Prefix Sum

Q1 Given an array size N & Q queries of the format s & e
start end

Return the sum of elements from indexes s to e for each query.
[s, e]

Eg

-3	6	2	4	5	2	8	-9	3	1
0	1	2	3	4	5	6	7	8	9

Q=3

s	e	
1	3	sum from [1,3] = 12
2	7	= 12
0	2	= 5

```
for(i=0 ; i<Q ; i++)
{    scan(s,e)
     int sum=0
     for(j=s ; j<=e ; j++)
     {
          sum += arr[j]
     }
     print(sum)
}
```

worst case O(N)

TC: O(N*Q)
SC: O(1)

Eg = Score of the last 10 overs in ODI

288	312	330	349	360	383	394	406	436
41	42	43	44	45	46	47	48	49

439
50

Runs scored in the last 5 overs.
no of runs scored in overs [46,50]

Final score − Runs at the end of 45th
R[50] R[45]

439 − 360 = 79

Runs scored in the 49th over.
[49, 49]

R[49] − R[48]
436 − 406 = 30

∘ Runs scored in overs [42,45]
R[45] − R[41] = 360 − 288 = 72

A : -3 6 2 4 5 2 8 -9 3 1

0 1 2 3 4 5 6 7 8 9

PS: -3 3 5 9 14 16 24 15 18 19

Prefix sum is sum of all values till that point.

3 6 2 1

3 9 11 12

PS[i] → Total sum of all values till index i.

$PS[0] = arr[0]$

$PS[1] = \underbrace{arr[0]}_{ps[0]} + arr[1]$

$PS[2] = \underbrace{arr[0] + arr[1]}_{ps[1]} + arr[2]$

$PS[3] = ps[2] + arr[3]$

$PS[i] = PS[i-1] + arr[i]$

$PS[0] = arr[0]$

// Build PS

```
int PS[N]
PS[0] = arr[0]

for(i=1 ; i<n ; i++)
{  PS[i] = PS[i-1] + arr[i]
}
```

TC : O(N)
SC : O(N)

What is the sum of all elem with indexes in the range [s, e]

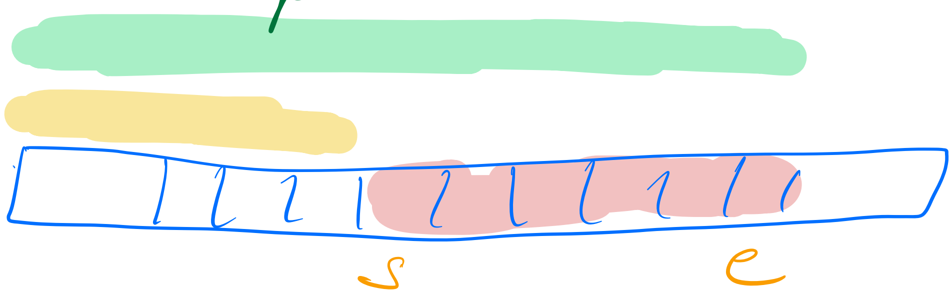
$a_s + a_{s+1} + a_{s+2} + \cdots + a_e$
$+ (a_0 + a_1 + a_2 + \cdots + a_{s-1})$
$- (a_0 + a_1 + a_2 + \cdots + a_{s-1})$

$x + \cancel{y} - \cancel{y} = x$

$ps[e] \quad - ps[s-1]$

$a_0 + a_1 + \cdots\ a_{s-1} + a_s + a_{s+1} + \cdots + a_e$

$ps[e]$



$ps[s-1] \quad sum[s:e] \quad = \quad ps[e]$

$sum[s:e] = ps[e] - ps[s-1]$

Answer Q Queries.

1) Build PS → $O(N)$
2) For each query, answer using formula → $O(Q)$

Total TC: $O(N+Q)$

SC: $O(N)$

$N = 10^5 \quad Q = 10^5$

Q2 Given an array, return true if there exist equilibrium index

EQ : Index for which

sum of elem on left side = sum of elem on right side (excluding that index)

A :	1	2	3	4	8	10
	0	1	2	3	4	5

$S_L = 10$ $S_R = 10$

If no elements on the left, $S_L = 0$
If no elements on the right, $S_R = 0$

Checking for index i

$S_L = a_0 + a_1 + a_2 + \dots a_{i-1} = ps[i-1]$

$S_R = a_{i+1} + a_{i+2} + \dots + a_{n-1}$

$sum[i+1, n-1] = ps[n-1] - ps[i]$

Edge Case

if (i == 0)
{ $sum_L = 0$
}

if (i == n-1)
$S_R = 0$

1) Build PS

2) For each i = 1 ; i ≤ n-2 ; i++
{ $sum_L = ps[i-1]$
$sum_R = ps[n-1] - ps[i]$

if ($sum_L == sum_R$)
return true.
}

return False.

TC: $O(N) + O(N) = O(N)$

$O(N) \downarrow$ PS

$O(N) \downarrow$ find eq idx

Eg	1	2	3	4	8	10
PS	1	3	6	10	18	28
idx	0	1	2	3	4	5
SL	0	1	3	6	10	
SR	27	25	22	18	10	

eq idx

TC : O(N)

SC : O(N)

Break (10:35)

Q3 Given an array & Q queries

DirectI
Flipkart

s, e, O ⇒ sum of all odd indexed elem in [s,e]

s, e, E ⇒ sum of all even indexed elem in [s,e]

A:	2	3	1	-1	0	8	5	4
	0	1	2	3	4	5	6	7

Q: 2

s	e	O/E	
3	6	O	a[3] + a[5] = 7
1	5	E	a[2] + a[4] = 1

PS even = Sum of all even idx elements till now

PS odd = Sum of all odd idx elements till now.

