

- TC-2
- 1) Time complexity & Space complexity
 - 2) Asymptotic analysis (Big O)
 - 3) Big O [Formal definition]
 - 4) TLE (Time Limit Exceeded)

- TC-1 :
- No of iterations
 - Big O (Basics)

1) $\log_2 N$

$$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \dots \rightarrow 1$$

number of times you divided is
 $\log_2 N$

$$\log_2 2^x =$$

$$2^x \rightarrow 2^{x-1} \rightarrow 2^{x-2} \rightarrow \dots \rightarrow 2^1 \rightarrow 2^0 = 1$$

x divisions.

$$2^3 \rightarrow 2^2 \rightarrow 2^1 \rightarrow 2^0 \quad \text{ans} = 3$$

$$2^0 = 1$$

$$3^0 = 1$$

$$4^0 = 1$$

$$1000^0 = 1$$

$$\frac{2^x}{2} = 2^{x-1}$$

$$2^x = \underbrace{2 \times 2 \times 2 \times 2 \dots \times 2}_{x \text{ times.}}$$

Ranges

$$[1, N]$$

$$= 1, 2, 3, 4, 5, \dots, N-1, N$$

$$[1, N]$$

$$2, 3, 4, \dots, N-1$$

[→ include
(→ exclude

$$[1, N) \quad 1, 2, 3, \dots, N-1$$

O [a, b] How many nos. in this range integers

a, a+1, a+2, ..., b-1, b

Count of nos = b-a+1

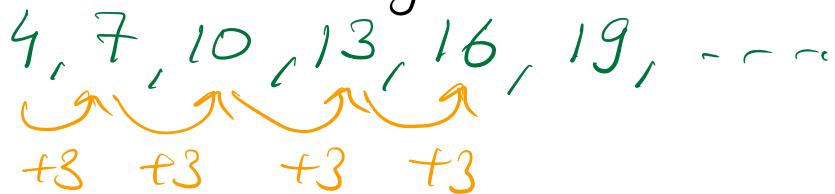
[4, 4]

$$\begin{matrix} 4 \\ 4-4+1 \end{matrix} = 1$$

[4, 5]

$$\begin{matrix} 4, 5 \\ \text{formula} = 5-4+1 = 2 \end{matrix}$$

Arithmetic Progression

4, 7, 10, 13, 16, 19, ...


a = Starting no / first term

d = common difference

$\begin{matrix} a, & a+d & a+2d & \dots & a+(n-1)d \\ 1^{\text{st}} & 2^{\text{nd}} & 3 & & n^{\text{th}} \end{matrix}$

$n=1$ a
a+(n-1)d

$\left. \begin{matrix} a+nd \\ a+d \end{matrix} \right\} n=1$

-



$$n^{\text{th}} \text{ term} = a + (n-1)d$$

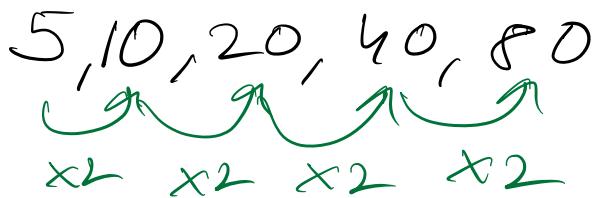
S_n = Sum of first n numbers
of sequence.

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

HW: Derive this.

