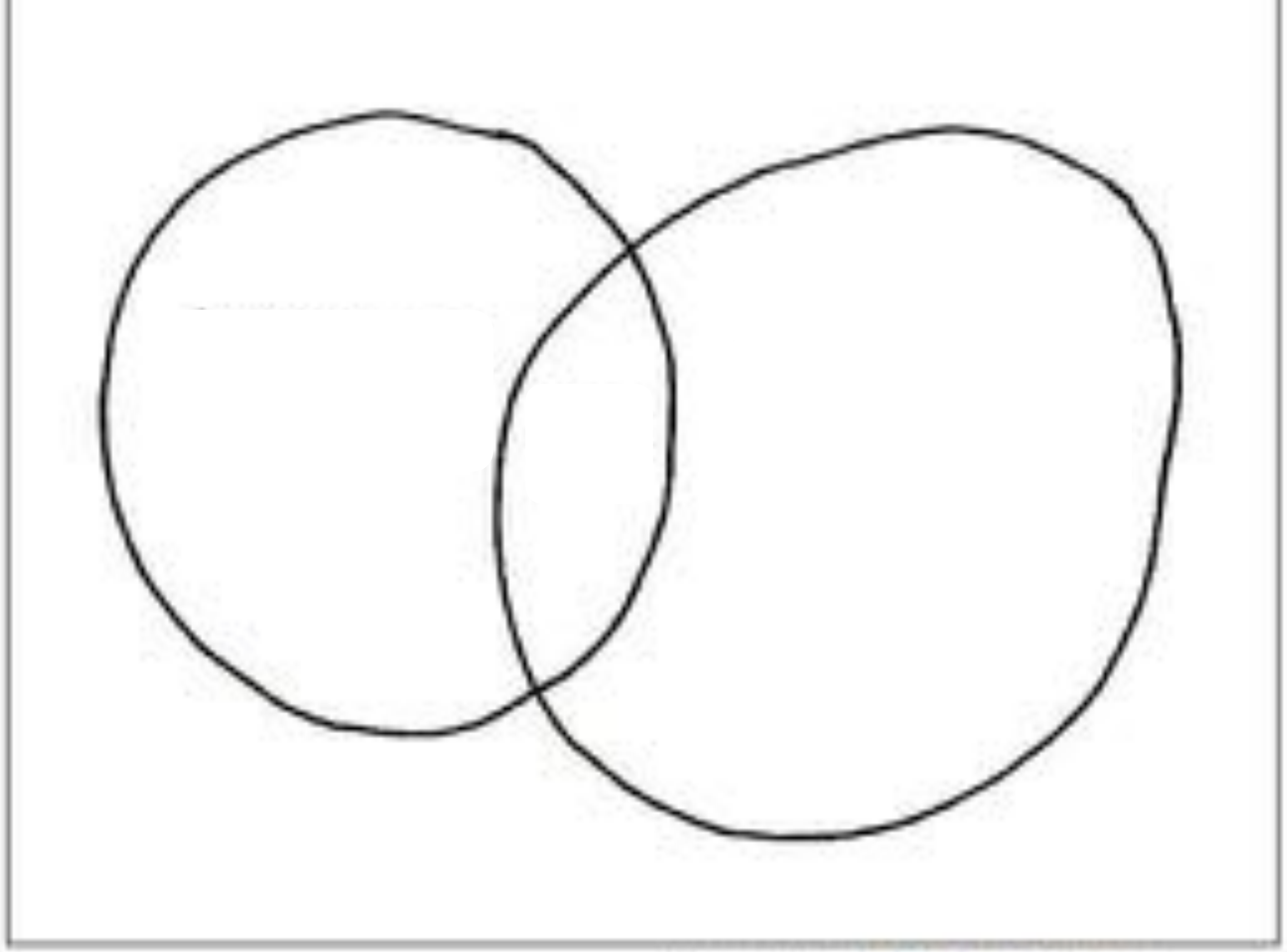
=

Gods of Science

Got something right

Attached his name to it

1881 John Venn's original diagram



Venn Diagrams

Download matplotlib_venn package   (link: notebook)

```
import matplotlib.pyplot as plt
import matplotlib_venn as venn
S = {1, 2, 3}
T = {0, 2, -1, 5}
venn.venn2([S, T], set_labels=('S','T'))
plt.show()
```


```
venn.venn3([S,T,U], set_labels=(’S’,’T’,'U'))
```

Venn Diagrams

Visualize sets

Help define, understand , and prove results about sets

Show in python (mostly for class)

