

UNIT - I

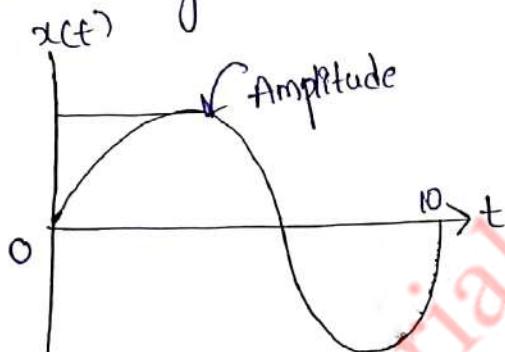
Digital system and binary numbers

Introduction :- In a real time basic signal is analog signal
analog signal is converted into digital signal

Analog signal :-

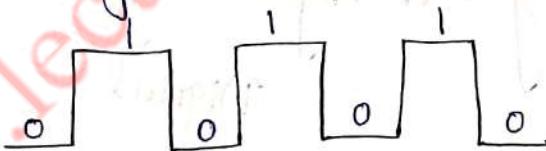
- * It is a continuous time signal with respect to the amplitude

Ex :- sinusoidal signal



Digital signal :- It is a discrete time signal. It's binary digits.

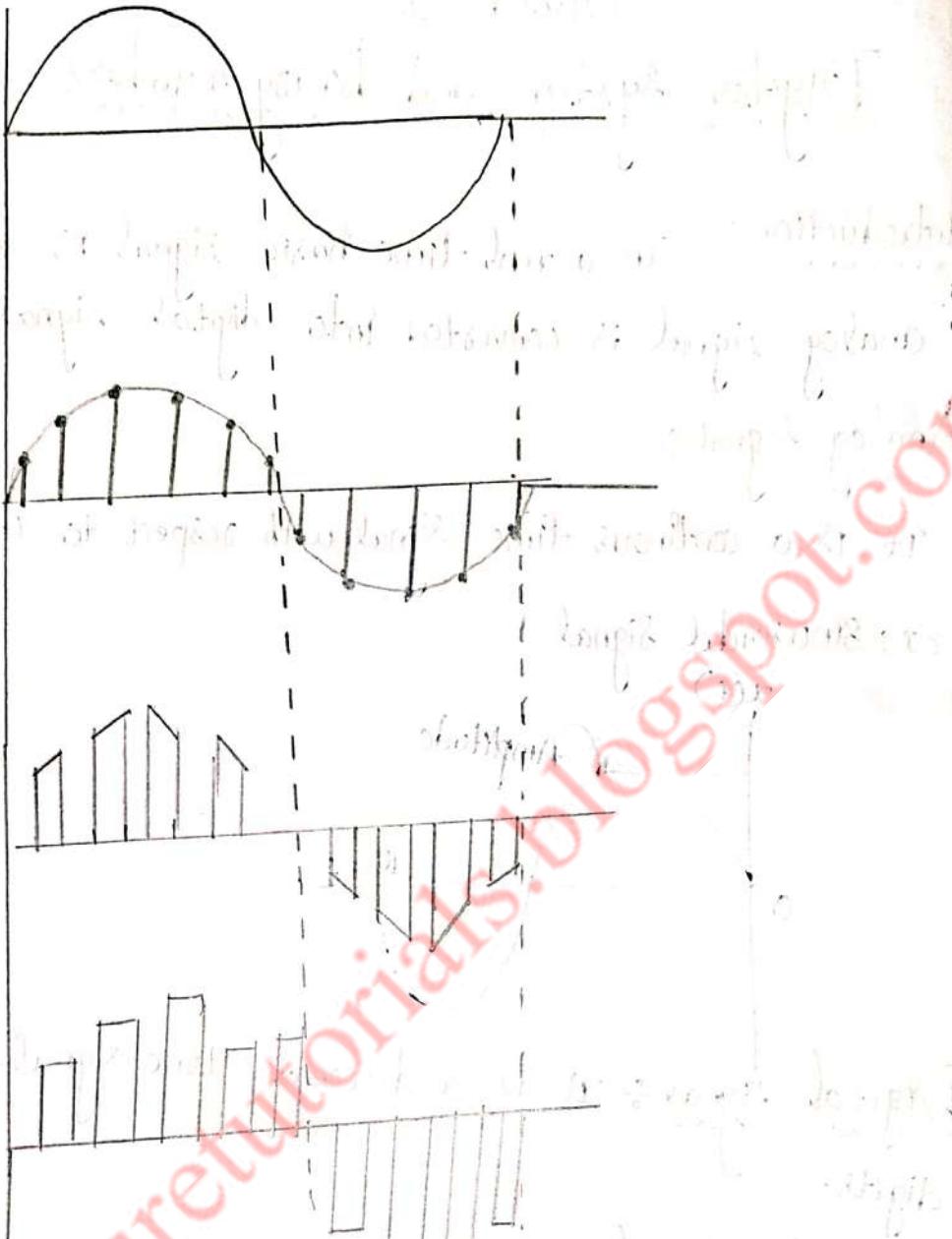
Ex :- pulse signal



Convert analog to digital signal :-

Basically analog to digital signal is conversion in a three steps.

1. Sampling
2. quantisation
3. coding



* Difference between analog and digital signal

Analog	Digital
* low accuracy	* more accuracy
* procedure is slow	* procedure is large
* size is large	* size is small
* Expensive (or) high cost	* low cost
* It is not upgraded	* It is easily upgradd device
* more noise occurs	* low noise occurs

logic gates:
Basically logic gates are three types. They are
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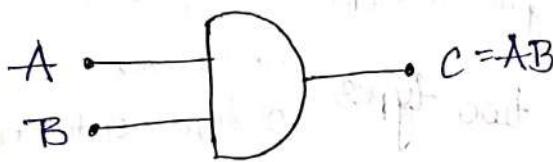
1) AND gate

2) OR gate

3) NOT gate

AND gate:

The AND gate symbol is given below,

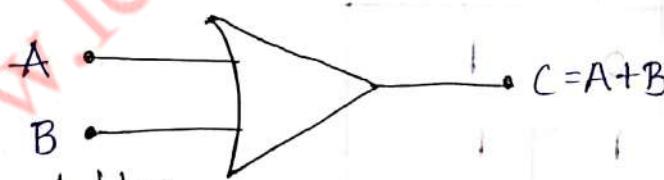


Truth table:

A	B	$C = AB$
0	0	0
0	1	0
1	0	0
1	1	1

OR gate:

The OR gate symbol is given below

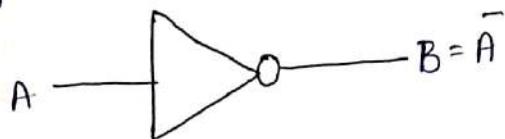


Truth table:

A	B	$C = A + B$
0	0	0
1	0	1
0	1	1
1	1	1

NOT gate:

NOT gate symbol is given below

Truth table:

A	B = \bar{A}
0	1
1	0

Universal gates: Universal gates are NAND, NOR. By using NAND and NOR we can construct any type of logic gate.

The universal gates are two types

NAND and NOR easily constructed Transistor Circuits.

1. NAND
2. NOR

1. NAND gate:

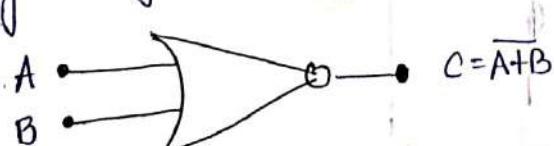
The NAND gate symbol is given below

Truth table:

A	B	C = \bar{A}B
0	0	1
0	1	1
1	0	1
1	1	0

NOR gate:

NOR gate symbol is given below



A	B	$C = A + \bar{B}$
0	0	1
0	1	0
1	0	0
1	1	0

Design :- Design means to reduce the logic gates to generate logic gates

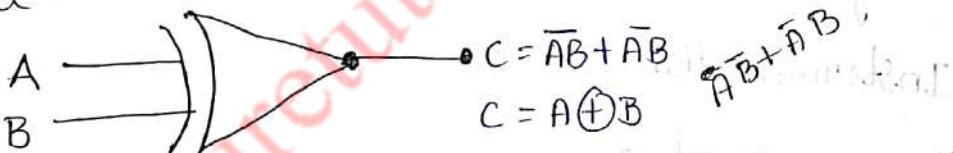
different components are connected to the sequential manner (or) sequence manner

Exclusive gates :-

X-OR gate :-

X-OR gate symbol is given below

Symbol :-



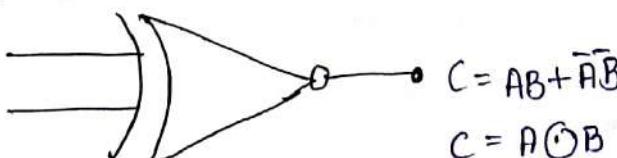
Truth table :-

A	B	$C = \bar{A}B + \bar{A}\bar{B}$
0	0	0
0	1	1
1	0	1
1	1	0

X-NOR gate :-

X-NOR gate symbol is given below

Symbol :-



A	B	$C = A \oplus B$
0	0	1
0	1	0
1	0	0
1	1	1

Feature scope of digital logic design :-

The digital logic design mainly use is a real time

- Tele communication
- Internet of thing (IOT)
- Cloud computing
- body area networking
- Information technology
- micro web point
- satelight & optical fibre technology
- Instrumentation
- Remote searching
- Signal processing
- Image processing

0	0	0
1	1	0
1	0	1

Digital system and Binary Numbers

Digital system or number system :-

- The basic digital decimal number system with qts 10 digits they are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
- In digital communication operate with binary numbers which use only the digits 0s and 1s.
- The number systems are basically four types they are:
 - i) Binary number system (2)
 - ii) Octal number system (8)
 - iii) Decimal number system (10)
 - iv) Hexadecimal number system (16)

(i) Binary number system (2) :-

- The binary number system are used qn digits 0s and 1s
- The binary Position Values as a power of '2' qts represented by

2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
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MSB - Most significance Bit

LSB -

Least significance
Bit

Ex:- Represent binary number 1101.101 qn power of

2. find qts decimal Equivalent

$$1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 1 \cdot 2^{-3}$$

2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
1	1	0	1	.	1	0

$$N = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$$

$$N = 8 + 4 + 0 + 1 + \frac{1}{8} + 0 + \frac{1}{8} \quad [\because a^0 = 1 \text{ and } 2^0 = 1]$$

$$N = (13.625)_{10}$$

Ex-2 $(1001101.10111)_2$

$$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \ 2^{-1} \ 2^{-2} \ 2^{-3} \ 2^{-4} \ 2^{-5} \dots$$

1	0	0	1	1	0	1	.	1	0	1	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---

$$N = 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + \\ 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} \times 1 \times 2^{-5}$$

$$N = 64 + 8 + 4 + 2 + 0.5 + 0.125 + 0.0625 + 0.03125$$

$$N = (78.71875)_{10}$$

* The octal number system uses first eight values as decimal number system. they are 0, 1, 2, 3, 4, 5, 6, 7. It's base is 8. Octal position value as a power of 8 represented is given by

8^3	8^2	8^1	8^0	.	8^{-1}	8^{-2}	8^{-3}
M8B				.			

* Represent octal number 567 in power of 8 and find its decimal equivalent.

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decimal Equivalent:

$$[8^0 = 1]$$

8^2	8^1	8^0
5	6	7

$$N = 5 \times 8^2 + 6 \times 8^1 + 7 \times 8^0$$

$$= 5 \times 64 + 6 \times 8 + 7 \times 1$$

$$N = (375)_{10}$$

* $(4271.635)_8$

8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	8^{-3}
4	2	7	1	.	6	3 5

$$N = 1 \times 8^0 + 7 \times 8^{-1} + 2 \times 8^{-2} + 4 \times 8^{-3} + 6 \times 8^{-4} + 3 \times 8^{-5} + 5 \times 8^{-6}$$

$$= 1 + 56 + 128 + 2048 + 6.125 + 0.1875 + 0.00976$$

$$= (2239.80)_{10}$$

* $(64562.1057)_8$

8^4	8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	8^{-3}	8^{-4}
6	4	5	6	2	.	1	0	5 7

$$N = 6 \times 8^4 + 4 \times 8^3 + 5 \times 8^2 + 6 \times 8^1 + 2 \times 8^0 + 1 \times 8^{-1} + 0 \times 8^{-2} + 5 \times 8^{-3} + 7 \times 8^{-4}$$

$$= 24576 + 2048 + 320 + 48 + 2 + 0.125 + 0.00625 + 0.00968 +$$

$$0.0017089$$

$$N = (26994.13639)_{10}$$

of P owing to only writing limited off all ←

Decimal number system :-

- In Decimal number system we can express any decimal number in units, tens, hundreds and thousands....
- In Decimal number system the numbers are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

→ The Decimal position values as a power of 10 is represented by

...	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	...
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MSB

LSB

Ex :- 6587.6

10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
6	5	8	7	.	6	

$$\begin{aligned}
 N &= 6 \times 10^3 + 5 \times 10^2 + 8 \times 10^1 + 7 \times 10^0 + 6 \times 10^{-1} \\
 &= 6000 + 500 + 80 + 7 + 0.6 \\
 &= 6587.6
 \end{aligned}$$

Hexa Decimal number system :-

→ The Hexa Decimal number system as a base of 16.

→ The Hexa Decimal number system having 16 digits they are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

→ The Hexa decimal position values as a power of 16 is represented by

16^3	16^2	16^1	16^0	16^{-1}	16^{-2}	16^{-3}	t.me/jntukonlinebits
MSB							LSB

Ex: Represents Hexadecimal number '3FD' in power of 16
and find Decimal equivalent.

$$\begin{array}{c}
 16^2 \quad 16^1 \quad 16^0 \\
 \hline
 3 \quad F \quad D
 \end{array}$$

$$\begin{aligned}
 N &= 3 \times 16^2 + F \times 16^1 + D \times 16^0 \\
 &= 3 \times 256 + 15 \times 16 + 13 \times 1 \\
 &= (1021)_{10}
 \end{aligned}$$

Ex: FDE42A.1DB9

16^5	16^4	16^3	16^2	16^1	16^0	16^{-1}	16^{-2}	16^{-3}	16^{-4}	
F	D	E	4	2	A	.	1	D	B	9

$$\begin{aligned}
 N &= 15 \times 16^5 + 13 \times 16^4 + 14 \times 16^3 + 4 \times 16^2 + 2 \times 16^1 + 10 \times 16^0 + \\
 &\quad 1 \times 16^{-1} + 13 \times 16^{-2} + 11 \times 16^{-3} + 9 \times 16^{-4} \\
 &= 16639018 + 851968 + 57344 + 1024 + 38 + 10 + 0.0625 + \\
 &\quad 0.0507 + 0.0026 + 0.00013732
 \end{aligned}$$

= (17549396.12)_{10}

Relation b/w binary, decimal, hex decimal in

below tables.

Decimal	Binary	Hexa Decimal
0	0000	0 = 0
1	0001	1 = 1
2	0010	2 = 2
3	0011	3 = 3

<u>Decimal</u>	<u>Binary</u>	<u>Hexa Decimal</u>
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Counting in Radix (Base) $r \div$

Radix (Base) r	Character in set
$r=2$	0, 1
$r=8$	0, 1, 2, 3, 4, 5, 6, 7
$r=10$	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
$r=16$	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, F

Ex: To find Decimal Values of 0 to 9 by using radix 5

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Decimal	Radix 5	
0	00	$0 \times 5^1 + 0 \times 5^0 = 0$
1	01	$0 \times 5^1 + 1 \times 5^0 = 1$
2	02	$0 \times 5^1 + 2 \times 5^0 = 2$
3	03	$0 \times 5^1 + 3 \times 5^0 = 3$
4	04	$0 \times 5^1 + 4 \times 5^0 = 4$
5	10	$1 \times 5^1 + 0 \times 5^0 = 5$
6	11	$1 \times 5^1 + 1 \times 5^0 = 6$
7	12	$1 \times 5^1 + 2 \times 5^0 = 7$
8	13	$1 \times 5^1 + 3 \times 5^0 = 8$
9	14	$1 \times 5^1 + 4 \times 5^0 = 9$

* find the decimal equivalent of 231.23 base 4

$$\begin{array}{c}
 4^2 \quad 4^1 \quad 4^0 \quad 4^{-1} \quad 4^{-2} \\
 \boxed{2 \quad 3 \quad 1 \quad . \quad 2 \quad 3}
 \end{array}$$

$$\begin{aligned}
 N &= 2 \times 16 + 3 \times 4 + 1 \times 4 + 2 \times 4^{-1} + 3 \times 4^{-2} \\
 &= 32 + 12 + 4 + 0.5 + 0.1875 \\
 &= (48.6875)_{10}
 \end{aligned}$$

Number base Conversion

→ The decimal, binary, octal and hexadecimal table 98
given below

Decimal	Binary	Octal	Hexa
0	0000	0	0
1	0001	1	1
2	0010	2	2

Decimal	Binary 8421	Octal	Hexadecimal
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Conversions:

- Basically binary, octal, hexadecimal conversions or 6 types
1. Binary to octal
 2. Octal to Binary
 3. Binary to Hexa
 4. Hexa to Binary
 5. Octal to Hexa
 6. Hexa to Octal

1. Binary to Octal

→ The binary numbers are 0 and 1. The octal number table is given below.

Decimal	Binary - 421
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Ex: Convert $(110101010.101,010)_2$ to octal

Sol:

$$\begin{array}{ccccc} \underline{110} & \underline{101} & \underline{010} & \cdot & \underline{101} & \underline{010} \\ \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow \\ 6 & 5 & 2 & & 5 & 2 \end{array}$$

$$\therefore (652.52)_8$$

Ex: $(1001010),0101)_2$

$$\begin{array}{ccccccccc} \underline{100} & \underline{101} & \underline{010} & \cdot & \underline{101} & \underline{010} & \underline{010} & \cdot & \underline{010} & \underline{100} \\ \downarrow & \downarrow & \downarrow & & \downarrow & & & & \downarrow & \\ 4 & 5 & 2 & & 5 & & & & 5 & \end{array}$$

$$(452.5)_8$$

Ex:

⑩ Octal to Binary Conversion

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Ex: $(643.27)_8$

$$\begin{array}{ccccccc} 6 & 4 & 3 & . & 2 & 7 \\ \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow \\ 110 & 100 & 011 & & 010 & 0111 \end{array}$$

$$\therefore (110100011.010111)_2$$

⑪ Binary to Hexa Conversion

Ex: $(1101100010011011)_2$

$$\begin{array}{cccc} 1101 & 1000 & 1001 & 1011 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ D & 8 & 9 & B \end{array}$$

$$(D89B)_{16}$$

$$\therefore (D89B)_H$$

⑫ Hexa to Binary Conversion

Ex: Convert $(3FD)H$

$$\begin{array}{ccc} 3 & F & D \\ \downarrow & \downarrow & \downarrow \\ 0011 & 1111 & 1001 \end{array}$$

$$(0011\ 1111\ 1101)_2$$

Ex: $(F9B0.1D8)H$

$$\begin{array}{ccccccccc} F & 9 & B & 0 & . & 1 & D & 8 \\ \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow \\ 1111 & 1001 & 1011 & 0000 & & 0001 & 0101 & 1000 \end{array}$$

$$(1111\ 1001\ 1011\ 0000\ 0001\ 1101\ 1000)_2$$

Ex: Convert $(5A89.B4)_{16}$ to binary

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5 A 8 9 B 4
↓ ↓ ↓ ↓ ↓ ↓
0101 1010 1000 1001 1011 0100

$(0101\ 1010\ 1000\ 1001\ 1011\ 0100)_2$

v) Octal to Hexadecimal:

→ The easiest way to convert octal number to Hexadecimal number in two steps.

i) convert octal number to binary form

ii) convert binary form to Hexadecimal

iii) convert binary form to its Hexadecimal equivalent.

Ex: Convert $(615)_8$ to its Hexadecimal equivalent.

(i) $(615)_8$

(ii) Binary - Hexa (110001101)

(iii) Octal - binary:

6 1 5
↓ ↓ ↓
 $(110\ 001\ 101)_2$

0001 000 1101
↓ ↓ ↓
1 8 D
 $\therefore (18D)_{16}$

(iv) Ex: $(7523.426)_8$

(i) Octal - binary

7 5 2 3 4 2 6
↓ ↓ ↓ ↓ ↓ ↓
111 101 010 011 100 010 110
 11101010011100010110

ii) Binary - Hexa

001 1110 1010 0111 0001 10110
↓ ↓ ↓ ↓ ↓ ↓
1 E A 7 1 6
 $\therefore (1EA716)_{16}$

vi) Hexa to octal conversion

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→ The Hexa decimal to octal conversion two steps is there

(i) Convert Hexa to binary

(ii) Convert binary - octal

Ex: convert $(25B)_H$ to its octal equivalent

$(25B)_H$

(i) Hexa to binary

2 5 B
↓ ↓ ↓
0010 0101 10110

(ii) binary - octal

0010 0101 1011
↓ ↓ ↓ ↓
1 1 3 3

$\therefore (1133)_8$

Ex: ② $(9DF6.C83)_H$

(i) Hexa to binary

9 D F 6 C 8 3
↓ ↓ ↓ ↓ ↓ ↓
1001 1101 1111 0110 1100 1000

(ii) binary - octal

(ii) binary - octal

1001101111011011001000

010 011 011 111 011 011 001 000
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
2 3 3 7 3 3 1 0

$(23373310)_8$

* Convert any Radix to Decimal

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In general the number can be represented as

$$N = A_{n-1}r^{n-1} + A_{n-2}r^{n-2} + \dots + A_1r^1 + A_0r^0 + A_{-1}r^{-1} + A_{-2}r^{-2} + \dots + C_mr^{-m}$$

where N = number in decimal

A = digit

r = radix or base of a number system

n = The number of digits in the integer portion of number

m = The no. of digits in the fractional portion of number

① $(1101.1)_2$ to convert Decimal

② $(475.25)_8$ to convert Decimal

③ $(9B2.1A)_4$ to convert Decimal

④ $(3102.12)_4$ to Decimal

⑤ $(614.15)_7$ to Decimal

① $(1101.1)_2$

2^3	2^2	2^1	2^0	2^{-1}
1	1	0	1	.

$$N = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times \frac{1}{2}$$

$$= (13.5)_{10}$$

② $(475.25)_8$

8^2	8^1	8^0	8^{-1}	8^{-2}
4	7	5	.	2

4	7	5	.	2	5
---	---	---	---	---	---

$$N = 4 \times 8^2 + 7 \times 8^1 + 5 \times 8^0 + 2 \times 8^{-1} + 5 \times 8^{-2}$$

$$= (317.328)_8$$

$$\textcircled{3} \quad (9B2.1A)_{16}$$

$$16^2 \quad 16^1 \quad 16^0 \quad 16^{-1} \quad 16^{-2}$$

9	B	2	.	1	A
---	---	---	---	---	---

$$N = 9 \times 16^2 + B \times 16^1 + 2 \times 16^0 + 1 \times 16^{-1} + A \times 16^{-2}$$

$$= (2482.101563)_{10}$$

$$\textcircled{4} \quad (3102.12)_4$$

$$4^3 \quad 4^2 \quad 4^1 \quad 4^0$$

3	1	0	2	.	1	2
---	---	---	---	---	---	---

$$N = 3 \times 4^3 + 1 \times 4^2 + 2 \times 1 + 1 \times 1/4 + 2 \times 1/16$$

$$= (210.375)_{10}$$

$$\textcircled{5} \quad (614.15)_7$$

$$7^2 \quad 7^1 \quad 7^0 \quad 7^{-1} \quad 7^{-2}$$

6	1	4	.	1	5
---	---	---	---	---	---

$$N = 6 \times 7^2 + 1 \times 7^1 + 4 \times 7^0 + 1 \times 1/7 + 5 \times 1/49$$

$$N = (301.844)_{10}$$

→ Basically the conversions of radix numbers are two type

1. successive division for integer part conversion

2. successive multiplication for fractional part conversion

1. successive division for integer part conversion

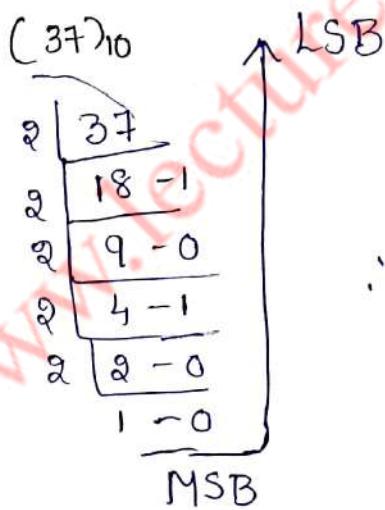
→ In this method we repeatedly divided the integer part of the decimal number by "r". until coeff. quotient is '0'.

→ The remainder of each division becomes the numerical in the new radix.

→ The reminders are taken in the reverse order to a new radix number.

→ This means that first remainder is the least significant bit and the last significant is the most significant bit in the new radix number.

Ex: Convert decimal number 37 to its binary equivalent



$$\therefore (1001011)_2$$

$$\therefore (37)_{10} = (1001011)_2$$

Verification:

$$2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\ 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1$$

$$= 1 \times 2^5 + 1 \times 2^2 + 1 \times 2^0$$

$$= 32 + 4 + 1 \times 1$$

$$= 37$$

AM

1) (625) to Binary

2) (739) to "

3) (523) to Octal

4) (649) "

Ex :- convert decimal to octal

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(Q14)

$$\begin{array}{r} 214 \\ \hline 8 | 26 - 6 \\ 8 | 3 - 2 \end{array}$$

$\therefore (326)_8$

Ex :- Convert decimal number $(3509)_{10}$ to Hexa decimal Equivalent

$$\begin{array}{r} 3509 \\ \hline 16 | 219 - 5 \\ 16 | 13 - 11 \end{array}$$

$\therefore (1311.5)_{10}$
 $\therefore (DB5)_{16}$

Ex :- convert decimal to Hexa $(4559)_{10}$

Ex :- convert decimal to Hexa $(5320)_{10}$

(i) $(4559)_{10}$

$$\begin{array}{r} 4559 \\ \hline 16 | 284 - 15 \\ 16 | 17 - 12 \\ 16 | 1 - 1 \end{array}$$

$(15\ 12\ 11)_{16}$
 $\therefore (4559)_{10} = (FCB)_{16}$

(ii) $(5320)_{10}$

$$\begin{array}{r} 5320 \\ \hline 16 | 332 - 8 \\ 16 | 20 - 12 \\ 16 | 1 - 4 \end{array}$$

$\therefore (5320)_{10} = (8\ 12\ 4\ 1)_{16}$

$\therefore (8C41)_{16}$

2. Successive multiplication for fractional part conversion
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→ convert 0.8125 to Binary

fractional	Radix	Result	Recorded carry	MSB
0.8125	$\times 2$	$= 1.625 = 0.625$	with carry 1	
0.625	$\times 2$	$= 1.25 = 0.25$	with carry 1	
0.25	$\times 2$	$= 0.5 = 0.5$	with carry 0	
0.5	$\times 2$	$= 1.0 = 0.0$	with carry 1	LSB

$$\therefore (0.8125)_{10} = (0.1101)_2$$

→ convert 0.95 to Decimal number to its binary equivalent

fractional	Radix	Result	Recorded carry
0.95	$\times 2$	$= 1.9 = 0.9$	with carry 1
0.9	$\times 2$	$= 1.8 = 0.8$	with carry 1
0.8	$\times 2$	$= 1.6 = 0.6$	" 1
0.6	$\times 2$	$= 1.2 = 0.2$	" 0
0.2	$\times 2$	$= 0.4 = 0.8$	" 0
0.4	$\times 2$	$= 0.8 = 0.8$	" 0
0.8	$\times 2$	$= 1.6 = 0.6$	" 0 1

$$(0.95)_{10} = (0.1110001)_2$$

i) 625 to Binary

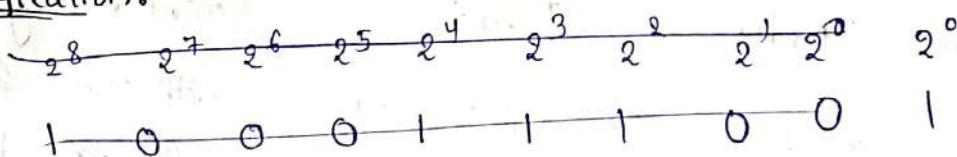
$$\begin{array}{r}
 2 | 625 \\
 2 | 312 - 1 \\
 2 | 156 - 0 \\
 2 | 78 - 0 \\
 2 | 39 - 0 \\
 2 | 19 - 1
 \end{array}$$

$$\begin{array}{r}
 2 | 9 - 1 \\
 2 | 4 - 1 \\
 2 | 2 - 0 \\
 2 | 1 - 0
 \end{array}$$

~~(1000111001)~~

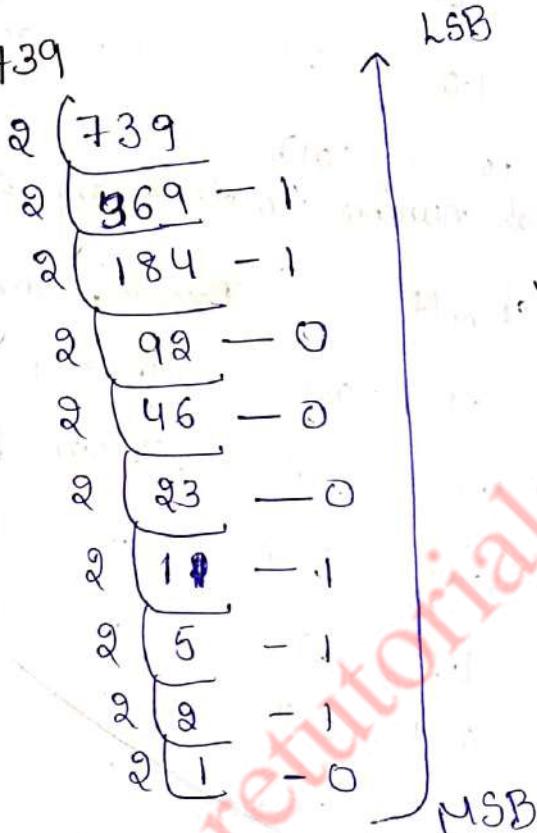
(1001110001)₂ t.me/jntukonlinebits

Verifications:



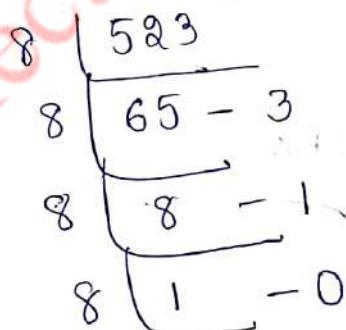
$$N = 1 \times 2^8 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2$$

② 739



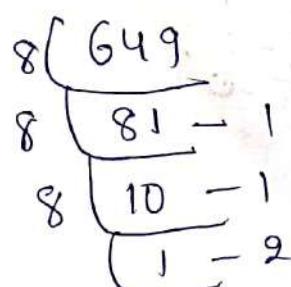
$$\therefore (1011100011)_2$$

③ 523



$$\therefore (1013)_8$$

④ 649



$$(1211)_8$$

Ex: Convert 0.640625 decimal number to 9 bits octal equivalent
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Fraction	Radix	Result	Recorded with carry
0.640625	x 8	= 5.125 = 0.125 with carry 5	MSB
0.125	x 8	= 1.0 = 0.0 with carry 1	LSB

$$\therefore (0.640625)_{10} = (0.51)_8$$

- (i) (0.925284) (ii) (0.752386)

Fraction	Radix	Result	Recorded with carry
0.925284	x 8	= 7.402272 = 0.402272 with carry 7	
0.402272	x 8	= 3.218176 = 0.218176 with carry 3	
0.2	x 8	= 0.25408 1.6 = 0.6 with carry 1	
0.6	x 8	4.8 0.8 with carry 4	
0.8	x 8	6.4 0.4 with carry 6	

$$\therefore (0.925284)_{10} = (0.73146)_8$$

- (i) (0.752386)

Fraction	Radix	Result	Recording with carry
0.752386	x 8	6.019088 = 0.01 with carry 6	
0.01	x 8	= 0.08 with carry 0	
0.08	x 8	= 0.08 0.64 with carry 0	
0.12	x 8	5.12 = 0.12 with carry 5	
0.96	x 8	0.96 with carry 0	
0.68	x 8	1.68 0.68 with carry 1	

* Convert 0.1289062 Decimal to Hexa Decimal
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fraction	Radix	Result	Recorded with carry
0.1289062	$\times 16$	2.0624992 = 0.0624992	with carry 2
<u>Wrong</u>			
0.0624992	$\times 16$	0.999872 = 0.999872	with carry 9
0.999872	$\times 16$	15.997952 = 0.997952	with carry 15
0.997952	$\times 16$	15	

fraction	Radix	Result	Recorded with carry
0.1289062	$\times 16$	2.0624992 = 0.0625	with carry 2 MSB
0.0625	$\times 16$	= 1.0 0.0	with carry 1

$$\therefore (0.1289062) = (0.21)_{16}$$

(iii) (0.15638) iv) (0.29725) Hexa

$$0.15638 \times 16 = 2.50208$$

$$0.50$$

$$0.8$$

* Convert decimal number 35.45 to octal number

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$$q) (35.45)_{10}$$

(i) Integer part

$$\begin{array}{r} 35 \\ \times 8 \\ \hline 4 \quad 3 \end{array}$$

$$\therefore (35)_{10} = (43)_8$$

$$\therefore (0.45)_{10} = (0.346314)_8$$

finally

$$\therefore (35.45)_{10} = (43.346314)_8$$

q) fractional part

Fraction	Radix	Result	Recorded with carry
0.45	$\times 8$	$= 3.6$ $= 0.6$	Carry 3
0.6	$\times 8$	$= 4.8$ $= 0.8$	Carry 4
0.8	$\times 8$	$= 6.4$ $= 0.4$	Carry 6
0.4	$\times 8$	$= 3.2$ $= 0.2$	Carry 3
0.2	$\times 8$	$= 1.6$ $= 0.6$	Carry 1
0.6	$\times 8$	$= 4.8$ $= 0.8$	Carry 4

