→ TLE (Time Limit Exceeded)

→ Big-O Notations O(N), O(log N)

→ Worst Case / Best Case / Average Case

Next class

23 Quizzes } → $\underbrace{\text{1000 Scaler Coins}}$

Code ⇒ # of Iterations

Q1: 20mins →

$\boxed{\log_2^N}$ →

Q2: [3, 10] : 3, 4, 5, 6, 7, 8, 9, 10

1 2 3 4 5 6 7 8

10−3 = 7 → 8 Numbers

[4, 17] : 17−4=13 14

[l, r] : r − l + 1

Square Brackets : All numbers in the range l to r including both l and r

$[l, r)$: All numbers in the range l to r, include l and exclude r

$[3, 11)$: $^{11-3=8}$ 3, 4, 5, 6, 7, 8, 9, 10 => 8

$[l, r)$: $r - l$

4, 5, 6

$(3, 7)$: Numbers in the range (3, 7) excluding 3 and 7

(l, r) : $r - l - 1$

Q3: Arithmetic Progressions

It is a series of numbers, difference between any 2 terms is same

Ex: $\underbrace{1 \quad 4}_{3} \quad \underbrace{\quad 7}_{3} \quad \underbrace{\quad 10}_{3} \quad \underbrace{\quad 13}_{3} \quad \underbrace{\quad 16}_{3}$ ✓

Ex: $\underbrace{7 \quad 19}_{12} \quad \underbrace{\quad 31}_{12} \quad \underbrace{\quad 44}_{13} \quad 56$ ✗

Generalize

AP: $\overset{1st}{a}$, $\overset{2nd}{a+d}$, $\overset{3rd}{a+2d}$, $\overset{4th}{a+3d}$ + ... k^{th}

a : First term of the series

d : Common difference

$2^{nd} \rightarrow a + d$

$3^{rd} \rightarrow a + 2d$

$4^{th} \rightarrow a + 3d$

⋮

$\boxed{k^{th} \rightarrow a + (k-1)d}$

Sum of N terms in an AP

$S =$ a, a+d, a+2d, a+3d + ... a+(N-1)d

$S =$ a+(N-1)d, a+(N-2)d, a+(N-3)d . . . a+d, a

2a+(N-1)d   2a+(N-1)d

$$S_N = \frac{N}{2}\left[2a + (N-1)d\right]$$

N: No. of terms

$$S = \frac{N}{2} \times [2a + (N-1)d]$$

Q4:

$\log_a a^x$

$\boxed{\log_y x}$: No. of times I have to divide x by y to make it 1

$2^{10} \xrightarrow{1} 2^9 \xrightarrow{2} 2^8 \xrightarrow{3} 2^7 \xrightarrow{4} 2^6 \xrightarrow{5} 2^5 \xrightarrow{6} 2^4 \xrightarrow{7} 2^3 \xrightarrow{8} 2^2 \xrightarrow{9} 2^1 \xrightarrow{10} 1$

$\log_a a^x$ $\qquad \dfrac{a \times a \times a \, . . . \, . . \; x \text{ times}}{a} = 1$

2^2

$2^4 = 16$

$2^{10} \longrightarrow 2^5$?

$\frac{2^4}{2} = \frac{16}{2}$ $\boxed{8}$

$\rightarrow \log_2 2^{10} = 10$

$\frac{2^{10}}{2} = 2^{10-1} = 2^9$ $\qquad \frac{2^a}{2} = 2^{a-1}$

Q5:

```
for(i = 1; i <= N; i++) {
    S = S + i;
}
```

$N \Rightarrow O(N)$

Range of i : $[1, N] \Rightarrow N - 1 + 1 = N$

$[l, r] = r - l + 1$

$\longrightarrow$

Q6:

i = i + 2

i = 1

for(i = 1 ; i ≤ N ; i = i + 2) {

i = i + 2

}

N = 10

i = 1 → iter1

i = 3 → iter2

i = 1, 3, 5, 7, 9 . . .

