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Q1) Given a number, find if it is prime.

Prime number \rightarrow Number with only 2 factors. 1 & itself.
Eg - 3, 5, 7

23 is a prime.

Q Is 1 prime.

Factors \rightarrow 1

Idea : Count the number of factors

```
bool is_prime(int n)
{
    int count = 0
    for (int i = 1; i <= n; i++)
    {
        if (n % i == 0)
            count++
    }
}
```

```

y
if (count == 2)
    return true;
else
    return false;

```

y

Iterations
= n

Eg 1

2 1

i = 1

i = 2

i = 3

4

5

6

7

...

i = 21

Count = 1

Count = 2

Count = 3

Count = 4

Eg 2

2 3

i = 1

i = 2 3

Count = 2

Eg 3 $n = 1$

$$i = 1$$

$$\text{count} = 1$$

Assumption \rightarrow 1 sec 10^8 op

$$n = 10^9 \rightarrow \frac{10^9}{10^8} = 10 \text{ sec}$$

$$1 \text{ sec } 10^8$$

$$10 \text{ sec } 10^9$$

$$n = 10^{18} \rightarrow \frac{10^{18}}{10^8} = 10^{10} \text{ sec} \\ \approx 317 \text{ yrs}$$

Obs: If $a * b = N$, then
the pair $\{a, N/a\}$
both are factors.

$$b = N/a$$

$$N = 24$$

i	N/i
<u>1</u>	<u>24</u>
<u>2</u>	<u>12</u>
3	8
<u>4</u>	<u>6</u>
6	4
8	3
12	2
24	1

$$\boxed{a} \leq 60$$

$$\underline{\underline{\max_a}} = ? \quad 60$$

$$i \leq N/i$$

$$i_{\min} = N/i_{\max} \Rightarrow i_{\max}^2 = N$$

$$i_{\max} = \sqrt{N}$$

```

bool is_prime(int n)
{
    int count = 0;
    for (int i = 1; i * i <= n; i++)
    {
        if (n % i == 0)
        {
            if (i * i == n)
                count++;
        }
    }
}

```

```

else
    count += 2
}
}

n = 9
{1, 3, 9}

if (count == 2)
    return true
else
    return false
}

Iterations =  $\sqrt{N}$ 

```

$n = 36$

$i = 1$

2
3
4
5
6

count
2
4
6
8
10

$n = 23$

$i = 1$

count
2

2
3
4

2
2
2

$$n = 10^8 \quad \text{iterations} = 10^9$$
$$= 10 \text{ sec}$$

Carl Friedrich Gauss

Find the sum of integers
from 1 to 100

$$S = 1 + 2 + 3 + \dots + 99 + 100$$

$$S = 100 + 99 + 98 + \dots + 2 + 1$$

$$2S = \underbrace{101 + 101 + 101 + \dots + 101}_{100 \text{ times.}}$$

$$2S = 101 \times 100$$

$$S = \frac{101 \times 100}{2} = 5050$$

$$S = 1 + 2 + 3 + \dots + n$$

$$S = n + n-1 + \dots + 1$$

$$2S = \underbrace{(n+1) + (n+1) + \dots + (n+1)}_{n \text{ times}}$$

$$2S = n(n+1)$$

$$S = \frac{n(n+1)}{2}$$

Q Given a number n , how many times can you divide by 2 till it reaches 1.

(Integer division)

$$7/2 = 3$$

2^0	1	\rightarrow	
2^1	2	\rightarrow	1
2^2	4	\rightarrow	2 \rightarrow 1
2^3	8	\rightarrow	4 \rightarrow 2 \rightarrow 1
2^4	16	\rightarrow	8 \rightarrow 4 \rightarrow 2 \rightarrow 1
2^5	32	\rightarrow	16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1
2^6	64	\rightarrow	32 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1

$$51 \rightarrow 25 \rightarrow 12 \rightarrow 6 \rightarrow 3 \rightarrow 1 \quad 5$$

$$60 \rightarrow 30 \rightarrow 15 \rightarrow 7 \rightarrow 3 \rightarrow 1 \quad 5$$

$$32 \quad 33 \quad 34 \dots 51 \dots 60 \dots 63$$

$$n \xrightarrow{\text{ans}} y$$

y is called the logarithm of n .

$$\log_2 n = y$$

$$\log_2 64 = 6$$

$$\log_2 51 = 5$$

$\log_3 n \rightarrow$ no of times you can divide by 3 till it reaches 1

```
int log (int n)
```

```
{ int count = 0;
  while (n != 1)
```

```
{
```

```
    n = n/2;
    count++;
```

```
}
```

```
return count;
```

```
}
```

$n = 64$

32

1

16
8
4
2
1

2
3
4
5
6

50
25
12
6
3
1

1
2
3
4
5

Break

\sqrt{n}

Back at 10:35

$\log(n)$

Q
Amazon

Given a perfect square no,
find the square root of
the number.