* Convert $(22.64)_{10}$ to Hexa decimal

(i) Convert $(24.6)_{10}$ to Binary

$$q) (22.64)_{10}$$

(i) Integer part $\div 16$

$$\begin{array}{r} 22 \\ \times 16 \\ \hline 16 \quad 6 \\ \hline 1 \quad 0 \end{array}$$

$$\therefore (22)_{10} = (106)_{16}$$

$$\begin{array}{r} 22.64 \\ \times 16 \\ \hline 16 \quad 6 \\ \hline 1 \quad 0 \end{array}$$

$$\begin{array}{r} 22.64 \\ \times 16 \\ \hline 16 \quad 6 \\ \hline 1 \quad 0 \end{array}$$

$$(10100)$$

(ii) fractional part $\div 0.64$

fraction	Radix	Result	Recorded time with carry
0.64	$\times 16$	= 10.24	with carry 10
		= 0.24	
0.24	$\times 16$	= 3.84	with carry 3
		= 0.84	
0.84	$\times 16$	= 13.44	with carry 13
		0.44	
0.44	$\times 16$	= 7.04	with carry 7
		0.04	
0.04	$\times 16$	= 0.64	with carry 0
		= 0.64	
0.64	$\times 16$	= 10.24	with carry 10
		= 0.24	

$\therefore (0.64)_{10} = (0.103137010)_H$

finally

$\therefore (24.6)_{10} = (106.103137010)_H$

(iii) $(24.6)_{10}$

(i) Integration part : 24

$$\begin{array}{r} 2 \left(\begin{array}{r} 24 \\ - 12 - 0 \\ \hline 6 - 0 \\ - 3 - 0 \\ \hline 1 - 1 \end{array} \right) \end{array}$$

$$\therefore (24)_{10} = (11000)_2$$

fraction	Radix	Result	Recorded with carry time/jntukonlinebits
0.6	x 2	= 1.2 = 0.2	with carry 1
0.2	x 2	= 0.4 = 0.4	with carry 0
0.4	x 2	= 0.8	with carry 0
0.8	x 2	= 1.6 = 0.6	with carry 1
0.6	x 2	= 1.2 = 0.8	with carry 1

$$(0.6)_{10} = (10011)_2$$

$$\therefore (24.6)_{10} = (11000.10011)_2$$

(Q) (0.29725) to Hexa

fraction	Radix	Result	Recorded with carry
0.29725	x 16	4.756 = 0.756	with carry 4
0.756	x 16	12.096 = 0.096	with carry 12
0.096	x 16	15.36 = 0.36	with carry 15
0.36	x 16	5.76 = 0.76	with carry 5
0.76	x 16	12.16 = 0.16	with carry 12
0.16	x 16	2.56 = 0.56	with carry 2
0.56	x 16	8.96 = 0.96	with carry 8

$$\begin{array}{r}
 0.96 \quad \times \quad 16 \\
 \hline
 0.36 \quad \times \quad 16 \\
 \hline
 \end{array}$$

$$\begin{aligned}
 &= 15.36 \\
 &= 0.36 \\
 &\quad 5.76 \\
 &= 0.76
 \end{aligned}$$

with carry 4
http://jatukononlinebits

$$\therefore (0.29725)_{10} = (4181551288155)_{16}$$

⑨ (0.15638)

fraction	Radix	Result	Recorded with carry
0.15638	$\times 16$	2.50208 $= 0.502$	with carry 2
0.502	$\times 16$	8.032 $= 0.032$	with carry 8
0.032	$\times 16$	0.512 $= 0.512$	with carry 0
0.512	$\times 16$	8.192 $= 0.192$	with carry 8
0.192	$\times 16$	3.072 $= 0.072$	with carry 3
0.072	$\times 16$	1.152 $= 0.152$	with carry 1
0.152	$\times 16$	2.432 $= 0.432$	with carry 2
0.432	$\times 16$	6.912 $= 0.912$	with carry 6
0.912	$\times 16$	14.592 $= 0.592$	with carry 4
0.592	$\times 16$	9.912 $= 0.912$	with carry 9
0.912	$\times 16$	15.152 $= 0.152$	with carry 15

$$\therefore (0.156387)_{10} = (0.808312614915)_H$$

$$= (28083.C6E9F)_H$$

Complement of numbers:

(i) 1's complement representation:

→ The 1's complement of a binary number is the number that results when we change all 1's to 0's and all 0's to

• 1's

Ex: find 1's complement of $(1101)_2$

$$(1101)_2$$

$$= 0010$$

Ex: find 1's complement of $(10111010111)_2$

$$(10111010111)_2$$

$$010001010000$$

(ii) 2's complement representation:

→ The 2's complement is the binary number that results when we add 1 to the 1's complement. It can be represented as

$$[2's \text{ complement} = 1's \text{ complement} + 1]$$

Addition operation:

— truth table

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

→ The 2's Complement form is used to represent a (-ve) numbers

Ex: Find 2's complement of $(1001)_2$

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$$\begin{array}{r} 1 - 0 \ 0 + \\ \quad \quad \quad | \\ \hline \quad \quad \quad 1 \\ \quad \quad \quad \cancel{+} \\ \hline \quad \quad \quad 0 \end{array} \quad (1001)_2 \text{ change to } 1001_2$$
$$= 0110 - 1^{\text{st}} \text{ complement}$$
$$= \begin{array}{r} 0 \ 1 \ 1 \ 0 \\ + 1 \rightarrow \text{adding } +1 \\ \hline \end{array}$$
$$0 \ 1 \ 1 \ 1$$

Ex: $(1010100110)_2$

$0101011001 \rightarrow 1^{\text{st}} \text{ complement}$

$$\begin{array}{r} 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \\ + 1 \rightarrow \text{adding } +1 \\ \hline \end{array}$$
$$0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0$$

$$\therefore (1010100110)_2 = (0101011010)$$

1^{st} Complement subtraction

→ For subtraction of two numbers we have two cases

(i) subtraction of smaller number from larger number

L - S

(ii) subtraction of larger number from smaller number

S - L

(i) smaller number from larger number

Steps:-
1. Determine the 1^{st} complement of the smaller number.

2. Add the 1^{st} complement of the larger number.

3. Remove the carry and adding to the result This is called
end around the carry

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Ex: Subtract ~~100~~ $(101011)_2$ from $(111001)_2$ using the 1's
Complement method

$(101011)_2$ from $(111001)_2$

(i) first u find
big number are
large number

$$\begin{array}{cccccc} 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ 1 & 0 & 1 & 0 & 1 & 1 \end{array}$$

$$= 43$$

$$\begin{array}{cccccc} 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}$$

$$= 57$$

L - 5

$$\begin{array}{r} 111001 \\ 010100 \\ \hline \end{array} \rightarrow \text{1st complement of } 101011$$

$$\begin{array}{r} 1001101 \\ \hline \end{array}$$

End around
carry

$$\begin{array}{r} +1 \\ \hline \end{array}$$

$$\begin{array}{r} 001110 \\ \hline \end{array}$$

$$\begin{array}{r} 57 \\ 43 \\ \hline 14 \end{array}$$

$$\therefore (001110)$$

Ex: $(101011)_2$ from $(111001)_2$

$(101011)_2$ from $(0111001)_2$

$$\begin{array}{cccccc} 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{array}$$

$$= 107$$

$$\begin{array}{cccccc} 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}$$

$$= 57$$

L - 5

$$\begin{array}{r} 1101011 \\ \hline \end{array}$$

$$\begin{array}{r} 1000110 \\ \hline \end{array}$$

$$\begin{array}{r} 111 \\ \hline \end{array}$$

$$\begin{array}{r} 0110001 \\ \hline \end{array}$$

\rightarrow 1st complement of
 0111001

$$\begin{array}{r}
 0110001 \\
 & \underline{-} \\
 010010 & \\
 & \underline{\quad\quad\quad} \\
 & 107
 \end{array}$$

$$\begin{array}{r}
 57 \\
 \underline{-} \\
 50
 \end{array}$$

(0110010)

$= 50_{11}$

(ii) Subtraction of large number from smaller number:

Steps: S-2

1. Determine the 1's complement of the larger number

2. Add the 1's complement of the smaller number

3. Answer is in 1's complement form to set the answer in true form take the 1's complement and assign (ve) sign to the answer.

Ex: Subtract $(111001)_2$ from $(101011)_2$ using the 1's complement

method

$$(111001)_2 \text{ from } (101011)_2$$

$$\begin{array}{r}
 101011 \\
 & \underline{-} \\
 000110 & \leftarrow 1's \text{ complement} \\
 & \underline{\quad\quad\quad} \\
 & 110001 \rightarrow \\
 & \underline{\quad\quad\quad} \\
 & 100110 \rightarrow 1's \text{ complement} \\
 & \underline{\quad\quad\quad} \\
 & -14 \quad \underline{1000110} \quad 1
 \end{array}$$

Ex: $(1111101)_2$ from $(1101011)_2$

$$\begin{array}{r} 2^6 2^5 2^4 2^3 2^2 2^1 2^0 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ \hline 125 & & & & & & \end{array}$$

$$\begin{array}{r} 2^6 2^5 2^4 2^3 2^2 2^1 2^0 \\ \hline 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ \hline 107 & & & & & & \end{array}$$

(a)

S-2

$$\begin{array}{r} 1101011 \\ 0000010 \rightarrow 1's \text{ complement of } (1111101) \\ \hline 1 \\ \hline 1101101 \\ - 0010000 \rightarrow 1's \text{ complement} \\ \hline \end{array}$$

-18 //

(b)

2-S

$$\begin{array}{r} 1111101 \\ 0000000 \leftarrow 1's \\ \hline 1111101 \\ 0000000 \\ \hline 0000000 \\ \hline 0010010 \\ = 18/1 \\ \hline \end{array}$$

(i) Subtraction of smaller number from larger number:

- Determine the 2's complement of a small number
- Add the 2's complement to the large number
- Discard the carry (or) remove the carry

Ex: Subtract $(101011)_2$ from $(111001)_2$

$\begin{array}{r} 111001 \\ - 101011 \\ \hline 001110 \end{array}$

(i) L - S

$$\begin{array}{r} 111001 \\ - 010100 \\ \hline 001110 \end{array}$$

remove

$$\begin{array}{r} 111001 \\ - 010100 \rightarrow 2's \text{ complement} \\ \hline 001110 \end{array}$$

2's complement

(ii) S - L

$$\begin{array}{r} 101011 \\ - 001110 \\ \hline 110010 \end{array}$$

remove

$$\begin{array}{r} 111001 \\ - 000111 \rightarrow 1's \text{ complement} \\ \hline 100111 \end{array}$$

$$\begin{array}{r} 001000 \\ - 001110 \\ \hline 110110 \end{array}$$

(ii) Subtraction of larger number from smaller number:

- Determine the 2's complement of the large number
- Add the 2's complement to the smaller one

→ Answer is in the 2's complement form to get the answer in the true form to take the 2's complement

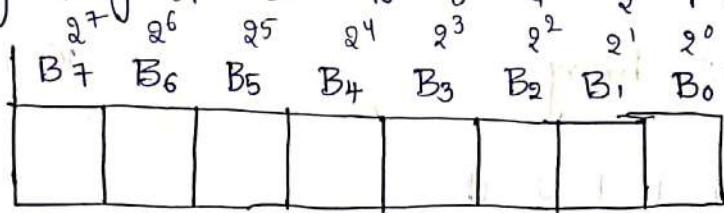
add assign (-ve) sign to the Answer

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Signed binary numbers :-

→ The sign magnitude formate for 8 bit signed number is

given by



Sign Bit

magnitude

$$\underline{\text{Ex:}} \quad 1 + 6 = 0000 \quad 0110$$

$$2) -14 = 1000 \quad 1110$$

$$3) +24 = 0001 \quad 1000$$

$$4) -64 = 1100 \quad 0000$$

$$5) +127 = 0111 \quad 1111$$

$$6) -128 = 1111 \quad 1111$$

→ The maximum (+ve) number +127 = 0111 1111

Binary Arithmetic

Rules for binary addition :-

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

From

0 = (1-1)

From

0 = (1-1)

From

0 = (1-1)

From

0 = (1-1)

Ex: Add $(1010)_2$ & $(0111)_2$

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1 0 1 0

0 0 1 1

1 1 0 1

Ex: Add 28 and 15 binary.

16 8 4 2 1

1 1 1 0 0

0 1 1 1 1

1 1 1 1 1
1 0 1 0 0 0

28
15
43

Ex: B.

Binary subtraction

A	B	Difference	Barrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Ex: Subtract $(0101)_2$ from $(1011)_2$

1 0 1 1
0 1 0 1
1 0 1 0

Hexa Decimal Arithmetic

$$\textcircled{1} \quad 9_{16} + 3_{16} = C_{16}$$

$$\textcircled{2} \quad 9_{16} + 7_{16} = (16 - 16) = 0 \quad \text{carry } 1$$

$$\textcircled{3} \quad A_{16} + 8_{16} = (18 - 16) = 2 \quad \text{carry } 1$$

① Add $(3F8)_{16}$ and $(5B3)_{16}$

(26 - 16) time/jntukonlinebits

-Add 1
 3 F 8

5 B 3

$$\overbrace{(q \ A \ B),_1}^{\sim}$$

② Add Hexadecimal numbers $(4FB)_{16}$, $(75D)_{16}$, $(A18)_{16}$

(C 39) 16 1 2

$$9) \begin{array}{r} 35 \\ - 16 \\ \hline 19 \end{array} \text{ carry } 1$$

$= (19 - 16) = 3$ carry 2

75D

$$9) 26 = (26 - 16) = 10 \text{ carry } 1$$

1 A 12

$$\text{iii) } 3A = (32 - 16) = 18 \text{ carry}$$

C 3

$$(18 - 16) = 2 \text{ carry } 2$$

$\mathcal{G} \ni g \mapsto$

• 150 •

Subtraction with 15's Complement

~~Subtraction with 15 complement~~

→ The 15th complement of a Hexadecimal number is formed by subtracting each digit from 15

Ex: find 15's complement of $(AQB)_{16}$

$$\begin{array}{ccccc} & 15 & 15 & 15 \\ (-) A & & q & B \end{array}$$

Ex: Use the 15's complement method of subtraction to compute $17 - 16 = 1$

$$(BO_2)_{16} \rightarrow (98F)_{16}$$

(+) Adding B/O 2

(L) 15 15 15 ^{oo} _{first row}
 6 7 0 bottom
 first row
 value tho
 (-ve)
 (-ve)
 (addition)
 (addition base 10)

Subtraction of 15's complement

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- find 15's complement of substrand
- Add to Hexadecimal numbers (1st number and 15's complement of 2nd number)
- If carry is produced. In the addition, add carry to the least significant bit of the sum, otherwise find 15's complement of the sum as a result with a (-ve) sign.

Ex: Use the 15's complement method of subtraction to

Compute $(69B)_{16} - (C14)_{16}$

$$\begin{array}{r} (20-16)=6 \text{ carry } 1 \\ 20-16=8 \text{ carry } 1 \\ \hline \end{array}$$

Step 1: $\begin{array}{r} 15 & 15 & 15 \\ (+) & C & 1 & 4 \\ \hline 3 & E & B \end{array}$

Step 2: $\begin{array}{r} 6 & 9 & B \\ (+) & 3 & E & B \\ \hline A & 8 & 6 \end{array}$

Step 3: $\begin{array}{r} 8 & 15 & 15 & 15 \\ -A & 8 & 6 \\ \hline -5 & 7 & 9 \end{array}$

$$\therefore (69B)_{16} - (C14)_{16} = (-579)_{16}$$

Subtraction of 16's complement

- The 16's complement of a hexadecimal number is found by subtracting each digit from 15 and add 1.
- Steps: steps for Hexadecimal subtraction using 16's complement method.

- find 16's complement of substrand
- Add to Hexadecimal numbers (1st number & 16's complement of the 2nd number)

→ If carry 9's produced in the addition it is discarded or removed, otherwise find 16's complement of the sum as a result with a (-ve) sign

Ex:- Find the 16's complement of $(A8C)_{16}$

$$\begin{array}{r}
 15 \quad 15 \quad 15 \leftarrow 15^{\text{'s}} \text{ complement} \\
 - A \quad 8 \quad C \\
 \hline
 5 \quad 7 \quad 3 \\
 + 1 \\
 \hline
 5 \quad 7 \quad 4
 \end{array}$$

2. Use the 16's complement method of subtraction to compute

$$(C\blacksquare B2)_{16} - (972)_{16}$$

Step 1:

$$\begin{array}{r}
 15 \quad 15 \quad 15 \leftarrow 15^{\text{'s}} \\
 - 9 \quad 7 \quad 8 \\
 \hline
 6 \quad 8 \quad D \\
 + 1 \\
 \hline
 6 \quad 8 \quad E \leftarrow 16^{\text{'s}} \text{ complement}
 \end{array}$$

Step 2:

$$\begin{array}{r}
 C \quad B \quad 2 \\
 + 6 \quad 8 \quad E \\
 \hline
 3 \quad 4 \quad 1
 \end{array}$$

$$\begin{aligned}
 (16-16) &= 1 \text{ carry} \\
 (20-16) &= 4 \text{ carry} \\
 (18-16) &= 3 \text{ carry}
 \end{aligned}$$

3. Use the 16's complement method of subtraction to compute

$$(387)_{16} - (854)_{16}$$

Step 1:

$$\begin{array}{r}
 15 \quad 15 \quad 15 \\
 - 8 \quad 5 \quad 4 \\
 \hline
 7 \quad A \quad B \\
 + 1 \\
 \hline
 7 \quad A \quad C \leftarrow 16^{\text{'s}} \text{ complement}
 \end{array}$$

$(19 - 16) = 3$ carry
 $(2 & - 16)$ time /intukonlinebits

Step 2: 3 B 7

$$(+) \quad 7 \quad A \quad C$$

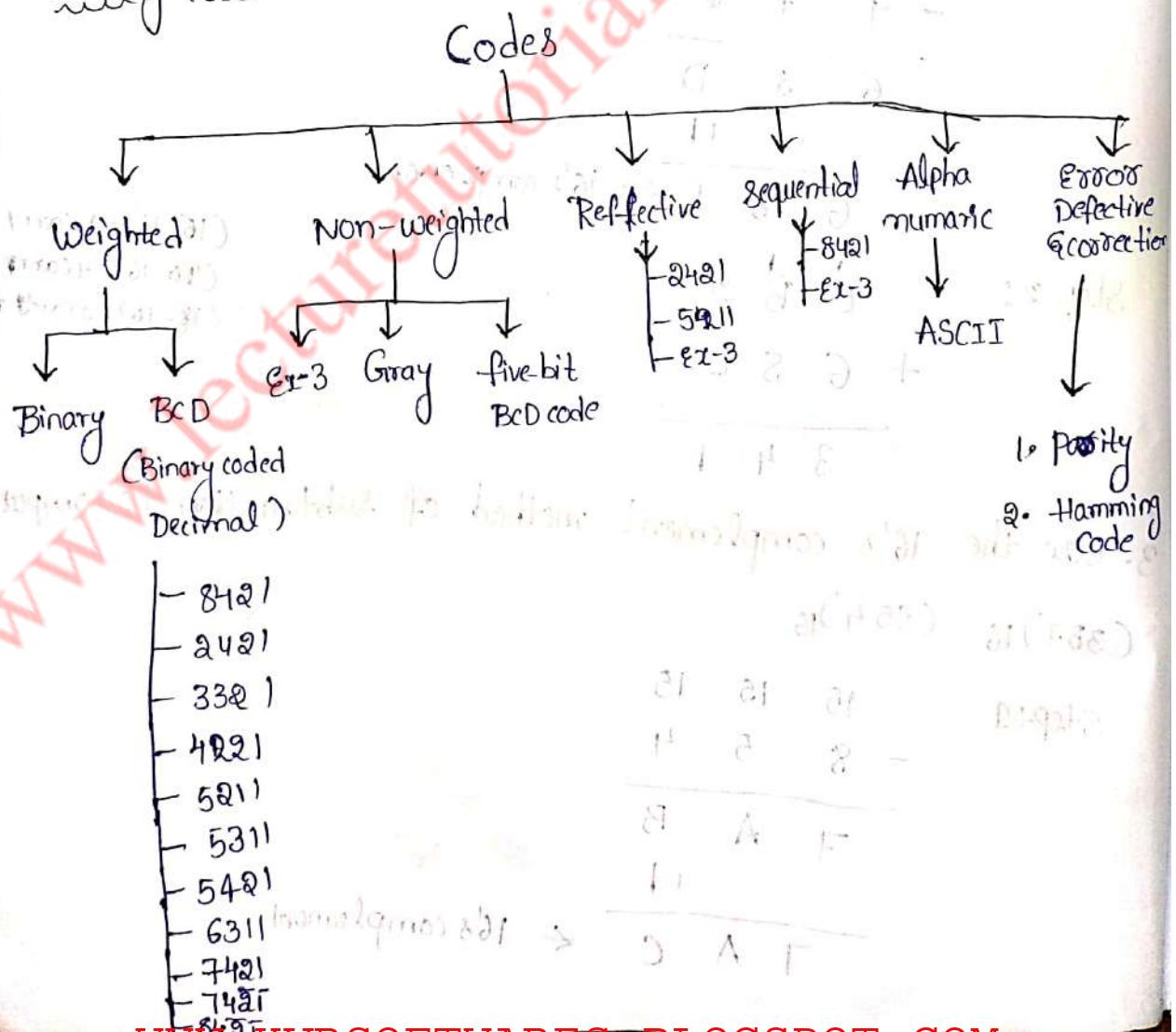
$$\begin{array}{r} \\ \\ \hline B \quad 6 \quad 3 \end{array}$$

Step 3: If NO carry

$$\begin{array}{r} 15 \quad 15 \quad 15 \\ - B \quad 6 \quad 3 \\ \hline 4 \quad 9 \quad C \\ \quad \quad \quad 1 \\ \hline -4 \quad 9 \quad D \end{array}$$

$$\therefore (3B7)_{16} - (854)_{16} = (-49D)_{16}$$

Binary codes :-



Excess 3 Code (E_2-3)

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Decimal	E_2-3	Binary code
0	$0+3=3$	0011
1	4	0100
2	5	0101
3	6	0110
4	7	0111

* To find binary code and Gray code the given binary number is 1101010101

truth table of Hexa X-OR Gate

$$A \oplus B = A\bar{B} + \bar{A}B$$

A	B	$C = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

binary-gray

$$\textcircled{1} \quad \begin{array}{ccccccc} & & & & & & \\ & & & & & & \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{array}$$



$$\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$$

$$101111111$$

$$\therefore 101111111$$

Gray code ÷ $\textcircled{1} \quad 1101010101$

$$\begin{array}{cccccccccc} 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ \downarrow & \oplus & \downarrow & \oplus & \downarrow & \oplus & \downarrow & \oplus & \downarrow & \oplus \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{array}$$

$$\therefore 1001100110$$

① BCD Addition:

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Ex: BCD of 58

$$\begin{array}{r} 5 \\ \downarrow \\ 0101 \end{array} \quad \begin{array}{r} 8 \\ \downarrow \\ 1000 \end{array}$$

② Sum equals "9" or less with carry "0"

Ex: Add 3 and 6 in BCD

$$\begin{array}{r} 3 \\ + 6 \\ \hline 9 \end{array} \quad \begin{array}{r} 0011 \\ 0110 \\ \hline 1001 \end{array} \rightarrow \text{Ans}$$

③ ~~Add~~ sum > 9 with carry "0"

Ex: Add 6 and 8

$$\begin{array}{r} 6 \\ + 8 \\ \hline 14 \end{array} \quad \begin{array}{r} 0110 \\ 1000 \\ \hline 1110 \end{array} \rightarrow 14$$

↓ 0110 ← Add 6

$$\begin{array}{r} 0001 \\ 0100 \\ \hline 14 \end{array} \rightarrow \text{BCD}$$

④ Sum equals to 9 or less with carry "1"

Ex: Add 8 + 9

$$\begin{array}{r} 8 \\ + 9 \\ \hline 17 \end{array} \quad \begin{array}{r} 1000 \\ 1001 \\ \hline 0001\ 0001 \end{array} \leftarrow \text{Invalid BCD}$$

$$\begin{array}{r} 0110 \\ 0001\ 0111 \\ \hline 1 \end{array}$$

* Perform each of the following decimal addition in 841 & 1 BCD
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① $24 + 18$

$$\begin{array}{r} 24 \\ - 18 \\ \hline 42 \end{array}$$

0010 0100
0001 1000
0011100
carry 0 110 ← Add 6
0011 0010 ← valid BCD

② $48 + 58$

$$\begin{array}{r} 48 \\ - 58 \\ \hline 106 \end{array}$$

0101 1000
0100 1000
10010000
1010 0000 ← Add 6
0110
10100110
10 A 6

③ $175 + 326$

$$\begin{array}{r} 175 \\ - 326 \\ \hline 501 \end{array}$$

0001 0111 0101
0011 0010 0110
010010011011
00100001 ← Add 6
001
1010

$$\begin{array}{r}
 & 1010 & \leftarrow \text{Add 6} \\
 \textcircled{1} & 0110 \\
 \hline
 0100 & \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 & 0101 & \leftarrow \text{Add 6} \\
 \textcircled{2} & 0101 & 0001 \\
 \hline
 0001 & \\
 \hline
 \end{array}$$

(4) $589 + 199$

$$\begin{array}{r}
 589 & 0101 & 1000 & 1001 \\
 199 & 0001 & 1001 & 1001 \\
 \hline
 788 & 1 & 0 & 1 \\
 0110 & 0001 & 0010 \\
 \hline
 0111 & 0010 & 0010 \\
 & 0110 & \leftarrow \text{Add 6} \\
 \hline
 & 1 & 1 \\
 & 1000 & \\
 \hline
 & 0010 \\
 & 0110 & \leftarrow \text{Add 6} \\
 \hline
 & 1 & 1 \\
 & 1000 & \\
 \hline
 & 0111 & 1000 & 1000 \\
 & 7 & 8 & 8
 \end{array}$$

BCD subtraction:

(9) subtraction with 9's complement

Digits	9's complement	Digits	9's complement
0	9		
1	8		
2	7	8	1
3	6	9	0
4	5		
5	4		
6	3		
7	2		

Regular subtraction

$$\begin{array}{r} 8 \\ - 2 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 8 \\ - 2 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 8 \\ - 2 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 9 \\ - 5 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 4 \\ - 8 \\ \hline 14 \end{array}$$

* perform each of the following decimal subtraction in 8421 BCD using 9's complement method.

$$(i) 79 - 26$$

$$\begin{array}{r} 79 \\ - 26 \\ \hline 53 \end{array}$$

$$\begin{array}{r} 0111 \quad 1001 \\ 0111 \quad 0011 \\ \hline 1110 \quad 1100 \end{array}$$

$[9-2=7]$
 $6-2=3]$

← 9's complement of 26)

0110 ← Add 6

$$\begin{array}{r} 1110 \quad 0010 \\ \hline 1110 \quad 0010 \end{array}$$

$$\begin{array}{r} 1111 \\ 0110 \\ \hline 0101 \end{array}$$

Add 6

0101 0010

0101 0011

9's complement subtraction

$$\begin{array}{r} 8 \\ - 2 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 9 \\ - 4 \\ \hline 3 \end{array}$$

(9's complement of result no
Carry indicates
that the answer
is negative and
completed from)

10's complement

$$\textcircled{a} \quad \begin{array}{r} 8 \\ - 2 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 8 \\ 8 \\ \hline \cancel{10} \end{array} \quad \begin{array}{l} \text{remove } 10 \\ \hline 6 \end{array}$$

 \rightarrow 10's complement ($10-2=8$)

$$\textcircled{b} \quad \begin{array}{r} 9 \\ - 5 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 9 \\ 5 \\ \hline \cancel{10} \end{array} \quad \begin{array}{l} \text{remove } 10 \\ \hline 4 \end{array}$$

 \rightarrow 10's complement ($10-5=5$)

$$\textcircled{c} \quad \begin{array}{r} 4 \\ - 8 \\ \hline -4 \end{array}$$

$$\begin{array}{r} 4 \\ 2 \\ \hline \cancel{10} \end{array}$$

 \rightarrow 10's complement of ($10-8=2$)

Result no carry indicate
that the answer is (-ve)
and in the 10's complement
form (or) incomplete form

Concept of Boolean Algebra

Introduction:

→ In 1854 George Boolean introduced a systematic treatment of logic and developed for this purpose of an algebraic system Now called Boolean Algebra.

→ In 1938 C.E. Shannon introduced a two-valued Boolean Algebra called a switching Algebra

Fundamental Postulates of Boolean Algebra

S.NO	Postulates	Comments
1.	→ Result of each operator is either 0 or 1	$1, 0 \in B$
2.	a) $0+0=0, 0+1=1, 1+0=1$ b) $1 \cdot 1 = 1; 0 \cdot 1 = 0, 1 \cdot 0 = 0$	Identity element "0" for "+" and "1" for "
3.	a) $(A+B) = (B+A)$ b) $(A \cdot B) = (B \cdot A)$	Commutative law
4.	a) $A \cdot (B+C) = (A \cdot B) + (A \cdot C)$ b) $A + (B \cdot C) = (A + B) \cdot (A + C)$	Distributive law
5.	a) $A + \bar{A} = 1, 0 + \bar{0} = 1, 1 + \bar{1} = 1$ b) $A \cdot \bar{A} = 0, \text{ if } \bar{0} \cdot \bar{0} = 0 \cdot 1 = 0$ $1 \cdot \bar{1} = 1 \cdot 0 = 0$	Complement

$$[\bar{0} = 1]$$

$$\bar{1} = 0]$$

Duality:

- The principle of Duality theorem says that starting with a boolean relation, we can derive another boolean relation by
1. changing each "OR" sign to an "AND" sign
 2. changing each "AND" sign to an "OR" sign
 3. Any 0 or 1 operating in the expression of boolean function.