Set Relations

set Relations

Relation Types

Human relations

Number relations

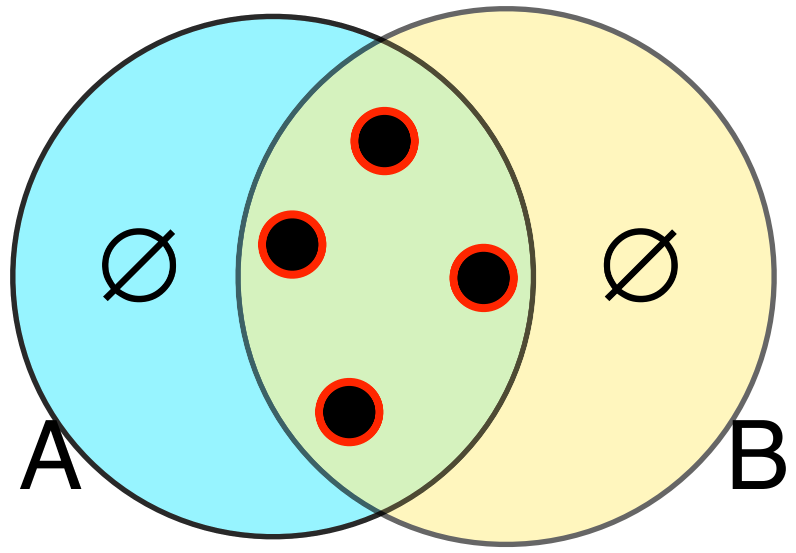
= ≤ <

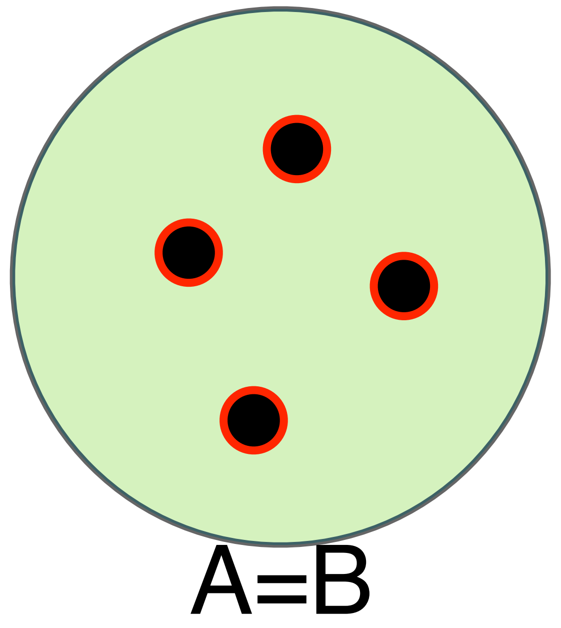
Generalize to sets

All men are created equal

generalize = of numbers

Sets A and B are equal, denoted A = B, if they have exactly the same elements

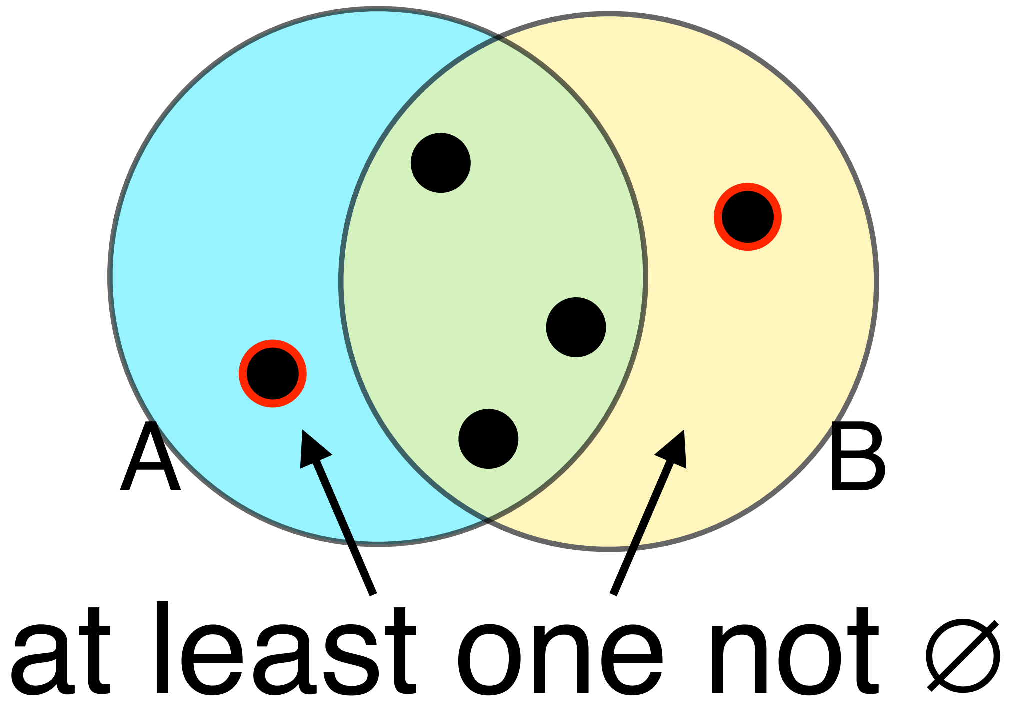




{0,1} = {1,0}

All sets are not created equal

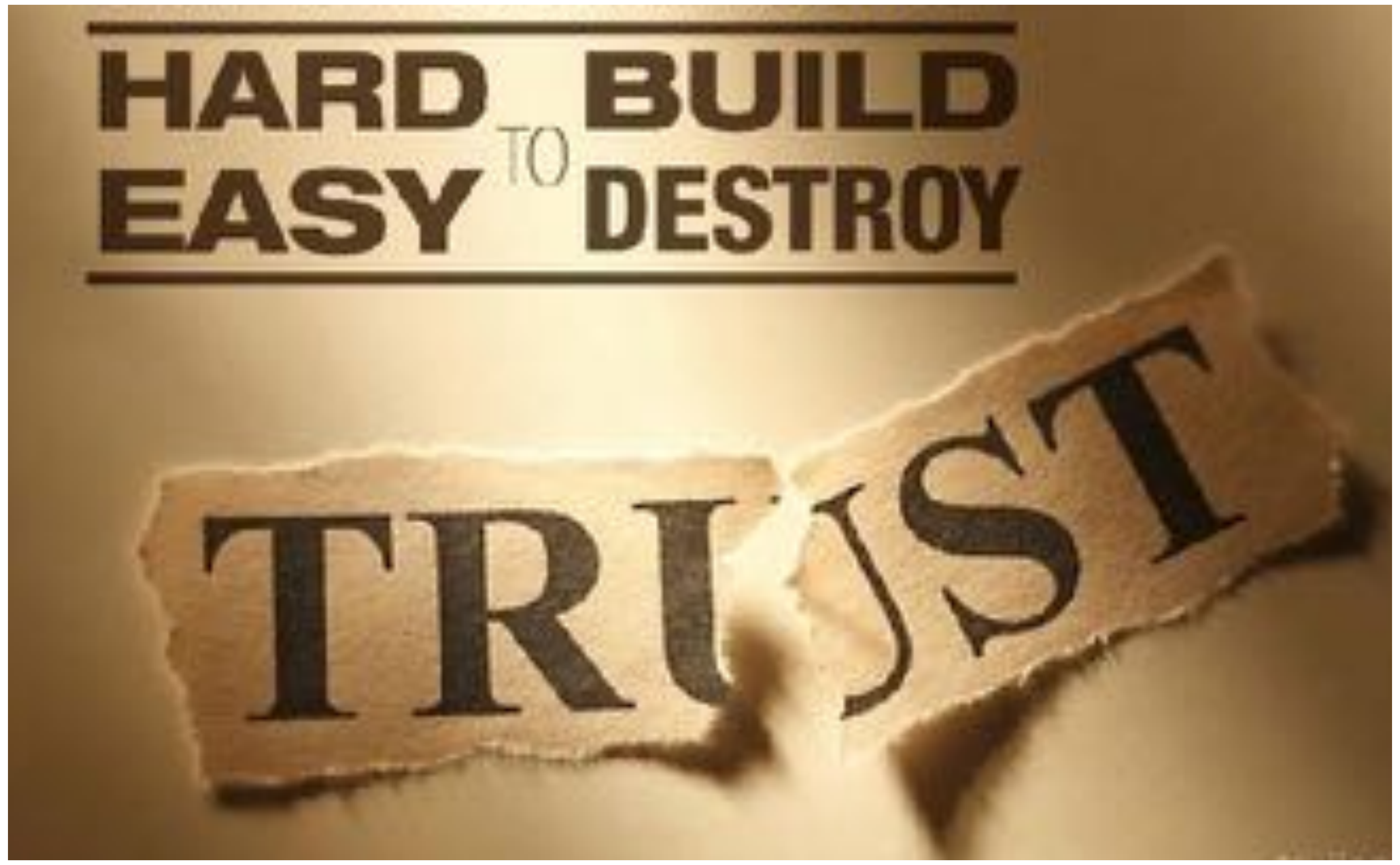
If A and B are not equal, they are different, denoted A ≠ B



{0,1} ≠ {1,2}

Equality QUIZ

What does set equality have in common with trust?



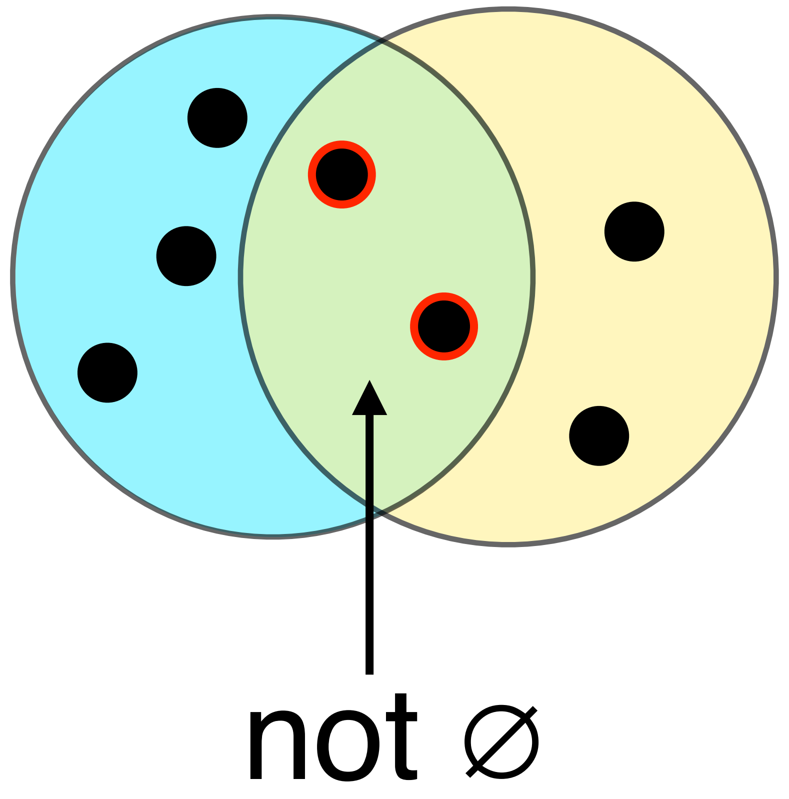
= All elements must be identical $\{1,2,4\} = \{4,1,2\}$

≠ One different element enough $\{1,2,4\} \neq \{1,2,4,8\}$

Intersection

Two sets **intersect** if they share at least one common element

$\exists x$
$x \in A \wedge x \in B$

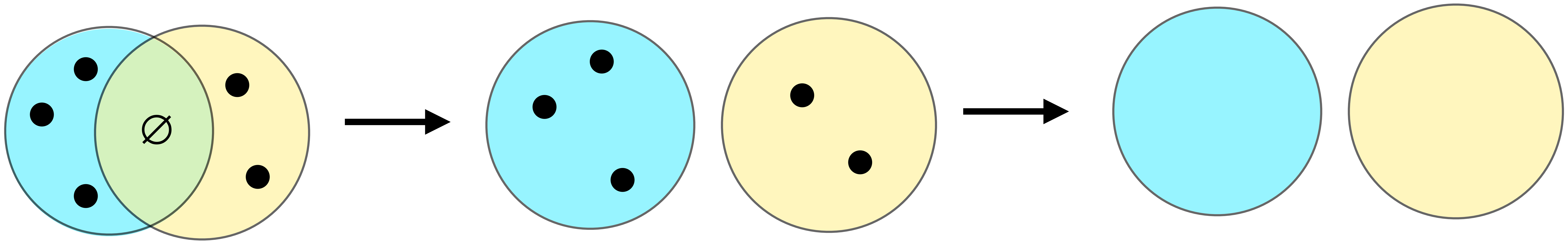


{0,1} {1,2} (1)   [3,4] [2,5] (3.5,..)

Two sets are **disjoint** if they share no elements

$\neg \exists x$
$x \in A \wedge x \in B$

{0,1} {2,3}   [3,4] (4,5]