for(i = 1 ; i ≤ 10 ; i++){

}

(1, 10)

[1, 10]

(1, 10)

i ⇒ All odd numbers in the range [1, N]

N = 4

i = 1, 3,

5

N = 5

i = 1, 3, 5

N = 12

1 – 12

$\frac{N}{2}$

N/2

$\boxed{\frac{N}{2}}$

$\frac{N}{2}$, $\frac{N+1}{2}$, N

N = 5

$\frac{5}{2}$ =

odd (1 – 4) : 2

odd (1 – 5) : 3

odd (1 – 6) : 3

odd (1 – 7) : 4

(1 – 8) : 4

(1 – 9) : 5

(1 – 10) 5

(1 – 11) 6

(1 – 12) 6

5, 6 ⇒ 3

7, 8 ⇒ 4

9, 10 ⇒ 5

11, 12 ⇒ 6

$\boxed{N/2}$

N = ~~5~~

2

$\boxed{\frac{N+1}{2}}$

N = 5, N = 6

N

$\left(\frac{8+1}{2}\right) ⇒ 4$

if N is even: $\Rightarrow \frac{N}{2}$

N is odd: $\frac{N+1}{2}$

$\frac{N-1}{2}+1 = \frac{N+1}{2}$

1, 2, 3, 4, 5, 6, 7

1, 2, 3, 4, 5, 6, 7, 8

$N = 1000$

500

1, 3, 5, 7, 9

$\left(\frac{10+1}{2}\right) = \frac{11}{2} = \boxed{5}$

Q7:

```
for(i = 0; i <= 100; i++) {

}
```

$100 \to O(1)$

Range of i : $[0, 100] \Rightarrow 100 - 0 + 1 = 101$

$[l, r] \to r - l + 1$

Q8:

```
for(i = 1; i * i <= N; i++) {

}
```

Prime

$\sqrt{i^2} = i$

$\sqrt{N^2} = \sqrt{N}$

$\boxed{i \times i \leq N} \Rightarrow i^2 \leq N$

$\Downarrow$

$\boxed{i \leq \sqrt{N}}$

Range of i : $[1, \sqrt{N}]$ $\quad \sqrt{N} - 1 + 1 = \boxed{\sqrt{N}}$

$[l, r] : r - l + 1$

$O(\sqrt{N})$

Q10:

```
i = N;
while (i > 1) {
    i = i/2;
}
```

$\log_2^N$

i Before	Iteration No.	i After
N	1	$N/2 \Rightarrow \frac{N}{2^1}$
$N/2$	2	$N/4 \Rightarrow \frac{N}{2^2}$
$N/4$	3	$\frac{N}{8} \Rightarrow \frac{N}{2^3}$
	K	$\boxed{\frac{N}{2^K}} \Rightarrow 1$

if $i \le 1$

After K iterations, $i = \frac{N}{2^K}$

$$\frac{N}{2^K} = 1$$

$$N = 2^K$$

Apply log on both the side

$$\log_2(N) = \log_2 2^K \Rightarrow K$$

$$\boxed{K = \log_2^N}$$

$O(\log_2^N)$

$$\underbrace{N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \frac{N}{8} \cdots\cdots 1}_{\boxed{\log_2^N} \text{ Iteration}}$$

$N = 10$

122

Q11 :

for(i = 0 ; i ≤ N; i = i * 2) {

}

Infinite Loop

i = 0

i = 0

i = i * 2

i = 0 * 2 = 0

i = i * 2

Q12:

for(i = 1; i ≤ N; i = i * 2) {

}

$\log_2^n$

1, 2, 4, 8 . . . . N

i = 1, 2, 4, 8 . . . . . . $\frac{N}{4}$, $\frac{N}{2}$, N

$N \to \frac{N}{2} \to \frac{N}{4} \to$ . . . . . . $4 \to 2 \to 1$ } $\log_2(N)$