Ex: Dual of relation $A + \bar{A} = 1$ is $A \cdot \bar{A} = 0$

$$A + \bar{A} = 1$$

$$A \cdot \bar{A} = 0$$

Theorems:

Laws of Boolean algebra:

Boolean Expression

$$\Rightarrow A + 1 = 1$$

$$A \cdot 0 = 0$$

$$\Rightarrow A + 0 = A$$

$$A \cdot 1 = A$$

$$\Rightarrow A + A = A$$

$$A \cdot A = A$$

$$\Rightarrow A \cdot A = A$$

$$\Rightarrow A = A$$

$$\text{NOT } A = A$$

B.A law (or) Rule

Annulation

Identity

Idempotent

Double Negation

Boolean Expression

BA Law (or) Rule
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$$\Rightarrow A + \bar{A} = 1$$

Complement

$$A \cdot \bar{A} = 0$$

$$\Rightarrow A + B = B + A$$

Commutative law

$$A \cdot B = B \cdot A$$

$$\Rightarrow \overline{A+B} = \bar{A} \cdot \bar{B}$$

De-morgan

$$\overline{AB} = \bar{A} + \bar{B}$$

Boolean Algebra functions

Function	Description	Expression
1.	NULL	0
2.	Identity	1
3.	INPUT A	A
4.	INPUT B	B
5.	NOT A	\bar{A}
6.	NOT B	\bar{B}
7.	A AND B (AND)	$A \cdot B$
8.	A AND NOT B	$A \cdot \bar{B}$
9.	NOT A AND B	$\bar{A} \cdot B$
10.	NOT AND (NAND)	$\overline{A \cdot B}$ $A \cdot \bar{A} = 0$ $A \cdot A = A$
11.	A OR B (OR)	$A + B$ $A + A = A$
12.	A OR NOT B	$A + \bar{B}$

Function	Description	Expression
13.	NOT A OR B	$A + B$
14.	NOT OR (NOR)	$\overline{A + B}$
15.	Exclusive OR (X-OR)	$AB + \overline{A}B$
16.	Exclusive NOR (X-NOR)	$\overline{AB + \overline{A}B}$

Theorems:

① a) $A + A = A$
 b) $A \cdot A = A$

② a) $A + 1 = 1$
 b) $A \cdot 0 = 0$

③ a) $\overline{\overline{A}} = A$

④ a) $A + AB = A$

b) $A(A+B) = A$

⑤ a) $A + \overline{A}B = A + B$

b) $A(\overline{A} + B) = AB$

① @ Proof:

L.H.S

$$A + A = (A+A) \cdot (1)$$

$$= (A+A)(A+\overline{A})$$

$$= AA + A\overline{A} + AA + A\overline{A}$$

$$= AA + A\overline{A}$$

$$= A(A+\overline{A})$$

$$= A //$$

$$\left\{ \begin{array}{l} \text{if } A = 0; 0+0=0 \\ \text{if } A = 1; 1+1=1 \end{array} \right.$$

$$[1 = A + \overline{A}]$$

② b) Proof:

L.H.S

$$A \cdot A = A \cdot A + (0)$$

$$= A \cdot A + A \cdot \overline{A}$$

$$= A(A+\overline{A})$$

$$= A //$$

$$\left\{ \begin{array}{l} A \cdot A = A \\ A = 0 \Rightarrow 0 \cdot 0 = 0 \\ A = 1; 1 \cdot 1 = 1 \end{array} \right.$$

$$[A \cdot \overline{A} = 0]$$

$$② @ A+1=1$$

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$$\underline{\text{Proof:}} \quad A+1 = 1 \cdot A + 1$$

$$= (A+\bar{A}) \cdot (A+1)$$

$$= A \cdot A + A + \bar{A} - A + \bar{A}$$

$$= A \cdot A + A + 0 + \bar{A}$$

$$= A + A + \bar{A}$$

$$= A + \bar{A}$$

$$= 1$$

$$\begin{aligned} A \cdot \bar{A} &= 0 \\ A + \bar{A} &= 1 \end{aligned}$$

$$\left\{ \begin{array}{l} \text{if } A=0; 0+1=1 \\ A=1; 1+1=1 \end{array} \right.$$

$$(b) A \cdot 0 = 0$$

$$\underline{\text{Proof:}} \quad A \cdot 0 = A \cdot (A \cdot \bar{A})$$

$$= (A \cdot 0)$$

$$= 0$$

$$\begin{aligned} A \cdot 0 &= 0 \\ A=0; 0 \cdot 0 &= 0 \\ A=1; 1 \cdot 0 &= 0 \end{aligned}$$

$$(3) \bar{\bar{A}} = A$$

$$\underline{\text{Proof:}} \quad \text{if } A=0; \bar{0} = \bar{1} = 0 \quad [\bar{0} = 1] \quad [\bar{1} = 0]$$

$$A=1; \bar{1} = \bar{0} = 1$$

$$(4) @ A+AB=A$$

$$\begin{aligned} \underline{\text{Proof:}} \quad A+AB &= A(1+B) \quad [\because 1+B=1] \\ &= A(1) \\ &= A \end{aligned}$$

$$(b) A(A+B)=A$$

$$\underline{\text{Proof:}} \quad A(A+B) = (A+AB)(A+B)$$

$$= AA+AB+AB+A \cdot B$$

$$= A + AB + A \cdot AB + AB$$

$$= A + AB + AB + AB$$

$$= A(1+B) + AB$$

$$= A + AB$$

$$= A(1+B)$$

$$= A \parallel$$

(5)(a) $A + \bar{A}B = A + B$

proof: $A + \bar{A}B = (A + \bar{A})(A + B)$

$$= A + B \quad [\because A + \bar{A} = 1]$$

(b) $A(\bar{A} + B) = AB$

proof: $A(\bar{A} + B) = A \cdot \bar{A} + AB$

$$= 0 + AB$$

$$AB = AB \parallel$$

** De-Morgan's theorem:

(i) $\bar{AB} = \bar{A} + \bar{B}$

(ii) $\bar{A+B} = \bar{A} \cdot \bar{B}$

(iii) $\bar{AB} = \bar{A} + \bar{B}$

A	B	\bar{A}	\bar{B}	\bar{AB}	$\bar{A} + \bar{B}$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

(iv) $\bar{A+B} = \bar{A} \cdot \bar{B}$

A	B	\bar{A}	\bar{B}	$\bar{A} \cdot \bar{B}$	$\bar{A+B}$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

→ Consensus theorem :-

→ In simplification of Boolean Expression, an Expression of the form $AB + \bar{A}C + BC$. The term BC is redundant and can be eliminated to form the equivalent expression $AB + \bar{A}C$.

→ The theorem is used for this simplification is known as Consensus theorem, and it is stated as

$$AB + \bar{A}C + BC = AB + \bar{A}C$$

Proof :- L.H.S $\Rightarrow AB + \bar{A}C + BC = AB + \bar{A}C + BC(1)$

$$= AB + \bar{A}C + BC(A + \bar{A})$$

$$= AB + \underline{\bar{A}C} + \underline{BC}A + \underline{BC}\bar{A}$$

$$= ABC(1 + C) + \bar{A}C(1 + B)$$

$$= AB + \bar{A}C$$

* Solve the given expression using Consensus theorem

$$(9) \bar{A}\bar{B} + Ac + B\bar{C} + \bar{B}C + AB$$

$$\Rightarrow \bar{A}\bar{B} + Ac + B\bar{C} + \bar{B}C(1) + AB \quad [A + \bar{A} = 1]$$

$$\Rightarrow \bar{A}\bar{B} + Ac + B\bar{C} + \bar{B}C(A + \bar{A}) + AB$$

$$\Rightarrow \underline{\bar{A}\bar{B}} + \underline{Ac} + \underline{B\bar{C}} + \underline{\bar{B}C}A + \underline{\bar{B}C}\bar{A} + AB$$

$$\Rightarrow \bar{A}\bar{B}(1 + C) + Ac(1 + \bar{B}) + B\bar{C} + AB \quad [HC = 1, 1 + B = 1]$$

$$\Rightarrow \bar{A}\bar{B} + Ac + B\bar{C} + AB$$

$$\Rightarrow \bar{A}\bar{B} + Ac + B\bar{C} + ABC(1)$$

$$\Rightarrow \bar{A}\bar{B} + Ac + B\bar{C} + AB(C + \bar{C})$$

$$\Rightarrow \bar{A}\bar{B} + \underline{Ac} + \underline{B\bar{C}} + \underline{ABC} + \underline{AB}\bar{C}$$

$$\Rightarrow \bar{A}\bar{B} + Ac(1 + B) + B\bar{C}(1 + A) \quad [1 + B = 1, 1 + A = 1]$$

$$\Rightarrow \bar{A}\bar{B} + A\bar{C} + B\bar{C}$$

Dual of consensus theorem:

The Dual form of consensus theorem is stated as

$$(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$$

$$(A\bar{A} + A(\bar{A}B + BC))(B+C) = (A\bar{A} + A(\bar{A}B + BC))$$

$$(0 + A(\bar{A}B + BC))(B+C) = (0 + A(\bar{A}B + BC)) \quad [A\bar{A}=0]$$

$$ABC + \bar{A}B \cdot B + BC \cdot B + AC \cdot C + \bar{A}BC + BC \cdot C = AC + \bar{A}B + BC \quad [BC + BC = BC]$$

$$BC(A+B) + \bar{A}B(B+C)$$

$$ABC + \bar{A}B \cdot B + BC + AC + \bar{A}BC + BC = AC + \bar{A}B + BC$$

$$\bar{A}B(1+C) + BC(1+A) + AC = AC + \bar{A}B + BC$$

$$\boxed{\bar{A}B + BC + AC = AC + \bar{A}B + BC}$$

Boolean function (or) switching function:

Boolean equations are constructed by connecting the boolean constants and variables with the boolean operation.

Boolean expressions are known as boolean formulas we use

This Boolean expressions are used to describe boolean functions.

Boolean expression to describe boolean functions.

for example: If Boolean Expression $(A+\bar{B})C$ is used to describe the function of f , then boolean function is written as, $f(A, B, C) = (A\bar{B})C$

$$f = (\bar{A} + B)C$$

Let us consider the whole four variable boolean function.

Product-terms:

$$f(A, B, C, D) = \boxed{A + (BC) + (ACD)} \quad \begin{array}{l} \text{product terms} \\ \downarrow \downarrow \downarrow \downarrow \\ \text{literals} \end{array}$$

Sum-Terms:

Sum terms

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$$f(A, B, C, D) = (B + \bar{D}) \cdot (A + \bar{B} + C) (\bar{A} + C)$$

↓ ↓ ↓ ↓ ↓ ↓
Literals

→ The literals and terms are arranged in the form as shown in two

1. sum of product (SOP)

2. product of sum (POS)

1. sum of products:

→ The sum of product is also called Disjunctive normal form (DNF)

Disjunctive normal formula

→ The words sum & product are described from the symbolic representation "OR" and "AND" function (+ and *)

Ex:- 1

$$f(A, B, C) = \overbrace{ABC + A\bar{B}\bar{C}}^{\substack{\text{sum terms} \\ \text{products}}}$$

Ex:- 2

$$f(P, Q, R) = \overbrace{\bar{P}Q + Q\bar{R} + \bar{R}S}^{\substack{\text{sum terms} \\ \text{products}}}$$

2. Product of sum:

→ The product of sum is also called conjunctive normal form (CNF) or conjunctive normal formula.

→ A product of sum is any group of sum terms ANDed

together.

Ex:-

$$f(A, B, C) = \overbrace{(A + B + \bar{C}) \cdot (\bar{A} + \bar{B} + C)}^{\substack{\text{product} \\ \text{sum}}}$$

$$f(P, Q, R) = (P+Q) \cdot (R+P) \cdot Q$$

↓ ↓
Sum product

* Canonical form \Leftrightarrow (standard SOP and POS forms)

→ Basically Canonical forms are two types

1. Standard SOP (or) minterm Canonical form

2. Standard POS (or) maxterm Canonical form

1. Standard SOP \Leftrightarrow (minterm Canonical form)

→ The SOP is given by $f(A, B, C) = (A\bar{B}C) + (\bar{A}BC) + (\bar{A}\bar{B}C)$

Each product term is consistent all literals in either completed form or uncompleted form

2. Standard POS \Leftrightarrow (maxterm)

$f(A, B, C) = (\bar{A}+\bar{B}+C) \cdot (\bar{A}+B+C) \cdot (\bar{A}+B+C)$

Each sum term consists of all literals in the completed form

* Converting expressions in standard SOP (or) POS form

Steps to convert SOP to standard SOP form \Leftrightarrow

1. find the missing literal in each product term if any

2. AND each product term having missing literals which term 1 formed by OR'ing the literals and its complement

3. Expand the terms by applying Distributive law and recorded

the literals in the product terms omitting (or) removing repeated product

4. Reduce the expression because $(A+A) = A$

Ex: Convert the given expression in standard SOP form

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$$f(A, B, C) = AC + AB + BC$$

Step 1: $f(A, B, C)$ = find the missing literal in each product

$$f(A, B, C) = AC + AB + BC$$



A literal is missing

C literal missing

B literal missing

Step 2: AND product terms with (missing literal + complement)

$$f(A, B, C) = AC(1 \oplus \bar{B}) + AB(1 \oplus \bar{C}) + BC(A + \bar{A})$$

$$\begin{aligned} A + \bar{A} &= 1 \\ B + \bar{B} &= 1 \\ C + \bar{C} &= 1 \end{aligned}$$

$$= AC(B + \bar{B}) + AB(C + \bar{C}) + BC(A + \bar{A})$$

Step 3: Expand the term and recorded the terms

Expand :-

$$f(A, B, C) = ACB + A\bar{C}\bar{B} + ABC + AB\bar{C} + BCA + B\bar{C}A$$

Recorded :-

$$f(A, B, C) = ABC + A\bar{B}C + ABC + AB\bar{C} + ABC + \bar{A}BC$$

Step 4: omitting the repeated product term

$$f(A, B, C) = \underline{ABC} + \underline{A\bar{B}C} + \underline{ABC} + \underline{AB\bar{C}} + \underline{ABC} + \bar{A}BC$$

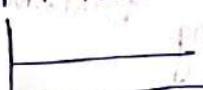
$$f(A, B, C) = ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC$$

Ex: Convert the given expression in standard SOP form

$$f(A, B, C) = A + ABC$$

Step 1: find

$$f(A, B, C) = A + ABC$$



B literal missing

C literal missing

Step 2: AND product term with (missing literal + complement)

$$f(A, B, C) = AC(1 \oplus \bar{B}) + ABC$$

$$= A(B + \bar{B})(C + \bar{C}) + ABC$$

Step 3:

$$f(A, B, C) = AB + \overline{A}B(C + \overline{C} + \overline{B}C + BC) + ABC$$

$$= \underline{ABC} + AB\overline{C} + A\overline{B}C + A\overline{B}\overline{C} + \underline{ABC}$$

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Step 4: Omitting the repeated product term

$$f(A, B, C) = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\overline{C}$$

Convert POS to standard POS form:

Step 5:

1. find the missing literals in each sum term if any.
2. OR each sum term having missing literal formed by ANDing the literal and its complement
3. Expand the terms by applying distributive law and recorded the literal in the sum terms
4. Reduce the Expression by omitting repeated sum term if any

$$A \cdot A = A$$

Ex: convert the given expression in standard POS form.

$$f(A, B, C) = (A+B) \cdot (B+C) \cdot (A+C)$$

$$\begin{aligned} A \cdot \overline{A} &= 0 \\ B \cdot \overline{B} &= 0 \\ C \cdot \overline{C} &= 0 \end{aligned}$$

Step 1: find the missing literal in each product

$$f(A, B, C) = (A+B) \cdot (B+C) \cdot (A+C)$$

→ B literal missing $[A+(BC)=A]$
 → A literal missing $(A+B)(A+C)$
 → C literal missing [distribut]

Step 2: OR product term with (missing literal + complement)

$$f(A, B, C) = [(A+B) + (C \cdot \overline{C})] \cdot [(B+C) + (A \cdot \overline{A})] \cdot [(A+C) + (B \cdot \overline{B})]$$

Step 3: expand the term and recorded the terms

Expand:

$$f(A, B, C) = [(A+B+C) \cdot (A+B+\overline{C})] \cdot [(B+C+A) \cdot (B+C+\overline{A})] \cdot [(A+C+B) \cdot (A+C+\overline{B})]$$

Recorded:

$$f(A, B, C) = [\underline{A+B+C}] \cdot [\underline{A+B+\bar{C}}] \cdot [\underline{\bar{A}+\bar{B}+C}] \cdot [\underline{\bar{A}+\bar{B}+\bar{C}}]$$

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Step 4: omitting the repeated product term $\underline{A+B+C}$

$$f(A, B, C) = (A+B+C)(A+B+\bar{C})(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$$

Ex: Convert the given expression in standard pos forms

$$f(A, B, C) = (A) \cdot (A+B+C)$$

Step 1: find the missing literal in each product

$$f(A, B, C) = A \cdot (A+B+C)$$

B, C literals missing

Step 2: OR product term with (missing literal + complement)

$$f(A, B, C) = [A + (B \cdot \bar{B}) + (\bar{C} \cdot \bar{C})] \cdot (A+B+C)$$

Step 3: Expand the term and recorded the terms

$$\begin{aligned} \text{Expand: } & [A + (B+C) + (B+\bar{C}) + (\bar{B}+C) + (\bar{B}+\bar{C})] \cdot (A+B+C) \\ & = (A+B+C) \cdot (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (A+\bar{B}+\bar{C}) \cdot (A+\bar{B}+C) \end{aligned}$$

Step 4: omitting the repeated product term

$$f(A, B, C) = (A+B+C) \cdot (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (A+\bar{B}+\bar{C})$$

M-Notations \div (minterms and maxterms)

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Decimal No.	Binary Numbers (4x3)	Minterm (m) (SOP)	Maxterm (M) (POS)
0	000	$\bar{A}\bar{B}\bar{C}$ (m ₀)	A + B + C (M ₀)
1	001	$\bar{A}\bar{B}C$ (m ₁)	A + B + \bar{C} (M ₁)
2	010	$\bar{A}B\bar{C}$ (m ₂)	A + \bar{B} + C (M ₂)
3	011	$\bar{A}BC$ (m ₃)	A + \bar{B} + \bar{C} (M ₃)
4	100	A $\bar{B}\bar{C}$ (m ₄)	\bar{A} + B + C (M ₄)
5	101	A $\bar{B}C$ (m ₅)	\bar{A} + B + \bar{C} (M ₅)
6	110	A B \bar{C} (m ₆)	\bar{A} + \bar{B} + C (M ₆)
7	111	A B C (m ₇)	\bar{A} + \bar{B} + \bar{C} (M ₇)

Onelinelet

Minterm = Complement of maxterm

$$\begin{aligned} \text{Example: } f(A, B, C) &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} \\ &= m_0 + m_1 + m_3 + m_6 \\ &= \sum m(0, 1, 3, 6) \end{aligned}$$

$$\begin{aligned} \text{Ex: } f(A, B, C) &= (A + B + \bar{C}) \cdot (\bar{A} + \bar{B} + \bar{C}) \cdot (\bar{A} + \bar{B} + C) \\ &= M_1 \cdot M_3 \cdot M_6 \\ &= \prod M(1, 3, 6) \end{aligned}$$

To find sum of product form to the given table

A B C	y
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0

[ABC] 98 a
Input variables
[SOP, K9 1's]

A. B C	y
1 0 1	1
1 1 0	0
1 1 1	0

$$f(A, B, C) = \overline{A}B\overline{C} + \overline{A}B\overline{C} + A\overline{B}\overline{C}$$

$$= m_2 + m_3 + m_6$$

$$= \sum m(2, 3, 6)$$

To find the product of sum from the given table.

[Product of
sum is zero's]

A+B+C	y
0 0 0	1
0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	1

$$f(A, B, C) = (A + \overline{B} + \overline{C}) \cdot (\overline{A} + B + \overline{C})$$

$$= M_2 \cdot M_5$$

$$= \prod M(2, 5)$$

Algebraic Simplifications:

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1. $A \cdot \bar{A}C = 0$
2. $ABCD + ABD = ABD$
3. $A(A+B) = A$
4. $AB + ABC + ABC(D+E) = AB$
5. $XY + XYZ + XY\bar{Z} + \bar{X}YZ$
6. $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C$
7. $ABC + A\bar{B}C + A\bar{B}\bar{C}$
8. $A + \bar{A}B + A\bar{B} = A + B$

$$\text{Ex: } \overline{\bar{A}\bar{B} + \bar{A} + AB}$$

$$[\bar{A}\bar{B} = \bar{A} + \bar{B}]$$

$$\Rightarrow \overline{\bar{A} + \bar{B} + \bar{A} + AB}$$

$$[\bar{B} + (AB) = \bar{B}A + (B \cdot B)]$$

$$\Rightarrow \overline{\bar{A} + \bar{B} + AB}$$

$$= [(\bar{B} + A) \cdot (\bar{B} + B)]$$

$$\Rightarrow \overline{\bar{A} + (\bar{B} + A) \cdot (B + B)}$$

$$[\bar{B} + B = 1]$$

$$\Rightarrow \overline{\bar{A} + \bar{B} + A \cdot (1)}$$

$$\Rightarrow \overline{(A + \bar{A}) + B}$$

$$[1 = 0]$$

$$\Rightarrow \overline{1 + \bar{B}}$$

$$\therefore = 0$$

$$\therefore = 0$$

$$\textcircled{1} \quad A \cdot \bar{A}C = 0$$

$$(A \cdot \bar{A})C = 0$$

$$(0)C = 0$$

$$A \cdot \bar{A}C = 0$$

$$\overline{\bar{A}\bar{B} + \bar{A} + AB}$$

$$\overline{\bar{A} + \bar{B} + AB}$$

$$= \overline{\bar{A} + \bar{B} + AB}$$

$$= \overline{A + \bar{A}\bar{B} + B\bar{B}}$$

$$= \overline{A + \bar{A}\bar{B}}$$

$$= \overline{(\bar{A} + A) \cdot (A + \bar{B})}$$

$$= \overline{1 + \bar{B}}$$

$$\textcircled{2} \quad ABCD + ABD = ABD$$

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[H.C = 1]

$$\Rightarrow ABD(1+C)$$

$$\Rightarrow ABD$$

$$\Rightarrow ABCD + ABD = ABD$$

$$\textcircled{3} \quad A(A+B) = A$$

$$\Rightarrow A \cdot A + AB$$

$$\Rightarrow A + AB$$

$$\Rightarrow A(1+B)$$

$$\Rightarrow A$$

$$\textcircled{4} \quad AB + ABC + ABC(D+E) = AB$$

$$\Rightarrow AB + ABC + AB(D+E)$$

$$\Rightarrow AB(1+C) + ABD + ABE$$

$$\Rightarrow AB + ABC(D+E)$$

$$\Rightarrow AB(1+D+E)$$

$$\Rightarrow AB[(1+D)+E]$$

$$\Rightarrow AB[1+E]$$

$$\Rightarrow ABC(1)$$

$$\Rightarrow AB$$

$$5. xy + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz = y(x+z)$$

$$\Rightarrow xy(1) + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz$$

$$\Rightarrow xy(1+\bar{z}) + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz$$

$$\Rightarrow xy + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz$$

$$\Rightarrow xy(z+\bar{z}) + x\bar{y}(z+\bar{z}) + \bar{x}yz$$

$$\Rightarrow xy + \bar{x}y + \bar{x}yz$$

$$\Rightarrow xy + \bar{x}yz$$

$$\Rightarrow y(x + \bar{x}z) \Rightarrow y((x + \bar{x}) \cdot (x + z)) \Rightarrow y(1) \cdot (x + z)$$

$$\Rightarrow y(x + z) //$$

$$⑥ A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}BC$$

$$\Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC$$

$$\Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}B(\bar{C} + C)$$

$$\Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}B$$

$$\Rightarrow \bar{A}C(\bar{B}\bar{C} + B)$$

$$\Rightarrow \bar{A}((\bar{B} + B), (\bar{C} + B))$$

$$\Rightarrow \bar{A}(1 \cdot (\bar{C} + B))$$

$$\Rightarrow \bar{A}(\bar{C} + B)$$

$$\Rightarrow \bar{A}\bar{C} + \bar{A}B$$

$$⑦ ABC + A\bar{B}C + A\bar{B}\bar{C}$$

$$ABC + A\bar{B}C + A\bar{B}\bar{C}$$

$$AC(C + \bar{B}) + A\bar{B}\bar{C}$$

$$AC + A\bar{B}\bar{C}$$

$$AC(C + B)$$

$$AC((C + B) \cdot (C + \bar{C}))$$

$$AC(C + B)$$

$$AC + AB$$

$$\begin{aligned} & A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}BC \\ & \bar{A}C(C + B) + \bar{A}BC \\ & = \bar{A}\bar{C} + \bar{A}BC \\ & = \bar{A}(C + BC) \\ & = \bar{A}((C + B) \cdot (C + \bar{C})) \\ & = \bar{A}\bar{C} + \bar{A}B \\ & ABC + A\bar{B}C + A\bar{B}\bar{C} \end{aligned}$$

$$AC((B + \bar{B}) + \bar{A}B\bar{C})$$

$$AC + A\bar{B}\bar{C}$$

$$AC(C + B)$$

$$AC((C + B) \cdot (C + \bar{C}))$$

$$AC(C + B)$$

$$AC + AB$$

$$8. A + \bar{A}B + A\bar{B} = A+B$$

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$$\Rightarrow A + \bar{A}B + A\bar{B}$$

$$\Rightarrow A(1 + \bar{B}) + \bar{A}B$$

$$\Rightarrow A(1) + \bar{A}B$$

$$\Rightarrow A + \bar{A}B$$

$$\Rightarrow (A + \bar{A})(A + B)$$

$$\Rightarrow (1)(A + B)$$

$$\Rightarrow A + B //$$

Ex: Simplify the following three variable expression by using Boolean algebra

$$Y = \Sigma m(1, 3, 5, 7)$$

$$\text{Given Data } Y = \Sigma m(1, 3, 5, 7) \\ = m_1 + m_3 + m_5 + m_7$$

$$= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$$

$$= \bar{A}C(\bar{B} + B) + AC(\bar{B} + B)$$

$$= \bar{A}C(1) + AC(1)$$

$$= C(\bar{A} + A)$$

$$= C //$$

Ex: Simplify the following three variable expression by using Boolean algebra

$$Y = \prod M(1, 3, 5, 7)$$

$$\text{Given that } Y = \prod M(1, 3, 5, 7)$$

$$= M_1 \cdot M_3 \cdot M_5 \cdot M_7$$

$$= (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$$

$$= [A \cdot A + (A \cdot \bar{B}) + (A \cdot \bar{C}) + (B \cdot A) + (B \cdot \bar{B}) + (B \cdot \bar{C}) + (\bar{C} \cdot A) + (\bar{C} \cdot \bar{B}) + (\bar{C} \cdot \bar{C})] \cdot$$

$$[(\bar{A} \cdot \bar{A}) + (\bar{A} \cdot \bar{B}) + (\bar{A} \cdot \bar{C}) + (B \cdot A) + (B \cdot \bar{B}) + (B \cdot \bar{C}) + (\bar{C} \cdot \bar{A}) + (\bar{C} \cdot \bar{B}) + (\bar{C} \cdot \bar{C})] \cdot$$

$$[(\bar{B} \cdot \bar{C}) + (C \cdot \bar{C})]$$

$$= ABC + A\bar{B}\bar{C} + A\bar{C} + \bar{A}\bar{B}\bar{C} + ABC + A\bar{B}\bar{C} + A\bar{C} + AB + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \\ BC + \bar{A}\bar{B}\bar{C} + B\bar{C} + ABC + A\bar{B}\bar{C} + AC + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C$$

$$+ B\bar{C} + \bar{A}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{C} + \bar{A}B + B\bar{C} + A\bar{C} + \bar{B}\bar{C} + \bar{C}$$

$$= ABC + A\bar{B}\bar{C} + A\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + B\bar{C} + \bar{A}\bar{B}\bar{C} + B\bar{C} + \bar{A}C + \bar{A}B + \bar{C}$$

$$= BC(A + \bar{A}) + \bar{B}\bar{C}(1 + A) + \bar{C}(A + \bar{A}) + \bar{A}B(1 + C) + \bar{C}(1 + B) + \bar{A}\bar{B}\bar{C}$$

$$= BC + \bar{B}\bar{C} + \bar{C} + \bar{A}B + \bar{C} + \bar{A}\bar{B}\bar{C}$$

$$= \bar{A}B + \bar{A}\bar{B}\bar{C} + \bar{C} //$$

Ex Simplify the following three variable expression. Convert the expression into minterm complementary form.

$$y = \Pi_N(1, 3, 5, 7)$$

Sol: The given expression is maxterm $y = \Pi_N(1, 3, 5, 7)$.

To convert given sequences to minterm complementary form

Minterm = Complementary of Maxterm

The minterm is given by

$$= \Sigma_m(0, 2, 4, 6)$$

$$y = \Sigma_m(0, 2, 4, 6) = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC \\ = m_0 + m_2 + m_4 + m_6 = \bar{A}((B + \bar{B})) + \bar{B}((A + \bar{A}))$$

$$= \bar{A}\bar{B}(\bar{B} + B) + \bar{A}\bar{C}(B + \bar{B}) = \bar{A}\bar{C} + \bar{A}\bar{C} = \bar{C}(A + \bar{A})$$

$$= \bar{A}\bar{C} + \bar{A}\bar{C} = \bar{C}(A + \bar{A}) = \bar{C} //$$

(M) * transform each of the following Canonical expression into its other Canonical form & its decimal notations.

$$(i) f(x, y, z) = \Sigma_m(1, 3, 5)$$

$$(ii) f(w, x, y, z) = \Pi_N(0, 2, 5, 6, 7, 8, 9, 11, 12)$$

$$(iii) f(x, y, z) = \Sigma_m(1, 3, 5)$$

$$f(x, y, z) = \Pi_N(0, 2, 4, 6, 7)$$

$$= M_0 \cdot M_2 \cdot M_4 \cdot M_6 \cdot M_7 (x + y + z) \cdot (x + \bar{y} + z) \cdot (\bar{x} + y + z) \cdot (\bar{x} + \bar{y} + z) \\ = (\cancel{x} + \cancel{y} + \cancel{z}) \cdot (\cancel{A} \cancel{B} + \cancel{E}) \cdot (\cancel{A} + \cancel{B} + \cancel{C}) \cdot (\cancel{A} + \cancel{B} + \cancel{C}) \cdot (\cancel{A} + \cancel{B} + \cancel{D})$$

$$f(\omega, x, y, z) = \text{IM}(0, 2, 5, 6, 7, 8, 9, 11, 12)$$

$$f(w, x, y, z) = \text{PI}_N(0, 8, 15, 6, 7, 8, 9, 11, 12)$$

$$= \overbrace{\sum m(1, 3, 4, 10, 14, 15)}$$

$$= m_1 + m_3 + m_4 + m_{10} + m_{13} + m_{14} + m_{15}$$

$$= \bar{w}\bar{x}\bar{y}z + \bar{z}\bar{y}\bar{x}w + \bar{z}\bar{y}\bar{x}\bar{z} + w\bar{x}yz + wxy\bar{z} + wxyz$$

$$= \bar{w}x\bar{z}(\bar{y}+y) + wxz(y\bar{y}+\bar{y}) + w\bar{y}\bar{z}(x+\bar{x}) + \bar{w}x\bar{y}\bar{z}$$

$$= \bar{w}\bar{x}z + wxz + \bar{w}\bar{y}\bar{z} + \bar{w}x\bar{y}\bar{z}$$

Decimal NO	Binary number	minterm <small>(m)</small>	Maxterm <small>t.me/jntukonlinebits</small> $A + B + C + D$
8	$\begin{array}{r} 1000 \\ 842) \end{array}$	$A \bar{B} \bar{C} \bar{D}$	$\bar{A} + B + C + \bar{D}$
9	$\begin{array}{r} 1001 \\ 842) \end{array}$	$\bar{A} \bar{B} \bar{C} D$ (m9)	$\bar{A} + B + C + D$
10	$\begin{array}{r} 1010 \\ 842) \end{array}$	$A \bar{B} \bar{C} \bar{D}$ (m10)	$\bar{A} + B + \bar{C} + D$
11	$\begin{array}{r} 1011 \\ 842) \end{array}$	$A \bar{B} C D$	$\bar{A} + B + \bar{C} + \bar{D}$
12	$\begin{array}{r} 1100 \\ 842) \end{array}$	$A B \bar{C} \bar{D}$	$\bar{A} + \bar{B} + C + D$
13	$\begin{array}{r} 1101 \\ 842) \end{array}$	$A B \bar{C} D$	$\bar{A} + \bar{B} + C + \bar{D}$
14	$\begin{array}{r} 1110 \\ 842) \end{array}$	$A B C \bar{D}$	$\bar{A} + \bar{B} + \bar{C} + D$
15	$\begin{array}{r} 1111 \\ 842) \end{array}$	$A B C D$	$\bar{A} + \bar{B} + \bar{C} + \bar{D}$

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UNIT - III

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Gate - Level Minimization

Map :-

→ The map method gives us a systematic approach for simplifying a boolean expression. The map method first proposed by Veitch and modified by Karnaugh, hence it is known as the Veitch-Karnaugh diagram or the Karnaugh map (K-map).

