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Chapter - I
Real numbers

⇒ Natural numbers → All counting no. starts from ~~one~~ 1 are called natural numbers.

⇒ Whole number → All counting numbers starts from 0 are called whole numbers.

⇒ Integers → The collection of all positive and negative whole numbers are called integers.

⇒ Rational numbers → The numbers which are in the form of p/q where ~~and~~ $q \neq 0$ and p & q are integers, are called rational number.

⇒ Irrational numbers → The numbers which are not in the form of p/q where $q \neq 0$ and ~~p~~ p & q are integers, are called irrational numbers.

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Exercise → 1.2

LCM × HCF = I × II

1 → (i) 140

2	140
2	70
5	35
7	7
	1

P.F = 2×2×5×7

⇒ $2^2 \times 5 \times 7$

(ii) 156

2	156
2	78
3	39
13	13
	1

P.F ⇒ 2×2×3×13

⇒ $2^2 \times 3 \times 13$

(iii) 3825

sol →

3	3825
3	1275
5	425
5	85
17	17
	1

P.F ⇒ 3×3×5×5

⇒ $3^2 \times 5^5$

(iv) 5005

sol →

5	5005
7	1001
11	143
13	13
	1

P.F ⇒ 5×...

(v) 7429

sol →

17	7429
19	437
23	23
	1

P.F. ⇒ 17×19×23

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4 ⇒ HCF (306, 657) = 9
LCM (306, 657) = ?

sol ⇒ ⇒ HCF × LCM = I × II

$\Rightarrow 9 \times LCM = 306 \times 657$

$\Rightarrow LCM \Rightarrow \frac{306 \times \cancel{657}^{\cancel{219}\ 73}}{\cancel{9}\ \cancel{3}}$

$\Rightarrow LCM \Rightarrow 22338$ Ans

2 ⇒ (i) 26 and 91

sol ⇒

2	26	7	91
13	13	13	13
	1		1

$26 \Rightarrow 2 \times 13$
$91 \Rightarrow 7 \times 13$
HCF ⇒ 13
LCM ⇒ $13 \times 2 \times 7 \Rightarrow 182$

(ii) 510 and 92

sol ⇒

2	510	2	92
3	255	2	46
5	85	23	23
17	17		1
	1		

$510 \Rightarrow 2 \times 3 \times 5 \times 17$
$92 \Rightarrow 2 \times 2 \times 23$
HCF ⇒ 2
LCM ⇒ $2 \times 3 \times 5 \times 17 \times 2 \times 23$
⇒ 23460

(iii) 336 and 54

sol ⇒

2	336	2	54
2	168	3	27
2	84	3	9
2	42	3	3
3	21		1
7	7		
	1		

$336 \Rightarrow 2 \times 2 \times 2 \times 2 \times 3 \times 7$
$54 \Rightarrow 2 \times 3 \times 3 \times 3$
HCF ⇒ 2×3 ⇒ 6
LCM ⇒ $2 \times 3 \times 2 \times 2 \times 2 \times 7 \times 3 \times 3$
⇒ 3024

3 → (i) 12, 15, 21

sol →

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

12 ⇒ 2×2×3

15 ⇒ 3×5

21 ⇒ 3×7

HCF ⇒ 3

LCM ⇒ 3×2×2×5×7

⇒ 420

(ii) 17, 23 and 29

sol →

$$\begin{array}{r|l} 17 & 17 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 23 & 23 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 29 & 29 \\ \hline & 1 \end{array}$$