$\log_2(N)$

$\log 2(N)$

$\log_2 N$

$\boxed{\log 2(N)}$

$O(\log_2^N)$

Q18:

$[l, r] = r - l + 1$

```
for (i = 1; i <= 10; i++) {
    for (j = 1; j <= N; j++) {
    }
}
```

i	j	# iteration
1	[1, N]	N
2	[1, N]	N
3		N
4		
⋮		
10		

[1, 10]

$10 - 1 + 1 = 10$

Total iteration = $\underbrace{10 \times N}$

$O(N)$

O

Q14:

```
for (i = 0; i < N; i++) {
    for( j = 0; j < N; j++) {

    }
}
```

[0, N]
[0, N)
[l, r) = r - l

i	j	# iterations
0	[0, N)	N
1	[0, N)	N

Range of i : [0, N)

=> N values of i

for(j = 1; j ≤ 1; j++) => Range of j : [1, 1] => N * N = N^2 → $O(N^2)$

N = 10

10 or $\frac{11}{2}$

iterations : 1

[1, 1]

for(j = 1; j ≤ 2; j++) { } → [1, 2]

[1, 3]

i = 1

[4-5 Maths

Q15:

```
for(i = 1 ; i ≤ N; i++) {
    for( j = 1 ; j ≤ i ; j++ ) {
        print("Hi")
    }
}
```

i = 1 i = 2

j = 1 . . . 2

Range of i : [1, N]

i	j	# iterations
1	[1, 1]	1
2	[1, 2]	2
3	[1, 3]	3
4	[1, 4]	4
N	[1, N]	N

$N * \left(\frac{N+1}{2}\right)$

$1 + 2 + 3 + 4 + \ldots N \Rightarrow \frac{N(N+1)}{2}$

$N - 1 + 1 = N$

$\Rightarrow \frac{N^2 + N}{2}$

$\to O(N^2)$

$\#\text{iterations} = 1 + 2 + 3 + 4 + \dots N$

$= \frac{N(N+1)}{2} = \frac{N^2+N}{2}$

$i = 1$ $(j = 1;\ j \le i;\ j{+}{+})$ $[1, 1]$

$\frac{N^2}{2} + \frac{N}{2} \to O(N^2)$

$i = i+2$

Q16:

```
for(i=1; i <= N; i = i+2){
    for( j = 1; j <= i; j++){
    }
}
```

N: very high $N = 1$

a) N^2

b) infinite

c) $(N^2+N)/2$

d) $(N^2+2N+1)/4$

e) $(N^2+2N+2)/4$

f) $(N^2+2N+2)/2$

g) $(N^2+2N+2)/8$

i	j	# iterations
1	[1, 1]	1
3	[1, 3]	3
5	[1, 5]	5
7	[1, 7]	7
N		N

$\#\text{iterations} = \underbrace{1 + 3 + 5 + \dots N}_{\text{sum of all odd Nos in } [1, N]}$

$a,\ a+d,\ a+2d,\ a+3d \dots$

$\frac{n}{2}$

Arithmetic Progression

$a = 1$

$d = 2$

$k = \frac{(N+1)}{2}$

$N=10 \Rightarrow [1 + 3 + 5 + 7 + 9]$

$N=11 \Rightarrow [1 + 3 + 5 + 7 + 9 + 11]$

$[1, N] \Rightarrow \frac{(N+1)}{2} \Rightarrow \left(\frac{11+1}{2}\right) = \boxed{6}$

?

