CSCE 643 Feburary 6th, 2025

Topic 1 – The Configuration Space

Prof. Dylan Shell / Dr. Steven Lavelle *Scribe: David Zhang*

1 Overview

Definition 1. ***Configuration Space:*** *The state space for motion planning is a set of possible transformations that could be applied to the robot*

2 Basic Topological Concepts

2.1 Topological Spaces

Definition 2. ***An open set is*** *a interval with no boundary points. That must follow this rule (TLDR)*

Definition 3. ***A topological space*** *is a set $\mathbb{X}$ together with a collection of open sets O that satisfy the following properties:*

1. *The union of any number of open sets is an open set.*
2. *The intersection of a finite number of open sets is an open set*
3. *Both $\mathbb{X}$ and $\emptyset$ are open sets*

Definition 4. ***Special Points:*** *A point on the border of an open or closed set*

2.1.1 Special Point scenarios

Consider a point x that in the topological space $\mathbb{X}$, and a set U that is a subset of X. The following terms capture the position of point x relative to U.

1. If x is in U, then x is an interior point of U
2. If x is not in U, then x is an exterior point of U
3. If x is neither an interior point nor an exterior point of U, then x is a boundary point of U
4. If x is an interior or boundary point, it is a limit point (which the set of all limit points of U is called the closure of U)

Important Note: The closure is always a closed set, because it contains all the boundary points. That also means the open set contains none of the boundary points, making it open.

2.2 The Ball analogy (Example 4.1)

Think of an open set as a ball in $\mathbb{R}^n$ space, and the points that fill the ball are the interior points centered at some point x.

1. *All open sets can be represented by a countable union of open balls*
2. *Any function constructed from primitives that use the $<$ relation are open.*

2.3 Subspace Topology

The subspace topology is a topology that have all of its representative open sets be every subset to a larger topological space. $U \subseteq \mathbb{X}$

2.3.1 Blah blah blah

Here is a subsubsection. You can use these as well.

2.4 Using Boldface

Make sure to use lots of boldface.

Question: *How would you use boldface?*

Example: *This is an example showing how to use boldface to help organize your topics.*

Some Formatting. *Here is some formatting that you can use in your notes:*

- Item One – *This is the first item.*
- Item Two – *This is the second item.*
- *. . . and here are other items.*

If you need to number things, you can use this style:

1. Item One – *Again, this is the first item.*
2. Item Two – *Again, this is the second item.*
3. *. . . and here are other items.*

Bibliography. *Please give real bibliographical citations for the papers that we mention in class. See below for how to include a bibliography section. If you use BibTeX, integrate the .bbl file into your .tex source. You should reference papers like this: "The FKS dictionary originates in a paper by Fredman, Komlós and Szemerédi [1]." In general, the name of the authors should appear in text at most once (for the first citation); further citations look like: "Our proof follows that of [1]".*

Take a look at previous topics (TeX files are available) to see the details. A excellent source for bibliographical citations is DBLP. Just Google DBLP and an author's name.

References

[1] *M. Fredman, J. Komlós, E. Szemerédi,* Storing a Sparse Table with $O(1)$ Worst Case Access Time*, Journal of the ACM, 31(3):538-544, 1984.*