Hint: Gauss metho

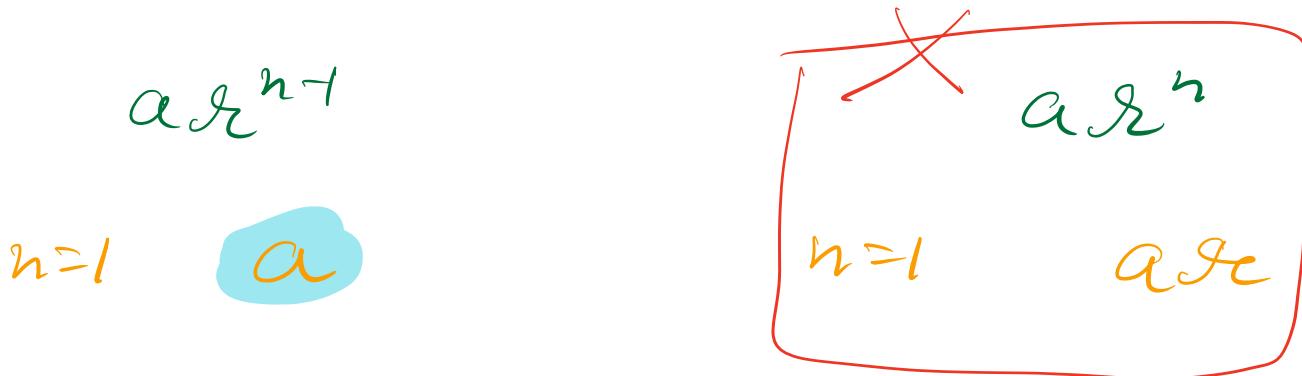
Geometric Progression

$$5, 10, 20, 40, 80$$


a = Starting no/ first term

r = common ratio

$$\begin{array}{ccccccc} a & ar & ar^2 & \dots & ar^{n-1} \\ \text{1st} & \text{2nd} & \text{3rd} & & \text{n}^{\text{th}} \end{array}$$



$$n^{\text{th}} \text{ term} = a r^{n-1}$$

S_N = Sum of first n numbers
of sequence.

$$\boxed{S_N = \frac{a(r^n - 1)}{r - 1}} \quad \text{when } r \neq 1$$

when $r = 1$, it is an AP
where all terms are same.
Hence, we use AP formula

$$\text{Eg. } 5, 5, 5, 5$$

is this AP	Yes
is this GP	Yes

AP

$n=4$

GP

$$a = 5$$
$$d = 0$$

$$a = 5$$
$$d = 1$$

$$\frac{n}{2} [2a + (n-1)d]$$

$$\frac{4}{2} [2 \times 5 + 3 \times 0] = \frac{4}{2} \times 10$$

$$= 20$$

5, 10, 20, 40

$$a = 5$$

$$r = 2$$

$$a \left(\frac{r^n - 1}{r - 1} \right) = 5 \frac{[2^4 - 1]}{2 - 1}$$

$$= 5 \frac{[16 - 1]}{1} = 75$$

Count number of iterations.

$$\theta = 0$$

\equiv for (int $i=1 ; i \leq N ; i++$)
 $S = S + i$

Num of iterations $= N$

$i = 1, 2, 3, \dots, N$

$[1, N]$
no of numbers $= \frac{b-a+1}{N-1+1} = N$
 $O(N)$

$\underline{\Omega}$ for ($i=1 ; i \leq N ; i++$) } N
print(i)

for ($i=1 ; i \leq M ; i++$) } M
print(i)

Num of iterations $= N+M$
 $O(N+M)$

$\underline{\Omega}$
 $S = 0$
for ($i=1 ; i \leq N ; i = 2$)
 $S += i$

3

$$\frac{N}{2}$$

put $N=1$

$$\frac{N}{2} = \frac{1}{2} = 0$$

$$i: 1, 3, 5, 7, 9, \dots$$

If n is odd, i will be equal to n

If n is even, i will be equal to $n-1$

n (odd)

$$i = 1, 3, 5, 7, \dots, n$$

How many nos

Last term = n

First term = 1

$d = 2$

m is

$$n = a + (m-1)d$$

1 2

total no
of terms

$$n = l + (m-1)2$$

$$\frac{n-1}{2} = m-1$$

$$m = \frac{n+1}{2}$$

$$m = \frac{n+1}{2}$$

is also

valid for even.

$$n = 19 \quad m = \frac{19+1}{2}$$

$1, 3, 5, 7, \dots, 17, 19$

$$= 10$$

$$n = 20 \quad \frac{20+1}{2}$$

$1, 3, 5, 7, \dots, 17, 19$

$$\dots, \frac{n}{2} = \frac{24}{2} = 10$$

Number of iterations

$$= \left(\frac{n+1}{2} \right) \quad O(N)$$

for ($i=2 ; i \leq N ; i++ = 2$)