$$S_K = \frac{K}{2}\left[2a + (K-1)d\right]$$

AP: $\underbrace{1, 3, 5, 7, \ldots \ldots}\ N$

$\boxed{a = 1, \quad d = 2, \quad K = \frac{N+1}{2}}$

$$\frac{N+1}{4}\left[2(1) + \left(\frac{N+1}{2} - 1\right)2\right]$$

$$= \frac{N+1}{4}\left[2 + \left(\frac{N+1-2}{\cancel{2}}\right) \times \cancel{2}\right]$$

$$= \frac{N+1}{4}\left[2 + N - 1\right]$$

$(a+b)^2 = a^2 + b^2 + 2ab$

$$= \frac{N+1}{4}(N+1) = \boxed{\frac{N^2 + 2N + 1}{4}}$$

$\frac{N^2}{4} + \frac{2N}{4} + \frac{1}{4} : O(N^2)$

Q:

$N = 2$ $\{1 + 3\} = 4$

$N = 3$ $\{1 + 3 + 5\} = 9$

$\boxed{N = 1 \quad = \{1\}}$

$\boxed{N = 1} = \{1\}$

a) N^2 $\Rightarrow 1^2 = 1$

b) $2*(N^2 + N)/2$ $\Rightarrow \frac{2(1+1)}{2} = 2$

c) $(N^2 + 2N)/4$ $\Rightarrow (1+2)/4 = 3/4$

d) $2N$

AP:

$$\underbrace{1 + 3 + 5 + 7 \ldots \ldots}_{\boxed{N} \text{ terms}}$$

1st, 2nd, 3rd, N^{th} odd

$a = 1$

$d = 2$

$K = N$

$a, a+d, a+2d, \ldots \ldots a+(k-1)d$

odd No.s in $[1, N] \Rightarrow \boxed{\frac{(N+1)}{2}}$

$$S_K = \frac{K}{2}[2a + (K-1)d]$$

$$= \frac{N}{2}[2(1) + (N-1)*2]$$

$$= \frac{N}{2}[\cancel{2} + 2N - \cancel{2}] = \frac{N}{\cancel{2}} * \cancel{2}N$$

$$= \boxed{N^2}$$

$O(N^2)$

$1, 2, 4, 8 \cdots \cdots \frac{N}{2}, N$

$\underbrace{\qquad\qquad}_{\log_2 N}$

Q:

```
for(i=1; i<=N; i++) {
  for( j=1; j<=N; j=j*2) {
  }
}
```

i=2

$1, 2, 4, 8 \cdots \cdots N$ }

i	j	# iterations
1		$\log_2 N$
2		$\log_2 N$
3		$\log N$
⋮		
N		

$\boxed{N \times \log_2 N}$

$O(N \log_2 N)$

i=1 $\log N$

i=2

```
for(i=1; i<=N; i++) {
  for(j=1; j<=N; j=j*2) {
  }
}
```

$\log_2 N$

i= 1 $\log n$

i = 2 $\log n$

i = 3 $\log n$

⋮

$\underbrace{1, 2, 4, 8 \cdots \cdots N}$

$\boxed{2^\wedge N = \quad 2^N}$ pow(2, N)

Q: for(i=1; $i \le 2^N$; i++) {

}

Range of i : $[1, 2^N]$

$[l, r] = r - l + 1$

$= 2^N - 1 + 1$

$= \boxed{2^N} \to O(2^N)$

Geometric Progression

2, 6, 18, 54 . . .

$a, \; ar, \; ar^2, \; ar^3, \; ar^4 + \ldots$

$$\frac{ar^2}{ar} = r \qquad \frac{ar^4}{ar^3} = r$$

GP: We get the same value if we divide consecutive terms.

a : First Term

r : Common Ratio

Sum of first K terms in a G P $\boxed{K>0}$

$$\boxed{S_K = \frac{a(r^K-1)}{r-1}}$$

$\boxed{r>1}$ $r>1$

$r<1$

$\boxed{r<1}$

$\boxed{r \neq 1}$

$a=1$, $r=\frac{1}{2}$

$r=\frac{1}{2}$ $\frac{1}{2}-1$

$1, \left(\frac{1}{2}\right), \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \left(\frac{1}{2}\right)^4 + \ldots\ldots$

