Numerical Linear Algebra March 3, 2025

Topic: QR Factorization and Least Squares

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1 Orthogonality

Definition 1. *A projector is a matrix P such that $P^2 = P$.*

An oblique projector is a matrix P such that $P^2 = P$ but $P^T P \neq I$.

From a geometric point of view, the projector might arise from the notion that if one were to shine a light onto the subspace $range(P)$, the light would then cast a shadow on Pv projected by an arbitrary vector v.

Definition 2. *A complimentary projector is a matrix $(I - P)$ such that $(I - P)^2 = I - P$. This is also the nullspace of P.*

In addition

$$range(I - P) = null(P)$$
$$null(I - P) = range(P)$$

A projector separates a vector space into two subspaces the nullspace and the range.

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References

[1] Lloyd N. Trefethen, David Bau III, *Numerical Linear Algebra*, Northwestern University.