Chapter-2 ; Polynomials

⇒ Polynomials → It is an special type of algabric expression which consist variable, cofficcient and degree should not be in negative form.

$a_0x + a_1x^2 + a_2x^3 + \text{----------} a_nx^n$

where a_0, a_1, a_2, a_3 ---------- ⇐ are constant

Classification of Polynomials

on the Basis of degree
- 1 → Linear
- 2 → Quadratic
- 3 → Cubic
- 4 → Biquadratic

on the basis of
- 1 → Monomial
- 2 → Binomial
- 3 → Trinomial
- 4 → Quadranomial

Quadratic Polynomial

General equation :→ $ax^2 + bx^2 + c$

zeroes → (α, β)

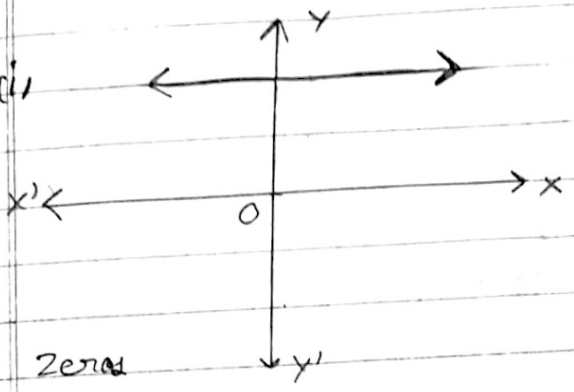
(i) $\alpha + \beta$ (sum of zeroes) ⇒ $-b/a$

(ii) $\alpha \cdot \beta$ (product of zeroes) ⇒ c/a

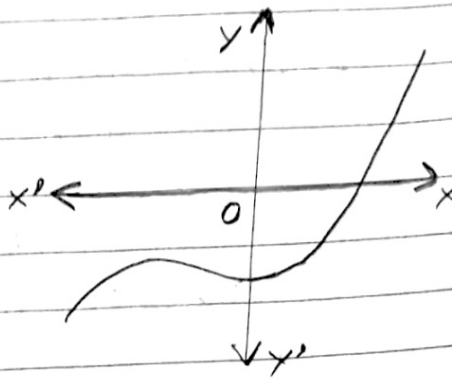
(iii) $\alpha - \beta$ (

Exercise → 2.1

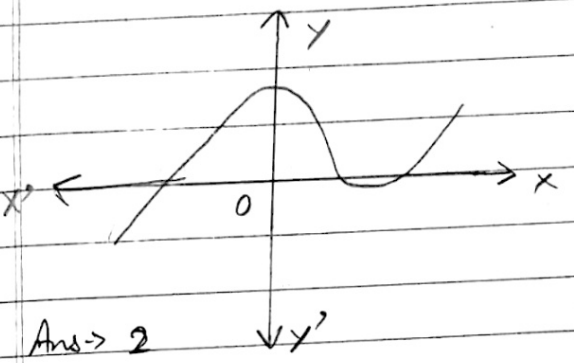
1 → (i)