$a=1$, $r=\frac{1}{2}$

$$S_K = \frac{1\left(\left(\frac{1}{2}\right)^K - 1\right)}{\frac{1}{2}-1} = \frac{\left(\frac{1}{2}\right)^K - 1}{-\frac{1}{2}}$$

$\left(\left(\frac{1}{2}\right)^K-1\right) \times \frac{-2}{1}$

$$S_K = \left[\left(\frac{1}{2}\right)^K - 1\right] \times (-2)$$

$$= \boxed{\left[1-\frac{1}{2^K}\right] \times 2}$$

$\boxed{K>0}$

$\boxed{K=-2}$

$\boxed{K \geq 0}$

$\left| 1 - \frac{1}{2^K} \right. \Rightarrow$ $\boxed{K=-2}$

$[0, 1]$

$K=1 \Rightarrow 1-\frac{1}{2} = 0.5$

$K=2 \Rightarrow \boxed{1-\frac{1}{4}} = 0.75$

$K=3 = 1-\frac{1}{2^3} = 0.875$

$\frac{1}{\left(\frac{1}{8}\right)} = 8$

$1 \times \frac{8}{1}$

Q: for(i = 1; i ≤ N; i++)

{ for(j = 1; j ≤ 2^i; j++) {

}

}

$2^1 - 1 + 1$

$2^2 - 1 + 1$

i : [1, N]

i	j	# iterations
1	$[1, 2^1]$	2^1
2	$[1, 2^2]$	2^2
3	$[1, 2^3]$	2^3
N	$[1, 2^N]$	2^N

×2

×2

×2

$2^1 + 2^2 + 2^3 + 2^4 + \dots \; 2^N$

$a, ar, ar^2, ar^3 \dots$

$a = 2$

$r = 2$

$K = N$

$$S_K = \frac{a(r^K - 1)}{r - 1}$$

$$= \frac{2(2^N - 1)}{2 - 1} = \boxed{2(2^N - 1)}$$

$2 \cdot 2^N - 1$

$\boxed{O(2^N)}$

Big-O Notation

3 steps

Input size

1) Find No. of iterations [Function of N]
2) Neglect all the lower order terms
3) Neglect the co-efficient of the higher order term

Iteration

$\Rightarrow 11N^2 + 1000N + 10^6$

2) $11N^2 + 1000N^1 + 10^6 \cdot N^0 = \boxed{O(N^2)}$

1) $100N^2 + 5N + 10^4 \Rightarrow O(N^2)$

2) $5N + 6N\log N + 8\sqrt{N}$: $O(N\log N)$

$\boxed{N > 10^5}$

$N > N^{1/2}$

$N \quad \boxed{N\log N}$

3) : $N^2 + 6N\sqrt{N} + 10^4 N\log N$

$O(N^2)$

4) $4N^2 + 3N + 10^6 \Rightarrow O(N^2)$

$\text{sqrt}(N)$ $\qquad \log_2(N)$

$\log_a a^x = x$ $\qquad 2^8$ $\qquad \log_2 2^{16} = 16$

$\boxed{N = 2^{16}}$

$\text{sqrt}(2^{16}) = 2^8$ $\quad 256$

$\log_2 2^{16} = 16$

2^8 $\qquad 16$

5) $4N + 3N\log N + 10^6 : O(N\log N)$

6) $4N\log N + 3N\sqrt{N} + 10^6 : O(N\sqrt{N})$

$\cancel{N}\log N \qquad \cancel{N}\sqrt{N}$

$\#\text{Iterations} = 100 = 100 \cdot N^0 = O(N^0)$

$= O(1)$

Doubts session

$F(n) = 2^N$ → #iteration $O(2^N)$

$\log_2 4 = 2$

$N = 4 \Rightarrow$ $\boxed{2^4 = 16}$

$O(\log(\log n))$

↓

Prime

$\boxed{\text{Sieve of Eratosthenes}}$

$\boxed{O(N.\log(\log N))}$

```
for(i=1 ; i <= N; i++){

    for(j=1 ; j <= i; j++){

    }
}
```

i = 1

for(i=1; i*i <= N; i++){

}

j=1;

while j*j <= N:

j = j+1;

