Order placement framework

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static strategy : Buy x in T (force exec = false)
Fire + Forget

1. Do nothing
2. Market at $t=0$ for price p_0^M
3. Market at $t=T$ for price p_T^M
4. Limit (then market) for price p^L

$U_E(p)$: payoff execution at price p

$U_{NE}(p)$: cost of no execution e.g. price p after T

Generalize 1-4 with limit order (buy) at p^L:

$$U(p^L) = \begin{cases} U_E(p), & \text{if order is executed} \\ U_{NE}(p_T^M), & \text{if not executed} \end{cases}$$

e.g. payoff is a random variable due to uncertainty of execution and price after T.

Expected payoff:

$$\mathbb{E}[U(p^L)] = \underbrace{P_E(p^L) \cdot U(p^L)}_{\text{Execution at } p^L} + \underbrace{[1-P_E(p^L)] \int_{-\infty}^{\infty} U_{NE}(p) \underbrace{f_{p_M^T \mid p^L}(p)}_{*} dp}_{\text{Non-execution} \quad \text{Distribution of price after } T}$$

* pdf of price after T given that p^L is not executed (eg. price > p^L)

variance of payoff:

$$V[U(p^L)] = P_E(p^L) \cdot \left(U_E(p^L) - E[U(p^L)]\right)^2 \quad \Big\}\ \text{variance of execution at } p^L$$

$$+ \underbrace{[1 - P_E(p^L)] \int_{-\infty}^{\infty} \left(U_{NE}(p^L) - E[U(p^L)]\right)^2 \cdot f_{P_M^T \mid p^L}(p)\, dp}_{\text{variance of non-execution}}$$

Replace expected value:

$$V[U(p^L)] = P_E(p^L) \cdot \left(U_E(p^L) - \left(P_E(p^L) U_E(p^L) + [1 - P_E(p^L)] \cdot \int_{-\infty}^{\infty} U_{NE}(p) \cdot f\_(p)\, dp\right)\right)^2$$

$$+ [1 - P_E(p^L)] \int_{-\infty}^{\infty} \left(U_{NE}(p^L) - P_E(p^L) U_E(p^L) + [1 - P_E(p^L)] \int_{-\infty}^{\infty} U_{NE}(p) \cdot f\_(p)\, dp\right)^2 \cdot f\_(p)\, dp$$

simpler:

$$V = P_E \cdot \left(U_E - \left(P_E U_E + [1 - P_E] \cdot \int_{-\infty}^{\infty}\right)\right)^2$$

$$+ [1 - P_E] \cdot \int \left(U_{NE} - \left(P_E U_E + [1 - P_E] \cdot \int_{-\infty}^{\infty}\right)\right)^2 \cdot f$$

fuck it:

$$V[U(p^L)] = [1 - P_E(p)] \left[P_E(p^L) \left(U_E(p^L) - \int_{-\infty}^{\infty} U_{NE}(p)\, f_{P_M^T \mid p^L}(p)\, dp\right)^2\right]$$

$$+ [1 - P_E(p^L)] \left[\int_{-\infty}^{\infty} [U_{NE}(p^*)]^2 f_{P_M^T \mid p^L}(p)\, dp - \left(\int_{-\infty}^{\infty} U_{NE}(p)\, f_{P_M^T \mid p^L}(p)\, dp\right)^2\right]$$

Mean variance optimization (utility function)

$$U_0(p^L) = E[U(p^L)] - \lambda V[U(p^L)]$$

$\uparrow$ Risk factor

$\lambda = 0$, trader only concerned about profit

$\lambda = 1$, trader equally concerned about profit from

1) better executions
2) risk of non-executions

Balance trade-off between profit and risk:

$$\hat{p} = \arg\max_{p^L} U_0(p^L)$$

Apply framework for liquidity traders

e.g. transact large order before deadline T

Trader has choices:

1. Execute at beginning for p_0^M
2. Execute at end for p_T^M
3. Limit p^L otherwise p_T^M

Determine best way to execute order to get most favourable price at specified risk

Order placement decision requires parameter specification

$$U_E(p^L) = p_0^M - \underbrace{p^L}_{p^L < p_0^M \text{ since limit order}}$$, payoff if executed is best ask market - limit

$$U_{NE}(p^L) = p_0^M - \underbrace{p_T^M}_{\text{can be } < \text{ or } > p_0^M}$$, payoff if non-execution is best ask market - market after T

Execution probability : Density estimation method

→ Execution estimated by empirical distribution of price fluctuations, or in case of non-execution, pdf of future closing price will be estimated by empirical distribution of asset price

$$M_T^B = \sup\{p_0 - p_t \, ; \, 0 \leq t \leq T\}$$ Difference of p_0 and lowest price reached during T

$$P_E(p^L | p_0) = Pr\{ \sup\{p_0 - p_t \,;\, 0 \le t \le T\} \ge p_0 - p \}$$

probability of execution with limit price p^L given market price p_0, e.g. $p^L < p_0$

$$= Pr\{ M_T^B \ge p_0 - p^L \}$$

$$= 1 - F_{M_T^B}(p_0 - p^L)$$

↑
Cummulative distribution of buy price fluctuation during T

Given a history of ~~buy~~ prices fluctuations $(\Delta_1, \dots, \Delta_n)$:

$$F_{M_T^B}(\Delta) = \frac{1}{N} \sum_{i=1}^{N} I\{\Delta_i < \Delta\}$$

$I = 1$, if true

$I = 0$, if false

Therefore the probability of the limit order execution:

$$P_E(p^L | p_0) = 1 - \frac{1}{N} \sum_{i=1}^{N} I\{\Delta_i < p_0 - p^L\}$$

Asset ~~return~~ ~~price~~ price distribution if non-execution after T derived from empirical distribution of asset returns

$$f_{r_T}(r) = \frac{1}{N} \sum_{i=1}^{N} I\{p_i - p_{i-1} = r\}$$

Asset price has to be higher than p^L for all time (otherwise execution). Hence the condition applied to the distribution of asset prices:

$$f_{p_T^M \mid p^L, p_0^M}(p_T^M) = f_{r_T}\left(p_T^M - p_0^M \mid \overbrace{p_T^M > p^L}\right)$$

$$= f_{r_T}\left(p_T^M - p_0^M \mid \underbrace{p_T^M - p_0^M}_{\text{return market price}} \geq \underbrace{p^L - p_0^M}_{\text{return limit price}}\right)$$

$$= \frac{\sum_{i=1}^{N} I\{p_i - p_{i-1} = p_T^M - p_0^M\}}{\sum_{i=1}^{N} I\{p_i - p_{i-1} \geq p^L - p_0^M\}}$$