TC-2
1) Time complexity & Space complexity
2) Asymptotic analysis (Big O)
3) Big O [Formal definition]
4) TLE (Time Limit Exceeded)

TC-1 : No of iterations
Big O (Basics)

1) $\log_2 N$

$$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \cdots \rightarrow 1$$

number of times you divided is $\log_2 N$

$$\log_2 2^x =$$

$$2^x \rightarrow 2^{x-1} \rightarrow 2^{x-2} \rightarrow \cdots \rightarrow 2^1 \rightarrow 2^0 = 1$$

x divisions.

$$2^3 \rightarrow 2^2 \rightarrow 2^1 \rightarrow 2^0$$ ans = 3

$2^0 = 1$ $\qquad$ $1000^0 = 1$

$3^0 = 1$

$4^0 = 1$

$$\frac{2^x}{2} = 2^{x-1}$$

$$2^x = \underbrace{2 \times 2 \times 2 \times 2 \ldots \ldots \times 2}_{x \text{ times.}}$$

Ranges $\qquad$ $[1, N]$

$= 1, 2, 3, 4, 5, \ldots\ldots, N-1, N$

$(1, N)$ $\qquad$ $2, 3, 4, \ldots\ldots\ldots, N-1$

$[\rightarrow$ include

$(\rightarrow$ exclude

$[1, N)$ $\qquad$ $1, 2, 3, \ldots, N-1$

Q $[a, b]$ How many no.s in this range

integers

$a, a+1, a+2, \text{-----} b-1, b$

count of nos $= b-a+1$

$[4,4]$ $\quad 4$

$4-4+1 = 1$

$[4,5]$ $\quad 4, 5$

formula $= 5-4+1 = 2$

Arithmetic Progression

$\underbrace{4, 7}_{+3}, \underbrace{7, 10}_{+3}, \underbrace{10, 13}_{+3}, \underbrace{13, 16}_{+3}, 19, \text{---}$

a = Starting no / first term

d = common difference

$\underset{1^{st}}{a}, \underset{2^{nd}}{a+d}, \underset{3}{a+2d}, \text{------}, \underset{n^{th}}{a+(n-1)d}$

$a+(n-1)d$

$n=1 \quad a$

$a+nd$

$n=1 \quad a+d$

— ___ ×

n^{th} term $= a + (n-1)d$

S_N = Sum of first n numbers of sequence.

$$\boxed{S_N = \frac{n}{2}\left[2a + (n-1)d\right]}$$

HW: Derive this.
Hint: Gauss metho

Geometric Progression

$5, 10, 20, 40, 80$

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2$

a = Starting no/ first term
r = common ratio

$a \quad ar \quad ar^2 \; - - - - - - - - \; ar^{n-1}$

$1^{st} \quad 2^{nd} \quad 3^{rd} \quad n^{th}$

ar^{n-1}

$n=1$ a

$\cancel{ar^n}$

$n=1$ ar

n^{th} term $= ar^{n-1}$

S_N = Sum of first n numbers of sequence.

$$\boxed{S_N = \frac{a(r^n-1)}{r-1}}$$ when $r \neq 1$

when $r=1$, it is an AP where all terms are same. Hence, we use AP formula

Eg: 5, 5, 5, 5

is this AP Yes
is this GP Yes

AP $n=4$ GP

$a = 5$ $\quad$ $d = 0$

$a = 5$ $\quad$ $r = 1$

$$\frac{n}{2}[2a+(n-1)d]$$

$$\frac{4}{2}[2\times5 + 3\times0] = \frac{4}{2}\times10$$

$$= 20$$

5, 10, 20, 40

$a = 5$ $\quad$ $r = 2$

$$a\left(\frac{r^n-1}{r-1}\right) = \frac{5[2^4-1]}{2-1}$$

$$= \frac{5[16-1]}{1} = 75$$

Count number of iterations.