	0	1	2	3	4	5
A =	2	3	1	6	4	5
PSeven	2	2	3	3	7	7
PSodd	0	3	3	9	9	14

Sum of even idx elements in [s, e] = PSeven [e] − PSeven [s−1]

Sum of odd idx elements in [s, e] = PSodd [e] − PSodd [s−1]

1) Build PS even → O(N)

2) Build PS odd → O(N)

3) Answer queries based on O/E → O(Q)

TC: O(N) + O(N) + O(Q)

O(2N) + O(Q)

O(N) + O(Q)

O(N + Q)

Q4 Given an array count the no of special indexes.

Amazon
Google
MS, Adobe
DirectI
Codenation

Special Index after removing
Sum of all ODD indexed elements
= Sum of all even index elements

in the resulting array

$A = 4 \quad 3 \quad 2 \quad 7 \quad 6 \quad -2$

$\quad\;\; 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$i=0 \qquad 3 \quad 2 \quad 7 \quad 6 \quad -2$

$\qquad\qquad 0 \quad 1 \quad 2 \quad 3 \quad 4$

$i=1 \qquad 4 \quad 2 \quad 7 \quad 6 \quad -2$

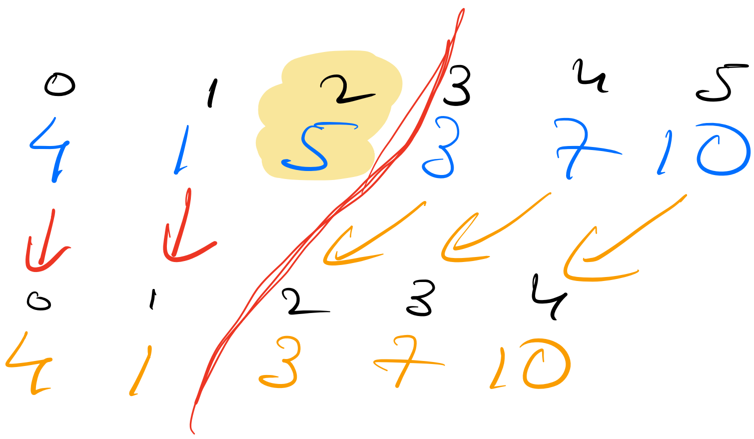
$\qquad\qquad 0 \quad 1 \quad 2 \quad 3 \quad 4$

Obs1: If I remove i^{th} index, all indexes $< i$ remain same.

Obs 2: If I remove i^{th} index all indexes $> i \Rightarrow$

even idx elem becomes odd idx
odd idx elem becomes even idx

1) Build PS even & PS odd



To the left of i	even idx	To the right of i
4		13

If removing i

Sum of even to the left

$= ps_{even}[i-1]$

Sum of even idx elem to the right =

Sum of all odd idx element the right (in orig array)

$Ps_{odd}[i+1 : n-1]$

$= Ps_{odd}[n-1] - Ps_{odd}[i]$

S_E (to the left) $= PS_{even}[i-1]$
S_E (to the right) $= Ps_{odd}[n-1) - PS_{odd}[i]$

Total sum of even $= PS_{even}[i-1] + PS_{odd}[n-1] - PS_{odd}[i]$

S_o (to the left) = PSodd [i-1]

S_o (to the right) = PSeven [n-1]
− PSeven [i]

Total sum of odd = PSodd [i-1]
+ PSeven [n-1] − PSeven [i]

int count = 0
// Build PSeven & PS odd

for (i=0 ; i<N ; i++)

{
S_E = PSeven [i-1]
+ PS odd (n-1) − PSodd [i]

S_O = PS odd [i-1]
+ PS even [n-1] − PSeven [i]

if (S_E == S_O)
count ++

}

// Handle

$i = 0$

return count seperately.

TC : $O(N)$ SC : $O(N)$

Doubts.

$i = N \; ; \; i > 0 \quad i /= 2$

$j = 0 \; ; \; j < i \quad j{+}{+}$

i	j
N	N
$N/2$	$N/2$
$N/4$	$N/4$
⋮	$N/8$
	⋮

$i = 1 \; ; \; i < N \; ; \; i{+}{+}$

$N-1$ iter

$[1, N]$

$b - a + 1$

$N - \cancel{1 + 1}$

$N + \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \ldots \ldots 1$

$$N\left[\underbrace{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\frac{1}{N}}_{\log_2 N}\right]$$

$$a\left(\frac{\gamma^{n}-1}{\gamma-1}\right)$$

$$1\times\left(\frac{\left(\frac{1}{2}\right)^{\log N}-1}{\frac{1}{2}-1}\right)$$

$$N\left(\frac{1-\left(\frac{1}{2}\right)^{\log n}}{1/2}\right)$$

$$\left(\frac{1}{2}\right)^{\log n}$$

max = 1

min = 0

$$\frac{1-0}{1/2}=2$$

$$\frac{1-1}{1/2}=0$$

$$N < x < 2N$$

$$O(N)$$