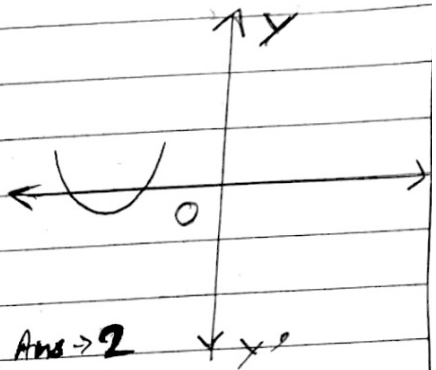
Ans → Zeros



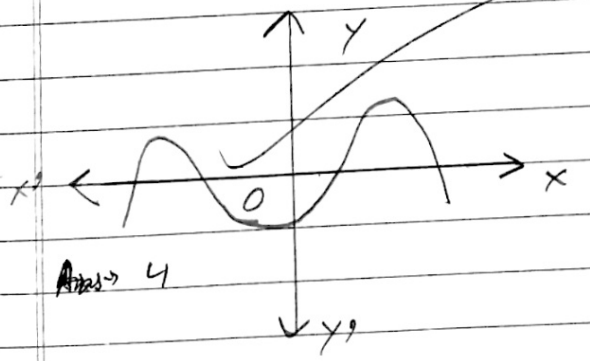
Ans → One 1



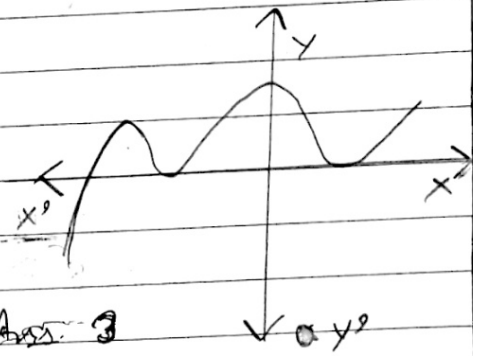
Ans → 2



Ans → 2



Ans → 4



Ans → 3

Exercise → 2.2

1 → (i) $x^2 - 2x - 8$

$\Rightarrow x^2 - (4x - 2x) - 8$

$\Rightarrow x^2 - 4x + 2x - 8$

$\Rightarrow x(x-4) + 2(x-4)$

$\Rightarrow (x-4)(x+2)$

Now, $x - 4 \Rightarrow 0$

$\alpha \Rightarrow 4$

$x + 2 \Rightarrow 0$

$\beta \Rightarrow 2$

Relationship

compare $p(x)$ with $ax^2 + bx + c$

$\Rightarrow a = 1, \; b = -2, \; c = -8$

Then, $\alpha + \beta \Rightarrow -b/a$

$\Rightarrow 4 + (-2) \Rightarrow -\frac{(-2)}{1}$

$\Rightarrow \boxed{2 \Rightarrow 2}$ verified

$\alpha\beta \Rightarrow \frac{c}{a}$

$\Rightarrow 4 \times 2 \Rightarrow \frac{8}{1}$

$\Rightarrow \boxed{8 = 8}$ verified

(v) $t^2 - 15$

sol⇒ $t^2 - 15 \Rightarrow p(x)$

put $p(x) \Rightarrow 0$

$\Rightarrow t^2 - 15 \Rightarrow 0$

$\Rightarrow t^2 \Rightarrow 15$

$\Rightarrow t = \pm\sqrt{15}$

$\Rightarrow \alpha \Rightarrow \sqrt{15}$, $\beta \Rightarrow -\sqrt{15}$

Relationship

compare $p(x)$ with $at^2 + bt + c$

$a = 1$, $b = 0$, $c = -15$

(i) $\alpha + \beta \Rightarrow -b/a$

$\Rightarrow \sqrt{15} + (-\sqrt{15}) \Rightarrow 0$

$\Rightarrow \boxed{0 = 0}$ verified

(ii) $\alpha \cdot \beta \Rightarrow c/a$

$\Rightarrow \sqrt{15} \times -\sqrt{15} \Rightarrow -15$

$\Rightarrow \boxed{-15 \Rightarrow -15}$ verified

(iii) $6x^2 - 3 - 7x$

sol⇒ $6x^2 - 7x - 03$

$\Rightarrow 6x^2 - 9x + 2x - 3$

$\Rightarrow 3x(2x - 3) + 1(2x - 3)$

$\Rightarrow (2x - 3)(3x + 1)$

now, $2x - 3 = 0$

$\alpha = \frac{3}{2}$

$\Rightarrow 3x + 1$, $\beta = \frac{-1}{3}$

compare $p(x)$ with $6x^2 - 7x - 3$

$a = 6, \; b = -7, \; c = -3$

$\Rightarrow \alpha + \beta \Rightarrow \frac{-b}{a}$

$\Rightarrow \frac{3}{2} + \left(\frac{-1}{3}\right) = -\left(\frac{-7}{6}\right)$

$\Rightarrow \frac{3}{2} - \frac{1}{3} = \frac{7}{6}$

$\Rightarrow \frac{9-2}{6} \Rightarrow \boxed{\frac{7}{6} = \frac{7}{6}}$ verified

$\Rightarrow$

$\alpha\beta = \frac{c}{a}$

$\Rightarrow \frac{3}{2} \times \frac{-1}{3} = \frac{-3}{6}$

$\Rightarrow \boxed{\frac{-1}{2} = \frac{-1}{2}}$ verified

(ii) $4s^2 - 4s + 1$

Sol → $4s^2 - 2s - 2s + 1$

$\Rightarrow 2s(2s-1) - 1(2s-1)$

$\Rightarrow (2s-1)(2s-1)$

$\Rightarrow 2s - 1 = 0$

$\Rightarrow \alpha = \frac{1}{2}$

$\Rightarrow 2s - 1 = 0$

$\beta = \frac{1}{2}$

compare $p(x)$ with $4s^2 - 4s + 1$, $a = 4, \; b = -4$,

$\alpha + \beta = \frac{-b}{a}$

$\Rightarrow \frac{1}{2} + \frac{1}{2} = -\left(\frac{-4}{4}\right)$

$\Rightarrow \boxed{1 = 1}$ verified

$\alpha\beta = \frac{c}{a}$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\Rightarrow \boxed{\frac{1}{4} = \frac{1}{4}}$ verified