One-variable, two-variable, three variable and four variable

Maps :-

$$1 \text{ Variable} = 2^1 = 2 \text{ cells}$$

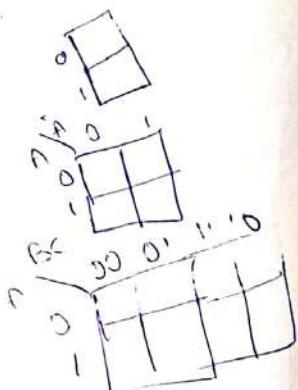
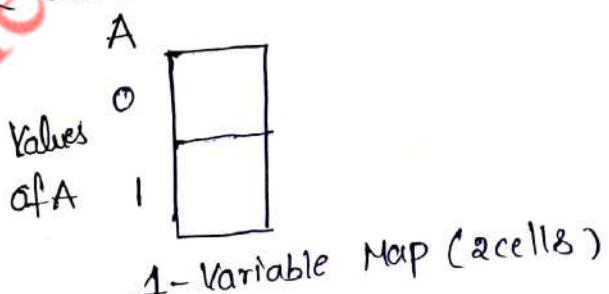
$$2 \text{ Variable} = 2^2 = 4 \text{ cells}$$

$$3 \text{ Variable} = 2^3 = 8 \text{ cells}$$

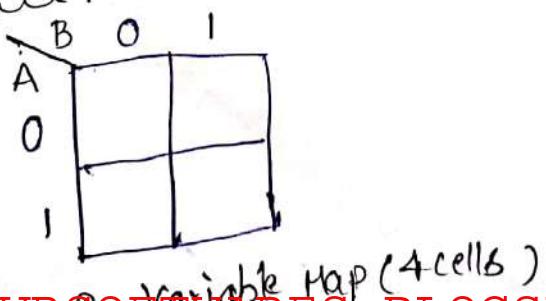
$$4 \text{ Variable} = 2^4 = 16 \text{ cells}$$

$$\begin{aligned}1 \text{ Var} &= 2^1 = 2 \text{ cells} \\2 \text{ Var} &= 2^2 = 4 \text{ cells} \\3 \text{ Var} &= 2^3 = 8 \text{ cells} \\4 \text{ Var} &= 2^4 = 16 \text{ cells}\end{aligned}$$

1 Variable K-Map :-



2 Variable K-Map :-



3-Variable K-maps

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	BC	00	01	11	10
A					
0					
1					

3-Variable K-map (8 cells)

4-Variable K-map

	CD	00	01	11	10
AB					
00					
01					
11					
10					

Values of CD

in gray code

4-Variable K-map (16 cells)

1-Variable

0	\bar{A}	$\bar{A} 0$
1	A	A 1

2-Variable

	B	\bar{B}	B
0	\bar{A}	$\bar{A} \bar{B}$	$\bar{A} B$
1	A	$A \bar{B}$	$A B$

3-Variable

	BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
0	\bar{A}	$\bar{A} \bar{B} \bar{C}$	$\bar{A} \bar{B} C$	$\bar{A} B \bar{C}$	$\bar{A} B C$
1	A	$A \bar{B} \bar{C}$	$A \bar{B} C$	$A B \bar{C}$	$A B C$

4-variable

$\bar{A}\bar{B}$	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}B$	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0	$\bar{A}B\bar{C}\bar{D}$ 1	$A\bar{B}C\bar{D}$ 3	$A\bar{B}CD$ 2
$\bar{A}B$	$\bar{A}BC\bar{D}$ 4	$\bar{A}B\bar{C}D$ 5	$\bar{A}BCD$ 7	$AB\bar{D}$ 6
AB	$A\bar{B}\bar{C}\bar{D}$ 12	$A\bar{B}C\bar{D}$ 13	$ABC\bar{D}$ 15	$ABC\bar{D}$ 14
$A\bar{B}$	$AB\bar{C}\bar{D}$ 8	$AB\bar{C}D$ 9	$ABC\bar{D}$ 11	$ABC\bar{D}$ 10

Representation of Truth table on k-map

two-variable k-map

→ The representation of two variable truth table on k-map is given below

A	B	y
0	0	0
0	1	1
1	0	1
1	1	0

A	B	0	1
0	0	0	1
1	1	1	0

A	B	\bar{B}	B
0	0	0	1
1	1	1	0

Three-variable k-map

A	B	C	y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

A	$\bar{B}\bar{C}$	00	01	11	10
0	0	0	0	0	1
1	1	1	1	0	0

A	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
0	0	0	0	1
1	1	1	1	0

four-variable K-maps

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A	B	C	D	y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

CD	00	01	11	10
AB	0000 1 0	0001 0 1	0011 1 3	0010 0
CD	0100 1 4	0101 1 5	0111 1 7	0110 0
CD	1100 1 12	1101 0 13	1111 1 15	1100 1 14
CD	1000 0 8	1001 0 9	1011 0 11	1000 1 10

CD	$\bar{C}D$	$\bar{C}\bar{D}$	CD	$C\bar{D}$
AB	1	0	1	0
$\bar{A}B$	0	1	3	2
$\bar{A}B$	1	4	5	6
AB	4	0	1	1
$\bar{A}B$	0	0	0	1
AB	8	9	11	10

Plot Boolean Expression $y = AB\bar{C} + ABC + \bar{A}\bar{B}C$ on the k-map

A	B	C	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

BC	00	01	11	10
0	0	1	0	0
1	0	0	1	1

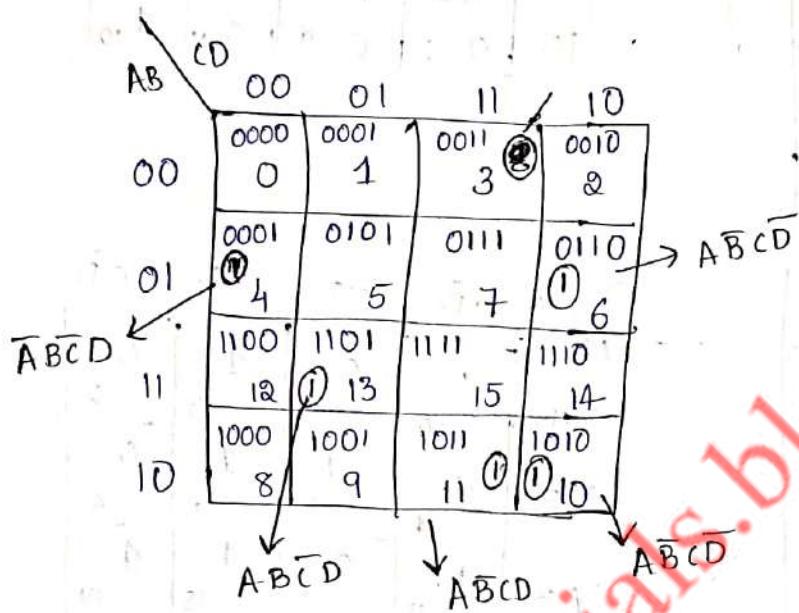
Ex: plot Boolean expression $y = \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}CD + ABC\bar{D}$ on the k-map.

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$$y = \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}CD + ABC\bar{D}$$

$$\hat{=} 0100 + 1010 + 0110 + 1011 + 1101$$

$$\hat{=} m_4 + m_{10} + m_6 + m_{11} + m_{13}$$



Representation of standard pos on k-map

Three variable k-map

A	B	C	Maxterm (M)
0	0	0	$\bar{A}+\bar{B}+C$ (M_0)
0	0	1	$\bar{A}+\bar{B}+\bar{C}$ (M_1)
0	1	0	$\bar{A}+\bar{B}+C$ (M_2)
0	1	1	$\bar{A}+\bar{B}+\bar{C}$ (M_3)
1	0	0	$\bar{A}+B+C$ (M_4)
1	0	1	$\bar{A}+B+\bar{C}$ (M_5)
1	1	0	$\bar{A}+\bar{B}+C$ (M_6)
1	1	1	$\bar{A}+\bar{B}+\bar{C}$ (M_7)

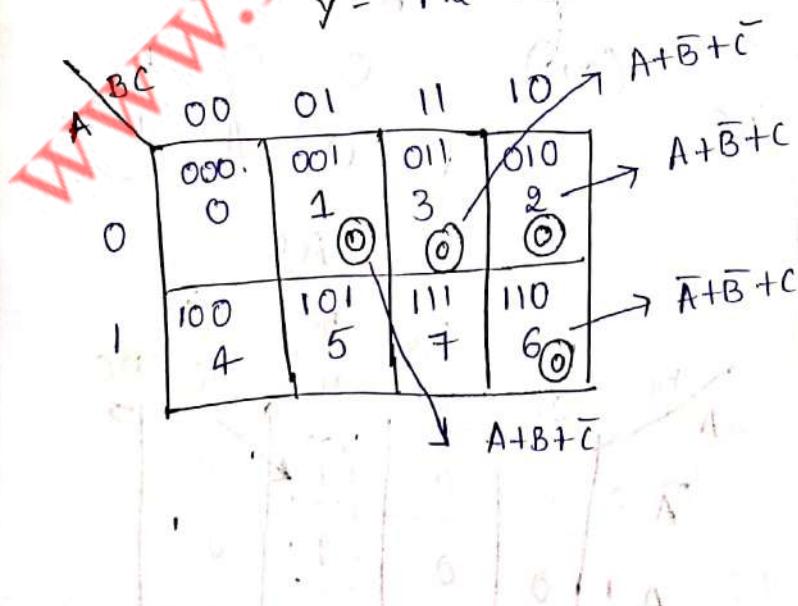
A	B	C	D	Maxterm (M)
0	0	0	0	$A+B+C+D$ (M_0)
0	0	0	1	$A+B+C+\bar{D}$ (M_1)
0	0	1	0	$A+B+\bar{C}+D$ (M_2)
0	0	1	1	$A+B+\bar{C}+\bar{D}$ (M_3)
0	1	0	0	$A+\bar{B}+C+D$ (M_4)
0	1	0	1	$A+\bar{B}+C+\bar{D}$ (M_5)
0	1	1	0	$A+\bar{B}+\bar{C}+D$ (M_6)
0	1	1	1	$A+\bar{B}+\bar{C}+\bar{D}$ (M_7)
1	0	0	0	$\bar{A}+B+C+D$ (M_8)
1	0	0	1	$\bar{A}+B+C+\bar{D}$ (M_9)
1	0	1	0	$\bar{A}+B+\bar{C}+D$ (M_{10})
1	0	1	1	$\bar{A}+B+\bar{C}+\bar{D}$ (M_{11})
1	1	0	0	$\bar{A}+\bar{B}+C+D$ (M_{12})
1	1	0	1	$\bar{A}+\bar{B}+C+\bar{D}$ (M_{13})
1	1	1	0	$\bar{A}+\bar{B}+\bar{C}+D$ (M_{14})
1	1	1	1	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$ (M_{15})

Ex: plot Boolean Expression $y = (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)(A+B+\bar{C})$

on the K-map:

$$y = M_2 \cdot M_3 \cdot M_6 \cdot M_1$$

[Maxterm k9 "1" represent
minter "0" represent]



Ex-5 plot Boolean Expression $y = (A+B+C+\bar{D})(A+\bar{B}+\bar{C}+\bar{D})(A+\bar{B}+\bar{C}+\bar{D})$
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$$(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{C}+\bar{D})$$

$$Y = (A+B+C+\bar{D}) \cdot (A+\bar{B}+\bar{C}+\bar{D}) (A+\bar{B}+\bar{C}+\bar{D}) (\bar{A}+\bar{B}+\bar{C}+\bar{D})$$

$$Y = M_1 \cdot M_6 \cdot M_3 \cdot M_{13} \cdot M_{14}$$

		CD		AB				(A+B+C+\bar{D})				(A+\bar{B}+\bar{C}+\bar{D})				(A+\bar{B}+\bar{C}+\bar{D})			
		00	01	11	10														
		00	0000 0	0001 4	0011 3	0010 2													
		01	0100 4	0101 5	0111 7	0110 6													
		11	1100 12	1101 13	1111 15	1110 14													
		10	1000 8	1001 9	1011 11	1010 10													

Grouping cells for simplification:

Grouping two adjacent ones (pair)

$$\begin{aligned} \underline{\text{Ex-6}} \quad Y &= \bar{A}\bar{B}C + \bar{A}B\bar{C} \\ &= \bar{A}C(\bar{B} + \bar{B}) \\ &= \bar{A}C \end{aligned}$$

		BC	$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC	$\bar{A}C$
		00	01	11	10	0	
		$\bar{A}0$	0	1	1	0	
		A 1	0	0	0	0	

(or)

A	B	C
0	0	1
0	1	1

		$\bar{B}\bar{C}$	$\bar{B}C$	$B\bar{C}$	BC	$\bar{B}C$
		00	01	11	10	0
		$\bar{A}0$	0	0	1	0
		A 1	0	0	1	0

$$\begin{aligned} \underline{\text{Ex-7}} \quad Y &= \bar{A}BC + ABC = BC \\ &= BC(A + \bar{A}) \\ &= BC \end{aligned}$$

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$$\text{Ex: } ③ Y = A\bar{B}\bar{C} + A\bar{B}C$$

$$= A\bar{C}(\bar{B} + B)$$

$$= A\bar{C}$$

	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	$B\bar{C}$ 10	BC 11
\bar{A}	0	0	0	0
A	1	1	0	1

$$\text{Ex: } ④ Y = \bar{A}\bar{B}\bar{C}D + A\bar{B}\bar{C}D$$

$$= \bar{B}\bar{C}D(A + \bar{A})$$

$$= \bar{B}\bar{C}D$$

	$\bar{C}D$ 00	$\bar{C}D$ 01	$\bar{C}D$ 10	CD 11	CD 10
$\bar{A}B$	0	1	0	0	0
$\bar{A}B$	0	0	0	0	0
$\bar{A}B$	0	0	0	0	0
$\bar{A}B$	0	1	0	0	0

$$\text{Ex: } ⑤ Y = \bar{A}\bar{B}C + \bar{A}\bar{B}C + ABC$$

$$= \bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}\bar{B}C + ABC$$

$$= \bar{A}C(\bar{B} + B) + BC(\bar{A} + A)$$

$$= \bar{A}C + BC$$

	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	$B\bar{C}$ 10	BC 11
\bar{A}	0	1	1	0
A	0	0	1	0

[OR]

A B C

Grouping four adjacent ones (quad)

	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	$B\bar{C}$ 10	BC 11
\bar{A}	0	0	0	0
A	1	1	1	1

Ex-2

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\overline{AB}	\overline{CD}	\overline{CD}	\overline{CD}	\overline{CD}
\overline{AB}	00	01	11	10
\overline{AB}	0	0	1	0
\overline{AB}	0	0	1	0
\overline{AB}	0	0	1	0
\overline{AB}	0	0	1	0

(b) $y = CD$

Ex-3

\overline{AB}	\overline{CD}	\overline{CD}	\overline{CD}	\overline{CD}
\overline{AB}	00	01	11	10
\overline{AB}	0	0	0	0
\overline{AB}	0	1	1	0
BD	0	1	1	0
AB	0	0	0	0
\overline{AB}	0	0	0	0

(3) $y = BD$

Ex-4

\overline{AB}	\overline{CD}	\overline{CD}	\overline{CD}	\overline{CD}
\overline{AB}	00	01	11	10
\overline{AB}	0	0	0	0
\overline{AB}	0	0	0	0
AB	1	0	0	1
\overline{AB}	1	0	0	1
AB	1	0	0	1
\overline{AB}	1	0	0	1

$y = \overline{AD}$

Ex-5

AB	CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	00	1	0	0	1
$\bar{A}B$	01	0	0	0	0
$A\bar{B}$	11	0	0	0	0
AB	10	1	0	0	1

$$\begin{array}{l}
 AB \quad CD \\
 00 \quad 00 \\
 00 \quad 10 \\
 10 \quad 00 \\
 10 \quad 10
 \end{array}$$

Ans: $y = \bar{B}\bar{D}$

$$\begin{aligned}
 &= \bar{B}\bar{C}\bar{D} + \bar{B}\bar{C}D \\
 &= \bar{B}\bar{D}(B + \bar{C})
 \end{aligned}$$

Ex-6

AB	CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	00	0	0	0	0
$\bar{A}B$	01	0	0	0	0
$A\bar{B}$	11	1	1	1	1
AB	10	0	1	1	1

Group 2 $\rightarrow AD$

(f) $y = AB + AD + AC$

Grouping eight adjacent under ones (Octet)

Ex-7

AB	CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	00	0	0	0	0
$\bar{A}B$	01	1	1	1	1
$A\bar{B}$	11	1	1	1	1
AB	10	0	0	0	0

(a) $y = B$

Ex-8

AB	CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	00	0	1	1	0
$\bar{A}B$	01	0	1	1	0
$A\bar{B}$	11	0	1	1	0
AB	10	0	1	1	0

(b) $y = D$

\bar{AB}	CD	\bar{CD}	$\bar{C}D$	CD	$C\bar{D}$
\bar{AB}	00	01	11	10	00
\bar{AB}	00	01	-1	-1	11
\bar{AB}	01	0	0	0	0
\bar{AB}	11	0	0	0	0
\bar{AB}	10	1	1	1	1

\bar{AB}	CD	\bar{CD}	$\bar{C}D$	CD	$C\bar{D}$
\bar{AB}	00	01	11	10	00
\bar{AB}	00	0	0	1	1
\bar{AB}	01	1	0	0	1
\bar{AB}	11	1	0	0	1
\bar{AB}	10	1	0	0	1

A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	1
0	0	1	0
0	0	0	0
1	0	0	0
0	0	0	1
0	0	1	1
1	0	1	0

A	B	C	D
0	0	0	0
0	1	0	0
1	1	0	0
0	0	0	0
0	0	1	0
0	1	1	0
1	0	1	0

Simplifications of SOP Expressions

→ from the above discussion we can outline generalized procedure to simplify Boolean Expressions as follows:

1. plot the k-map and place 1's in those cells corresponding to the 1's in the truth table. or sum of product expression place 0's in other cells.

2. check the k-map for adjacent 1's and encircle those 1's which are not adjacent to any other 1's. these are called isolated 1's.

3. check for those 1's which are adjacent to only one other 1 and encircle such pairs.

4. check for quads and octets of adjacent 1's even if it contains some 1's that have already been circled. while doing this make sure that there are minimum no. of groups.

5. combine any pairs necessary to include any 1's that have not yet been grouped.

6. From the simplified expression by summing product terms of all groups.

Ex: minimize the expression $Y = A\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$$\begin{aligned} Y &= 101 + 001 + 011 + 100 + 000 \\ &= m_5 + m_1 + m_3 + m_6 + m_0 \\ &= \Sigma m(5, 3, 4, 0) \end{aligned}$$

Step 1: The k-map for three variables and its plotted according to the given expression.

Step 2: There are no isolated 1's

Step 3: 1 in the cell 3 adjacent only to 1 in the cell 4. This pair is combined and referred to as group 1

		$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
		00	01	11	10
$A\bar{B}$	0	① 0	① 1	① 3	2
	1	100	101	111	110
A	0	① 4	① 5	7	6
	1	8	8	8	8

Step 4: There is no octet, but there is a quad cells 0, 1, 4 and 5 from a quad.

This quad is combined and referred to as a group.

Step 5: All ~~the~~^{4x} have already been grouped

		$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
		00	01	11	10
$A\bar{B}$	0	1	1	1	0
	1	1	1	0	0
A	0	1	1	1	0
	1	1	1	1	0

Step 6: Each group generates a term

In expression for y . In group 1 B variable is eliminated and in group 2 Variables A and C are eliminated and we get

$$y = \overline{A}C + \overline{B}$$

	BC	$\overline{B}\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$	time/jntukonlinebits
A	00	01	01	10	10	AC
B	0	0	1	1	0	
1	1	1	1	1	0	
2	1	1	1	1	0	
3	1	1	1	1	0	
4	1	1	1	1	0	
5	1	1	1	1	0	
6	1	1	1	1	0	
7	1	1	1	1	0	
8	1	1	1	1	0	
9	1	1	1	1	0	
10	1	1	1	1	0	
11	1	1	1	1	0	
12	1	1	1	1	0	
13	1	1	1	1	0	
14	1	1	1	1	0	
15	1	1	1	1	0	
16	1	1	1	1	0	

Ex: minimize the Expression

$$y = \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + A\overline{B}\overline{C}\overline{D} + AB\overline{C}D + A\overline{B}\overline{C}D + \overline{A}\overline{B}\overline{C}\overline{D}$$

$$= 0100 + 0101 + 1100 + 1101 + 1001 + 0010$$

$$= m_4 + m_5 + m_{12} + m_{13} + m_9 + m_2$$

$$= \sum m(4, 5, 12, 13, 9, 2)$$

		CD	$\overline{C}\overline{D}$	$\overline{C}D$	CD	$\overline{C}\overline{D}$	Group ③	$\overline{A}B\overline{C}\overline{D}$	G_{11}	G_{12}	G_{13}
		AB	00	01	11	10		A	B	C	D
$\overline{A}\overline{B}$	00	0	1	3	2	①		0	1	0	0
$\overline{A}\overline{B}$	01	4	5	7	6			0	1	0	0
$\overline{A}B$	11	10	13	15	14			1	0	0	0
$A\overline{B}$	10	8	9	11	10	①		1	0	1	1

$\overline{A}B\overline{C}\overline{D} + A\overline{C}D + B\overline{C}$

Group ① Group ② Group ③

$$y = \overline{A}\overline{B}\overline{C}\overline{D} + A\overline{C}D + B\overline{C}$$

* Simplify the logic function specified by the truth table by using the Karnaugh map method. y is the output variable and A, B and C are the input variables.

A	B	C	y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$\begin{aligned}
 y &= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC \\
 &= m_0 + m_3 + m_4 + m_7 \\
 &= \sum m(0, 3, 4, 7)
 \end{aligned}$$

		BC		A	
		00	01	11	10
		0	1	3	2
0	0	4		1	2
1	0	4	5	7	6
1	1	1		1	0

Group 1			Group 2		
<u>A</u>	<u>B</u>	<u>C</u>	<u>A</u>	<u>B</u>	<u>C</u>
0/0	0	0	0/1	1	1
	0	0	1/1	1	1
			<u>BC</u>	<u>BC</u>	<u>BC</u>

$$\therefore y = \bar{B}\bar{C} + BC$$

Ex: Reduce the following function using K-map technique.

$$f(A, B, C, D) = \sum m(0, 1, 4, 8, 9, 10)$$

$$= m_0 + m_1 + m_4 + m_8 + m_9 + m_{10}$$

$$\begin{aligned}
 &= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + \\
 &= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D}
 \end{aligned}$$

AB\CD	00	01	11	10
00	0 1 4 12	1 5	3	2
01			7	6
11			15	14
10	8 9	11	10	1

Product of sum simplification:

1. plot the k-map and place 0s in those cells corresponding to the 0s in the truth table or minterms in the product of sum expression.
 2. check the k-map for adjacent 0s and encircle those 0s which are not adjacent to any other 0s. These are called isolated 0s.
 3. check for those 0s which are adjacent to only one other 0 and encircle such pairs.
 4. check for quads and octets of adjacent 0s even if it contains some 0s that have already been encircled. while doing this make sure that there are minimum no. of groups.
 5. Combine any pairs necessary to include any 0s that have not yet been grouped.
 6. From the simplified pos expression for F by taking product of sum terms of all the groups.
- To get familiar with these steps we will solve some Examples.

Ex minimize the Expression

$$y = (A+B+C)(A+\bar{B}+C)(\bar{A}+\bar{B}+C)(\bar{A}+B+C)(A+B+C)$$

$$= (001)(011)$$

$$= (A+B+C) = M_1, \quad (A+\bar{B}+C) = M_3, \quad (\bar{A}+\bar{B}+C) = M_7$$

$$= (\bar{A}+B+C) = M_4, \quad (A+B+C) = M_0$$

$$(\bar{A}+\bar{B}+C) = M_5$$

A	B	C	D
1	0	0	0
1	0	0	0

$$Y = A\bar{B}\bar{D} + \bar{B}\bar{C} + \bar{A}B\bar{C}D$$

A	B	C	D
0	0	0	0
0	0	0	1
1	0	0	0

A	B	C	D
1	0	0	1
1	0	1	0

A	B	C
0	1	0
1	0	0

Step 1: (a) shows the k-map for three Variable A, B, C and its plotted according to given minterms

	BC	00	01	11	10
A	0	0	0	0	0
	4	5	7	6	
1	0	0	0	0	0

Step 2: three are no isolated 0s

Step 3: 0 in the cell 4, 98 adjacent only to 0 in the cell 0 and 0 in cell 7 98 adjacent only to 0 in the cell 3. These two pairs are combined and referred to as group 1 and group 2 respectively.

	BC	00	01	11	10
A	0	0	1	3	2
	0	0	0	0	0
1	0	0	0	0	0

Step 4: There are no quads and octets

Step 5: The 0 in the cell 1 can be combined with 0 in the cell 3 to be from a pair. This pair is referred to as group 3

Step 6: In group 1 and in group 2, A is eliminated, where in group 3 variable B is eliminated and we get

	BC	00	01	11	10
A	0	0	1	3	2
	0	0	0	0	0
1	0	0	0	0	0
	0	0	0	0	0

Group 1			Group 2			Group 3		
A	B	C	A	B	C	A	B	C
0	0	0	0	1	1	0	0	1
1	0	0	1	1	1	1	0	1
			<u>BC</u>			<u>BC</u>		<u>AC</u>

$$\bar{Y} = \overline{B\bar{C}} + B\bar{C} + \bar{A}C$$

$$\bar{Y} = Y = \overline{B\bar{C}} + B\bar{C} + \bar{A}C \quad [\text{According to DeMorgan's}]$$

$$Y = \overline{\bar{B}\bar{C}} + B\bar{C} + \bar{A}C$$

$$= (\bar{B}\bar{C}) \cdot (\bar{B}\bar{C}) \cdot (\bar{A}C)$$

$$= (\bar{B} + \bar{C}) \cdot (\bar{B} + \bar{C}) \cdot (\bar{A} + C)$$

$$Y = (B + C)(\bar{B} + \bar{C})(\bar{A} + C)$$

$$a \quad d \quad A$$

$$0 \quad 0 \quad 0 \quad 0 \quad 3$$

$$0 \quad 0 \quad 0 \quad 1$$

$$1 \quad 1 \quad 1 \quad 1$$

$$0 \quad 1 \quad 1 \quad 1$$

$$0 \quad 1 \quad 1 \quad 1$$

Ex: Minimize the following expression in the pos form t.me/jntukonlinebits

$$Y = (\bar{A} + \bar{B} + C + D)(\bar{A} + \bar{B} + \bar{C} + D)(\bar{A} + \bar{B} + \bar{C} + \bar{D})(\bar{A} + B + C + D)(A + \bar{B} + \bar{C} + D) \\ (\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + B + C + D)(\bar{A} + \bar{B} + C + \bar{D})$$

Sol: $(\bar{A} + \bar{B} + C + D) = M_{12}$, $(\bar{A} + \bar{B} + \bar{C} + D) = M_{14}$, $(\bar{A} + \bar{B} + \bar{C} + \bar{D}) = M_{15}$

$$(\bar{A} + B + C + D) = M_8, (A + \bar{B} + \bar{C} + D) = M_6, (A + \bar{B} + \bar{C} + \bar{D}) = M_7$$

$$(A + B + C + D) = M_0 \text{ and } (\bar{A} + \bar{B} + C + \bar{D}) = M_{13}$$

Step 1: Shows the k-map for four variable and it plotted according to given max terms.

Step 2: There are no isolated 1's

Step 3: 0 in the cell 0 & 8 adjacent

Only to 0 in the cell 11 this pair is combined and referred to as group 1.

Step 4: There are two quads cells 12, 13, 14 and 15 form a quad

1 and cells 6, 7, 14, 15 forms a quad 2. These two quads are referred to as group 2 and group 3 respectively.

Step 5: All 0s have already been grouped

Step 6: In group 1, Variable A is eliminated

In group 2, Variable C and D are eliminated and 01

in group 3 variable A and D are eliminated.

∴ we get simplified pos expression

		CD	C+D	C+D̄	C̄+D	C̄+D̄
		AB	00	01	11	10
A+B	00	0	1	3	2	
		0	4	5	7	6
A+B̄	01	12	13	15	14	
		0	0	0	0	0
A+B	10	8	9	11	10	
		0	0	0	0	0

		CD	Group 1		
		AB	00	01	11
	00	1	0	1	3
	01	0	4	5	7
	11	12	0	13	15
	10	8	0	9	11

		CD	Group 2		
		AB	00	01	11
	00	1	0	1	3
	01	0	4	5	7
	11	12	0	13	15
	10	8	0	9	11

		CD	Group 3		
		AB	00	01	11
	00	1	0	1	1
	01	0	0	1	0
	11	1	1	1	1
	10	0	1	1	0

Group 1		
A	B	CD
0	0	00
1	0	00
		<u><u>B̄ C̄ D̄</u></u>

Group 2		
A	B	CD
1	1	00
1	1	01
		<u><u>1111</u></u>
		<u><u>1110</u></u>

Group 3		
A	B	CD
0	1	11
0	1	10
1	1	11
1	1	10

$$\bar{Y} = \overline{BCD} + AB + BC$$

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$$y = \bar{Y} = \overline{BCD} + AB + BC$$

$$y = (B+C+D) \cdot (\bar{A}+\bar{B}) \cdot (\bar{B}+\bar{C})$$

Incompletely specified functions (Don't Care terms or conditions)

e.g.

A	B	C	y
0	0	0	0
1	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	X
1	1	1	X

Sol:

A	BC		00	01	11	10
	00	01	10	11	01	00
0	0	1	1	1	0	0
1	4	5	7	X	X	6

Don't Care Conditions

Group 1

A	BC		00	01	11	10
	00	01	10	11	01	00
0	0	1	1	1	0	0
1	4	5	7	1	0	6

Group 1

$$\therefore Y = C$$

A	B	C
0	0	1
0	1	1
1	0	1
1	1	1

Describing Incomplete Boolean function

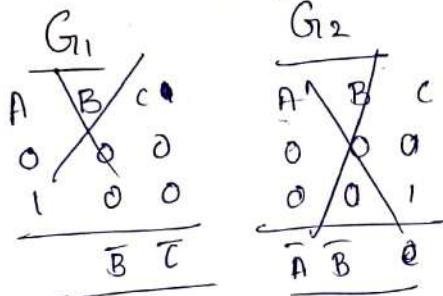
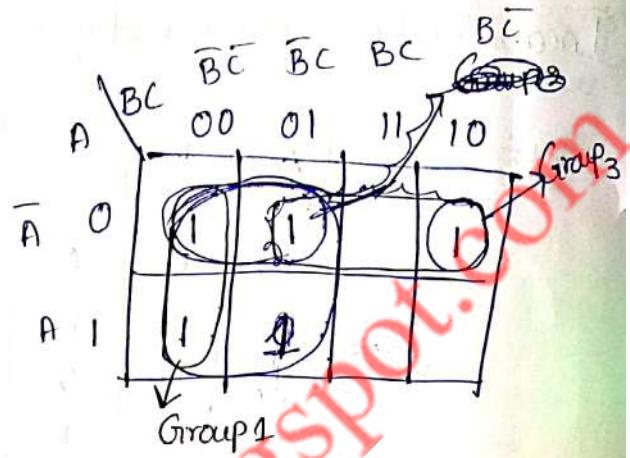
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$$\text{Ex: } f(A, B, C) = \sum m(0, 2, 4) + d(1, 5)$$

$$f(A, B, C) = \prod M(2, 5, 7) + d(1, 3)$$

$$f(A, B, C) = \sum m(0, 2, 4) + d(1, 5)$$

		BC	00	01	11	10
		A	0	1	3	2
0	0	①	X			①
	1	4	5	7		6

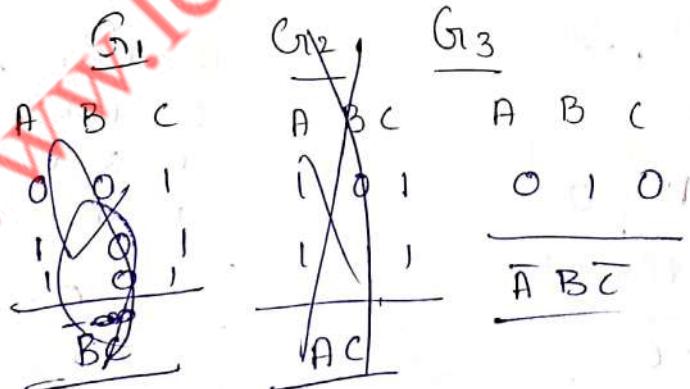
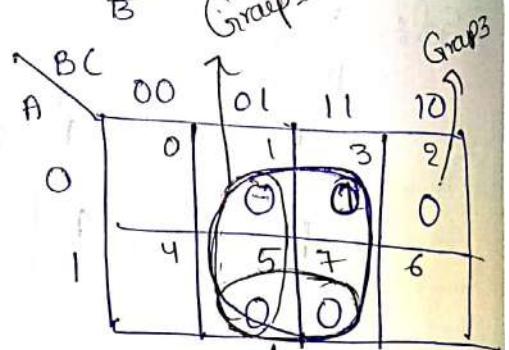


$$y = \bar{B} \bar{C} + \bar{A} \bar{B} + \bar{B} \bar{C} //$$

$\begin{array}{l} \text{Group 1} \\ \hline A \quad B \quad C \\ 0 \quad 0 \quad 0 \\ 0 \quad 0 \quad 1 \\ 1 \quad 0 \quad 0 \\ \hline \bar{B} \end{array}$
 $y = \bar{B} + \bar{B} \bar{C}$

$$f(A, B, C) = \prod M(2, 5, 7) + d(1, 3)$$

		BC	00	01	11	10
		A	0	1	3	2
0	0	0	X	X		0
	1	4	5	7		6



$$\begin{array}{l} G_1 \\ \hline A \quad B \quad C \\ 0 \quad 0 \quad 1 \\ 0 \quad 1 \quad 1 \\ 1 \quad 0 \quad 1 \\ 1 \quad 1 \quad 1 \\ \hline C \end{array}$$

$$y = \bar{A} \bar{B} \bar{C} + \bar{B} \bar{A} \bar{C} //$$

Don't care conditions in logic Design

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	A	B	C	D	P
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	-
11	1	0	1	1	-
12	1	1	0	0	-
13	1	1	0	1	-
14	1	1	1	0	-
15	1	1	1	1	-

$$P = \Sigma_m(1, 2, 4, 7, 8) + \Sigma_d(10, 11, 12, 13, 14, 15)$$

: find the reduced SOP form of the following function.