Eg 25 \rightarrow 5

$$\sqrt{49} \rightarrow 7$$

N is perfect-square means that an integer x exists st

$$x * x = N$$

Idea: Square root will lie between 1 & N .

```

int sqrt(int n)
{
    for (int i=1; i< n; i++)
    {
        if (i*i == n)
            return i;
    }
}

```

$$n = 36$$

```

i = 1
i = 2
i = 3
i = 4
i = 5
i = 6

```



Number of iterations = \sqrt{N}

Binary search.

Obs: If n increases, the square root also increases.

625

1 - 152 - 313 - - - 625
↑ ↑ ↑
start end end

$$\text{mid} = \frac{(1 + 625)}{2} = \underline{313}$$

$$313 * 313 = 625 \quad \times$$

$$313 * 313 > 625$$

$$\text{mid} = \frac{(1 + 313)}{2} = 152$$

$$152 * 152 > 625$$

$$\left(\frac{1 + 152}{2} \right) = 79$$

$$79 * 79 > 625$$

$$\frac{1 + 79}{2} = 40$$

$$40 * 40 > 625$$

$$\left[\frac{1 + 40}{2} \right] = 20 \quad 1 \quad 40$$

$$20 * 20 < 625$$

20

40

20 40

$$\left(\frac{20+40}{2}\right) = 30$$

$$30 * 30 > 625$$

$$20 \quad 30$$

$$\frac{20+30}{2} = 25$$

$$25 * 25 = 625$$

1 - - - - 625

1 313

1 252

1 79

1 - - - - n



625

313

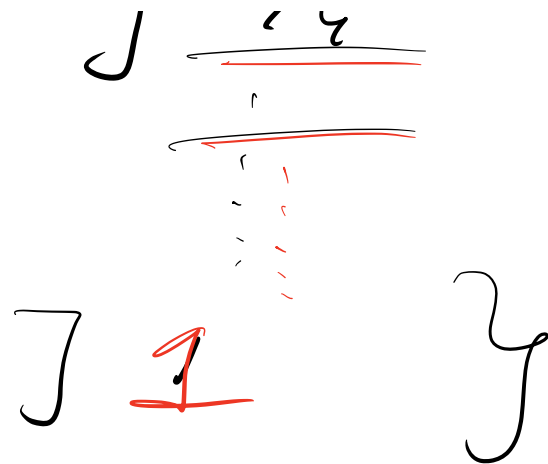
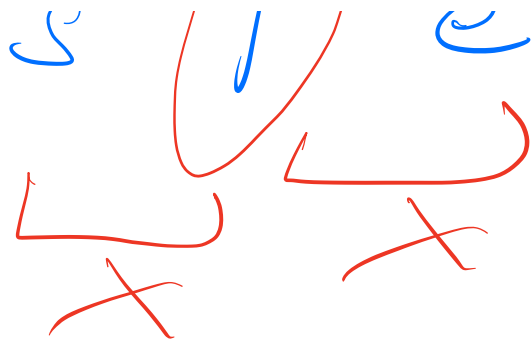
152

79

} n no. 4

} n/2

} n/1



Number of iterations in binary search = $\log_2 n$

$$n = 10^{18}$$

$$\log_2 10^{18} \approx 60$$

$$\sqrt{10^{18}} = 10^9$$

end ! = start

while (start \leq end)

{ mid = start + end

$$\frac{10^8}{10^9} \rightarrow \underline{1 \text{ sec}}$$

$$= \underline{10} \times \underline{10^8} \rightarrow 10 \text{ sec}$$

$$\begin{array}{lcl} 10^8 & \rightarrow & 1 \\ 10 & \rightarrow & 1/10^8 \times 10 = 10^{-7} \end{array}$$

$$\sqrt{n} \rightarrow x$$

$$x * x = n$$

$$\log(n) \rightarrow \boxed{x}$$

$$n \rightarrow n/2 \rightarrow n/4 \rightarrow \dots \rightarrow 1$$

x times.