(iv) $4u^2 + 8u$

sol $\rightarrow$ $4u^2 + 8u = p(x)$

$p(x) = 0$

$4u^2 + 8u = 0$

$\Rightarrow 4u^2 = -8u$

$\Rightarrow u^2 = \frac{-8u}{4}$

$\Rightarrow u \times u = -2u$

$\Rightarrow u = \frac{-2u}{u} \Rightarrow u' = -2$

$\Rightarrow 4u(u+2) \Rightarrow p(x)$

$\Rightarrow p(x) \Rightarrow 0$

$\Rightarrow 4u = 0, \; u+2=0$

$\Rightarrow u = 0, \; u = -2$

$\Rightarrow \alpha = 0, \; \beta = -2$

Relationship

compare $p(x)$ with $4u^2 + 8u$

$a = 4, \; b = 8, \; c = 0$

$\alpha + \beta = -\frac{b}{a}$

$0 + (-2) = -\left(\frac{8}{4}\right)$

$\boxed{-2 = -2}$

$\alpha\beta = \frac{c}{a}$

$0 \times -2 = \frac{0}{4}$

$\Rightarrow \boxed{0 = 0}$ verified

(vi) $3x^2 - x - 4$

sol→ $3x^2 - 4x + 3x - 4$

$\Rightarrow x(3x-4) + 1(3x-4)$

$\Rightarrow (3x-4)(x+1)$

$\Rightarrow 3x - 4 \Rightarrow 0$

$\Rightarrow x = \frac{4}{3}$,

$\Rightarrow x + 1$

$\Rightarrow \beta = -1$;

Relationship

compare $p(x)$ with $3x^2 - x - 4$

$a = 3, b = -1, c = -4$

$\Rightarrow \alpha + \beta = \frac{-b}{a}$

$\Rightarrow \frac{4}{3} + \frac{-1}{01} = -\left(\frac{-1}{3}\right)$

$\Rightarrow \frac{4-3}{3} = \frac{1}{3}$

$\Rightarrow \boxed{\frac{1}{3} = \frac{1}{3}}$ verified

$\Rightarrow \alpha\beta = \frac{c}{a}$

$\Rightarrow \frac{4}{3} \times -1 = \frac{-4}{3}$

$\Rightarrow \boxed{\frac{-4}{3} = \frac{-4}{3}}$ verified

2 → (i) $\frac{1}{4}$, -1

sol → Let a quadratic equation $ax^2 + bx + c$

$\Rightarrow \alpha + \beta = \frac{-b}{a} = \frac{1}{4}$

$\Rightarrow \alpha\beta = \frac{c}{a} = \frac{-1 \times 1}{1 \times 1} \Rightarrow \frac{-1}{4}$

$\Rightarrow a = 4, \; -b = 1 \Rightarrow b = -1, \; c = -4$

$\Rightarrow \boxed{4x^2 - x - 4}$ Ans

(ii) $\sqrt{2}$, $\frac{1}{3}$

sol Let a quadratic equation $ax^2 + bx + c$

$\Rightarrow \alpha + \beta = \frac{-b}{a} = \frac{\sqrt{2}}{1}$

$\Rightarrow \alpha\beta = \frac{1}{3}$

Here we find that $a = 3$ so we multiply 3 from $\frac{-b}{a}$

$\Rightarrow \frac{\sqrt{2} \times 3}{1 \times 3} \Rightarrow \frac{3\sqrt{2}}{3}$

so, $a = 3, \; -b = 3\sqrt{2} \Rightarrow -3\sqrt{2}, \; c = 1$

$\Rightarrow \boxed{3x^2 - 3\sqrt{2}x + 1}$ Ans

(iii) 0, $\sqrt{5}$

Let a quadratic equation $ax^2 + bx + c$

$\Rightarrow \alpha + \beta = -\frac{b}{a} = \frac{0}{1}$

$\Rightarrow \alpha\beta = \frac{c}{a} = \frac{\sqrt{5}}{1}$

so, $a = 1, \; b = 0, \; c = \sqrt{5}$

$\Rightarrow \boxed{x^2 + \sqrt{5}}$ Ans

Exercise - 2.2

(iv) $\frac{1}{4}$, 1

Sol→ Let the zeroes α and β

A.T.Q, $\alpha + \beta = \frac{1}{4}$, $\alpha\beta = 1$

Required Polynomials

$\Rightarrow p(x) = x^2 - (\alpha+\beta)x + (\alpha\beta)$

$\Rightarrow x^2 - (1)x + 1$

$\Rightarrow \boxed{x^2 - x + 1}$ Ans

(v) $\frac{-1}{4}$, $\frac{1}{4}$

sol. Let the zeros α and β

A.T.Q., $\alpha + \beta = \frac{-1}{4}$, $\alpha\beta = \frac{1}{4}$

Required Polynomials

$\Rightarrow x^2 - (\alpha+\beta)x + (\alpha\beta)$

$\Rightarrow p(x) = x^2 - (\frac{-1}{4})x + (\frac{1}{4})$

$\Rightarrow x^2 + \frac{1}{4}x + \frac{1}{4}$

$\Rightarrow \frac{1}{4}(4x^2 + x + 1)$

$\Rightarrow \boxed{4x^2 - x + 1}$ Ans

(vi) 4, 1

sol→ Let the zeroes α and β

A.T.Q; $\alpha + \beta = 4$, $\alpha\beta = 1$

Required polynomials

$\Rightarrow p(x) = x^2 - (\alpha+\beta)x + (\alpha\beta)$

$\Rightarrow \boxed{x^2 - 4x + 1}$ Ans

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