$$f(A, B, C, D) = \Sigma_m(1, 3, 7, 11, 15) + \Sigma_d(0, 2, 4)$$

$$f(A, B, C, D) = \Sigma_m(1, 3, 7, 11, 15) + \Sigma_d(0, 2, 4)$$

	AB	CD		
AB	00	01	11	10
00	X	1	1	X
01	X	5	7	6
11	12	13	15	14
10	8	9	14	10

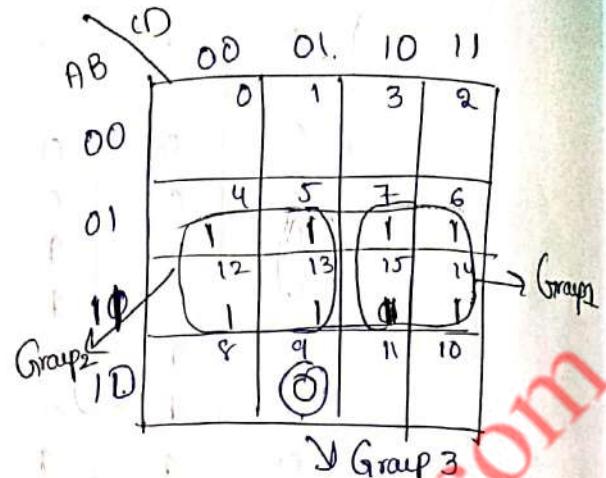
AB	CD	00	01	11	10		Group 2	G ₁₁	G ₂
00		1	1	1	1			A B C D	A B C D
01		0		1				0 0 0 0	0 0 0 1
11				1				0 0 1 1	0 0 1 1
10					1			1 0 1 1	0 0 1 0
								1 0 1 1	
									AB
									CD

$$\therefore Y = \bar{A}\bar{B} + CD$$

Ex: Reduce the following function using k-map technique t.me/jntukonlinebits

$$f(A, B, C, D) = \sum m(5, 6, 7, 12, 13) + \sum d(4, 9, 14, 15)$$

		CD	00	01	10	11
		AB	00	01	10	11
00	01	0	1	3	2	
		4	5	7	6	
10	11	X	1	1	1	
		12	13	15	14	
10	11	1	1	X	X	
		8	9	11	10	
		X				



G₁

G₂

$$\begin{array}{l} A \ B \ C \ D \\ \hline 0 \ 1 \ 0 \ 0 \\ 0 \ 1 \ 0 \ 1 \\ 1 \ 1 \ 0 \ 0 \\ 1 \ 1 \ 0 \ 1 \end{array}$$

$$\begin{array}{l} A \ B \ C \ D \\ \hline 0 \ 1 \ 1 \ 0 \\ 0 \ 1 \ 1 \ 1 \\ 1 \ 1 \ 1 \ 0 \\ 1 \ 1 \ 1 \ 1 \end{array}$$

$$A \ B \ C \ D$$

$$1 \ 0 \ 0 \ 1$$

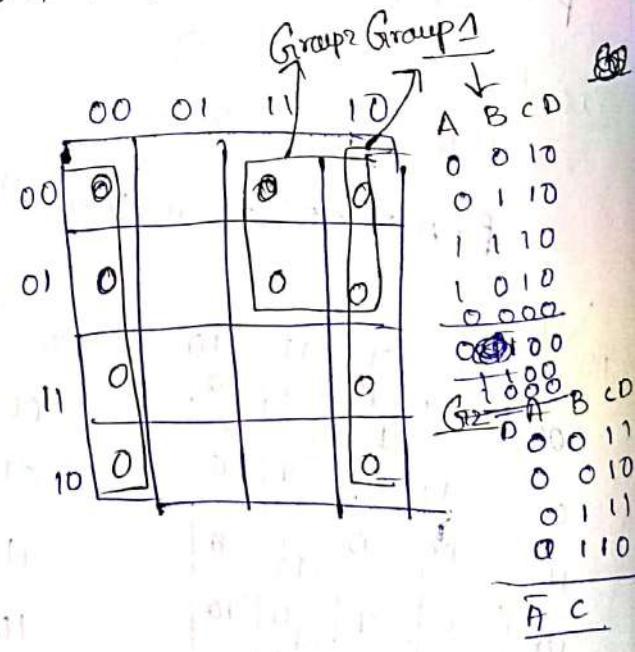
$$\overline{A \ B \ C \ D}$$

$$Y = \overline{A \ B \ C \ D} + \overline{B \ C} + BC$$

Ex: Reduce the following function by using k-map technique

$$f(A, B, C, D) = \sum m(0, 3, 4, 7, 8, 10, 12, 14) + \sum d(2, 6)$$

		CD	00	01	11	10
		AB	00	01	11	10
00	01	0	1	0	X	2
		4	5	0	X	6
11	10	0	0	0	0	
		12	13	15	14	
10	11	0	0	0	0	
		8	9	11	10	
		X				

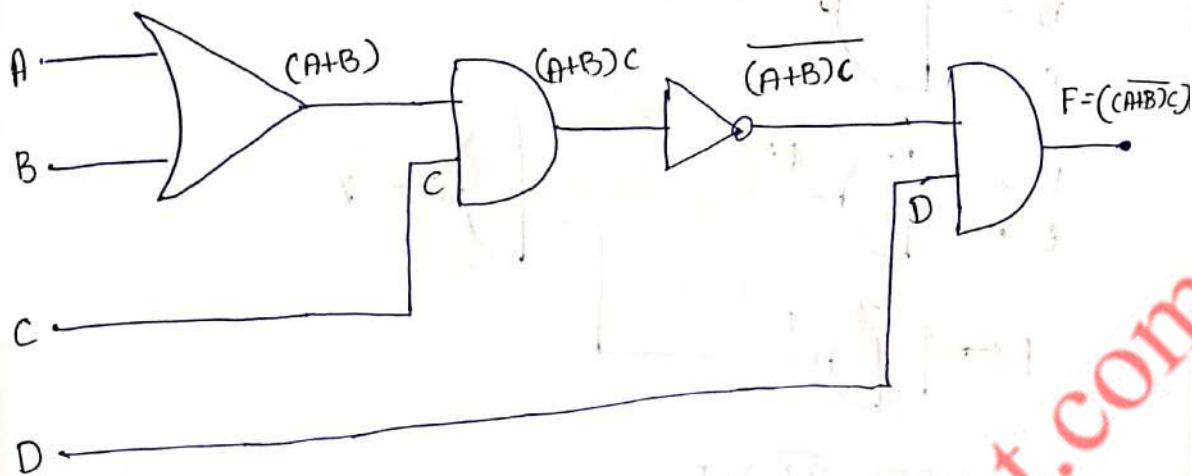


$$X = \overline{D} + \overline{A}C$$

$$\begin{aligned} Y &= \overline{Y} = \overline{\overline{D} + \overline{A}C} \\ &= (\overline{D}) \cdot (\overline{\overline{A}C}) \\ &= D \cdot (A + \overline{C}) \end{aligned}$$

$$* f(A+B, C, D) =$$

Ex: $F = \overline{(A+B)}C D$ to implement logic Design?



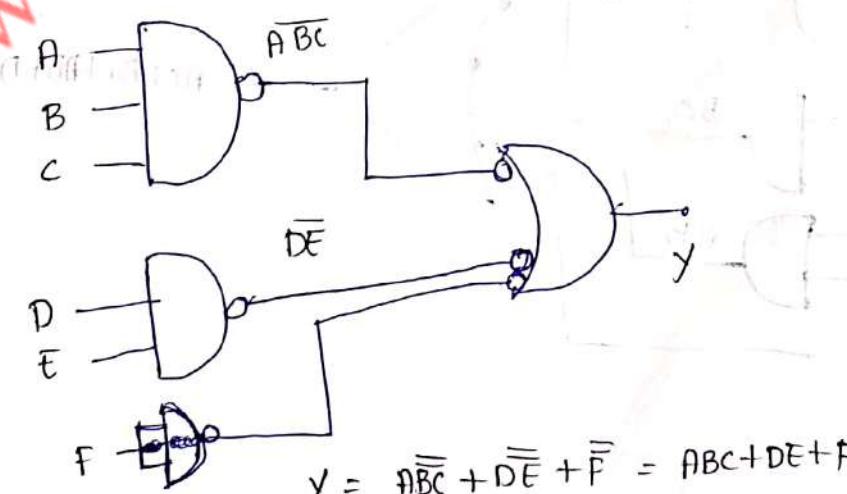
NAND to NAND Implementation:

$$Y = ABC + DEF$$

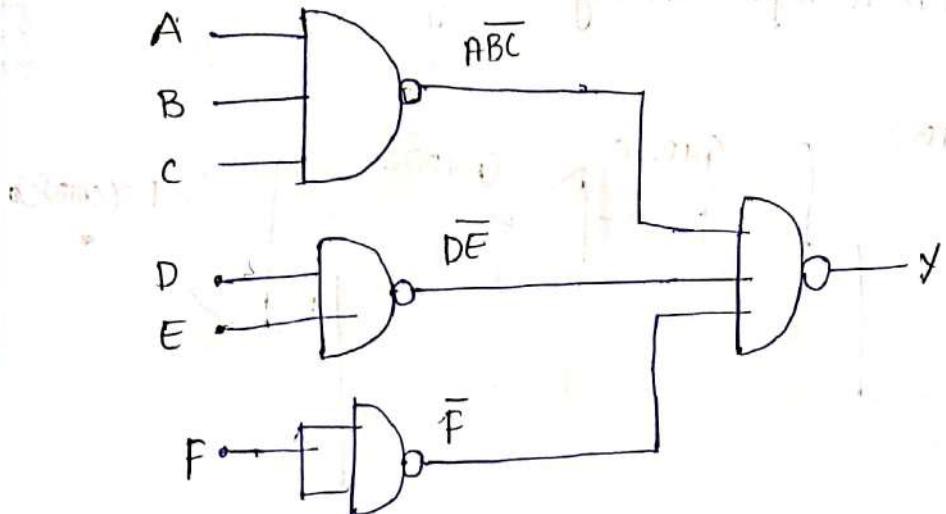
(a) AND - OR



(b) NAND - Bubble OR



$$Y = \overline{ABC} + \overline{DE} + \overline{F} = ABC + DEF$$

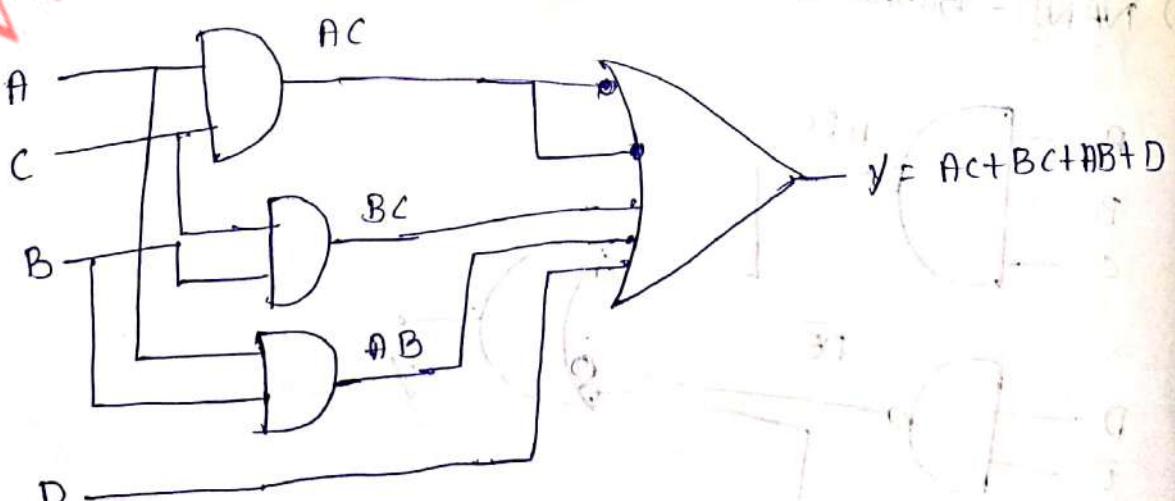
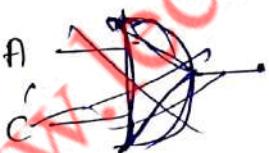


$$\begin{aligned}
 Y &= (\overline{ABC}) \cdot (\overline{DE}) \cdot \overline{F} \\
 &= (\overline{\overline{ABC}}) + (\overline{\overline{DE}}) + \overline{F} \\
 &= ABC + DE + F
 \end{aligned}$$

Ex: $y = AC + ABC + \overline{ABC} + AB + D$ Implement the following Boolean function with NANA to NAND logic.

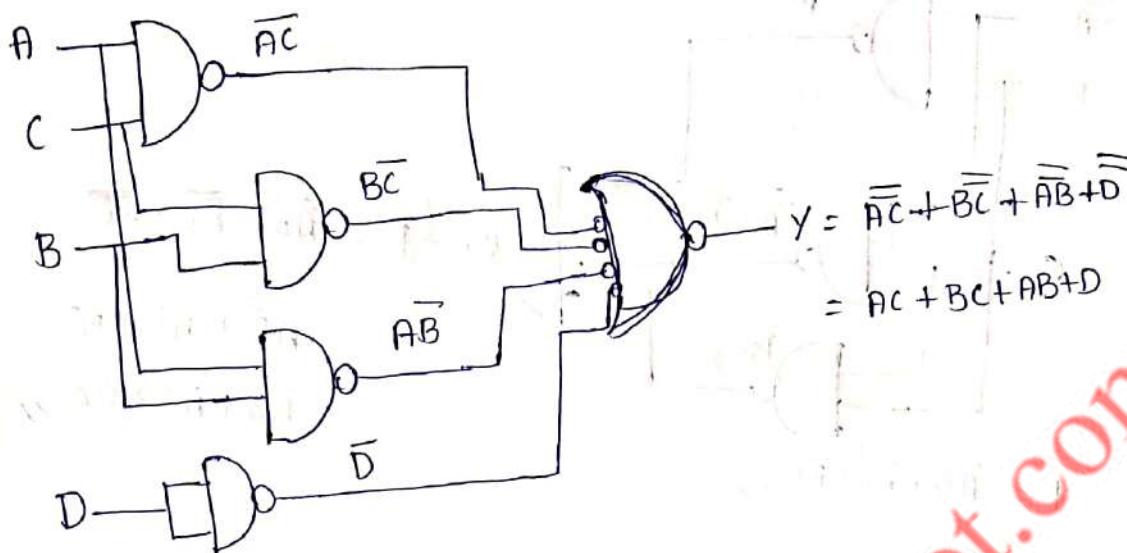
$$\begin{aligned}
 y &= AC + ABC + \overline{ABC} + AB + D \\
 &= AC + BC(A + \overline{A}) + AB + D \\
 y &= AC + BC + AB + D
 \end{aligned}$$

(Q) AND - OR



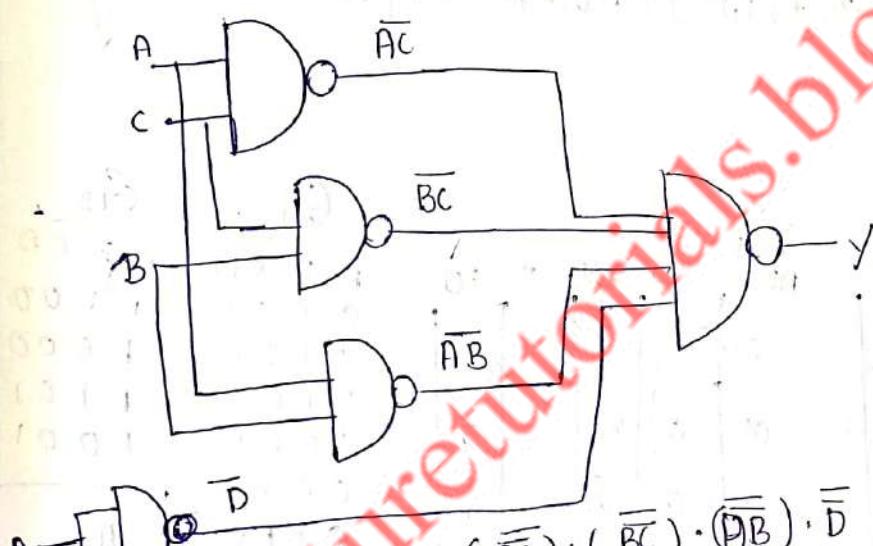
(q) NAND - Bubble OR

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$$\begin{aligned}Y &= \bar{AC} + \bar{BC} + \bar{AB} + \bar{D} \\&= AC + BC + AB + D\end{aligned}$$

(q) NAND - NAND



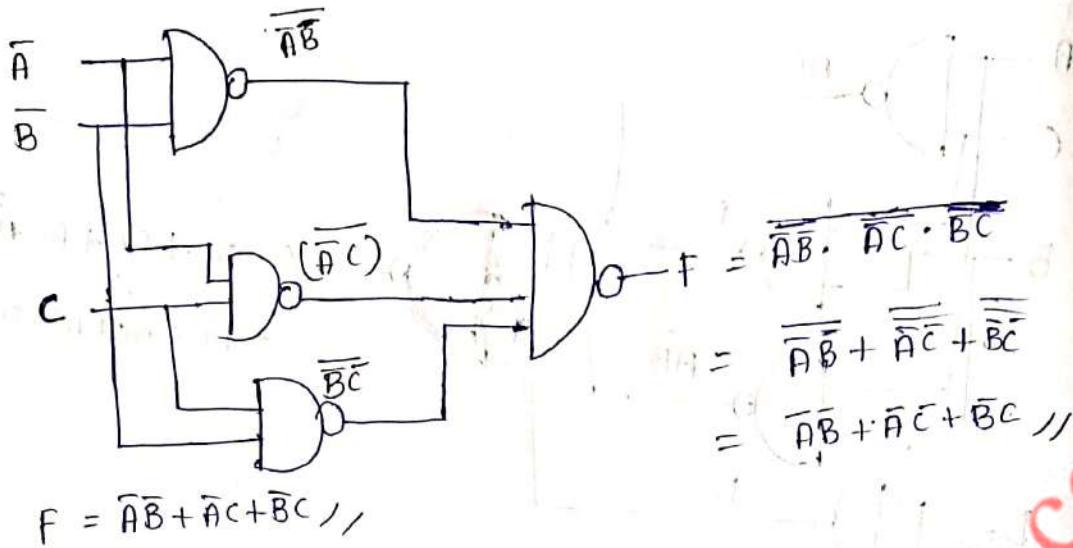
$$\begin{aligned}x &= (\bar{AC}) \cdot (\bar{BC}) \cdot (\bar{AB}) \cdot \bar{D} \\&= \bar{AC} + \bar{BC} + \bar{AB} + \bar{D} \\&= AC + BC + AB + D\end{aligned}$$

Eg: Implementation the following Boolean Expression with NAND-NAND logic.

$$F(A, B, C) = \sum m(0, 1, 3, 5)$$

		G1		G2			
		00	01	11	10		
A	0	0	1	1	0	G3	
	1	4	5	7	6		

$$F(A, B, C) = \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}$$



Ex: ① Find the reduced pos-form of the following Equation
 $f(A, B, C, D) = \sum m(1, 3, 7, 11, 15) + d(0, 2, 5)$ Implement and using
 NAND logic?

CD	00	01	11	10	
AB	00	X ₀	1 ₁	1 ₃	X ₂
AB	01	0 ₄	X ₅	1 ₇	0 ₆
AB	11	0 ₁₂	0 ₁₃	1 ₁₅	0 ₁₄
AB	10	0 ₈	0 ₉	1 ₁₁	0 ₁₀

CD	00	01	11	10
AB	00	0	1	0
AB	01	0	X	0
AB	11	0	0	0
AB	10	0	0	0

Group 2 Group 1

$$\begin{array}{l} G_1 \\ \hline \begin{array}{lll} A & B & CD \\ 0 & 0 & 00 \\ 0 & 1 & 00 \\ 1 & 1 & 00 \\ 1 & 0 & 00 \\ 0 & 0 & 10 \\ 0 & 1 & 10 \\ 1 & 1 & 10 \\ 1 & 0 & 10 \end{array} \end{array} \quad \begin{array}{l} G_2 \\ \hline \begin{array}{lll} A & B & CD \\ 1 & 1 & 00 \\ 1 & 0 & 00 \\ 1 & 1 & 01 \\ 1 & 0 & 01 \\ 0 & 0 & 10 \\ 0 & 1 & 10 \\ 1 & 1 & 10 \\ 1 & 0 & 10 \end{array} \end{array}$$

$$\overline{D}$$

$$\bar{x} = \bar{D} + A\bar{C}$$

$$Y = \bar{\bar{x}} = (\bar{D})(\bar{A}\bar{C})$$

$$\therefore Y = D \cdot (\bar{A} + \bar{C})$$

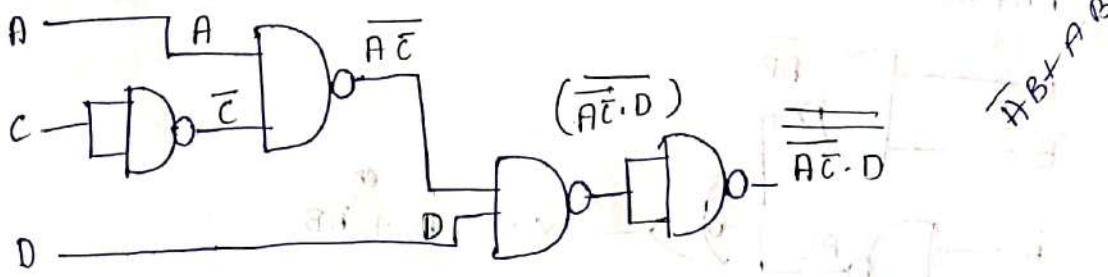
Ex: ② Using k-map, determine the minimal SOP expression and realize the simplified expression using NAND logic?

$$f(w, x, y, z) = \overline{\text{ITM}}(0, 2, 3, 7, 8, 9, 10)$$

$$\text{at: } f(w, x, y, z) = \sum m(1, 4, 5, 6, 11, 12, 13, 14, 15)$$

$$\therefore Y = D \cdot (\bar{A} + C)$$

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$$Y = (\bar{A}\bar{C} \cdot D)$$

$$= (\bar{A} + \bar{C}) \cdot D$$

$$Y = (\bar{A} + C) \cdot D //$$

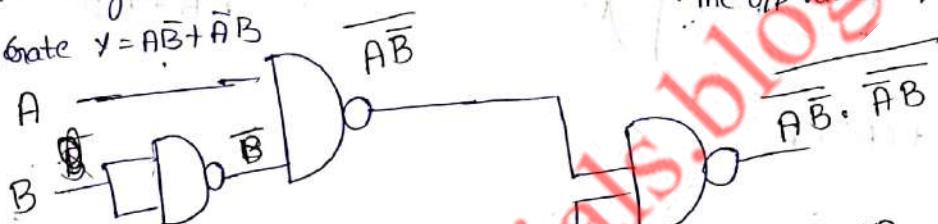
Eg: To implement X-OR gate by using, NAND-NAND logic gate

$$Y = A\bar{B} + \bar{A}B$$

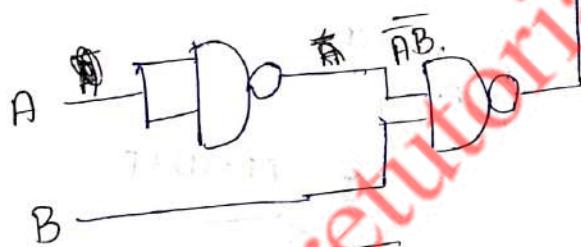
The X-OR gate function is two inputs and one output. The i/p variables are A and B.

The o/p variable Y

$$\text{X-OR gate } Y = A\bar{B} + \bar{A}B$$



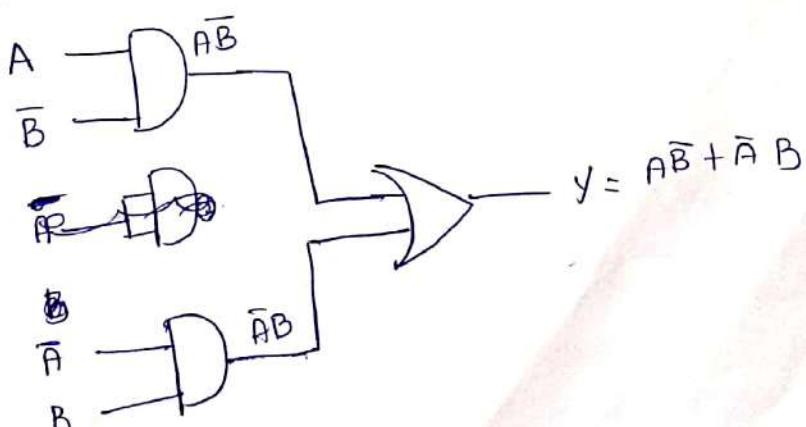
NAND-NAND



$$Y = \bar{\bar{A}\bar{B}} + \bar{\bar{A}}B$$

$$Y = A\bar{B} + \bar{A}B //$$

(a) AND-OR

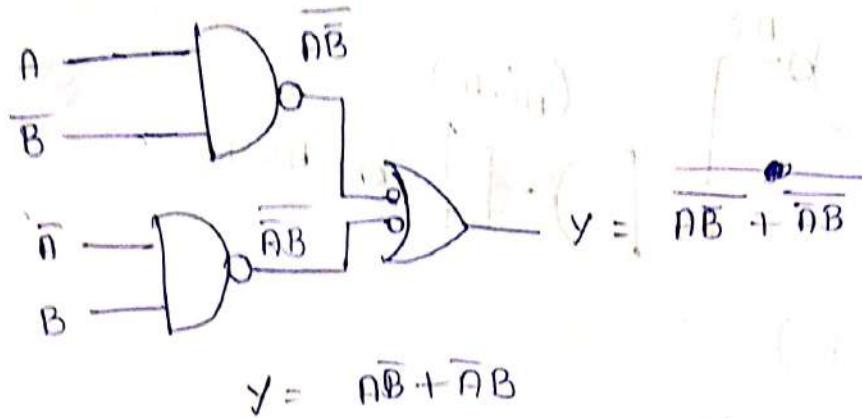


$$Y = A\bar{B} + \bar{A}B$$

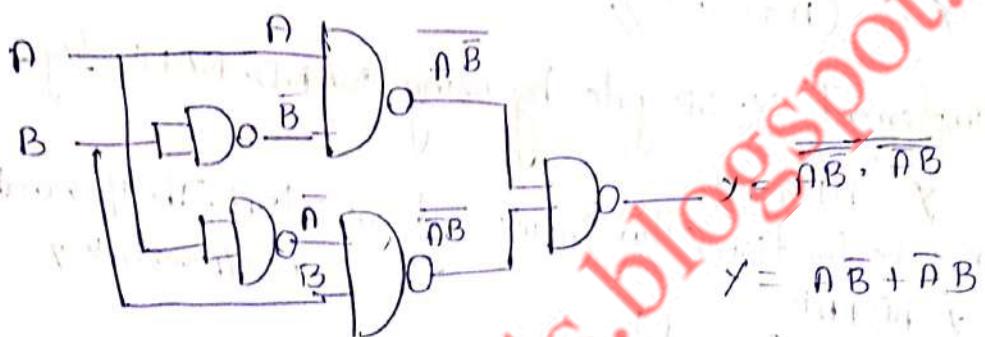
(ii) NAND - Bubbled OR

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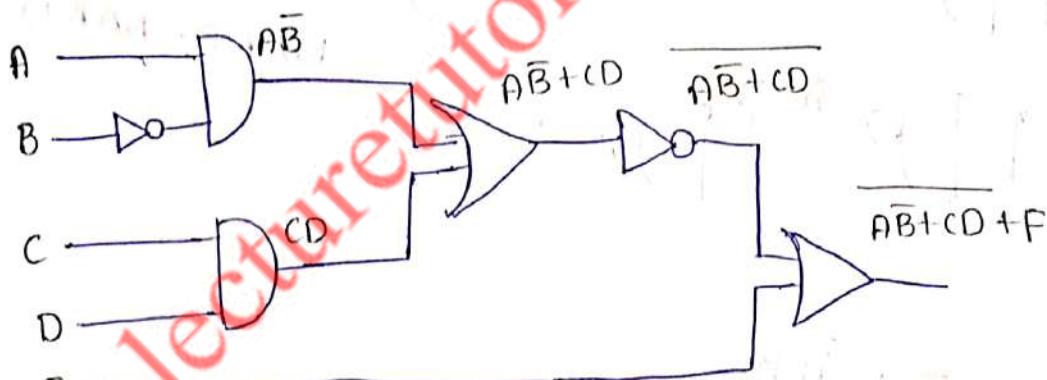
$$Y = A\bar{B} + \bar{A}B$$



(iii) NAND-NAND



$$\text{Ex: } \overline{(AB + CD) + F}$$



Ex: Implement the following function with NAND-to-NAND logic

$$f(A, B, C, D) = \sum_m(0, 1, 3, 5)$$

$$Y = AC + BC + AB + D$$

NOR - NOR Implementation:

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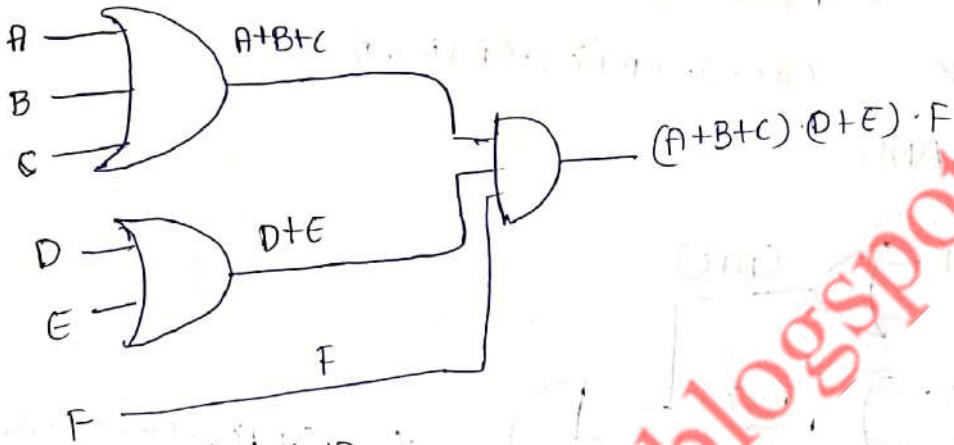
(i) OR-AND

(ii) NOR-Bubbled AND

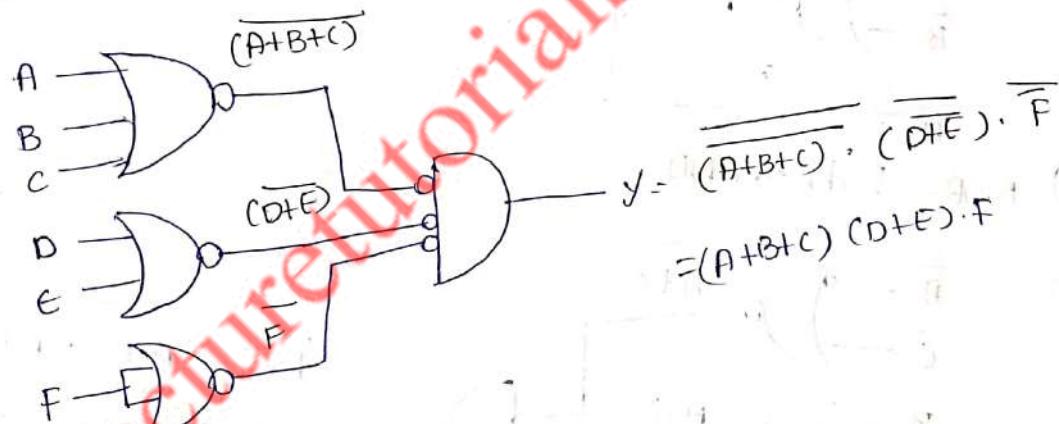
(iii) NOR-NOR

$$\text{Ex: } y = (A+B+C) \cdot (D+E) \cdot F$$

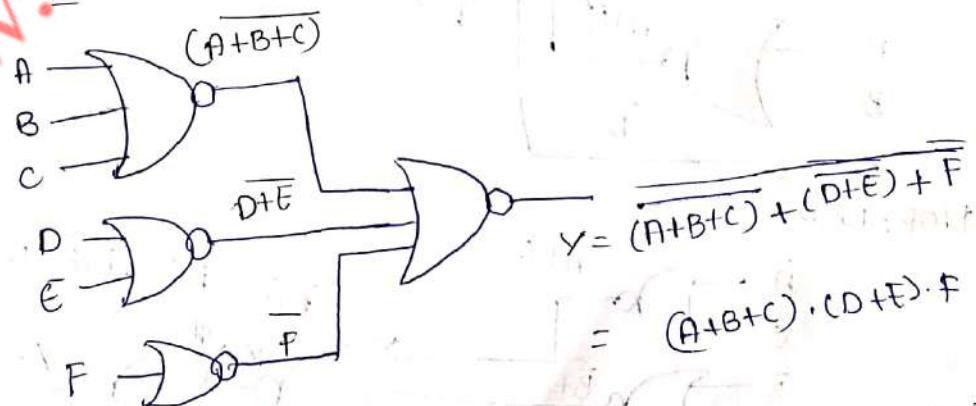
(i) OR-AND



(ii) NOR-Bubbled AND



(iii) NOR-NOR



$$[\because \overline{A+B} = \overline{A} \cdot \overline{B}]$$
$$[\overline{\overline{A}} = A]$$