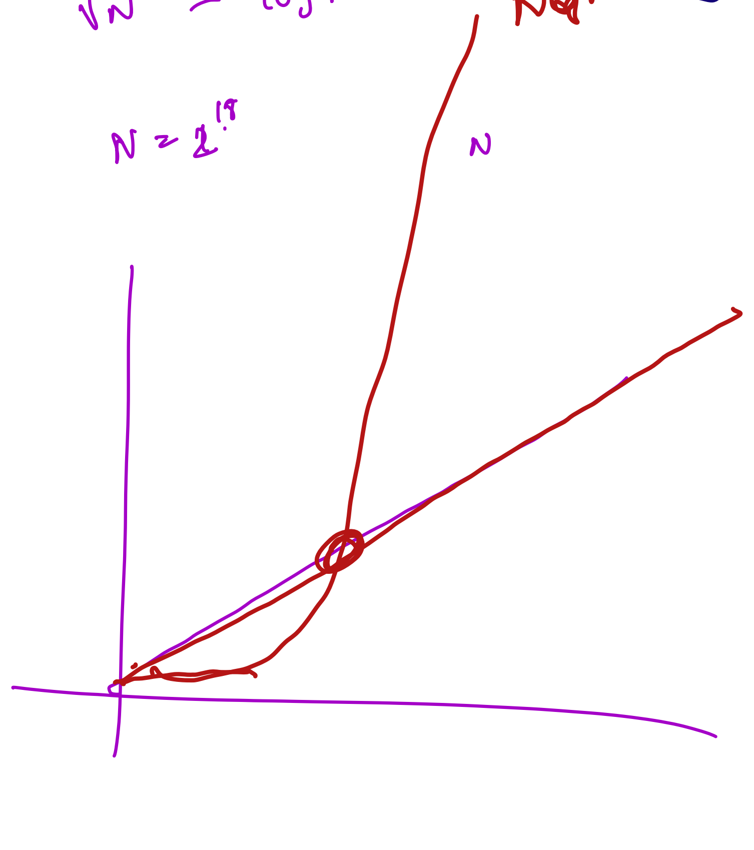
NlogN , N√N , N

√N > log N

NlogN

N = 1, 2

N = 2^{19}



O(N)

N=10

```
for(i=1; i <= N; i++){
    for(j=1; j <= N; j++){

    }
}
```

i	j	iteration(i)	iteration

i → 10 iteration

1st iteration of i → 10 iterations

: content

2nd

[l, r]

[l, r) → for(i=0; i<n; i++)

(l, r)

[0, n-1]

[0, n)

[l, r]

$a, ar, ar^2, ar^3, ar^4, ar^5 + \dots$

for( ){

for(j=1; j ≤ 2^i; j++){

1

3

?

$\boxed{G.P}$

$\left(\frac{N+1}{2}\right)^2$

Case 1: Even $\Rightarrow \boxed{\frac{N}{2}}$

1. 2 3 4 5 . . . . ?

Case 2: N. odd:

$\boxed{1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6} \quad 7$

$N-1$

$\left(\frac{N-1}{2}\right) + 1 = \frac{N-1+2}{2} = \boxed{\frac{N+1}{2}}$

$a + ar + ar^2 + ar^3 + ar^4 + \dots \quad a \cdot r^{n-1}$

1st 2 3 4 5 n^{th}

$a(1 + r + r^2 + r^3 + \dots r^{n-1})$

$$\boxed{(r^n - 1)} = (r-1)\left(r^{n-1} + r^{n-2} + r^{n-3} + \dots r^3 + r^2 + r + 1\right)$$

$$\frac{r^n - 1}{r - 1}$$