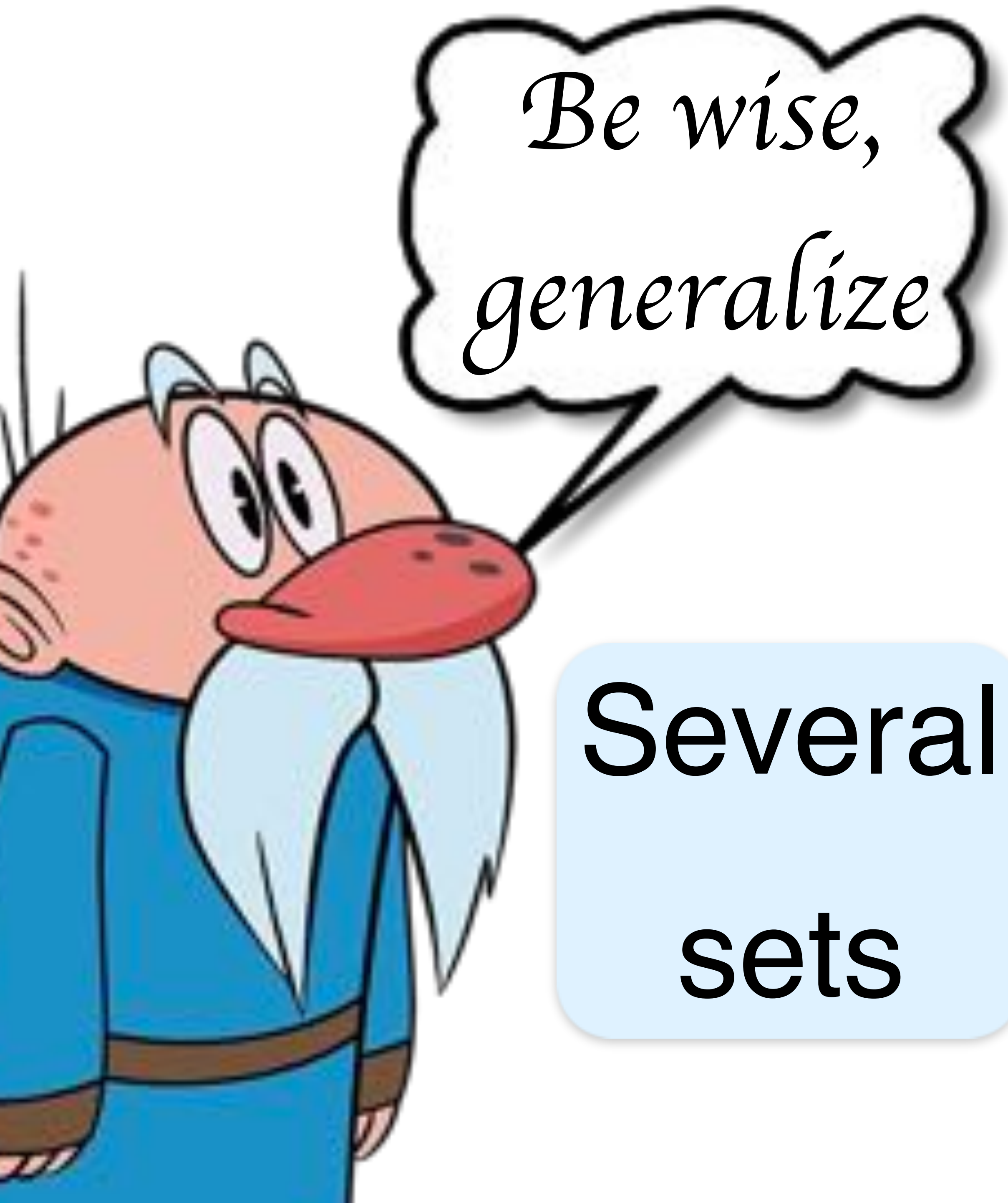


Intersection

∅ disjoint from any set

Non-empty Ω intersects every set

A set intersects itself iff it is non-empty

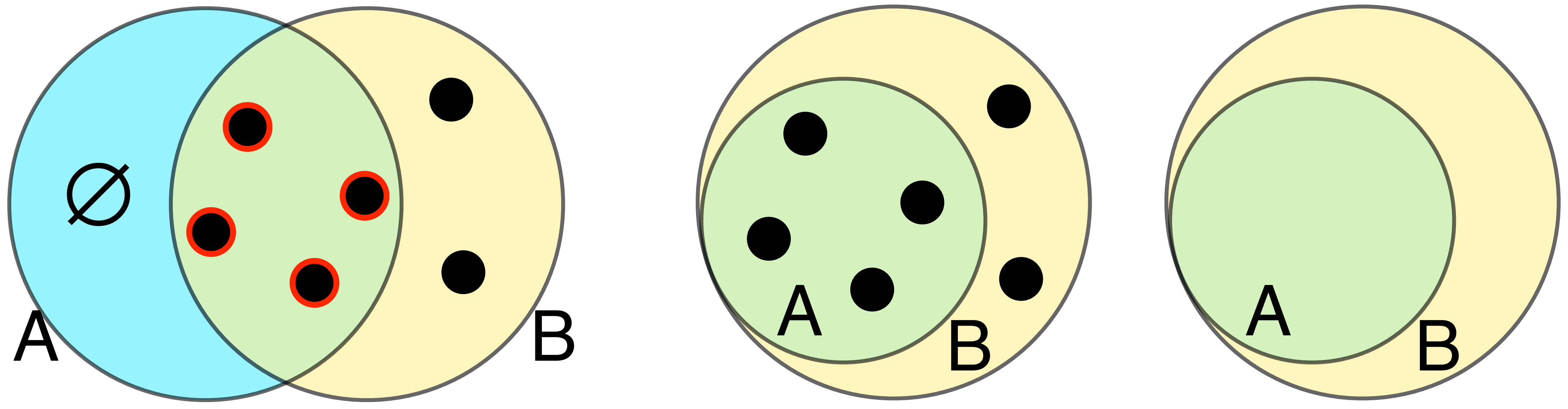


Several sets

- intersect if all share a common element
- mutually disjoint if every two are disjoint

Subsets

generalize $\leqslant$



If every element in A is also in B, then A is a subset of B, denoted $A \subseteq B$

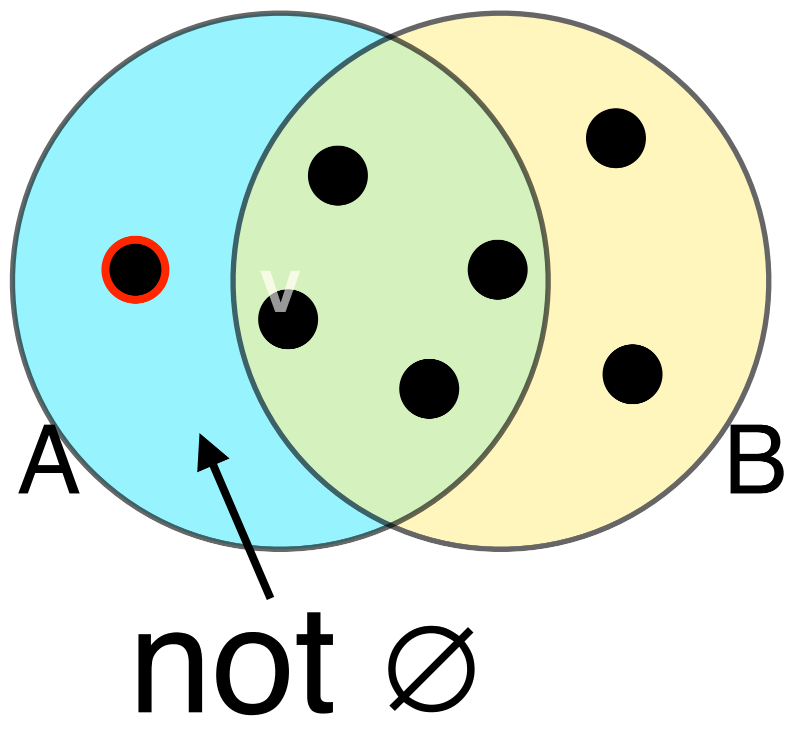
$\{0\} \subseteq \{0,1\}$

$\{0\} \subseteq \{0\}$

Equivalently, B is a superset of, or contains, A, denoted $B \supseteq A$

$\{0,1\} \supseteq \{0\}$

If A has an element that's not in B, then A is not a subset of B, denoted $A \nsubseteq B$, or $B \nsupseteq A$