Ex :- Implement the following Boolean function with NOR-NOR logic

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$$Y' = AC + BC + AB + D$$

The given Boolean Expression

$$Y = AC + BC + AB + D$$

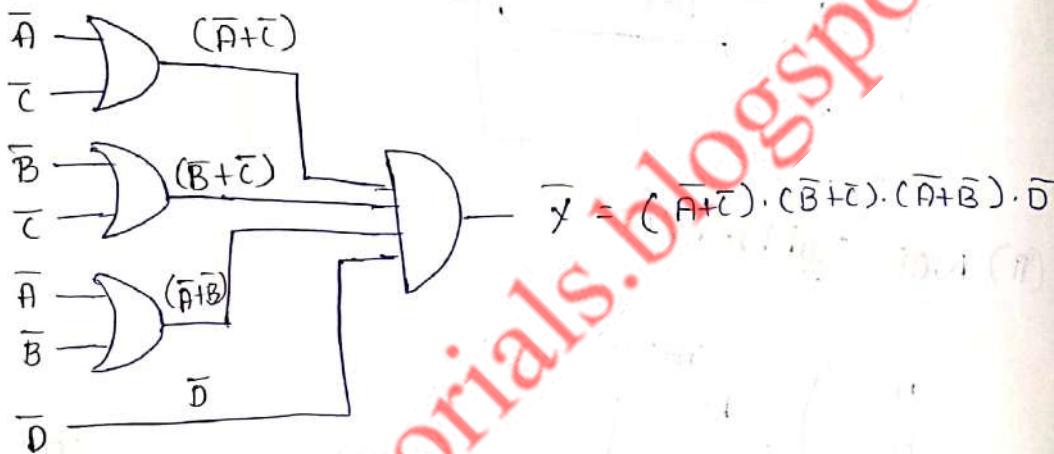
Assumption . By using duality theorem

$$\bar{Y} = \overline{AC + BC + AB + D}$$

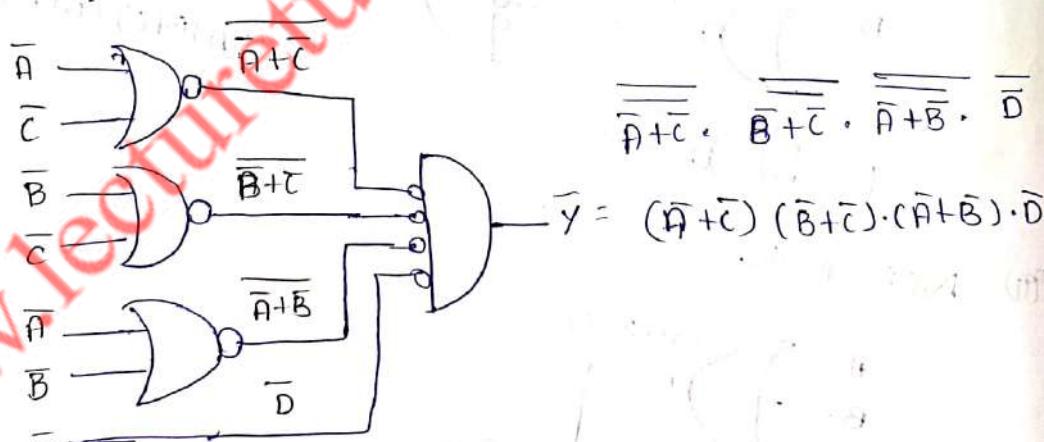
$$= (\bar{A}\bar{C}) \cdot (\bar{B}\bar{C}) \cdot (\bar{A}\bar{B}) \cdot \bar{D}$$

$$\bar{Y} = (\bar{A} + \bar{C}) \cdot (\bar{B} + \bar{C}) \cdot (\bar{A} + \bar{B}) \cdot \bar{D}$$

(i) OR-AND

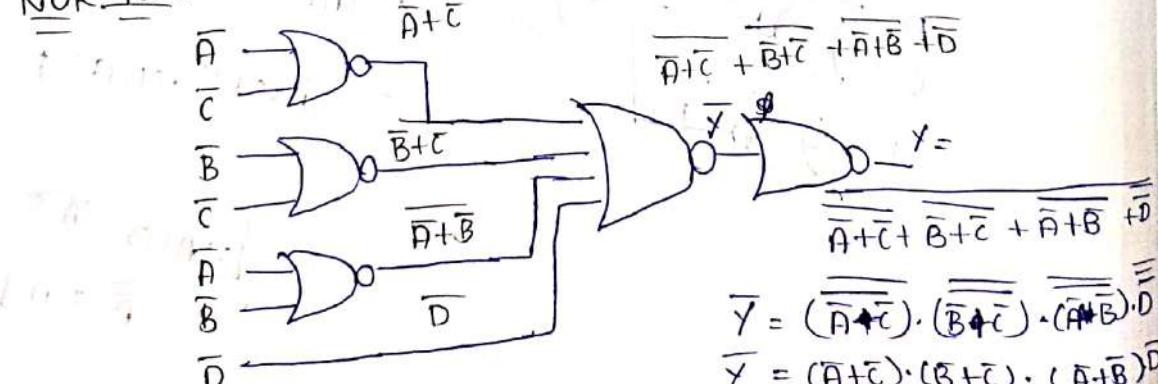


(ii) NOR-Bubbled AND



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NOR=NOR



$$\bar{Y} = (\bar{A} + \bar{C}) \cdot (\bar{B} + \bar{C}) \cdot (\bar{A} + \bar{B}) \cdot \bar{D}$$

$\therefore \bar{A} + \bar{B} = \bar{A} \cdot \bar{B} = AB$
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$$Y = AC + BC + AB + D$$

Ex: Implement the following Boolean function with NOR-NOR logic

$$f(A, B, C) = \overline{\prod_M}(0, 2, 4, 5, 6)$$

Ex: Implement the X-NOR gate by using only NOR gate ($AB + \bar{A}\bar{B}$)

	BC	00	01	11	10
A	0	0	2	3	0
	1	0	0	5	7
G ₁					
G ₂					
G ₃					

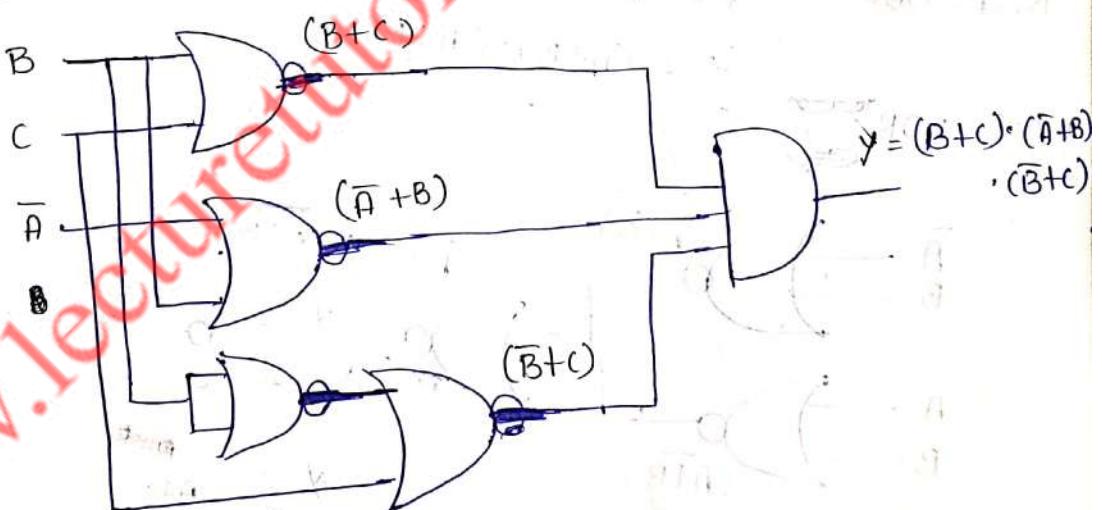
$$\begin{array}{c} G_1 \\ \hline A & B & C \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \\ \hline \bar{B}\bar{C} \end{array} \quad \begin{array}{c} G_2 \\ \hline A & B & C \\ \hline 0 & 0 & 1 \\ 1 & 0 & 1 \\ \hline A\bar{B} \end{array} \quad \begin{array}{c} G_3 \\ \hline A & B & C \\ \hline 0 & 1 & 0 \\ 1 & 1 & 0 \\ \hline \bar{B}\bar{C} \end{array}$$

$$\bar{Y} = \overline{\bar{B}\bar{C} + A\bar{B} + BC}$$

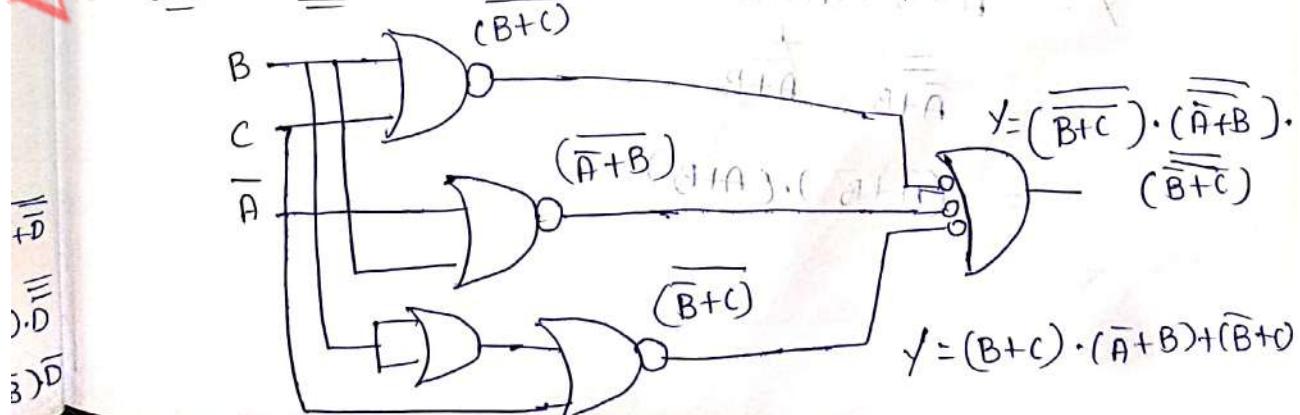
$$= (\bar{B} + \bar{C}) \cdot (\bar{A} + \bar{B}) \cdot (\bar{B} + \bar{C})$$

$$Y = (B + C) \cdot (\bar{A} + B) \cdot (\bar{B} + C)$$

(i) OR-AND



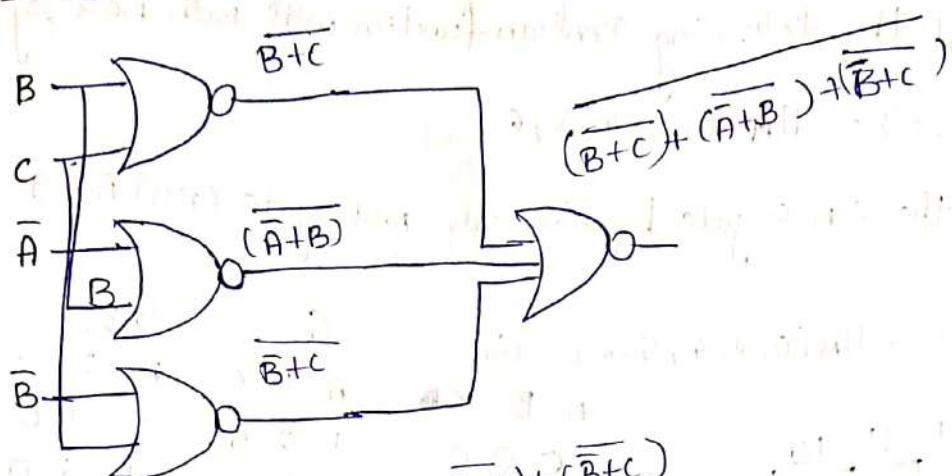
(ii) NOR-Bubbled AND



(iii) NOR-NOR

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$$Y = (B+C) \cdot (\bar{A}+B) \cdot (\bar{B}+C)$$



$$Y = (\overline{B+C}) + (\bar{A}+B) + (\overline{B+C})$$

$$= (\overline{B+C}) \cdot (\bar{A}+B) \cdot (\overline{B+C})$$

$$Y = (\overline{B+C}) \cdot (\bar{A}+B) \cdot (\overline{B+C})$$

- ② Implement the X-NOR gate by using only NOR gate
 The X-NOR gate function has two inputs and one output. The input variables are A and B and the output variable is Y.

$$Y = AB + \bar{A}\bar{B}$$

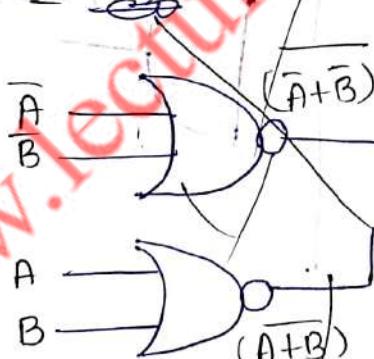
NOR-NOR

$$Y = \overline{AB + \bar{A}\bar{B}}$$

$$Y = (\bar{A}+\bar{B}) \cdot (A+B)$$

NOR+NOR

$(\bar{A}+\bar{B})$



$$Y = AB + \bar{A}\bar{B}$$

$$\bar{Y} = \overline{\bar{A}+\bar{B}} + \overline{(A+B)}$$

$$\bar{Y} = \overline{\bar{A}+\bar{B}} \cdot \overline{A+B}$$

$$(Simplifying) \quad \bar{Y} = ((\bar{A}+\bar{B}) \cdot (A+B)) + \overline{(A+B)}$$

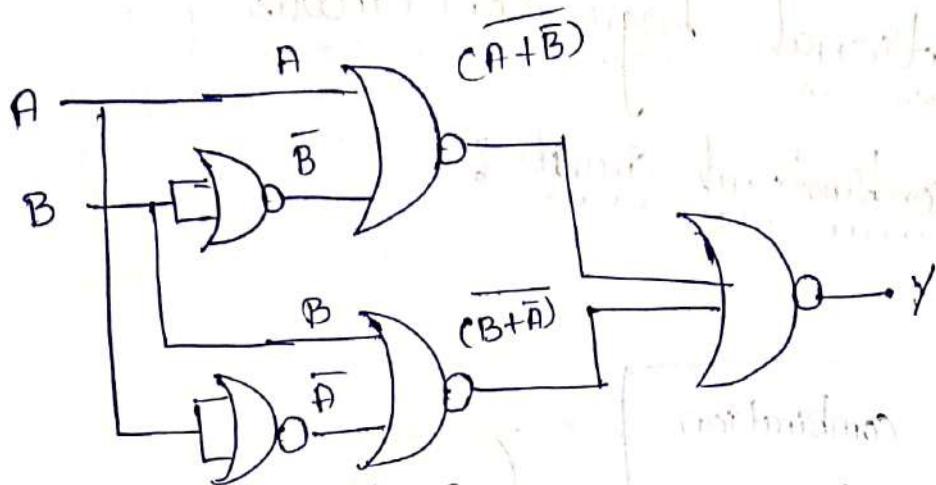
$$Y = \overline{A+B}$$

$$= ((\bar{A}+\bar{B}) \cdot (A+B)) + \overline{(A+B)}$$

Q1: Implement the $\bar{A} \oplus B$

A-TING

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$$y = (\bar{A} + B) + (A + \bar{B})$$

$$y = (\bar{A} + B) \cdot (A + \bar{B})$$

$$y = (\bar{A} + B)(A + \bar{B})$$

$$= AB + \bar{B}B + A\bar{A} + \bar{A}\bar{B}$$

$$= AB + \bar{A}\bar{B}$$

$$\begin{cases} \text{if } B\bar{B} = 0 \\ \text{if } A\bar{A} = 0 \end{cases}$$

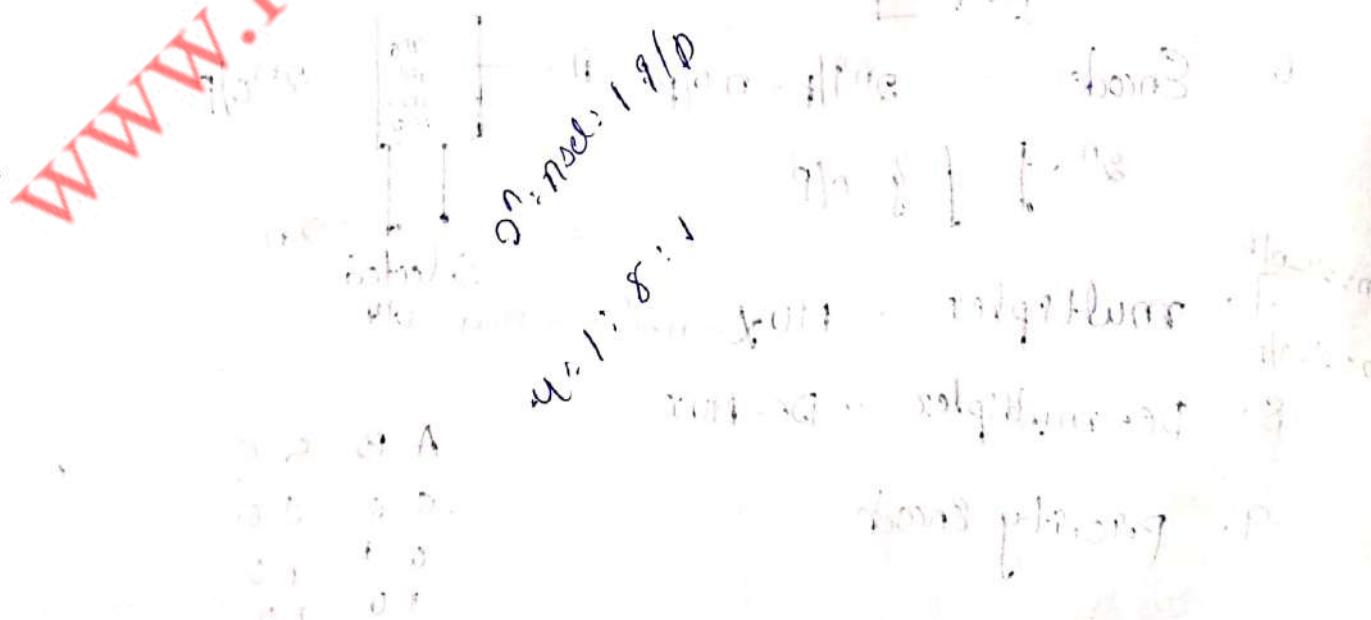
open for Q1P 5 min

for Q2P 5 min

when 30s

Q2P 5 min

last 5 min

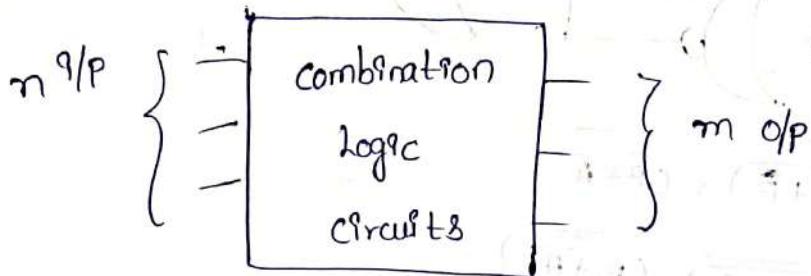


UNIT-4

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Combinational logic (or) Circuits

Block diagram of combinational circuits



Topics:

1. Half - Adder

- 2 - I/P - 2 O/P

2. Half - Sub

- 2 - I/P - 2 O/P

3. Full - adder

- 3 - I/P - 2 O/P

4. Full - Sub

- 3 - I/P - 2 O/P

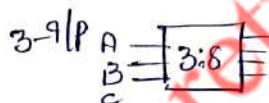
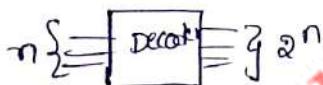
5. Decoder -

n - I/P - 2^n O/P

$$Ex: 2 - I/P - 2^2 = 4 O/P$$

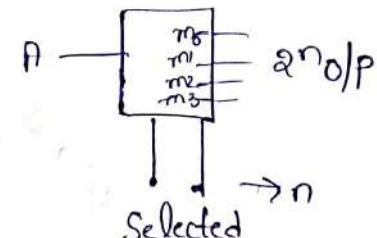
$$Ex: 3 - I/P - 2^3 = 8 O/P$$

3:8 Decoder



6. Encode

- 2^n I/P - n O/P



n: 2^n - O/P

n: 2^n - I/P

7. Multiplexer - MUX - n selection lines

- 2^n I/P

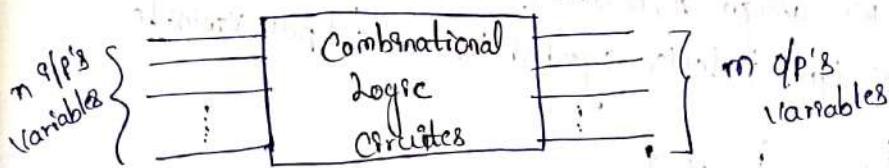
8. De-multiplexer - De-MUX

9. Priority Encoder

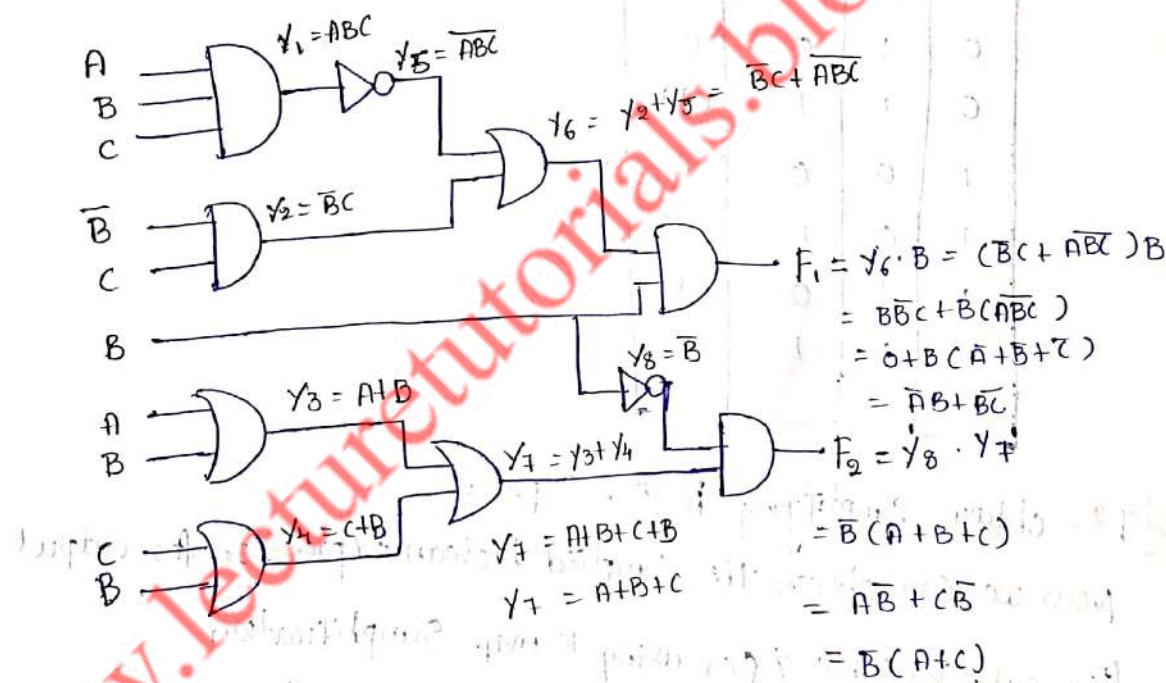
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Combinational Circuits

- When logic gates are connected together to produce a specified output for given input variables, with no storage involved, certain specified combinations of input variables, with no storage involved.
- The resulting circuit is called as Combinational logic circuit. In combinational logic the output variables are at all times dependent on the combination of input variables.
- The block diagram of input vs combination circuit is given below



Analysis Procedure :-
To obtain the boolean function for output of the given circuit



$$F_1 = B(C + \bar{A}C), F_2 = \bar{B}(A + C)$$

ABC	\bar{B}	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	F_1	F_2
0 0 0	1	0	0	0	0	1	1	0	1	0	0
0 0 1	1	0	1	0	1	1	1	1	0	0	0
0 1 0	0	0	0	1	1	1	1	0	1	1	0
0 1 1	0	0	0	1	1	1	1	1	0	0	1
1 0 0	1	0	0	1	0	1	1	1	1	0	0
1 0 1	1	0	1	1	1	1	1	0	0	1	0
1 1 0	0	0	0	1	1	1	1	0	0	0	0
1 1 1	0	1	0	1	1	0	0	0	0	0	0

Design procedure :-

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Example

- * Design a combinational logic circuit with three input variables that will produce a logic one output when more than one input variables are logic one.

Step 1: Derive the truth table for given statement

The given problem specifies that there are three input variables and only one output variable. We assign A, B and C letter symbols to three input variables and assign "Y" letter symbol to one output variable.

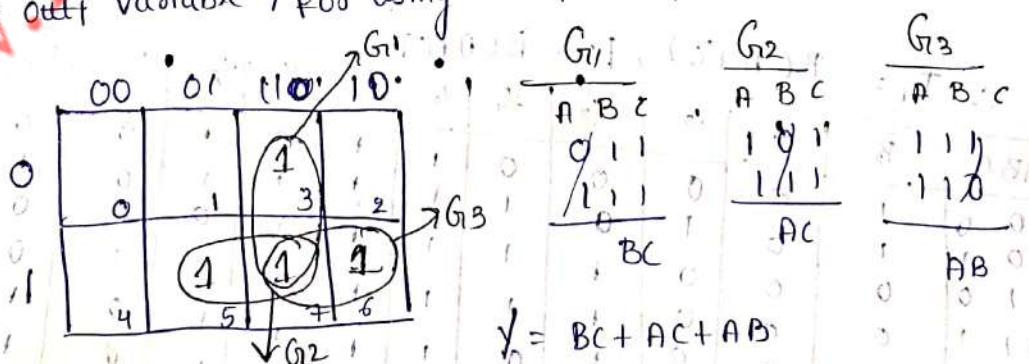
Step

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Step 2: Obtain Simplifying Boolean Expression

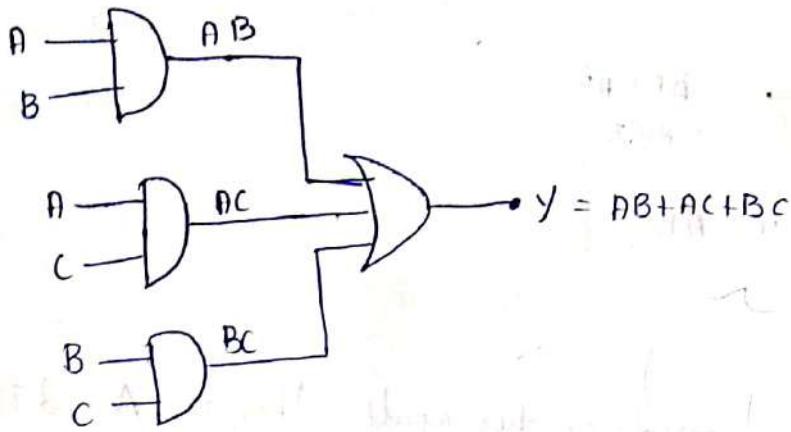
Now we can obtain the simplified boolean expression for output

for out variable Y for using K-map simplification



Step 3: Draw the logic diagram

We are going to draw various combinational circuits to the above boolean expression

Design of Adders

→ Digital computers perform various arithmetic operations like the addition of two binary digits. This simple addition consists of four elementary operations, namely

$$\begin{aligned} 0+0 &= 0 \\ 0+1 &= 1 \\ 1+0 &= 1 \\ 1+1 &= (10)_2 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

Half adder

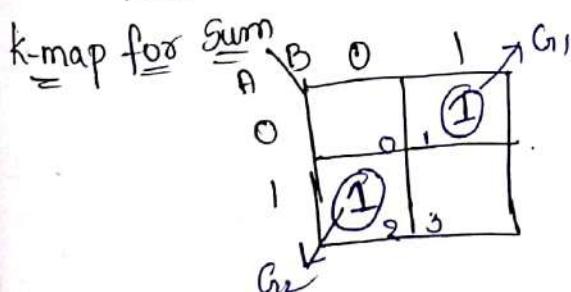
→ The Half adder consists of two input variables and two output variables. Assign A and B as input variables and sum (S) and carry (C) as output variables.

$$A, B = 1/p \text{ variables}$$

$$S, C = 0/p \text{ variables}$$

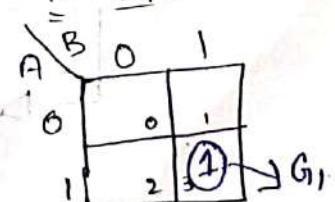
Truth table

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



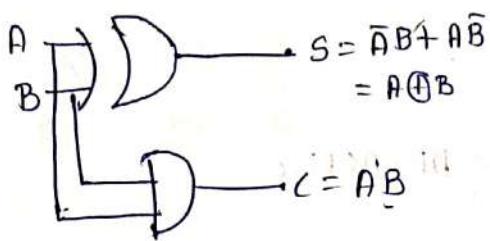
$$\begin{aligned} S &= \overline{AB} + \overline{A}B \\ &= A \oplus B \end{aligned}$$

K-map for carry



$$C = AB$$

for S

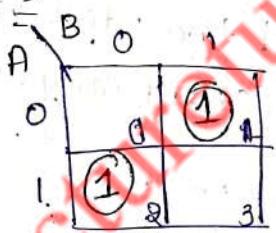
⑨ Half - subtraction:

→ The Half subtraction consists of two inputs they are. A and B and two outputs they are difference (D) and borrow (B).