17 ⇒ 17

23 ⇒ 23

29 ⇒ 29

HCF ⇒ 1

LCM ⇒ 17×23×29

⇒ 11339

(iii) 8, 9, and 25

sol →

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

8 ⇒ 2×2×2

9 ⇒ 3×3

25 ⇒ 5×5

HCF ⇒ 1

LCM ⇒ 2×2×2×3×3×5×5

⇒ 1800

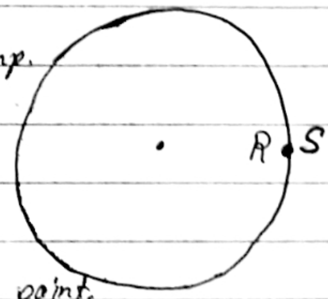
5 → In this question we have to check that can 6^n ends with 0 or not.
According to the prime factorisation of 6 which is 2×3 and we know that if any number can ends with 0 so, there

be 2 as well as 5 in the prime factorisation but in the prime factorisation of 6 there is 2 but not 5. so, clearly we can say that 6^n can not ends with -0.

6→ We know that composite number are those numbers which have more than 2 factors. and in this question we have $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ numbers have more that two factors so that's why these numbers are composite.

7→ Given → Sonia (S) takes 18 minutes to comp. one round. Ravi (R) takes 12 minutes to complete 1 round.

Find, - After how many mainutes will they meet again at the starting point.



Sol.→

2	18, 12
2	9, 6
3	9, 3
3	3, 1
	1, 1

L.C.M. ⇒ $2 \times 2 \times 3 \times 3$

⇒ 36

⇒ 36 minutes

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Q-> Prove that $\sqrt{2}$ is a irrational number.

sol-> ~~Prove~~ Let $\sqrt{2}$ be a natural number

$\sqrt{2} = \frac{a}{b}$, $b \neq 0$, a and b are integers

squarring both sides

$\Rightarrow (\sqrt{2})^2 = \frac{a^2}{b^2}$

$\Rightarrow 2 = \frac{a^2}{b^2}$

$\Rightarrow \boxed{2 b^2 \Rightarrow a^2}$ ——— ①

a^2 is divisible by 2

also a is divisible by 2

now, Let $a = 2c$

$\Rightarrow a^2 = 4c^2$

$\Rightarrow 2b^2 = 4c^2$

$\Rightarrow b^2 = 2c^2$

$\Rightarrow b^2$ is divisible by 2

$\Rightarrow b$ is also divisible by 2.

According to above discussion a and b have 2 as a common factor other than 1. so our assumption is wrong. It is a contradiction

$\sqrt{2}$ is an irrational number

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Exercise → 1.3

Q→ $3+2\sqrt{5}$, where $\sqrt{5}$ is an irrational

Let $3+2\sqrt{5}$ is an irrational

$\Rightarrow 3+2\sqrt{5} = \frac{a}{b}$

$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$

$\Rightarrow 2\sqrt{5} = \frac{a-3b}{b}$

$\Rightarrow \sqrt{5} = \frac{a-3b}{2b}$

Now, $\sqrt{5}$ is an irrational and irrational ≠ rational so our assumption is wrong. It is contradiction. $3+\sqrt{5}$ is an irrational.

1→ Prove that $\sqrt{5}$ is irrational

Sol→ Let $\sqrt{5}$ be a rational number

so, $\sqrt{5} \Rightarrow \frac{a}{b}$, $b \neq 0$, a and b are integers.

squarring both sides

$\Rightarrow (\sqrt{5})^2 \Rightarrow (\frac{a}{b})^2$

$\Rightarrow 5 = \frac{a^2}{b^2}$

$\Rightarrow \boxed{5b^2 \Rightarrow a^2}$ ——— (1)

a^2 is divisible by 5, so a is also by 5

, Let $a = 5c$

$\Rightarrow a^2 = 25c^2$

$\Rightarrow 5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

$\Rightarrow b^2$ is divisible by 2

$\Rightarrow b$ is also divisible by 2

According to above discussion a and b have 5 as a common factor ~~is a contradiction~~

other than 1. so over assumption is wrong. It a contradiction $\sqrt{5}$ is a rational number

Q→ prove that $\sqrt{3}$ is irrational number.