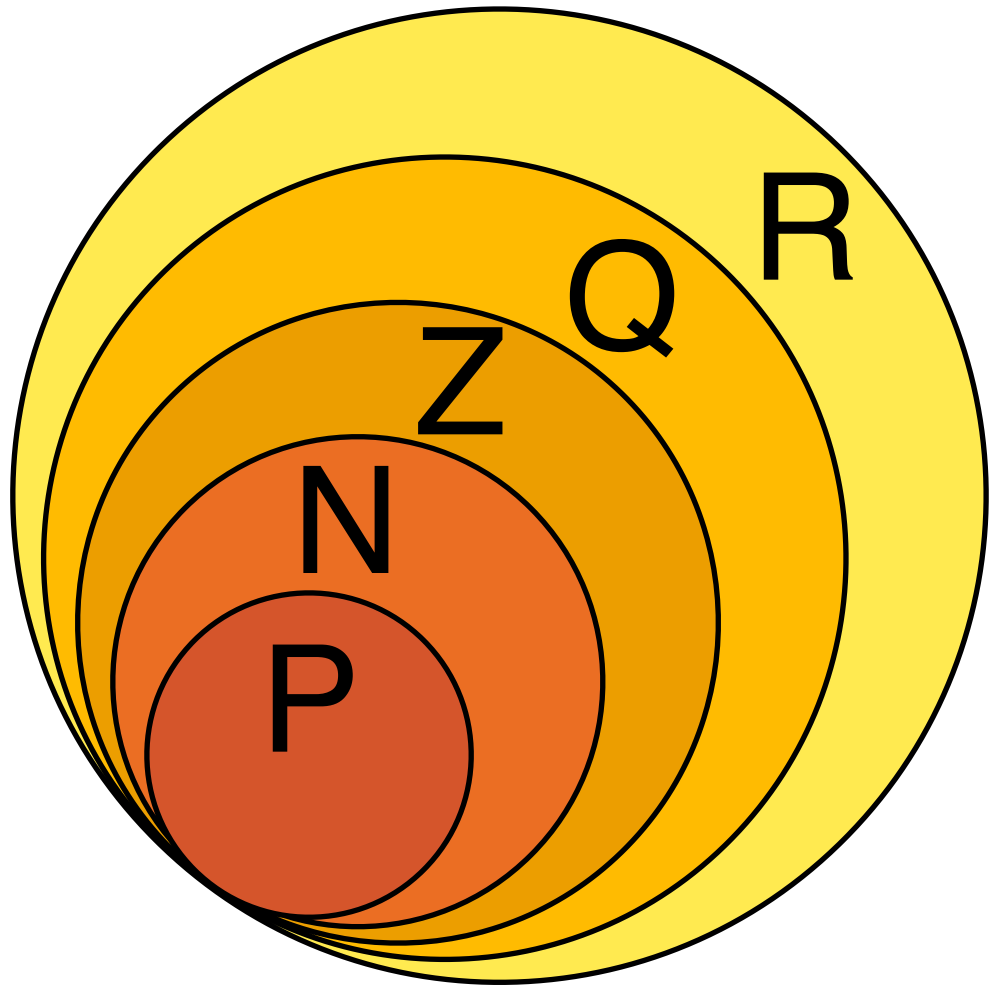


$\{0,1\} \nsubseteq \{1,2\}$

$\{1,2\} \nsupseteq \{0,1\}$

Subsets

$P \subseteq N \subseteq Z \subseteq Q \subseteq R$

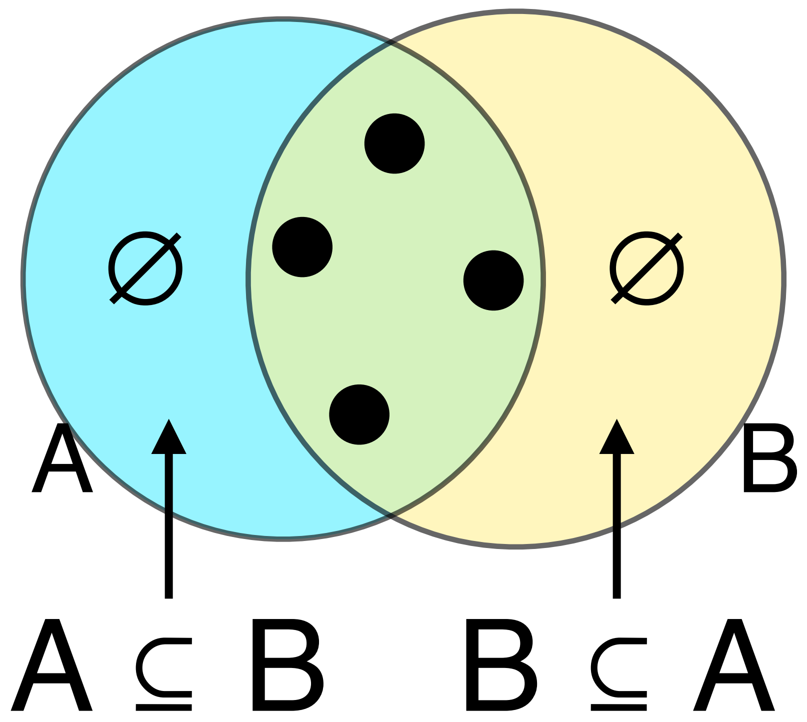


$\varnothing \subseteq A \subseteq A \subseteq \Omega$

$A \subseteq B$ and $B \subseteq C \rightarrow A \subseteq C$

$\subseteq$ is transitive

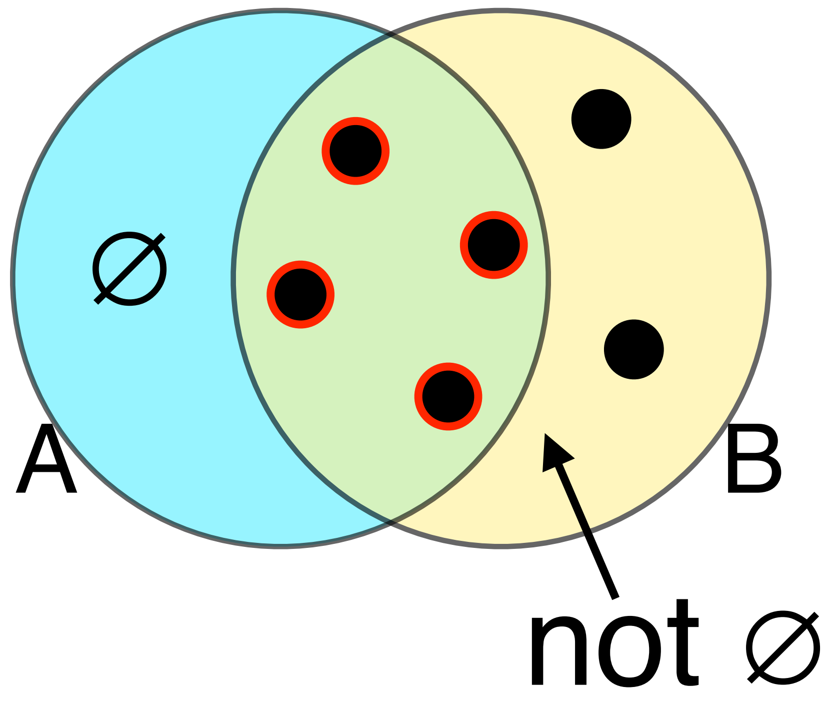
$A \subseteq B$ and $B \subseteq A \rightarrow A = B$



Strict Subsets

generalize $<$

If $A \subseteq B$ and $A \neq B$, A is a strict subset of B, denoted $A \subset B$, and B is a strict superset of A, denoted $B \supset A$



$\{ 0 \} \subset \{ 0, 1 \}$

$\{ 0,1 \} \supset \{ 0 \}$

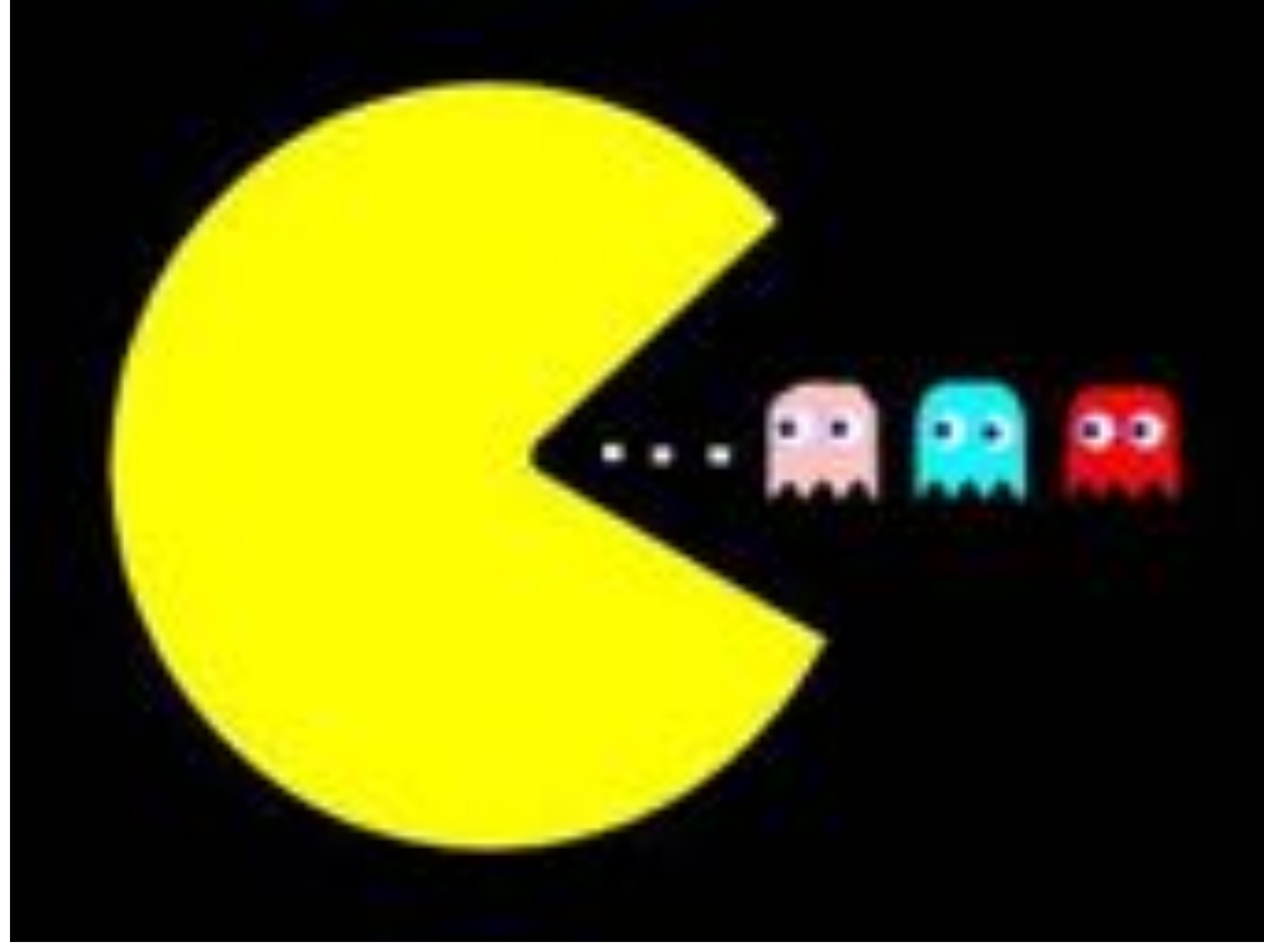
If A is not a strict subset of B, we write $A \not\subset B$ or $B \not\supset A$

Two possible reasons

$A \nsubseteq B$ $\{ 0 \} \not\subset \{ 1 \}$

$A = B$ $\{ 0 \} \not\subset \{ 0 \}$

belongs to $\in$ **VS.** $\subseteq$ subset of



$\in$ Relation between an element and a set

$x \in A$: element x belongs to, or is contained in, set A

$\{0,1\}$ has two elements: 0 and 1

$\boxed{0 \in \{0,1\}}$ $\boxed{\{0\} \notin \{0,1\}}$

$\subseteq$ Relation between two sets

$A \subseteq B$: set A is a subset of set B

$\{0,1\}$ two elements: 0 and 1

$\{0\}$ one elt: 0

$\boxed{\{0\} \subseteq \{0,1\}}$

0 is an element of $\{0,1\}$, but 0 is not a set

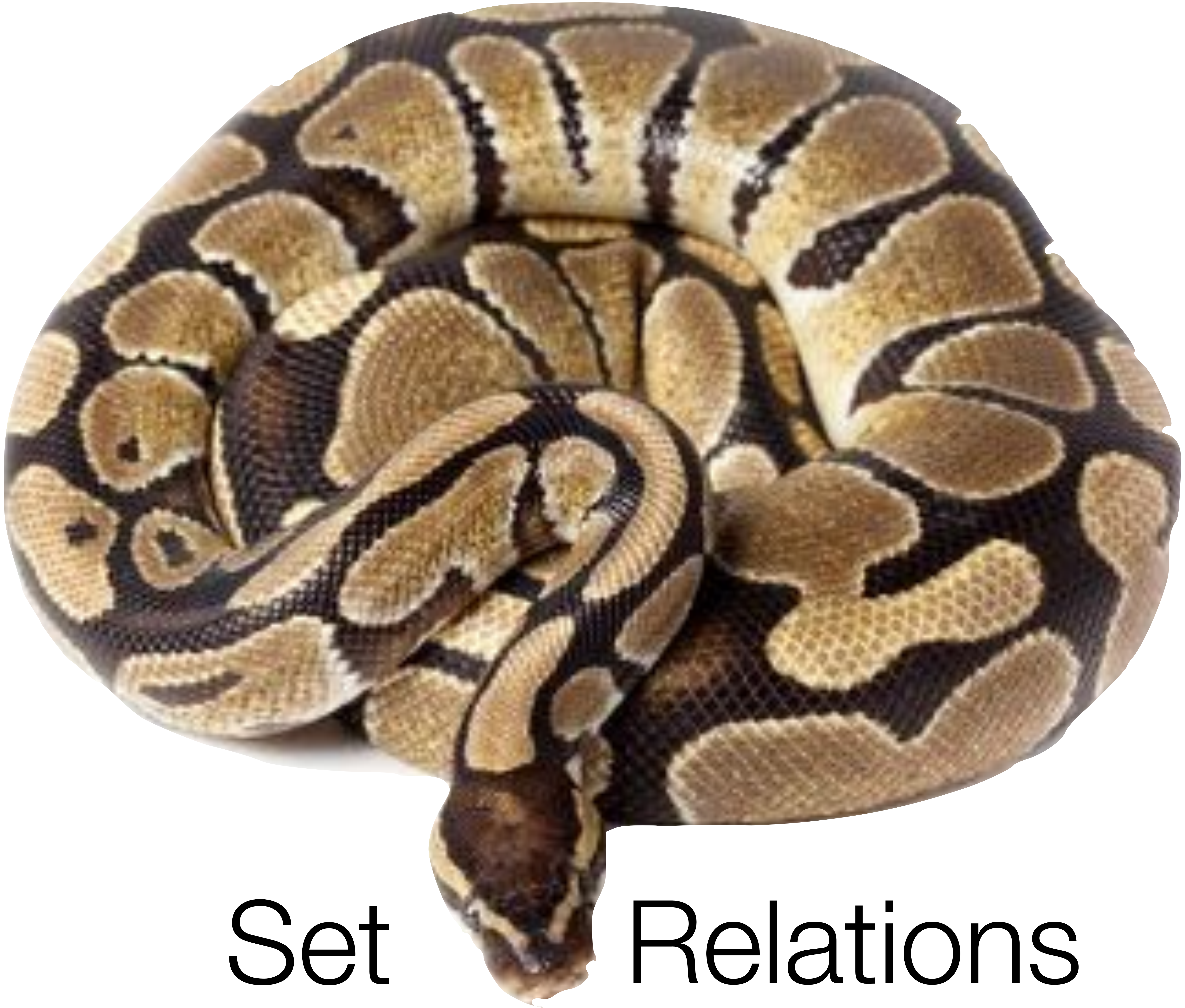
$\boxed{0 \not\subseteq \{0,1\}}$

P U Z Z L E

Are there sets A and B such that
A is both an **element** and a **subset** of B?