$i = 2, 4, 6, 8, \dots, n$ even
 $n-1$ odd

O for ($i=1 ; i \leq 100 ; i++$)

print (i)

$i = 1, 2, 3, \dots, 100$

Num of iterations = 100 $O(1)$

O for ($i=1 ; i*i \leq N ; i++$)

for (i=1; i<=N; i=i*2)
 print(i)

$i = 1, 2, 3, \dots, \sqrt{N}$

$$\begin{aligned}i * i &\leq N \\i^2 &\leq N \\i &\leq \sqrt{N}\end{aligned}$$

Num of iterations = \sqrt{N} $O(\sqrt{N})$

O
 $i = N$
while ($i \geq 1$)
 $i = i/2$
}

$i = N \rightarrow N/2 \rightarrow \frac{N}{4}, \dots, 1$

$\log_2(N)$ $O(\log_2 N)$

O
 $S = 0$
for ($i=0$; $i \leq N$; $i=i+2$)

$\{ \quad s+t = i$

y

Before After

$$i=0$$

$$0$$

$$i = i*2 = 0$$

$$i=0$$

$$0$$

$$i = i*2$$

$$= 0$$

$$0$$

$$0$$

$$0$$

:

:

$$0$$

Infinite loop

$\overline{\text{D}}$

$$s = 0$$

for ($i=1 ; i \leq N ; i = i*2$)

$\{$

$$s+t = i$$

y

Before

After

Iter

1

$2 (2^1)$

1

2

$4 (2^2)$

2

4

$8 (2^3)$

3

8

$16 (2^4)$

4

.

.

.

.

.

:

2^K

K

- - - - -

$2^K > N$

$K \geq \log_2(N)$

Num iterations = $\log_2(N)$

$i = 1, 2, 4, 8, 16$ (For $n=16$)
 $\log_2(16) = 4$

5

$O(\log N)$

Break

10:50

for ($i=1; i \leq 10; i++$)

 for ($j=1; j \leq N; j++$)

 print ($i*j$)

y

y

i
1
2
3
:

j
[1, N] → n
[1, N] → n
[1, N] → n

10

\vdots i \vdots j
 10 $[1, N]$ n \downarrow
 $O(N)$ $10n$

```

 $\{$     for ( $i=1; i \leq N; i++$ )
 $\{$      for ( $j=1; j \leq N; j++$ )
 $\{$        print ( $i*j$ )
    }
  
```

y
 y

i
 1 $[1, N]$ \rightarrow N
 2 $[1, N]$ \rightarrow N
 3 \vdots \rightarrow N
 .
 .
 .
 N $[1, N] \rightarrow \frac{N}{N^2}$

Iterations = N^2

$O(N^2)$

~~for (i=1; i≤N; i++)~~

~~for (j=1; j ≤N; j=j+2)~~

~~print (i*j)~~

y

y

$\log n$

i

1

2

3

.

{

}

N

j

$\log n$

$\log n$

$\log n$

:

:

j

$\log n$

$\frac{N \log N}{2}$

?

$\log n$

~~x₁~~

N times

$O(N \log N)$

Note: sqrt & log are both functions

$$\text{sqrt}(n) = n$$

$$n * n = n$$

$$\log_2(n) = n$$

$$n \xrightarrow{\text{1/2}} \frac{n}{2} \xrightarrow{\text{1/2}} \frac{n}{4} \dots \xrightarrow{\text{1/2}} 1$$

Number of divisions = 2^N

$$1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \rightarrow \dots >n$$

for (i=1; i <= 2^N ; i++)

{
 print(i)
}

$$i = \{ 2, 3, \dots, 2^N \}$$

Num of iterations = 2^N

O for ($i=1 ; i \leq N ; i++$)

X for ($j=1 ; j \leq 2^i ; j++$)

X print (i,j)

y

y

i	$\underline{2^i}$
1	$\underline{2^1}$
2	$\underline{2^2}$
3	$\underline{2^3}$
4	$\underline{2^4}$
⋮	⋮
N	<u>2^N</u>

$$2^1 + 2^2 + 2^3 + \dots + 2^N$$

This is GP

$$\begin{array}{rcl} a & = & 2 \\ r & = & 2 \\ n & = & N \end{array}$$

$$\begin{aligned} \text{Sum} &= \frac{a(r^n - 1)}{r - 1} \\ &= 2 \frac{(2^N - 1)}{2 - 1} = 2(2^N - 1) \end{aligned}$$

Num of iterations

$$\begin{array}{l} 2^{n+1} - 2 \\ 2^{n+1} = 2 \cdot 2^n \\ \hline O(2^n) \end{array} = 2(2^N - 1)$$

How to calculate Big O from number of iterations.