① $\quad$ $s = 0$

=

for (int i=1 ; i≤N; i++)
S = S+i

Num of iterations = N

i = 1, 2, 3, ------- N

[1, N]

no of numbers = b-a+1
N-1+1 = N

O(N)

Q

for (i=1 ; i≤N; i++)
print(i)
} N

for (i=1 ; i≤M ; i++)
print (i)
} M

Num of iterations = N+M
O(N+M)

Q

S = 0
for (i=1; i≤N ; i+=2)
{
S += i

}

$\frac{N}{2}$ put $N = 1$

$\frac{N}{2} = \frac{1}{2} = 0$

$i : 1, 3, 5, 7, 9, - - - - -$

If n is odd, i will be equal to n

If n is even, i will be equal to $n-1$

n (odd)

$i = 1, 3, 5, 7, - - - - -, n$

How many no.s

Last term $= n$
First term $= 1$
$d = 2$

... m is

$$n = \underset{1}{a} + (m-1)\underset{2}{d} \quad \} \text{ total no of terms}$$

$$n = 1 + (m-1)2$$

$$\frac{n-1}{2} = m-1$$

$$m = \frac{n+1}{2}$$

$m = \frac{n+1}{2}$ is also valid for even.

$n = 19$ $\qquad m = \frac{19+1}{2}$

1, 3, 5, 7, ... 17, 19

$= 10$

$n = 20$ $\qquad \frac{20+1}{2}$

1, 3, 5, 7, ... 17, 19

$= \frac{21}{2} = 10$

Number of iterations

$= \left(\frac{n+1}{2}\right)$ $O(N)$

for ($i=2$; $i \le N$; $i += 2$)

$i = 2, 4, 6, 8, \ldots\ldots n$ even

$n-1$ odd

Q for ($i=1$; $i \le 100$; i++)

{

print (i)

}

$i = 1, 2, 3, \ldots\ldots 100$

Num of iterations $= 100$ $O(1)$

Q for ($i=1$; $i*i \le N$; i++)

{

print(i)
}

$i = 1, 2, 3, \ldots\ldots \sqrt{N}$

$i * i \le N$

$i^2 \le N$

$i \le \sqrt{N}$

Num of iterations $= \sqrt{N}$ $O(\sqrt{N})$

Q

```
i = N
while (i >= 1)
{  i = i/2
}
```

$i = N \rightarrow N/2 \rightarrow \frac{N}{4} \rightarrow \ldots\ldots 1$

$\log_2(N)$ $O(\log_2 N)$

Q

```
S = 0
for (i=0; i <= N; i=i*2)
```

{ s += i

}

Before After

i = 0   0   i = i*2 = 0

i = 0   0   i = i*2 = 0

0   0

0

⋮

0

Infinite loop

Q

s = 0

for(i=1; i ≤ N; i=i*2)

{ s += i

y

Before	After	Iter
1	2 (2^1)	1
2	4 (2^2)	2
4	8 (2^3)	3
8	16 (2^4)	4
⋮		
	2^K	K

$$2^K > N$$

$$K \geq \log_2(N)$$

Num iterations = $\log_2(N)$

$i = 1, 2, 4, 8, 16$ (for $n=11$)

$\log_2(11) = 4$

5

$O(\log N)$

Break 10:50

Q

```
for(i=1; i≤10; i++)
{   for(j=1; j≤N; j++)
    {   print(i*j)
    }
}
```

i	j	
1	$[1, N]$ →	n
2	$[1, N]$ →	n
3	$[1, N]$ →	n
⋮	⋮	⋮

10

$\vdots \quad\quad \vdots \quad\quad \vdots$

$10 \quad\quad [1, N] \quad\quad \underline{n}$

$O(N) \quad\quad 10n$

Q
```
for(i=1; i<=N; i++)
{   for(j=1; j<=N; j++)
    {   print(i*j)
    }
}
```

i	j		
1	[1, N]	$\rightarrow$	N
2	[1, N]	$\rightarrow$	N
3	$\vdots$	$\rightarrow$	N
$\vdots$	$\vdots$		
N	[1, N]	$\rightarrow$	$\underline{N}$