$A \in B$ and $A \subseteq B$

Next lecture



Set Relations

Equality, Inequality, Disjoint

= `==`

```
S1 = {0,1}
S2 = set({0,1})
S3 = {1,0,1}
T  = {0,2}

S1 == T
False

S1 == S2
True

S1 == S3
True
```

≠ `!=`

```
S1 != S2
False

S1 != T
True
```

disjoint **`isdisjoint`**

```
S1.isdisjoint(T)
False
S1.isdisjoint({2})
True
```

Subsets and Supersets

```
zero = {0}
```

```
zplus = {0,1}
```

```
zminus = {0,-1}
```

$\subseteq$ **`<= or issubset`**

```
zminus <= zplus
False
zminus.issubset(zero)
True
```

$\subset$ **`<`**

```
zplus < plus
False
zero < zminus
True
```

$\supseteq$ **`>= or issuperset`**

```
zplus >= zminus
False
zplus.issuperset(zero)
True
```

$\supset$ **`>`**

```
zminus > zminus
False
zplus > zero
True
```

Set Relations

Equality and inequality $=$ $\neq$

Intersection and disjointness

Subsets and Supersets $\subseteq$ $\subset$ $\supseteq$ $\supset$ negations

Python `==` `!=` `isdisjoint` `<=` `<` `>=` `>`

Set Operations

set Operations

$\mathcal{P}$ $\mathcal{U}$ $\mathcal{Z}$ $\mathcal{Z}$ $\mathcal{L}$ $\mathcal{E}$

Are there sets A and B such that
A is both an **element** and a **subset** of B?

$A \in B$

$A \subseteq B$

Yes!

$\varnothing \subseteq$ any set

$A = \varnothing$

need $\varnothing \in B$

$B = \{\varnothing\}$

$\varnothing \in \{\varnothing\}$ and $\varnothing \subseteq \{\varnothing\}$

Also solutions with nonempty sets, but this is simplest

set

Operations

Plan

Again generalize numbers

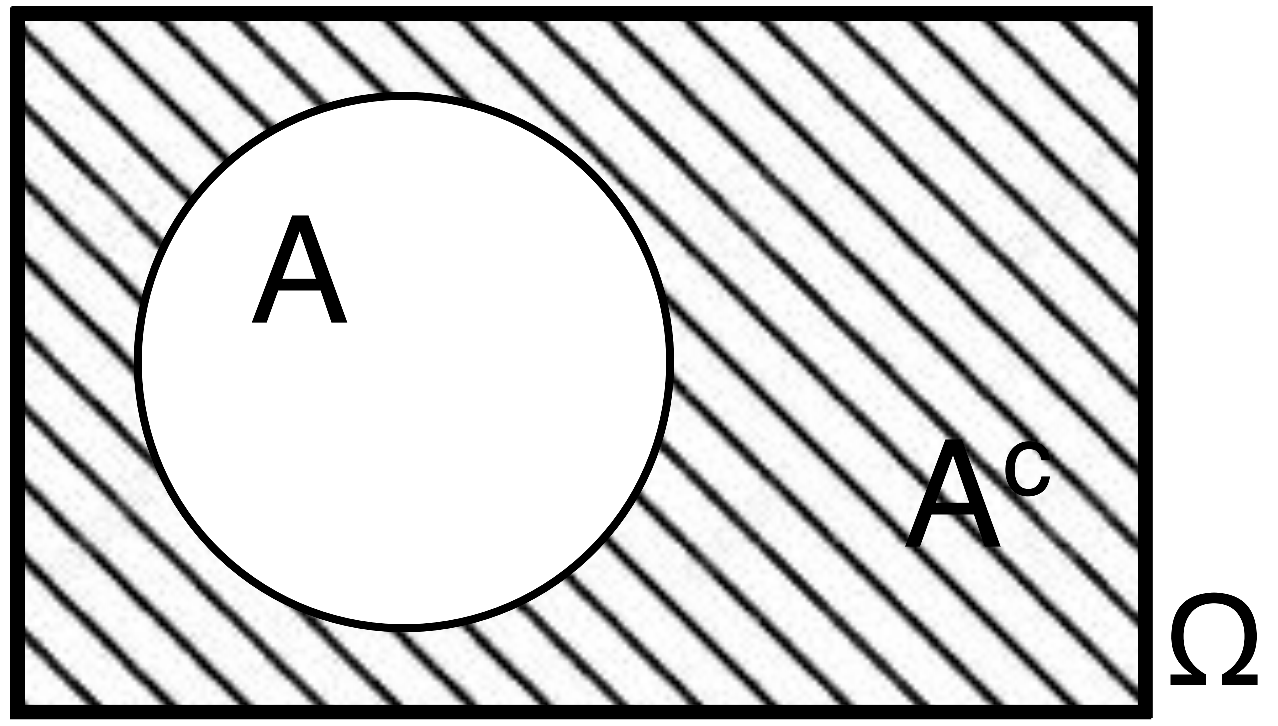
	Relations			Operations		
Number	=	≤	<	+	-	x
Set	=	⊆	⊂	∪	-	x
				This lecture		Next lecture
				∩		

Complement

Recall: Universal set Ω contains all elements

The complement A^c of A is the set of Ω elements not in A

$A^c = \{ x \in \Omega : x \notin A \}$



$\Omega = \{0,1\}$ $\{0\}^c = \{1\}$ $\{0,1\}^c = \varnothing$ $\varnothing^c = \{0,1\}$

$\Omega = \{0,1,2\}$ $\{0\}^c = \{1,2\}$ ← A^c depends on both A and Ω

$\Omega = \mathbb{Z}$ $\{\ldots,-2,-1\}^c = \mathbb{N}$

$\mathbb{E}$ - even, $\mathbb{O}$ - odd $\mathbb{E}^c = \mathbb{O}$

Set Identities

Relations that hold for all sets

$\varnothing^c = \Omega$

$\Omega^c = \varnothing$

A and A^c are disjoint

$(A^c)^c = A$

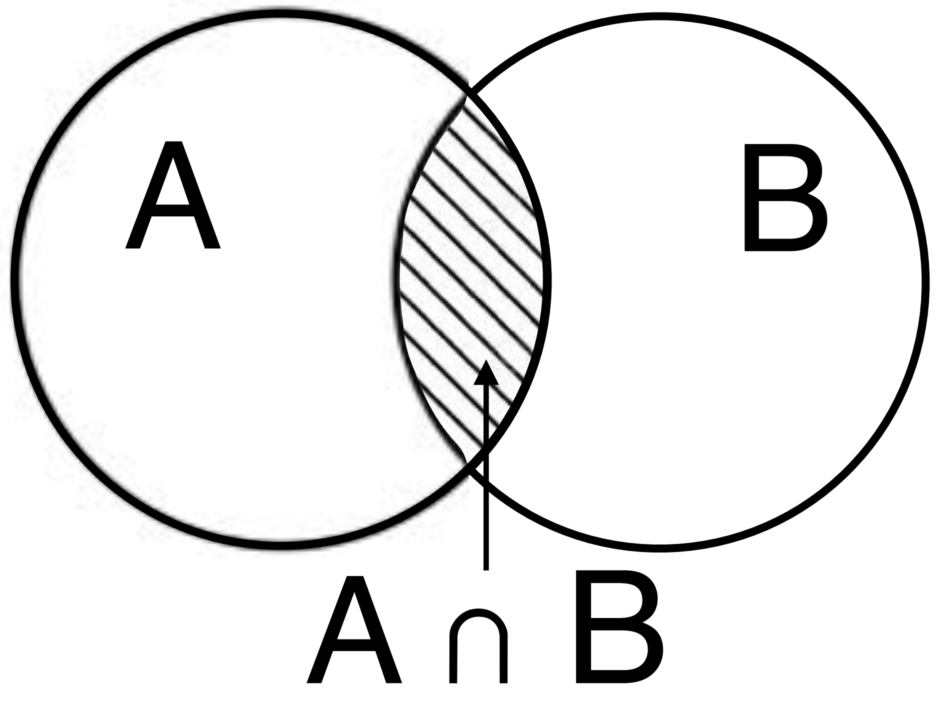
"involution"

$A \subseteq B \rightarrow A^c \supseteq B^c$

Intersection

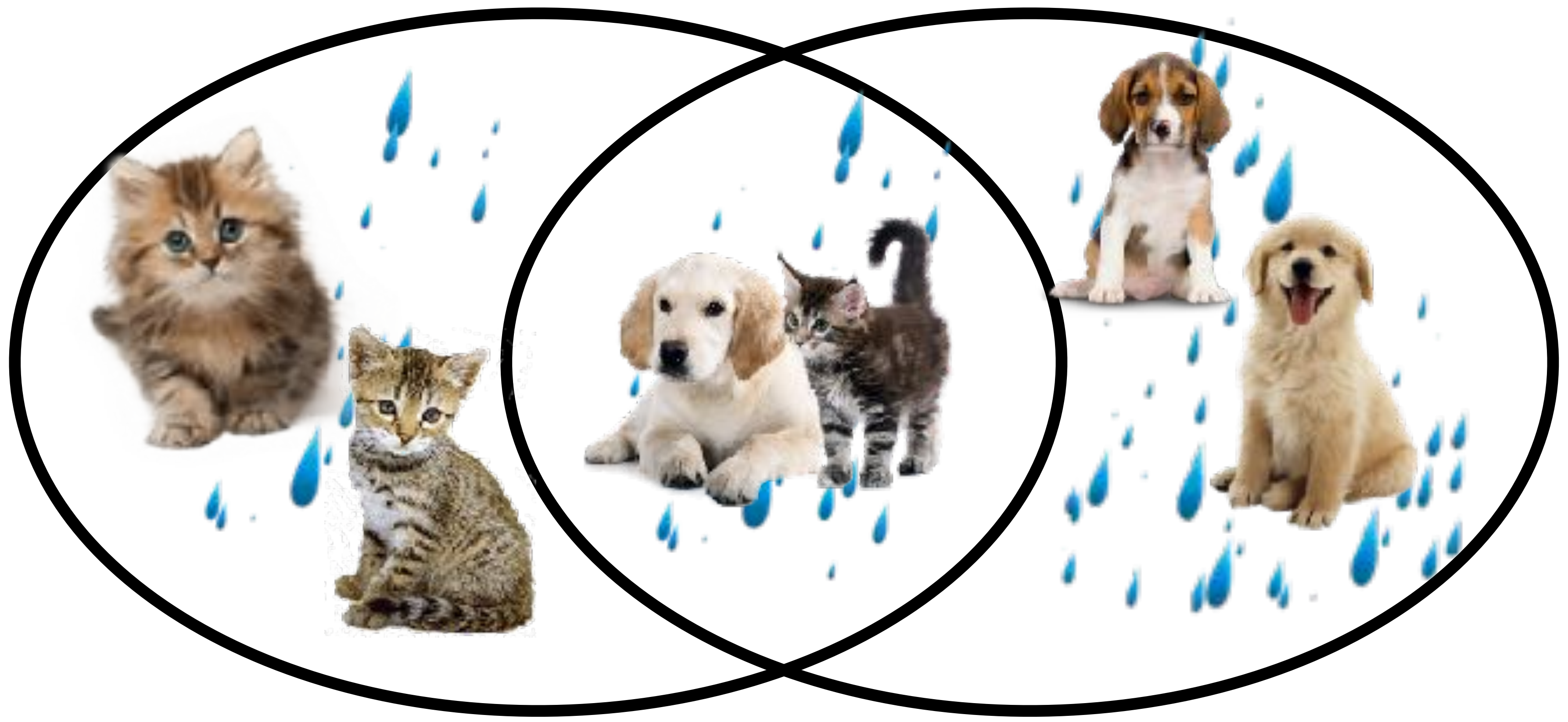
The intersection $A \cap B$ is the set of elements in both A and B