sol→ Let $\sqrt{3}$ be a rational number

so, $\sqrt{3} = \frac{a}{b}$, $b \neq 0$ and a and b are integers

squarring both sides

$\Rightarrow (\sqrt{3})^2 = \left(\frac{a}{b}\right)^2$

$\Rightarrow 3 = \frac{a^2}{b^2}$

$\Rightarrow \boxed{3b^2 = a^2}$ ——— ①

$\Rightarrow b^2 = \frac{a^2}{3}$

so, a^2 is divisible by 3

also a is divisible by 3

now, let $a = 3c$

$a^2 = 9c^2$ (squarring both sides)

$\Rightarrow 3b^2 = 9c^2$ (by using eq- 1)

$\Rightarrow b^2 = 3c^2$

$\Rightarrow \frac{b^2}{3} \Rightarrow c^2$

so, b^2 is divisible by 3

also b is divisible by 3

According to above discussion a and b have a common factor 3 as a common factor other than 1. so, our assumption is wrong. It is a contradiction. $\sqrt{2}$ is an irrational number.

Q-1 → Prove that $\sqrt{7}$ is irrational.

sol → Let $\sqrt{7}$ be a rational number

so, $\sqrt{7} = \frac{a}{b}$, $b \neq 0$, a and b are integers.

$\Rightarrow (\sqrt{7})^2 = \left(\frac{a}{b}\right)^2$ (squarring both sides)

$\Rightarrow 7 = \frac{a^2}{b^2}$

$\Rightarrow \boxed{7b^2 \Rightarrow a^2}$ —— ①

$\Rightarrow b^2 = \frac{a^2}{7}$

so, a^2 is divisible by 7

also a is divisible by 7

now, let $a = 7c$

$\Rightarrow a^2 = 49c^2$ (squarring both sides)

$\Rightarrow 7b^2 = 79c^2$ (using eq. ①)

$\Rightarrow b^2 = 7c^2$

$\Rightarrow \frac{b^2}{7} = c^2$

so b^2 is divisible by 7

also b is divisible by 7

According to above discussion a and b have 7 as a common factor other than 1. So, our assumption is wrong. It is a contradiction. $\sqrt{7}$ is a irrational number.

3→ Prove that the following are irrationals where $\sqrt{5}$ and $\sqrt{2}$ are irrationals.

(i) $\frac{1}{\sqrt{2}}$

Sol→ let $\frac{1}{\sqrt{2}}$ is rational

so, $\frac{1}{\sqrt{2}} = \frac{a}{b}$, $b \neq 0$, a & b are integers.

$\Rightarrow 1 = \frac{\sqrt{2}a}{b}$

$\Rightarrow \frac{a}{b} \Rightarrow \sqrt{2}$

but $\sqrt{2}$ is an irrational number and irrational $\neq$ rational. so, our assumption is wrong. It is contradiction. $\frac{1}{\sqrt{2}}$ is an irrational number.

(ii) $7\sqrt{5}$

sol→ let assume that $7\sqrt{5}$ is a rational number

so, $7\sqrt{5} = \frac{a}{b}$, $b \neq 0$, a and b are integers.

$\Rightarrow \sqrt{5} \Rightarrow \frac{a}{7b}$

but $\sqrt{5}$ is an irrational number and irrational $\neq$ rational. so, our assumption is wrong. It is contradiction. $7\sqrt{5}$ is an irrational number.

(iii) 6+

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(iii) $6+\sqrt{2}$

Sol→ Let assume that $6+\sqrt{2}$ is a rational number

so, $6+\sqrt{2} = \frac{a}{b}$; $b \neq 0$, a and b are integers ~~and coprime~~

$\Rightarrow \sqrt{2} \Rightarrow \frac{a}{b} - 6$

but $\sqrt{2}$ is an irrational number and irrational $\neq$ rational. So, our assumption is wrong. It is a contradiction. $6+\sqrt{2}$ is a irrational number.