→ The truth table

A	B	D (difference)	B (borrow)
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

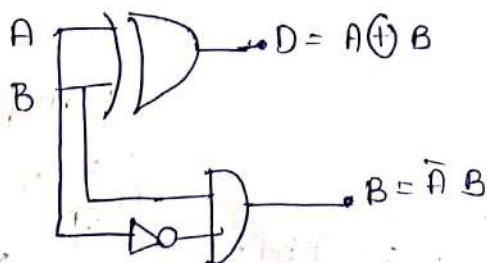
K-map for D



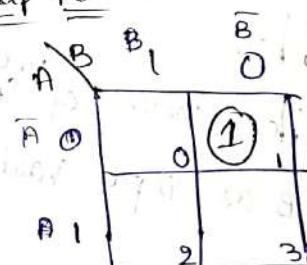
$$D = A\bar{B} + \bar{A}B$$

$$= A \oplus B$$

Logic Diagram



K-map for B



$$B = \bar{A}B$$

Full-Adder:

In full-adder consists of three inputs they are A, B, C and outputs are sum and carry

A	B	C	S (sum)	C (carry)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

K-map for C

$$C = AB + BC + CA$$

K-map for S

$$S = A \oplus B \oplus C$$

		BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
		00	01	11	10	01
A	B	0	1			1
		1	0	1	1	0
A	B	0	1	1	0	1
		1	0	0	1	0
		4	5	7	6	

$$S = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

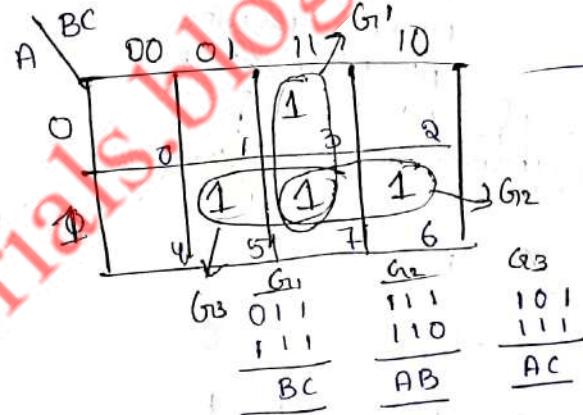
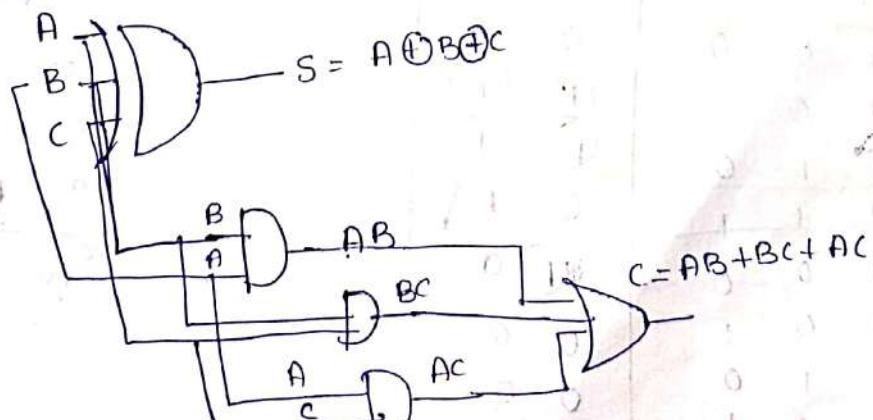
$$= C(\bar{A}\bar{B} + AB) + \bar{C}(\bar{A}B + \bar{A}\bar{B})$$

$$S = C(A \oplus B) + \bar{C}(A \oplus B)$$

$$S = C(\bar{A} \oplus B) + \bar{C}(A \oplus B)$$

$$S = A \oplus B \oplus C$$

Logic Diagram:

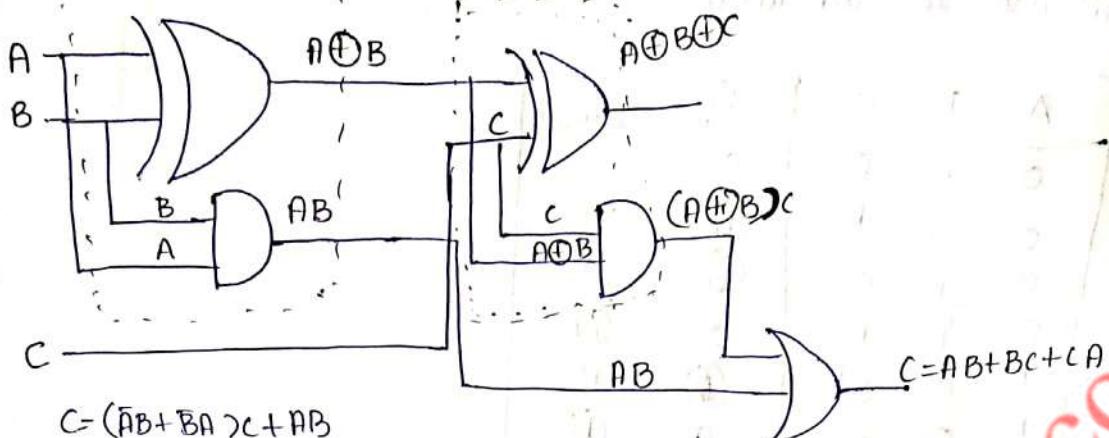


$$C = AB + BC + CA$$

$$= \cancel{ABC} + \cancel{ABC} + \cancel{ABC} + \cancel{ABC}$$

$$= \cancel{(ABC + \bar{A}\bar{B}\bar{C})} +$$

* To Design full Adder by using two half adders and OR gate
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$$\begin{aligned}
 C &= (\bar{A}B + BA)C + AB \\
 &= ABC(1+C) + C(\bar{A}B + A\bar{B}) \\
 &= AB + ABC + \bar{A}BC + A\bar{B}C \\
 &= AB + B(C(A + \bar{A})) + A\bar{B}C \\
 &= AB + BC + A\bar{B}C \\
 &= AB(C + 1) + BC + A\bar{B}C \\
 &= AB + ABC + BC + A\bar{B}C \\
 &= AB + BC + AC(B + \bar{B}) \\
 &\boxed{C = AB + BC + AC}
 \end{aligned}$$

$$\begin{aligned}
 C &= (A \oplus B)C + AB \\
 &= (\bar{A}B + A\bar{B})C + AB
 \end{aligned}$$

$$\begin{aligned}
 &= \bar{A}BC + A\bar{B}C + AB \\
 &= B(\bar{A}C + A) + A\bar{B}C \\
 &= BC + A\bar{B}C \\
 &= C(A)
 \end{aligned}$$

Subtraction operations:

A	B	D	B
0	-0	=	0 0
0	-1	=	1 1
1	-0	=	1 0
1	-1	=	0 0

Full-subtraction

→ It consists of three inputs two outputs they are A,B,C & the inputs D,B & the out put is

A	B	C	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
1	0	1	0	1
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0

$$\begin{array}{r}
 0-1 \\
 \hline
 11 \\
 -0 \\
 \hline
 11 \\
 -0 \\
 \hline
 01 \\
 -0 \\
 \hline
 01 \\
 -0 \\
 \hline
 10 \\
 -0 \\
 \hline
 10 \\
 -0 \\
 \hline
 00
 \end{array}$$

K-map for D

	$\bar{B}C$	$B\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	00	01	11	10	
A	0	1	3	2	
	4	5	7	6	

$$D = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

$$= ((\bar{A}\bar{B} + AB) + \bar{C}(\bar{A}B + A\bar{B})$$

$$= ((A \oplus B) + \bar{C}(A \oplus B))$$

$$= C(\overline{A \oplus B}) + \bar{C}(A \oplus B)$$

$$= A \oplus B \oplus C$$

K-map for B

	$\bar{B}C$	00	01	11	10
\bar{A}	0	1	1	1	0
A	1	0	1	0	1
	4	5	7	6	

$$B = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}B\bar{C} + ABC$$

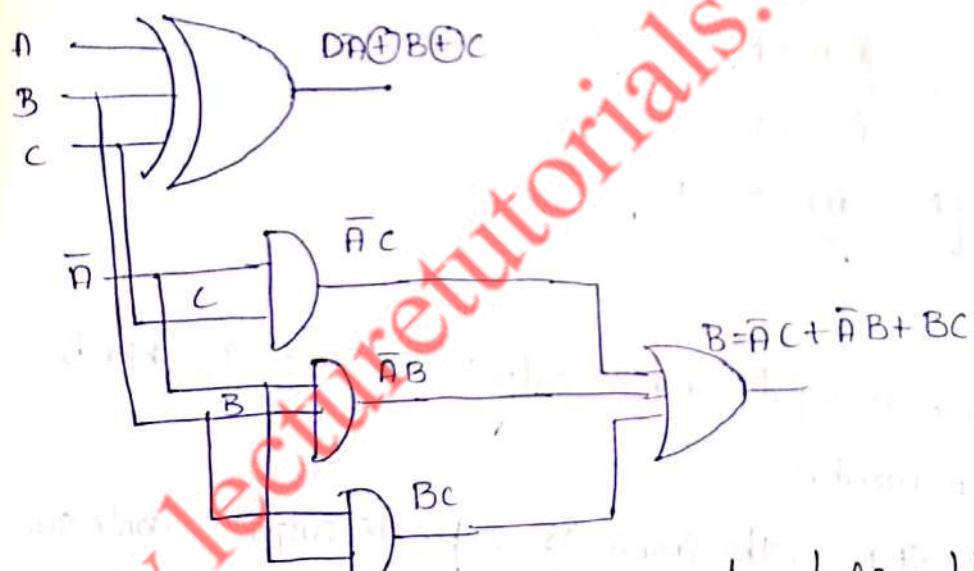
$$= \frac{G_1}{\begin{matrix} A \\ \bar{B}C \end{matrix}} \quad \frac{G_2}{\begin{matrix} \bar{A} \\ B\bar{C} \end{matrix}} \quad \frac{G_3}{\begin{matrix} \bar{A} \\ B \\ \bar{C} \end{matrix}}$$

$$\begin{array}{r} G_1 \\ \hline \begin{matrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{matrix} \\ \hline \overline{AB} \end{array}$$

$$\begin{array}{r} G_2 \\ \hline \begin{matrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} \\ \hline \overline{BC} \end{array}$$

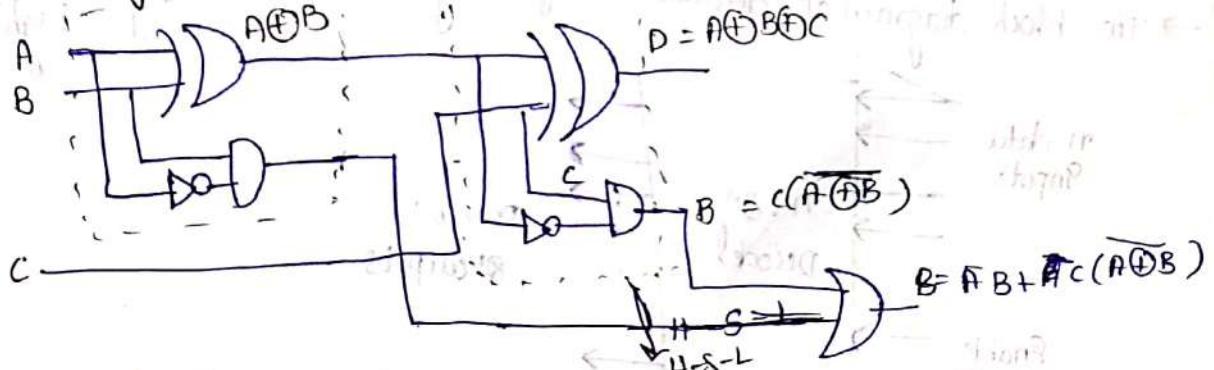
$$B = \bar{A}C + \bar{A}B + BC$$

Logic diagram



To Design full subtraction by using two half subtraction and one OR gate

one OR gate



$$B = C(\overline{A \oplus B}) + \bar{A}B$$

$$= C(\overline{A} \oplus \overline{B}) + \bar{A}B$$

$$= C(\overline{A}\bar{B} + \overline{B}A) + \bar{A}B$$

$$= \bar{A}\bar{B}C + ABC + \bar{A}B$$

$$B = \bar{A}B + BC + \bar{A}C$$

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$$B = \bar{A}B + C(\overline{A \oplus B})$$

$$= \bar{A}B + C(A \oplus B)$$

$$= \bar{A}B + C(\overline{AB} + AB)$$

$$= \bar{A}B + (\overline{A}\bar{B} + ABC)$$

$$= \bar{A}BC(1+C) + \bar{A}\bar{B}C + ABC$$

$$= \bar{A}B + \underline{\bar{A}BC} + \underline{\bar{A}\bar{B}C} + ABC$$

$$= \bar{A}B + BC(\bar{A} + A) + \bar{A}BC$$

$$= \bar{A}B + BC + \bar{A}BC$$

$$= \bar{A}BC(1+C) + BC + \bar{A}\bar{B}C$$

$$= \bar{A}B + \bar{A}BC + BC + \bar{A}\bar{B}C$$

$$= \bar{A}B + BC + \bar{A}C(B + \bar{B})$$

$$\boxed{B = \bar{A}B + BC + \bar{A}C}$$

Decoder

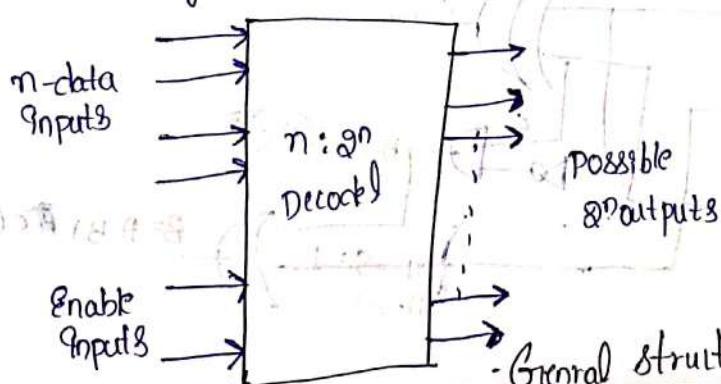
→ The Decoder has n inputs and 2^n outputs the output is depends upon enable input variable.

→ The Enable is (EN), the Enable is zero the output is don't care condition.

→ The Enable is '1' the output is on state

→ The block diagram of decoder is given by

[0 - Low
1 - high]

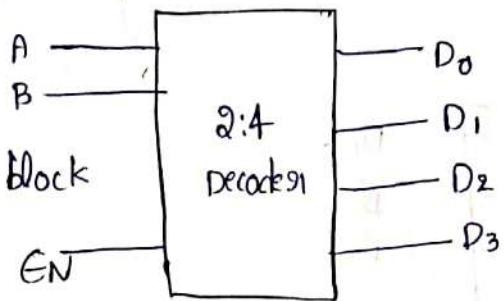


Binary Decoder

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2 : 4 Decoder :-

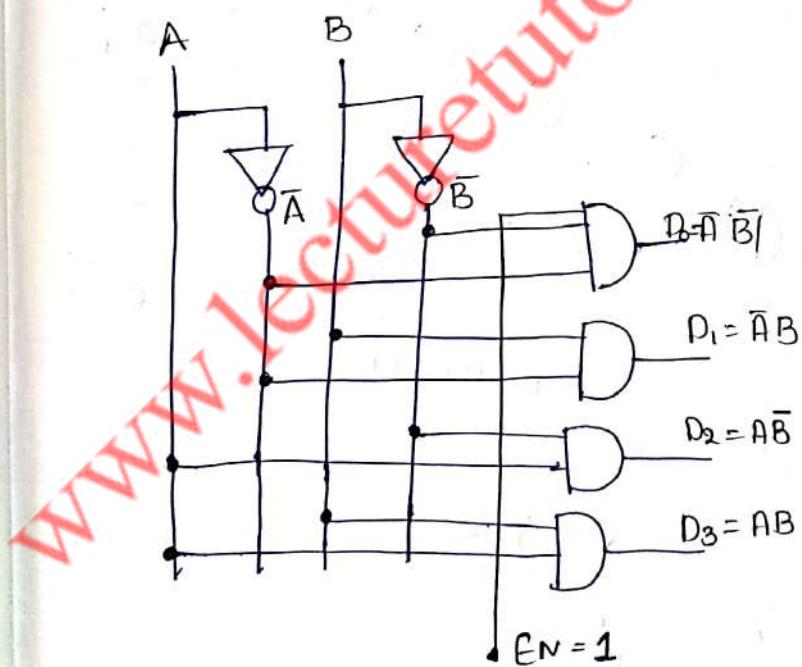
Truth table :-
The 2:4 Decoder block diagram is given by



2:4 Decoder

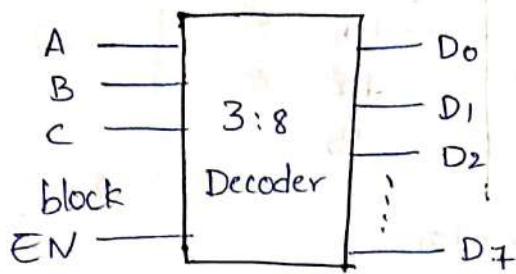
Truth table :-

Inputs			outputs			
EN	A	B	D ₃	D ₂	D ₁	D ₀
0	X	X	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0



3:8 Decoder \div The 3:8 Decoder is given by

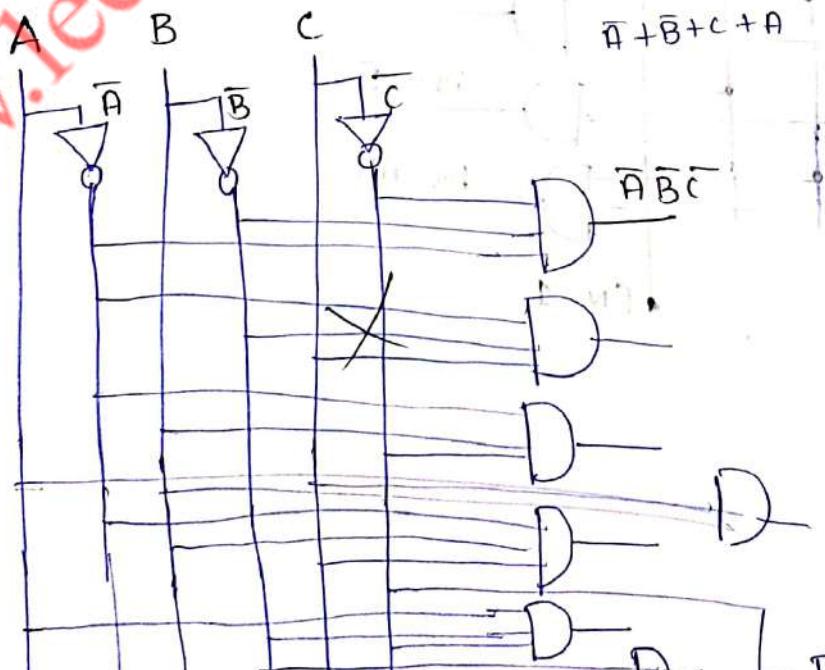
t.me/jntukonlinebits

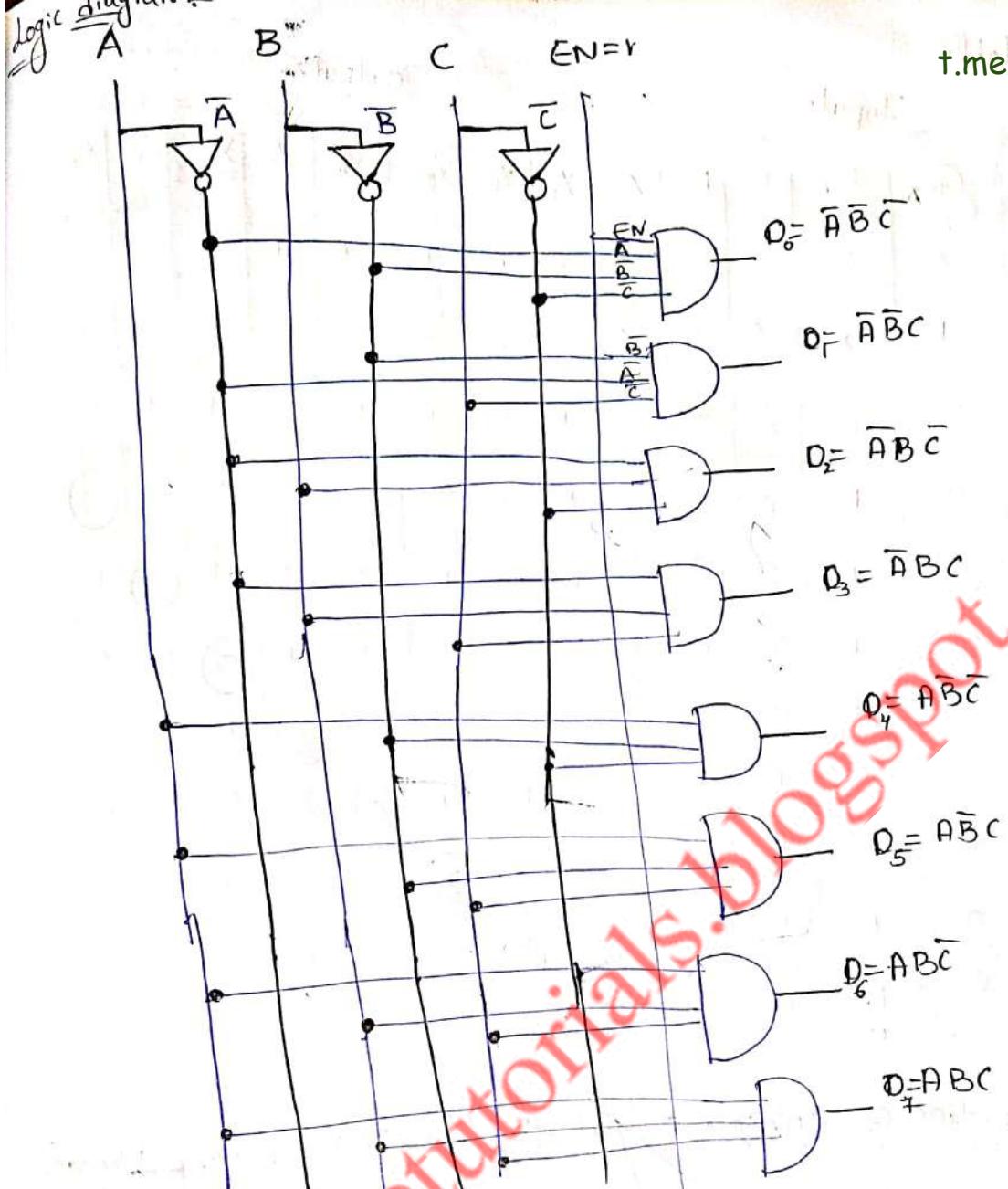


Truth table

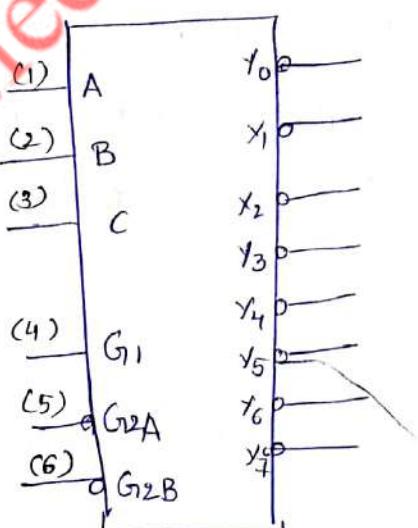
Input				Output							
EN	A	B	C	D ₇	D ₆	D ₅	D ₄	D ₃	D ₂	D ₁	D ₀
0	X	X	X	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0	1
1	0	1	0	0	0	0	0	0	0	0	1
1	0	1	1	0	0	0	0	0	0	1	0
1	1	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	1	0	0	0
1	1	1	0	0	0	1	0	0	0	0	0
1	1	1	1	1	0	0	0	0	0	0	0

$$\overline{A} + \overline{B} + C + A$$





The 74x138 3 to 8 Decoder
= =



logic symbol

Truth Table

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Inputs						Outputs							
$G_{12}A$	$G_{12}B$	G_1	C	B	A	\bar{y}_7	\bar{y}_6	\bar{y}_5	\bar{y}_4	\bar{y}_3	\bar{y}_2	\bar{y}_1	y_0
1	x	x	x	x	x	1	1	1	1	1	1	1	1
x	1	x	x	x	x	1	1	1	1	1	1	1	1
x	x	0	x	x	x	1	1	1	1	1	1	1	1
0	0	1	0	0	0	1	1	1	1	1	1	1	0
0	0	1	0	0	1	1	1	1	1	1	1	1	0
0	0	1	0	1	0	1	1	1	1	1	1	0	1
0	0	1	0	1	1	1	1	1	1	1	1	0	1
0	0	1	1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	1	0	1	1	1	1	1	1	1	1
0	0	1	1	1	1	0	1	1	1	1	1	1	1
0	0	1	1	1	1	1	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	0	1	1	1	1	1
0	0	1	1	1	1	1	1	1	0	1	1	1	1

Realization of Multiple output function using Binary Decodes,

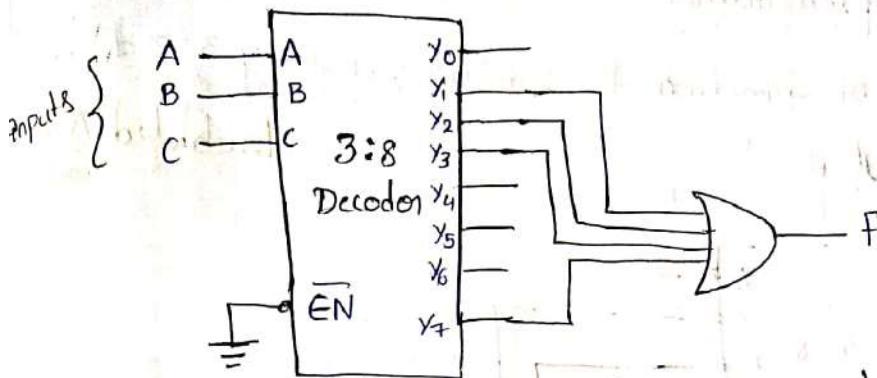
For Active high output :-

Sop function implementation

When decoder output is active high, it generates minterms (product terms) for input variables i.e. it makes selected output logic 1.

In such case to implement Sop function we have to take sum of selected product terms generated by decoder. This can be implemented by ORing the selected decoder outputs, as shown in the fig 5.62. The fig 5.62 shows the implementation of function $f = \sum M(1, 2, 3, 7)$ using 3:8 decoder with active high outputs.

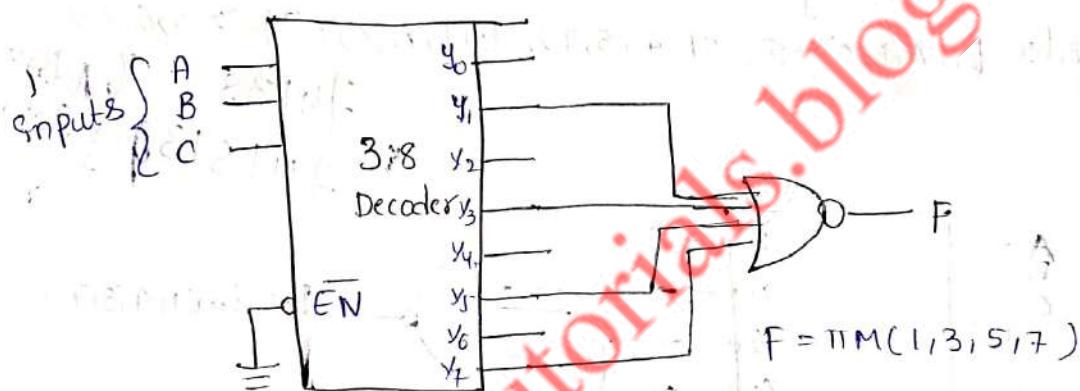
0 - low - off state
1 - high - on



Single output function implementation using decoder and gate

POS Function Implementation

The implementation of a function $f = \prod M(1, 3, 5, 7)$ using 3:8 decoder with active high outputs.



Implementation of pos function using decodes

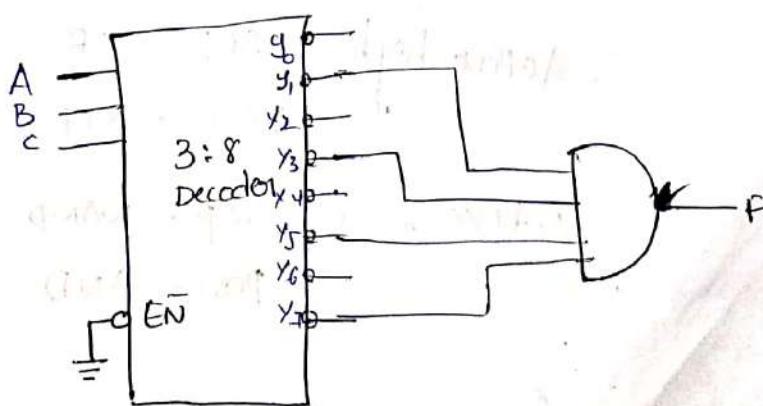
for Active Low Output

=
pos function implementation

The implementation of function $f = \prod M(1, 3, 5, 7)$ using 3:8 decoder

with active low outputs

Bubbled AND



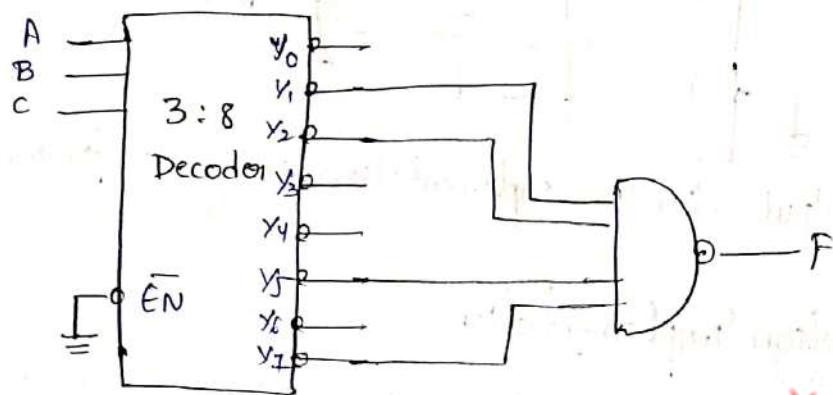
Implementation of pos function using decoder

SOP function implementation

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the implementation of function $f = \sum m(1, 2, 5, 7)$ using 3:8 decoder with active low outputs

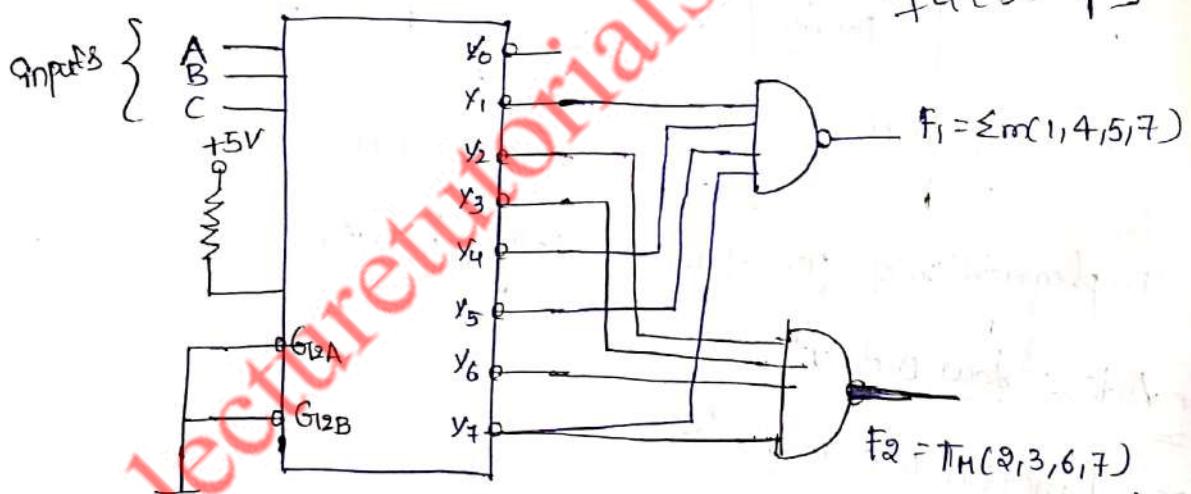
Bubbled NAND Gate



Implementation of SOP function using decoder

Ex:- Implement following multiple output function using 74LS138 and external gates. $F_1(A, B, C) = \sum m(1, 4, 5, 7)$, $F_2(A, B, C) = \overline{\sum m(2, 3, 6, 7)}$

74138 {Active
74LS138 } Low



Active high - SOP - OR
POS - NOR

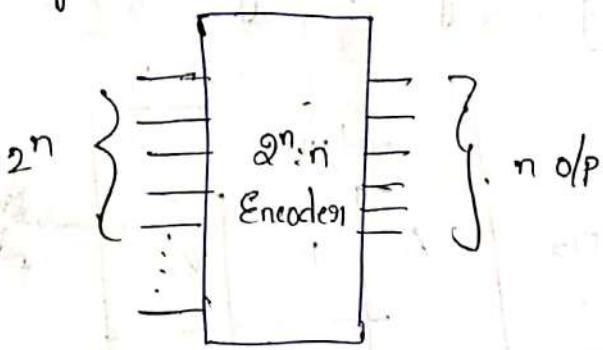
Active low - SOP - NAND
POS - AND

Encoder ($2^n q/p - n o/p$)

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→ The Encoder qt consists of 2^n inputs and n outputs

Block diagram:



Ex 4

Octal - Binary Encoder

truth-table

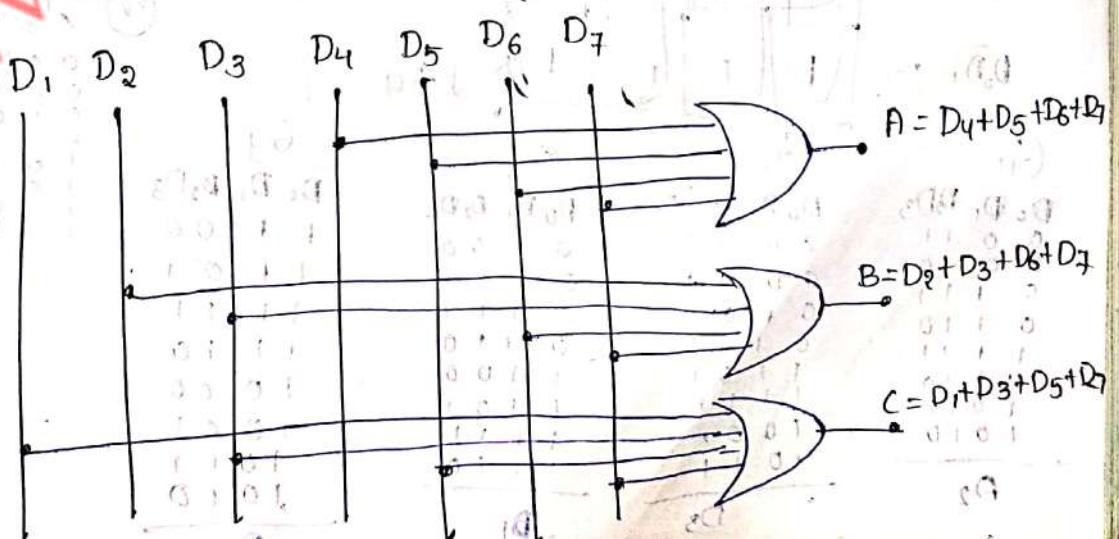
→ The truth table of octal to binary converter

Inputs								Outputs		
D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇	A	B	C
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	1	0	1
0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	1	0	0	1	0
0	0	0	0	0	0	0	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0

$$A = D_4 + D_5 + D_6 + D_7$$

$$B = D_2 + D_3 + D_6 + D_7$$

$$C = D_1 + D_3 + D_5 + D_7$$



Priority Encoder:

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A priority encoder is an encoder circuit that includes the priority function. In priority encoder, if two or more inputs are equal to 1 at the same time, the input having the highest priority will take precedence.

truth

Table:

Inputs				outputs		
D ₀	D ₁	D ₂	D ₃	A	B	V
0	0	0	0	x	x	0
1	0	0	0	0	0	1
x	1	0	0	0	1	1
x	x	1	0	1	0	1
x	x	x	1	1	1	1

→ Valid
indicator

four variable k-map - "V"

$D_0 \bar{B}_1$	$D_1 \bar{B}_1$	$D_2 \bar{B}_1$	$D_3 \bar{B}_1$	G_{11}
00	00	00	00	1111
01	01	01	01	1111
11	11	11	11	1111
10	10	10	10	1111

$$\begin{array}{c}
 \text{G}_1 \\
 \hline
 D_0 D_1 D_3 \\
 0 \quad 0 \quad 1 \quad 1 \\
 0 \quad 0 \quad 1 \quad 0 \\
 0 \quad 1 \quad 1 \quad 1 \\
 0 \quad 1 \quad 1 \quad 0 \\
 1 \quad 1 \quad 1 \quad 1 \\
 1 \quad 1 \quad 1 \quad 0 \\
 1 \quad 0 \quad 1 \quad 1 \\
 1 \quad 0 \quad 1 \quad 0 \\
 \hline
 D_2
 \end{array}$$

<u>G₁₂</u>			
<u>A₀</u>	<u>B₁</u>	<u>D₂</u>	<u>D₃</u>
0 0	0 1		
0 0	1 1		
0 1	0 1		
0 1	1 1		
1 1	0 1		
1 1	1 1		
1 0	0 1		
1 0	1 1		