$A \cap B = \{ x : x \in A \wedge x \in B \}$



$\{0,1\} \cap \{1,3\} = \{1\}$

$\{0\} \cap \{1\} = \varnothing$



Raining cats and dogs

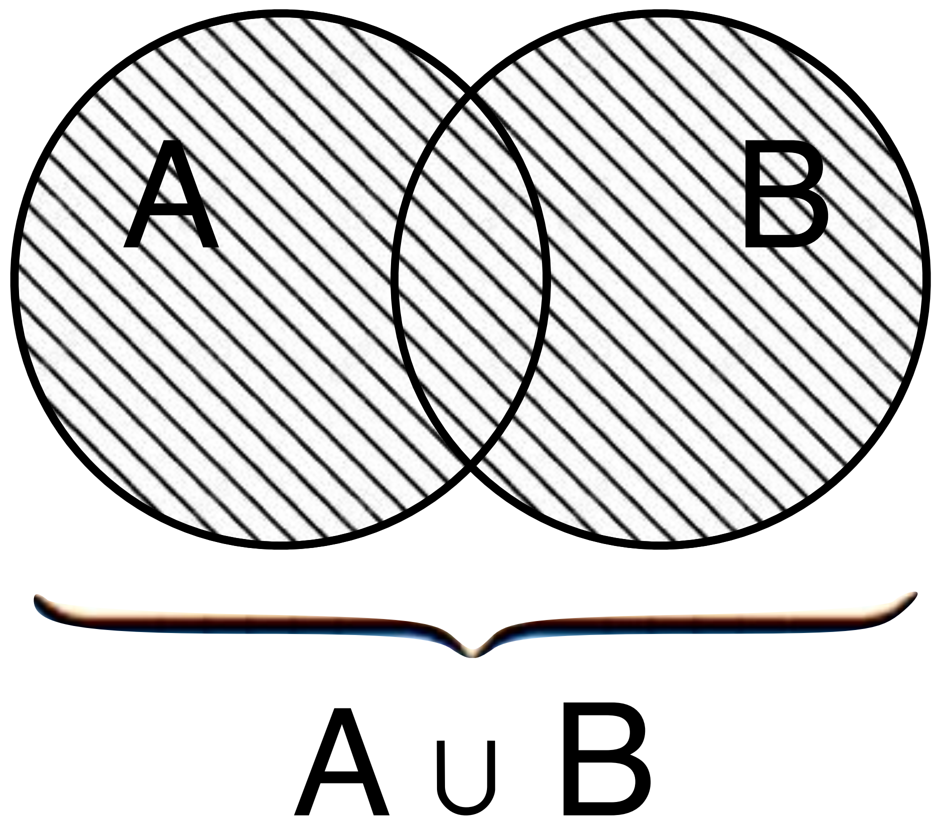
$[0,4) \cap [3,6] = [3,4)$

$[0,2] \cap (2,5] = \varnothing$

Union

The union $A \cup B$ is the collection of elements in A, B, or both

$A \cup B = \{ x : x \in A \vee x \in B \}$



$\{0,1\} \cup \{1,2\} = \{0,1,2\}$

$\{0,1\} \cup \{2\} = \{0,1,2\}$

$[0,2] \cup [1,3] = [0,3]$

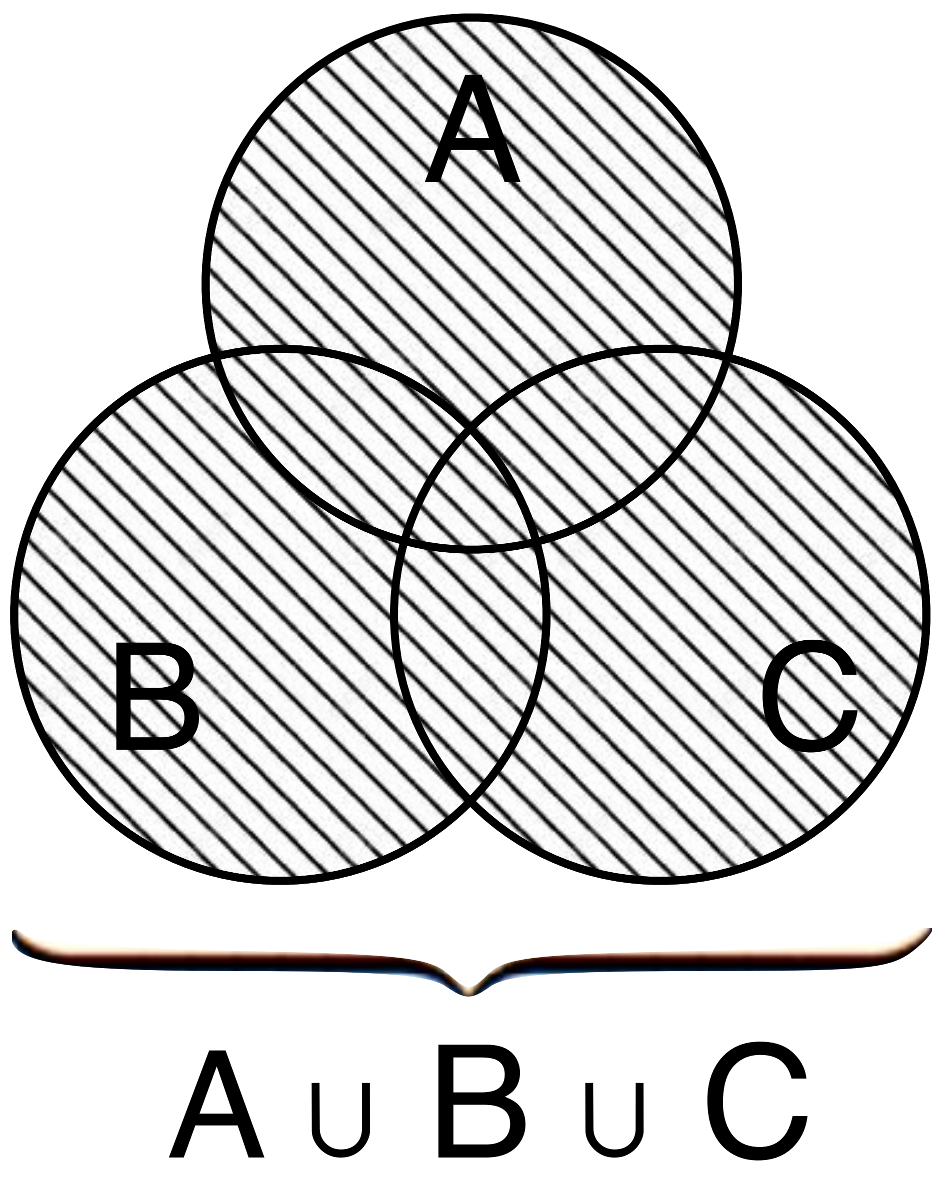
$(0,1) \cup \{1\} = (0,1]$

$\mathbb{E} \cup \mathbb{O} = \mathbb{N}$

Multiple Sets

A ∪ B ∪ C = { x∈Ω : x∈A ∨ x∈B ∨ x∈C }

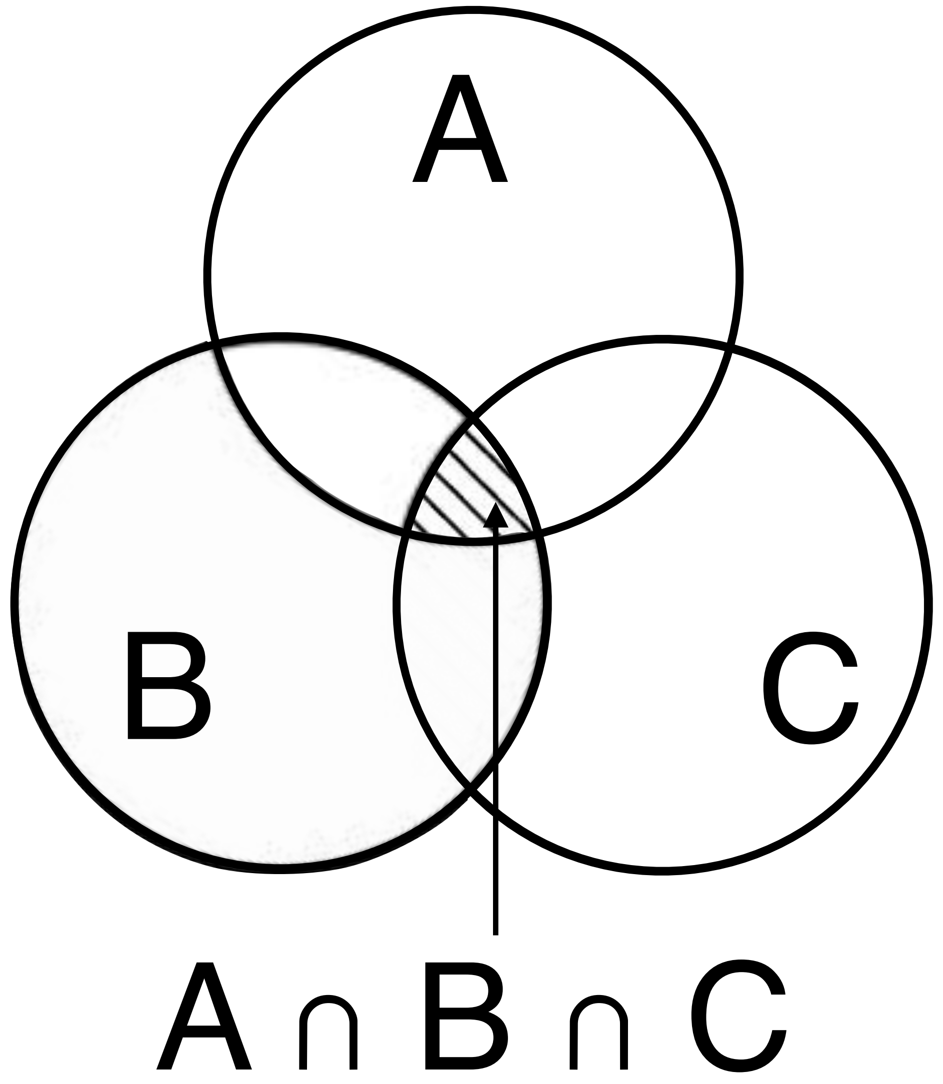
{0,1} ∪ {1,2} ∪ {2,3} = {0,1,2,3}



Generally $\bigcup_{i=1}^{t} A_i = \{x : \exists 1 \le i \le t, x \in A_i\}$

$\bigcup_{i=-\infty}^{\infty} \{i\} = \mathbb{Z}$

Similarly for intersection



Identities - One Set

Identity	$A \cap \Omega = A$	$A \cup \Omega = \Omega$