N

$\underline{N^2}$

Iterations = N^2

$O(N^2)$

Q

```
for ( i=1; i ≤ N ; i++)
{   for ( j=1 ; j ≤ N ; j=j*2)
    {   print ( i*j )
    }
}
```

log n

log n $\times$ 1

i	j
1	log n
2	log n
3	log n
⋮	⋮
N	log n

N times

$N \log_2 N$

$O(N \log N)$

<u>Note</u>: sqrt & log are both functions

$$\mathrm{sqrt}(n) = x$$

$$x * x = n$$

$$\log_2(n) = x$$

$$n \xrightarrow{/2} \frac{n}{2} \xrightarrow{/2} \frac{n}{4} \dots \xrightarrow{/2} 1$$

Number of divisions $= x$

$$1 \longrightarrow 2 \longrightarrow 4 \longrightarrow 8 \longrightarrow \dots > n$$

Q for $(i=1;\ i \le 2^N;\ i{+}{+})$

```
{   print(i)
}
```

$$i = \underbrace{1, 2, 3, \dots 2^N}$$

Num of iterations $= 2^N$

v

Q for (i=1 ; i≤N ; i++)

{ for (j=1; j ≤ 2^i; j++)

{ print (i+j)

}

}

i	j
1	2^1
2	2^2
3	2^3
4	2^4
⋮	⋮
N	2^N

$$2^1 + 2^2 + 2^3 + \ldots\ldots 2^N$$

This is GP

$a = 2$
$r = 2$
$n = N$

$$\text{Sum} = \frac{a(r^n - 1)}{r-1}$$

$$= 2\frac{(2^N-1)}{2-1} = 2(2^N-1)$$

Num of iterations

$2^{n+1} - 2^x$

$2^{n+1} = 2 \cdot 2^n$ $= 2(2^N - 1)$

$O(2^n)$

How to calculate Big O from number of iterations.

First find no of iterations.

1) Neglect lower order terms

$\boxed{N^2}$ $\cancel{N}$ $\cancel{1}$

2) Neglect the constant co-efficient.

Q $4N^2 + 3N + 1$

$\rightarrow$ $4N^2$

$\rightarrow$ N^2 $O(N^2)$

order of N^2

Q $4N^2 + 6N + 1331$

$4N^2$

$\rightarrow$ $O(N^2)$

Q $4N^2 + 2N^2 + 50N$

$6N^2 + 50N$

$6N^2$

$O(N^2)$

Q $\quad N^2 + \log N$

$O(N^2)$

Q $\quad 100 \rightarrow O(1)$

$$O(1) < O(\log_2 N) < O(\sqrt{N}) < O(N)$$

$$< O(N \log N) < O(N\sqrt{N}) < O(N^2)$$

$$< O(2^N) < \underbrace{O(N!)}_{\text{factorial } (1\times 2\times 3\times \ldots N)} < O(N^N)$$

$5N^2 + \log N$

$5N^2$

$O(N^2)$

Doubts

$\boxed{10000\,N}$ $\quad 10^5$ $\quad N = 10$

$O(N)$ $\quad 10$

20

$1 \qquad N$

$1\,N$

$1000 \qquad 5000$

$N \rightarrow \frac{N}{2}$

$i = 1 \; ; \; i \le N \; ; \; i{+}{+}$

$j = 1 \; ; \; j \le i \; ; \; j{+}{+}$

i	j
1	1
2	2
3	3
...	...
N	N

$$\boxed{\frac{N(N+1)}{2}}$$

$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \cdots \rightarrow 1$

$N = 16$

$16 \longrightarrow 8 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1$

$\log_2 16 = 4$

int a = 1

if (a == 2)

else

i = i * 2

i = 1 ; i ≤ N ; i *= 2

$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \cdots \rightarrow 1$

$1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \cdots\cdots \rightarrow N$

$\log n$

$\log n - 1$

$O(\log n)$

$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$1 \quad 4 \quad 9 \quad 16 \quad 25 \quad 36$

$\mathrm{sqrt}(n) < n$

$x < n$