First find no of iterations.

1) Neglect lower order terms

$\overbrace{1 \quad 2}^{\dots}$

N N 1
 ~~N~~ ~~N~~ ~~1~~

2) Neglect the constant
co-efficient.

$$\underline{\underline{O}} \quad 4N^2 + 3N + 1$$

$$\rightarrow \quad \textcolor{pink}{4N^2} \quad \times \quad \times$$

$$\rightarrow \quad N^2 \quad \quad \quad O(N^2)$$

order of N^2

$$\underline{\underline{O}} \quad 4N^2 + 6N + 1331$$

~~$4N^2$~~ ~~$6N$~~ ~~1331~~

$$\rightarrow \quad \textcolor{pink}{4N^2} \quad \quad \quad O(N^2)$$

$$\underline{\underline{O}} \quad 4N^2 + 2N^2 + 50N$$

$$6N^2 + 50N$$

$\cancel{O(N^2)}$

$O(N^2)$

$\underline{\underline{O}} \quad N^2 + \log N$
 $O(N^2)$

$\underline{\underline{O}} \quad 100 \rightarrow O(1)$

$O(1) < O(\log_2 N) < O(\sqrt{N}) < O(N)$

$< O(N \log N) < O(N\sqrt{N}) < O(N^2)$

$< O(2^N) < O(N!) < O(N^N)$

$\underbrace{\qquad}_{\text{factorial}}$
 $(1 \times 2 \times 3 \times \dots \times N)$