Universal bound	$A \cap \emptyset = \emptyset$	$A \cup \emptyset = A$
Idempotent	$A \cap A = A$	$A \cup A = A$
Complement	$A \cap A^c = \emptyset$	$A \cup A^c = \Omega$

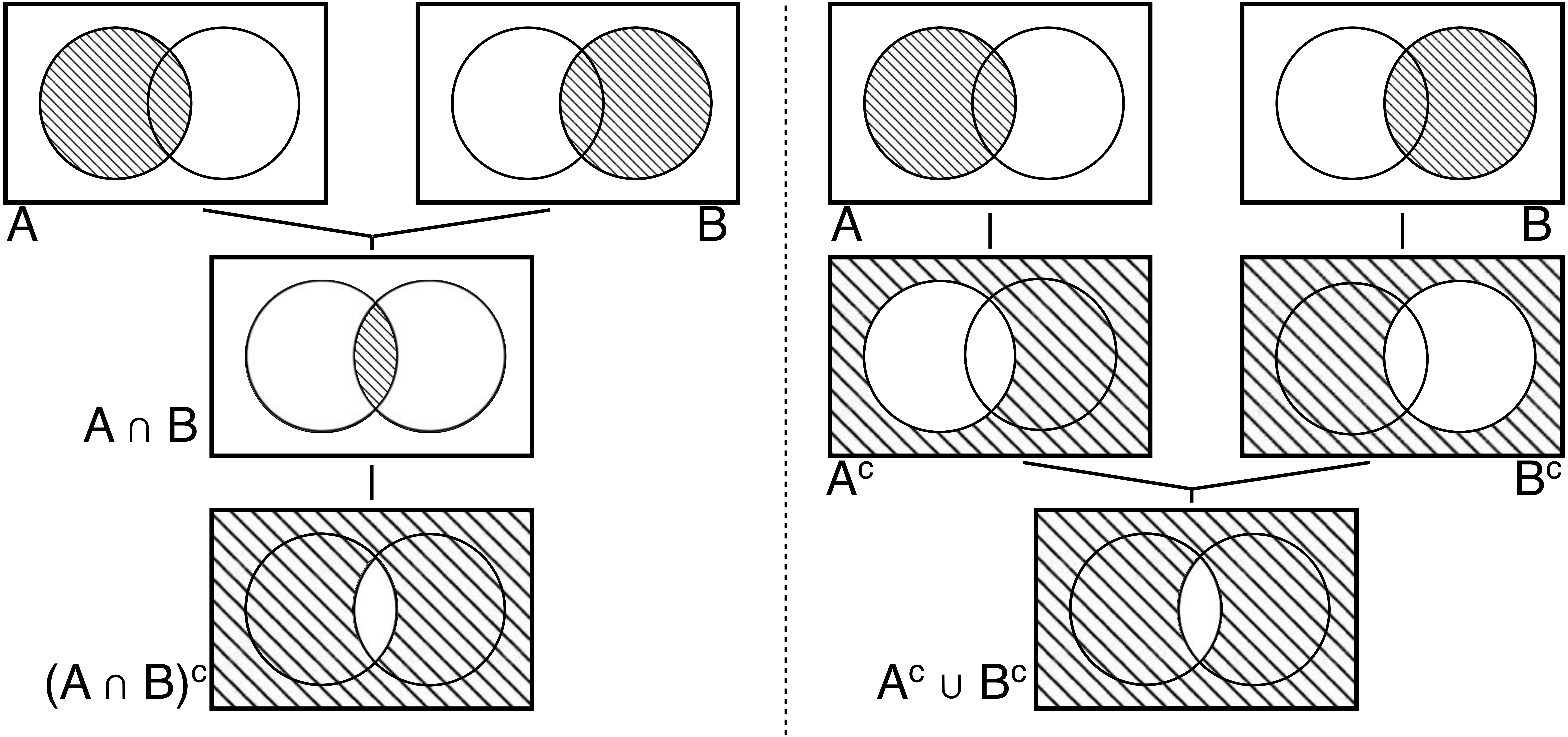
Often called laws

Laws - Two and Three Sets

Commutative	$A \cap B = B \cap A$	$A \cup B = B \cup A$
Associative	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
De Morgan	$(A \cap B)^c = A^c \cup B^c$	$(A \cup B)^c = A^c \cap B^c$

De Morgan's Law

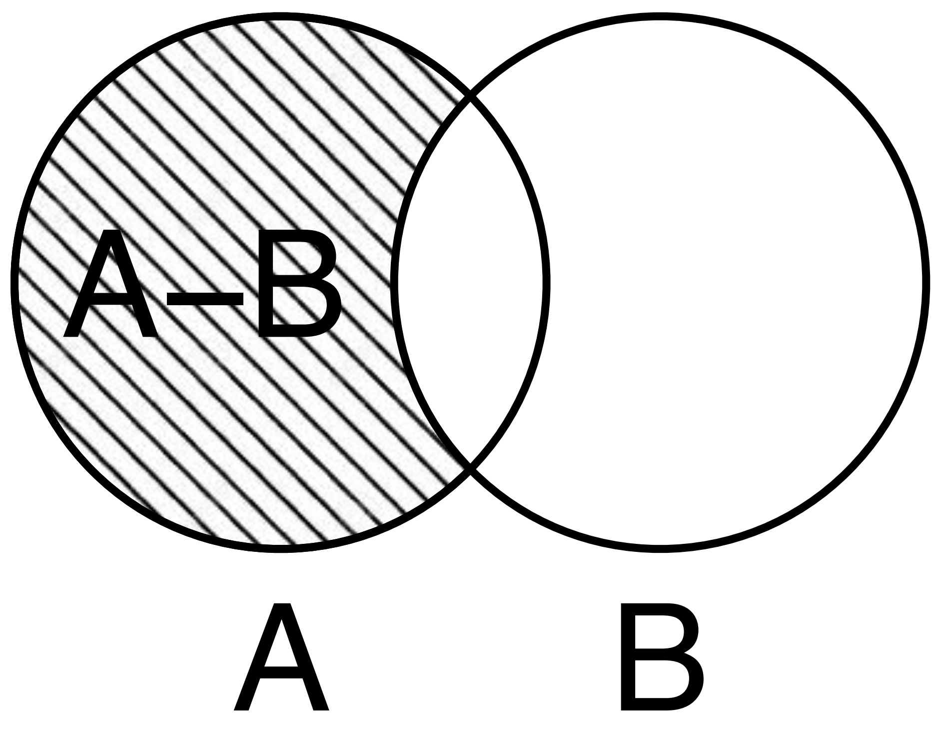
$(A \cap B)^c = A^c \cup B^c$



Set Difference

The difference A–B is the set of elements in A but not in B

$$A - B = \{ x : x \in A \wedge x \notin B \}$$



$\{0,1\} - \{1\} = \{0\}$

$\{0,1\} - \{0,1,2\} = \varnothing$

Notetation

Also
A \ B

$[1,3] - [2,4] = [1,2)$

$[1,3] - (1,3) = \{1,3\}$

$A - B = A \cap B^c$

Symmetric Difference

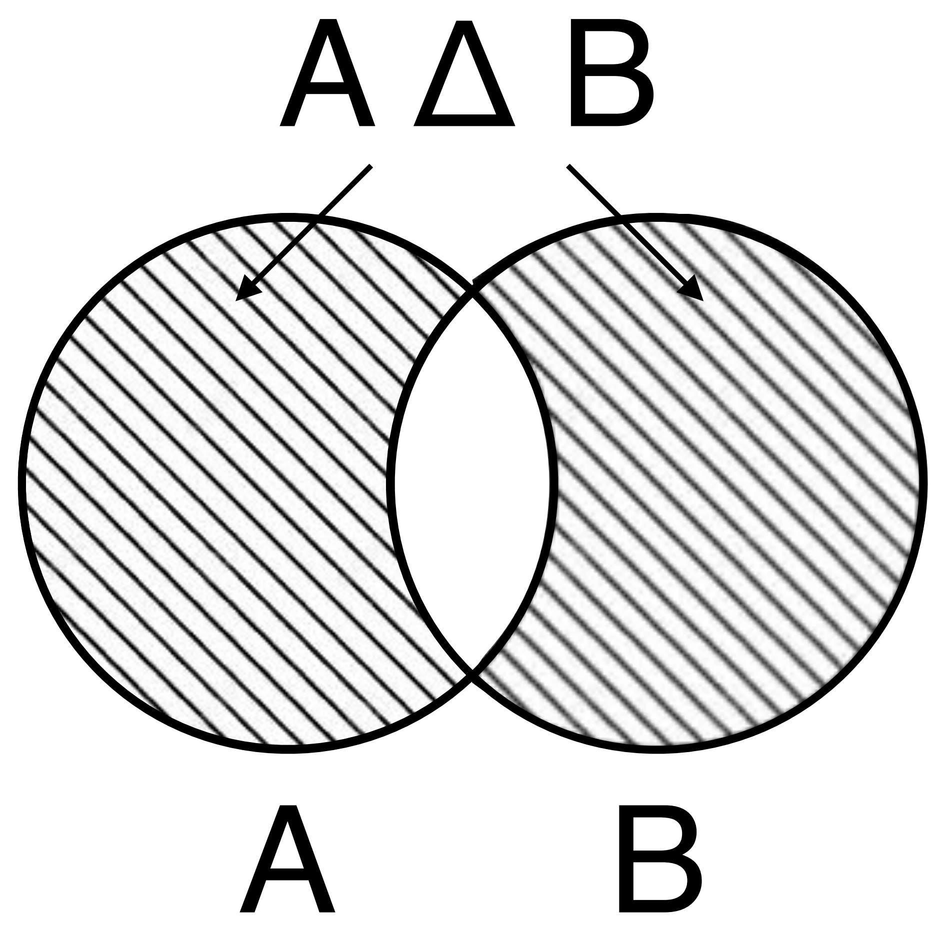
The symmetric difference of two sets is the set of elements in exactly one set