$$5N^2 + \log N$$

$$5N^2$$

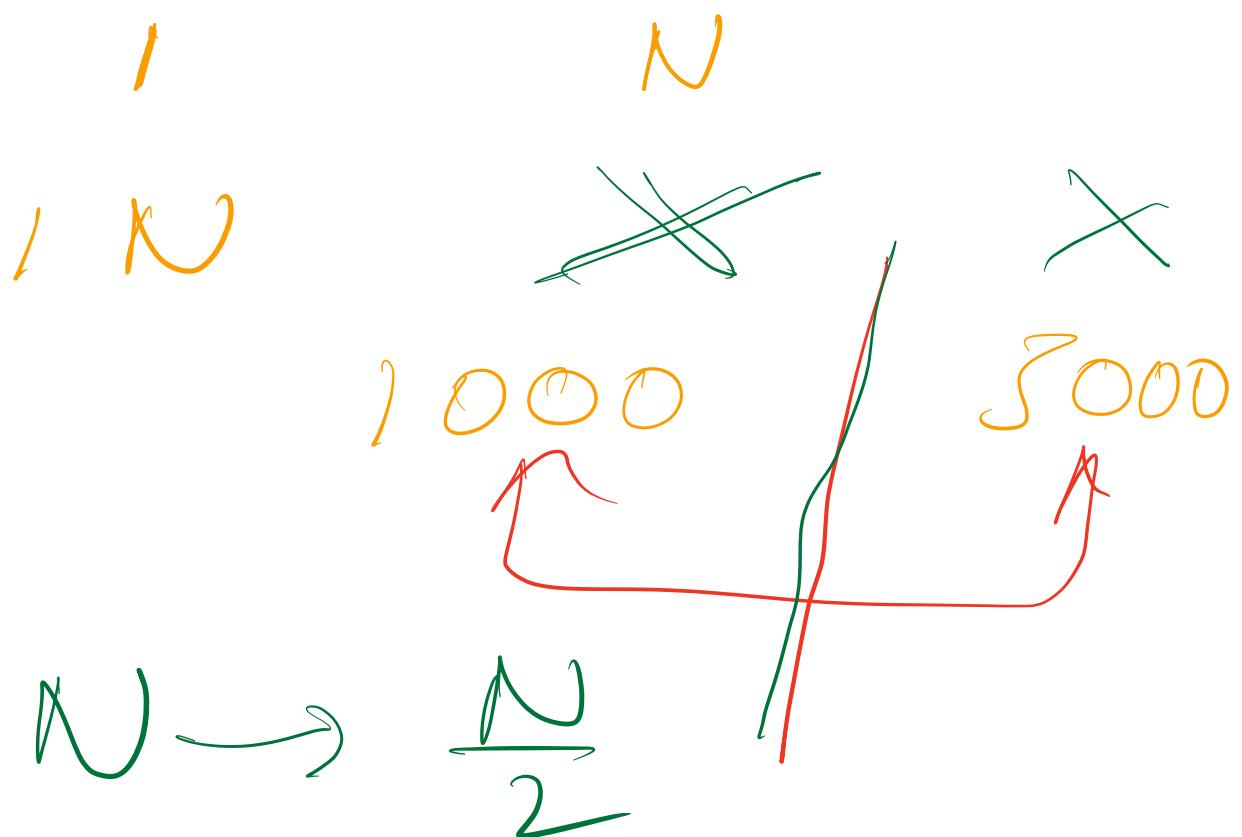
$$O(N^2)$$

Doubts

$$\boxed{10000 N} \quad 10^5 \quad N=10$$

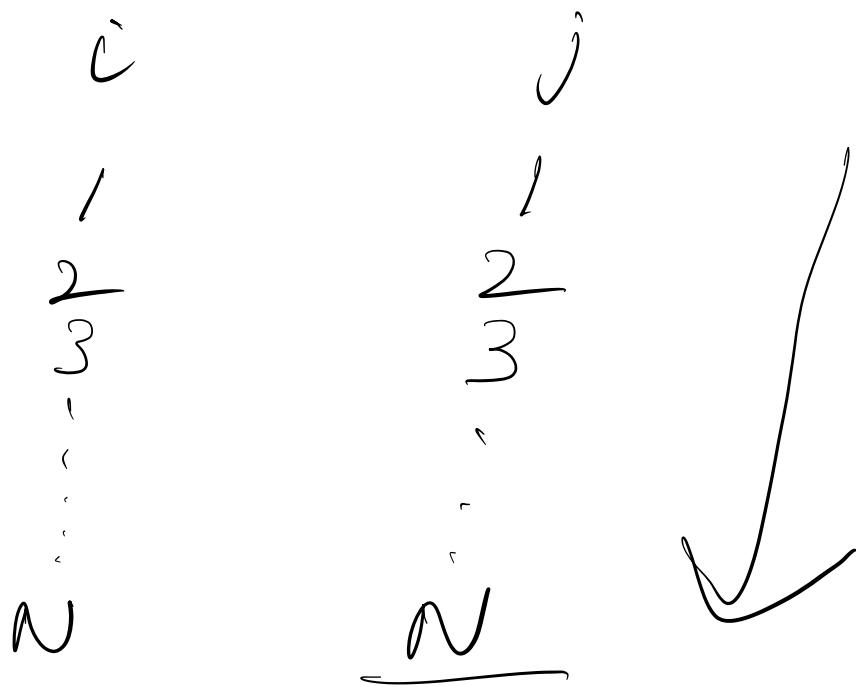
$$O(N)$$

$$10 \\ 20$$



$i = 1 ; i \leq N ; i++$

$j = 1 ; j \leq i ; j++$



$$\frac{N(N+1)}{2}$$

$$N \rightarrow \frac{N}{2} + \frac{N}{4} \rightarrow \dots \rightarrow 1$$

$$N = 16$$

$$16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$\log_2 16 = 4$$

int a = 1

if (a == 2)

else

i = i * 2

i = 1 ; i ≤ N ; i * = 2

$$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \dots \rightarrow 1$$

$$1 \rightarrow 2 \rightarrow 4 \rightarrow \dots \rightarrow N$$

$\log n$

$\log n - 1$

$O(\log n)$

1 2 3 ↘ ↙ 6

1 4 9 16 25 36



$\sqrt{n} < n$

$x < n$