$A \,\Delta\, B =$
$\{x : x \in A \wedge x \notin B \;\vee\; x \notin A \wedge x \in B\}$

$\{0,1\} \,\Delta\, \{1,2\} = \{0,2\}$

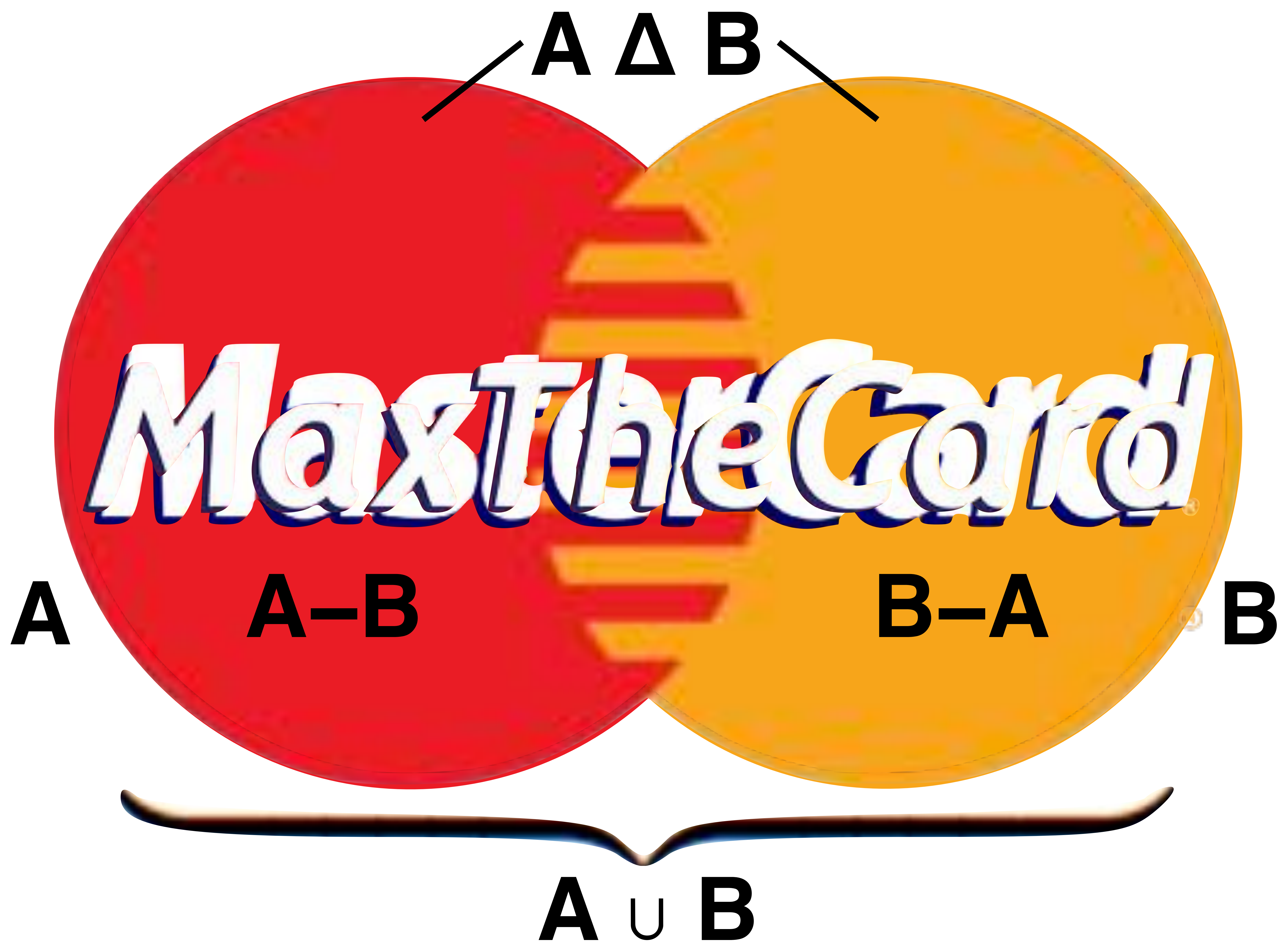
$[0,2] \,\Delta\, [1,4] = [0,1) \cup (2,4]$

$A \,\Delta\, B = (A-B) \cup (B-A)$

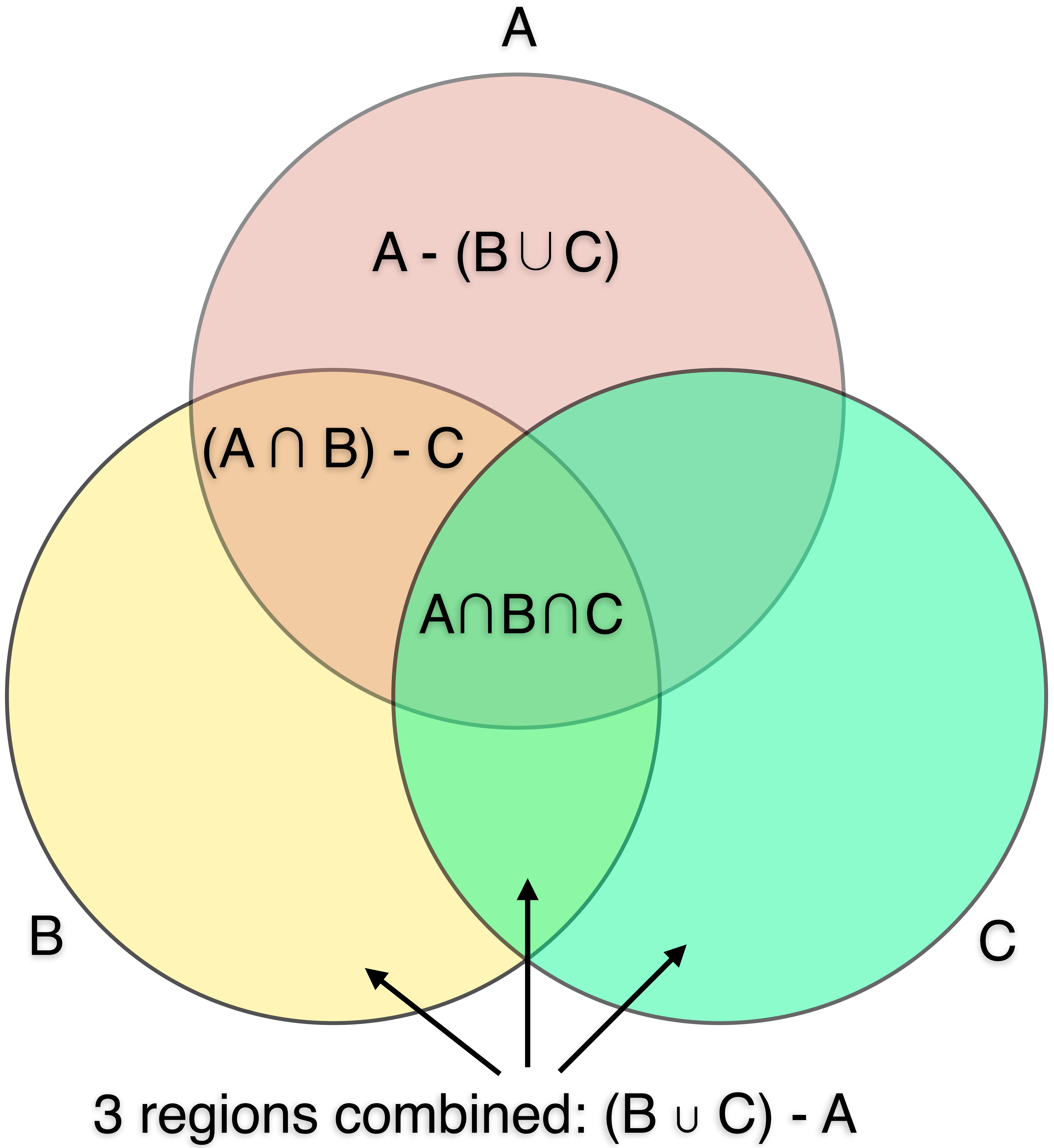


Venn's Master Chart

QUIZ



3 Sets



set Operations

Union and Intersection

```
A = {1,2}
B = {2,3}
```

∪ `|` or **`union`**

```
A | B
{1, 2, 3}
C = A.union(B)
print(C)
{2, 1, 3}
```

∩ **`&`** or **`intersection`**

```
A & B
{2}
C = A.intersection(B)
print(C)
{2}
```

Set- and Symmetric-Difference

```
A = {1,2}
B = {2,3}
```

Set difference

`-` or **`difference`**

```
A - B
{1}
B.difference(A)
{3}
```

Symmetric difference

`^` or **`symmetric_difference`**

```
A ^ B
{3,1}
B.symmetric_difference(A)
{3,1}
```

Set Operations

Operations

Complement A^c

Union ∪

Intersection ∩

Set difference -

Symmetric difference Δ

Laws

Commutative

Associative

Distributive

De Morgan

Python

`|` or `union`

`&` or `intersection`

`-` or `difference`

`^` or `symmetric_difference`

Cartesian Products