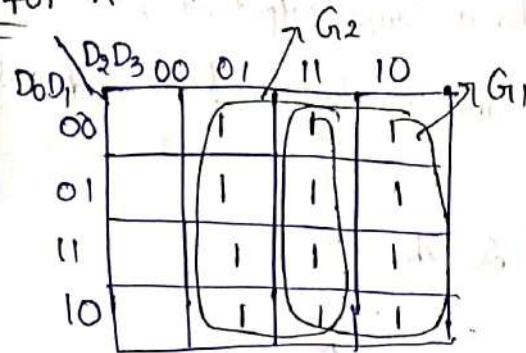
G ₃			
D ₀	B ₁	D ₂	D ₃
0	1	0	0
0	1	0	1
0	1	1	1
0	1	1	0
1	1	0	0
1	1	0	1
1	1	1	1
1	1	1	0

G4		
B ₀	B ₁	B ₂ D ₃
1	1	00
1	1	01
1	1	11
1	1	10
1	0	00
1	0	01
1	0	11
1	0	10
<u>B₃</u>		

⑧ 1 0 0 0
 ⑨ x 1 0 0
 0 1 0 0
 1 0 0 0
 ⑩ x x 1 0
 0 0 1 0
 0 1 1 0
 1 0 1 0
 1 1 1 0
 ⑪ x x x 1
 0 0 0 1
 0 0 1 1
 0 1 0 1
 0 1 1 1
 1 0 0 1
 1 0 1 1
 1 1 0 1
 1 1 1 1

$$V = D_0 + D_1 + D_2 + D_3$$

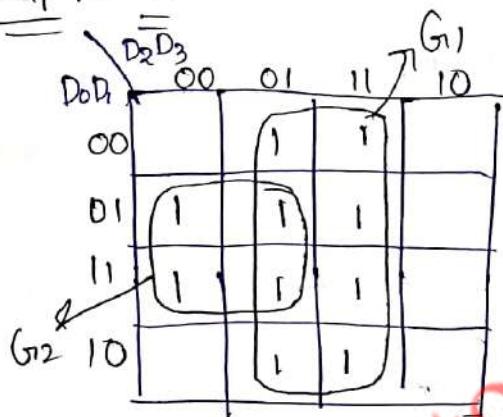
K-map for "A"



G ₁	G ₂	\oplus	x x 1 0
D ₀ D ₁ D ₂ D ₃	D ₀ D ₁ D ₂ D ₃	D ₀ D ₁ D ₂ D ₃	D ₀ D ₁ D ₂ D ₃
0000	0001	0001	0010
0010	0011	0011	0110
0110	0101	0101	1010
1110	1101	1101	1110
1011	1001	1001	0001
1010	1011	1011	0000

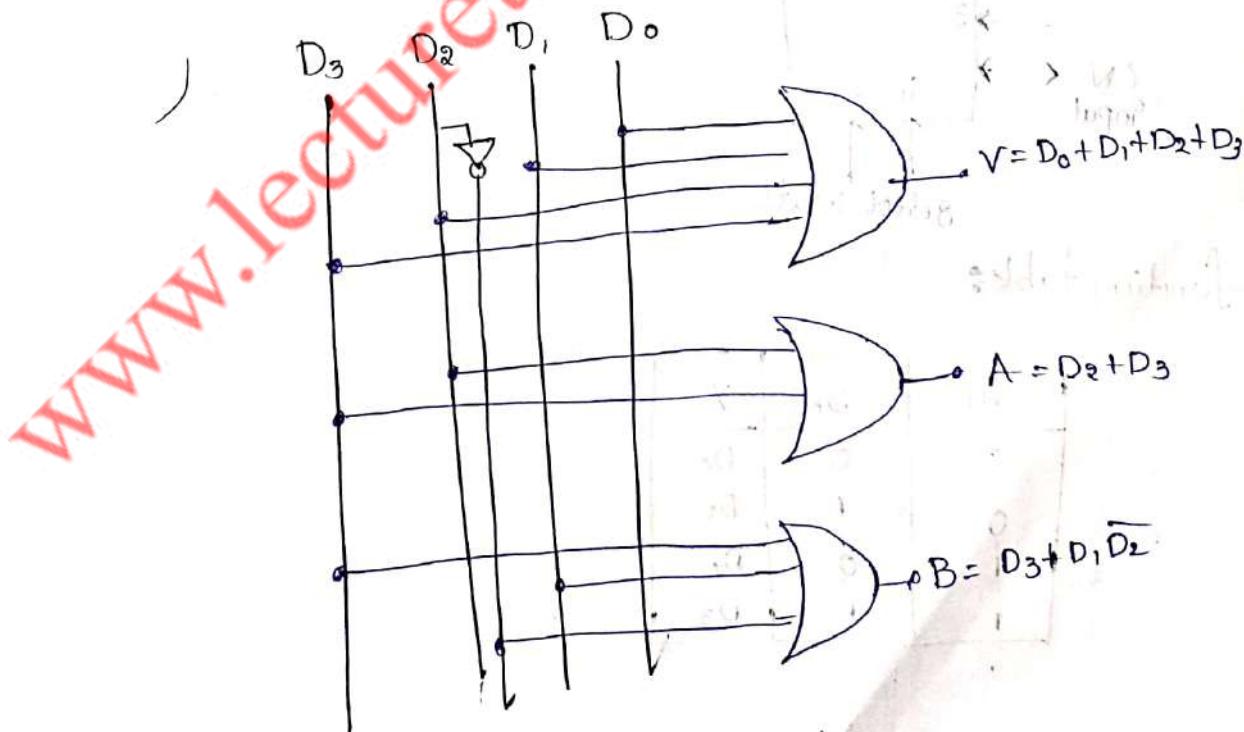
$$A = D_2 + D_3$$

K-map for "B"



$$B = D_3 + \overline{D_1} \overline{D_2}$$

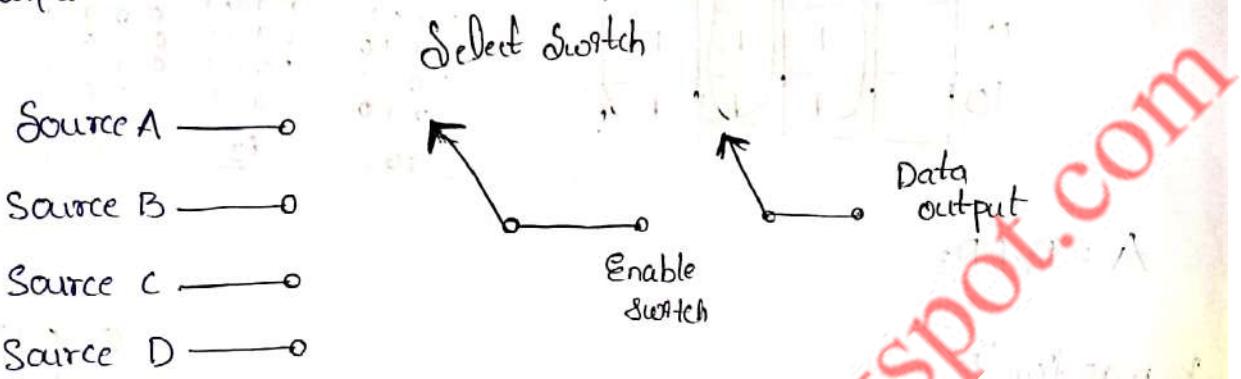
G ₁	G ₂	\oplus	Q
D ₀ D ₁ D ₂ D ₃	D ₀ D ₁ D ₂ D ₃	D ₀ D ₁ D ₂ D ₃	D ₀ D ₁ D ₂ D ₃
0000	0001	0001	1000
0010	0011	0011	0100
0101	0100	0100	1100
0110	0111	0111	1001
1101	1100	1100	0010
1110	1111	1111	1101
1001	1000	1000	0001
1010	1011	1011	0000



Multiplexers (2^n inputs - n selection lines) → 1 output
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Multiplexer is a digital switch. It allows digital information from several sources to be routed onto a single output line.

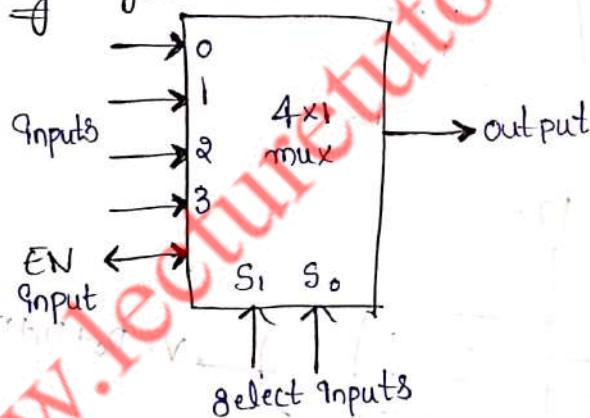
It consists of 2^n input lines and n selection lines and only one output.



Analog selector switch

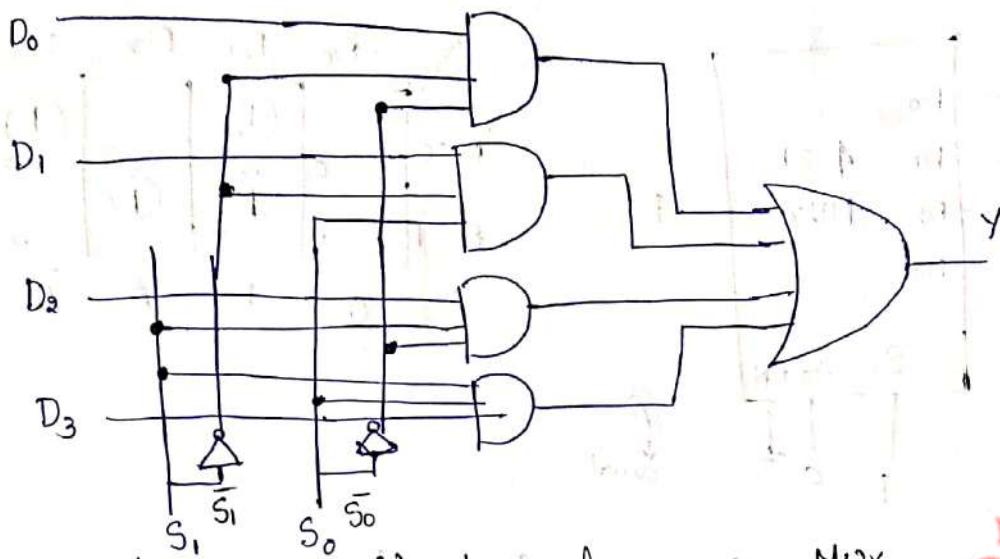
4 to 1 line multiplexer

Logic symbol:



function table:

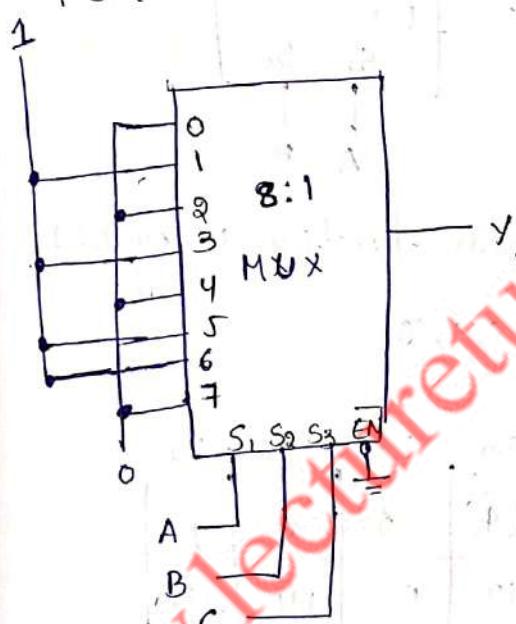
S_1	S_0	y
0	0	D_0
0	1	D_1
1	0	D_2
1	1	D_3



Implementation of combinational logic using MUX

Ex: Implement the following Boolean function using 8:1 multiplexer

$$F(A, B, C) = \sum m(1, 3, 5, 6)$$



Boolean function after implementation using MUX

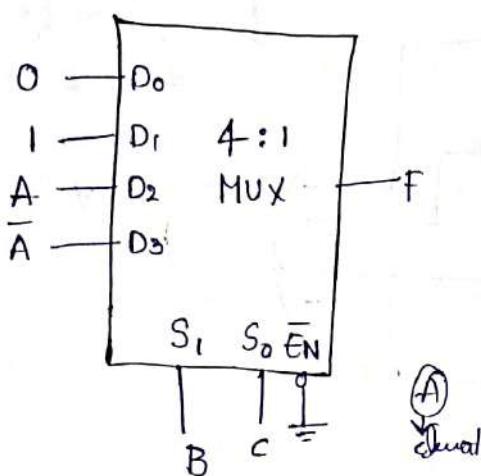
Ex: Implement the following Boolean function 4:1 multiplexer

$$F(A, B, C) = \sum m(1, 3, 5, 6)$$

truth table

minterm	A	B	C	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Multiplexer Implementation



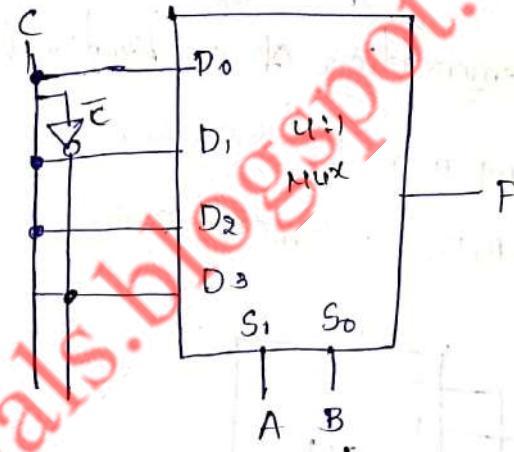
Implementation table
 $F = \sum m(1, 3, 5, 6)$

	D ₀	D ₁	D ₂	D ₃
$\bar{A} = 0$	1	0	1	0
$\bar{A} = 1$	0	1	1	1

Output F: 0, 1, A, \bar{A}

Case 2

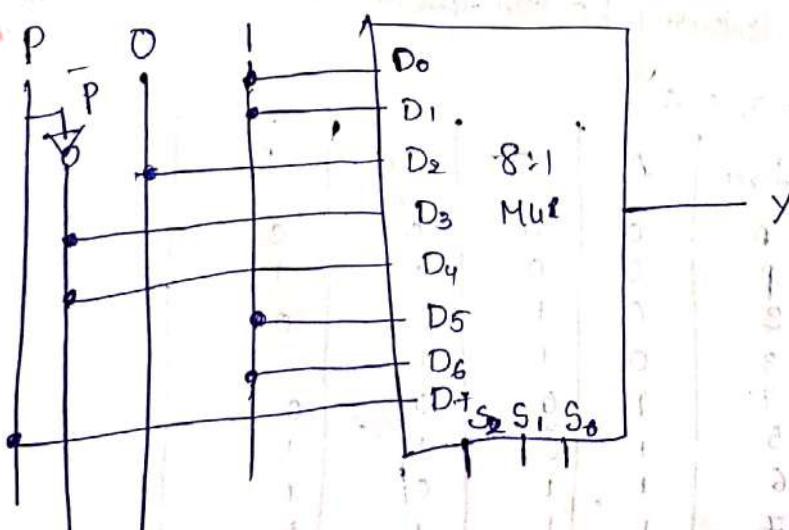
C	\bar{C}	C
0	1	0
2	3	1
4	5	0
6	7	1



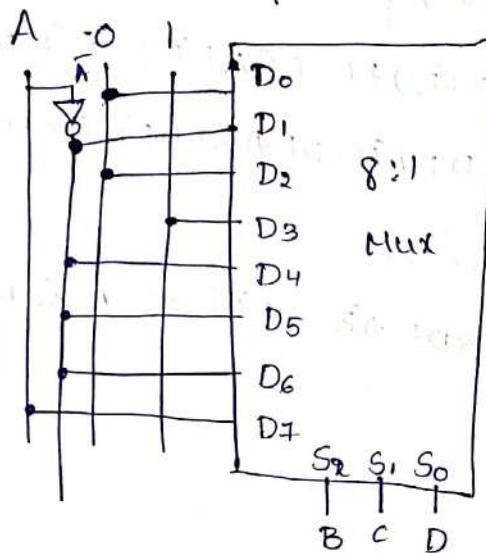
Ex: Implement the following Boolean function using 8:1 Mux. $F(P, Q, R, S) = \sum m(0, 1, 3, 4, 8, 9, 15)$

	D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
\bar{P}	0	1	0	1	0	1	0	1
P	1	0	1	0	1	0	1	0

Control inputs: P, Q, R



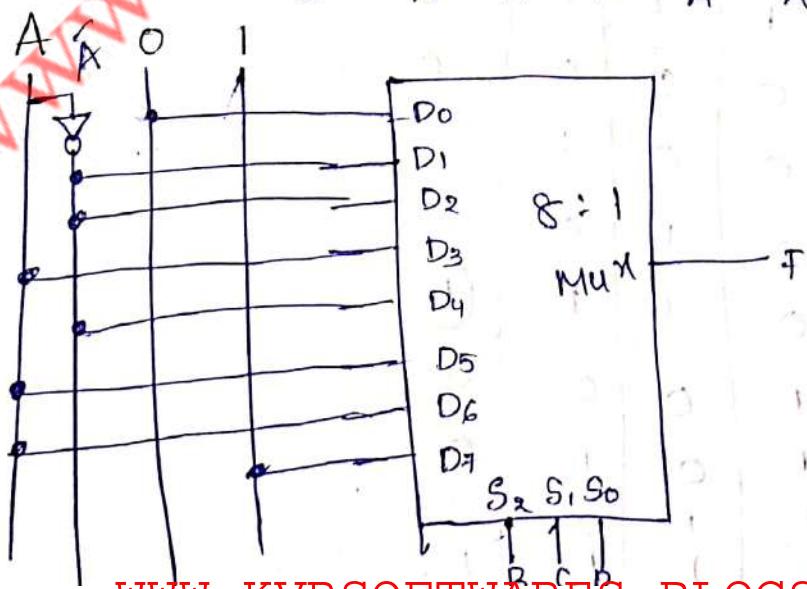
D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
0	1	2	3	4	5	6	7
8	9	10	11	12	13	14	15
0	A	0	1	A	A	A	A

Implementation tableMultiplexer Implementation

* Implement the following Boolean function using 8:1 multiplexer
 $F(A, B, C, D) = \prod M(0, 3, 8, 5, 9, 10, 12, 14)$

Here, instead of minterms, maxterms are specified
 Thus, we have to circle maxterms which are not included
 in the Boolean function shown below

D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇	
Ā	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15
0	Ā	Ā	A	Ā	A	Ā	A	



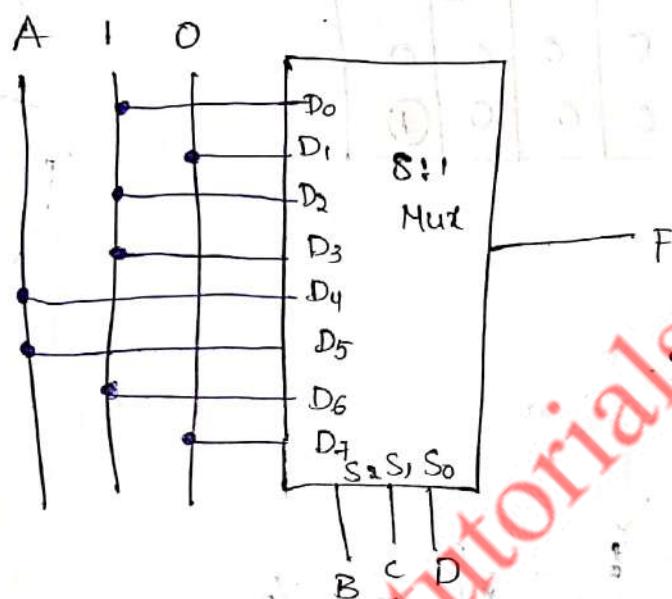
Implement the following Boolean function with 8:1 Multiplexor
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$$F(A, B, C, D) = \sum m(0, 2, 6, 10, 11, 12, 13) + d(3, 8, 14)$$

Sol:

	D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
Ā	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15

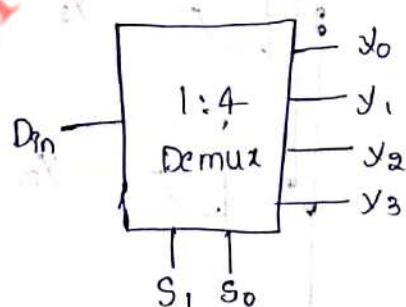
Here don't care are treated.



Demultiplexer (1 input - n selection lines - 2ⁿ outputs)

1 : 4 Demux

= logic symbol:



Enable (0) - Inputs are Don't care
outputs are zero

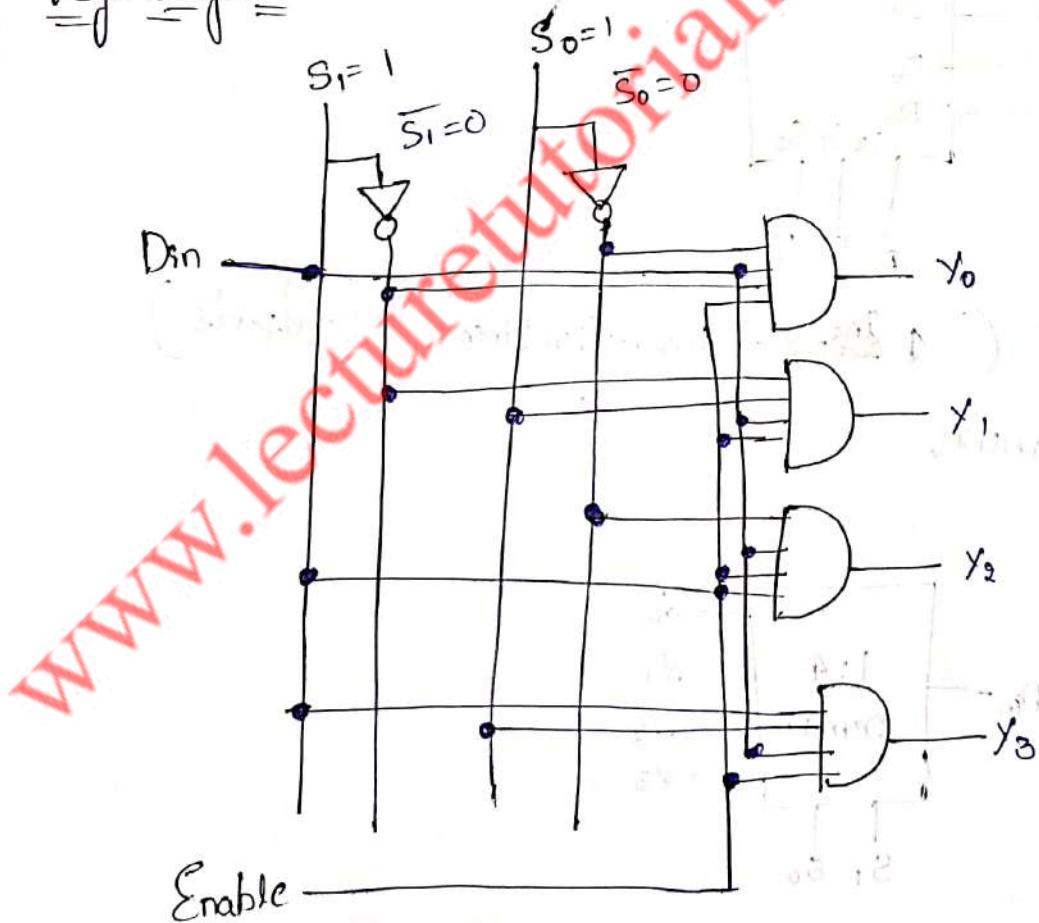
truth table:

t.me/jntukonlinebits

Enable	s_1	s_0	D_{in}	y_0	y_1	y_2	y_3
0	x	x	x	0	0	0	0
1	0	0	0	0	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	0
1	1	1	0	0	0	0	0
1	1	1	1	0	0	0	1

1:4 demultiplexer

Logic diagram



Enable

Eg: Implement the full subtractor by using demultiplexor

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A	B	C	D Difference	B Borrow
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	0	0	0
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

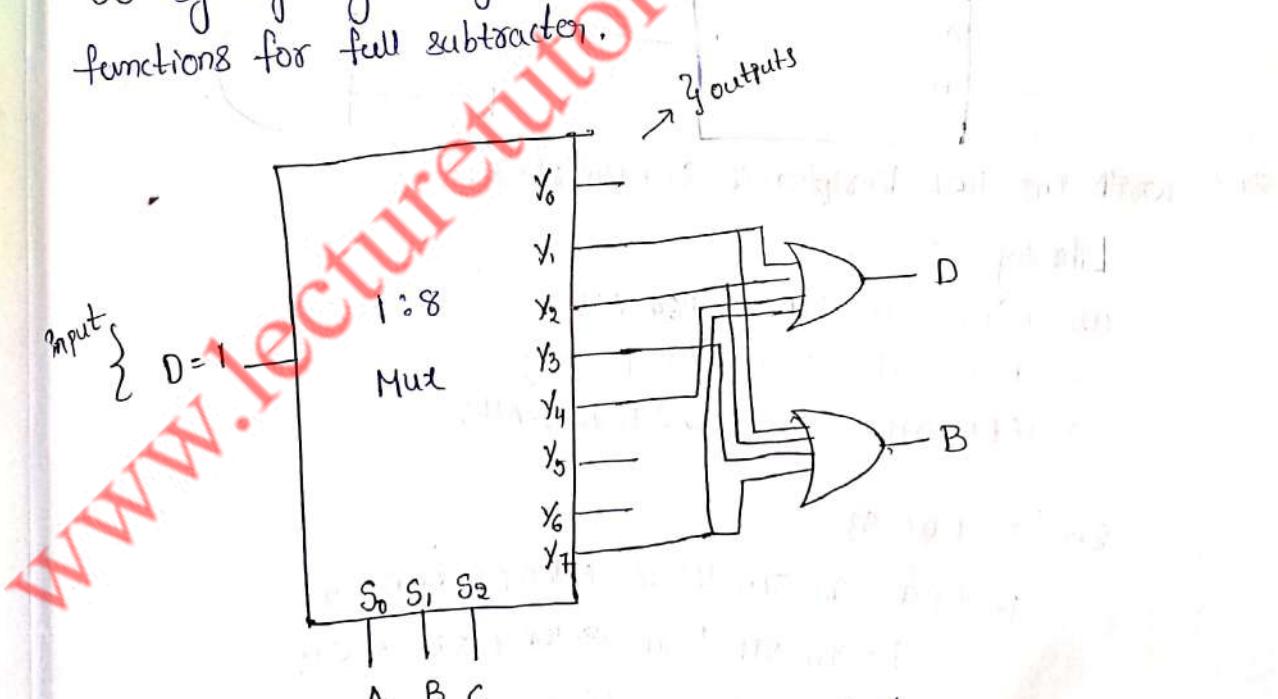
A	B	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

truth table of full subtractor

$$D = \Sigma_m(1, 2, 4, 7)$$

$$B = \Sigma_m(1, 2, 3, 7)$$

With D as input 1, demultiplexer gives minterms at the output
So by logically ORing required minterms we can implement Boolean functions for full subtractor.



Full subtractor using 1:8 demultiplexer

Ex: write the HDL description of AND gate

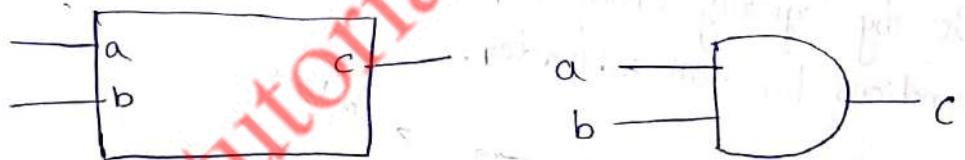
```
Library IEEE;  
use IEEE.STD_LOGIC_1164.ALL;  
use IEEE.STD_LOGIC_ARITH.ALL;  
use IEEE.STD_LOGIC_UNSIGNED.ALL;
```

```
entity andgate is  
port (a,b : in STD_LOGIC;  
      c : out STD_LOGIC);  
end andgate;
```

architecture Behavioral of andgate is

```
begin  
c<=a and b;  
end Behavioral;
```

Output:



Ex: write the HDL description of 8:1 Multiplexer

```
Library IEEE;  
use IEEE.STD_LOGIC_1164.ALL;  
use IEEE.STD_LOGIC_ARITH.ALL;  
use IEEE.STD_LOGIC_UNSIGNED.ALL;
```

```
entity MUX is  
port (a : in STD_LOGIC_VECTOR (7 downto 0);  
      s : in STD_LOGIC_VECTOR (2 downto 0);  
      v : out STD_LOGIC);
```

end MUX;

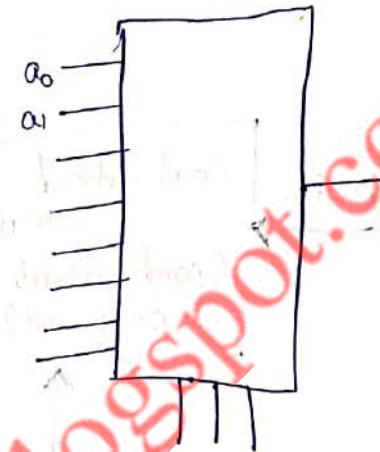
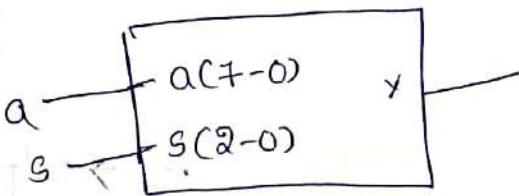
architecture Behavioral of MUX is

```
begin  
v <= a(s) when s = "000" else  
      ac1 when s = "001" else  
      ac2 when s = "010" else  
      ac3 when s = "011" else  
      ac4 when s = "100" else  
      ac5 when s = "101" else  
      ac6 when s = "110" else  
      ac7 when s = "111" else  
      ac8;
```

- a(2) when $s = "010"$ else
- a(3) when $s = "011"$ else
- a(4) when $s = "100"$ else
- a(5) when $s = "101"$ else
- a(6) when $s = "110"$ else
- a(7) when $s = "111"$;

end Behavioral;

Output:



Date
5/8/19

Unit - V

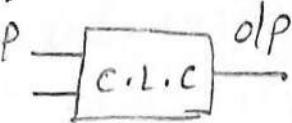
Synchronous Sequential Logic

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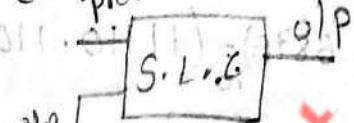
Logic Circuit (ckt)

Combinational logical ckt Sequential logic ckt

Ex:



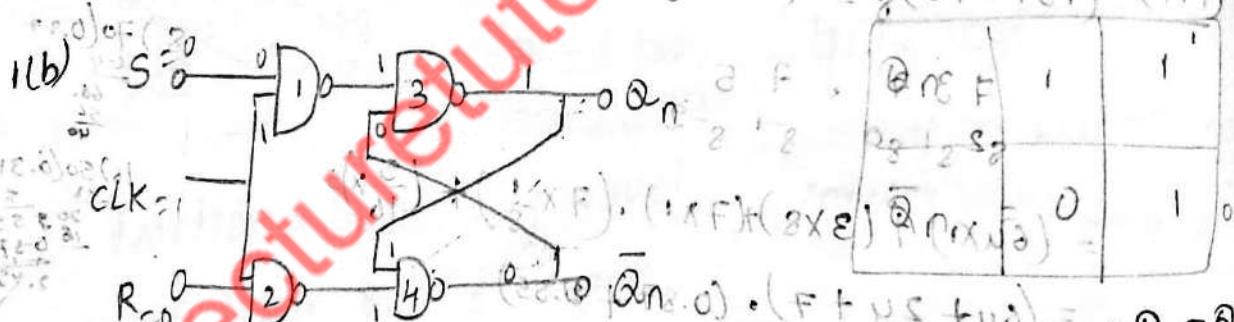
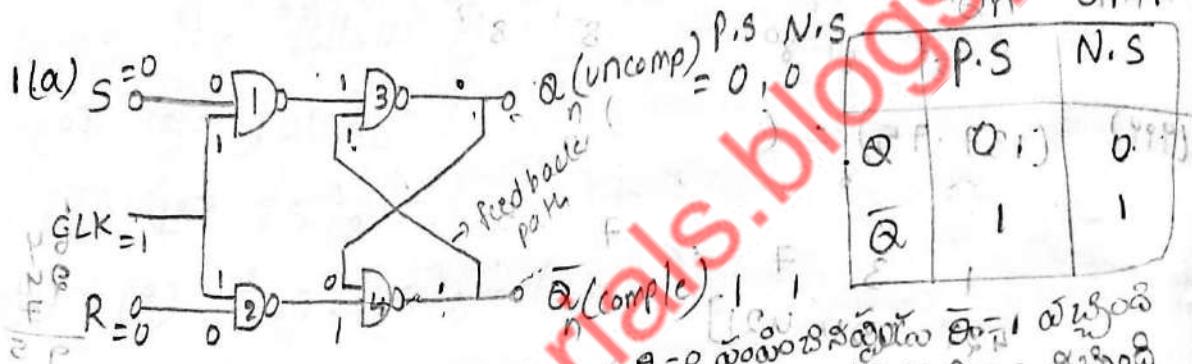
Ex: previous



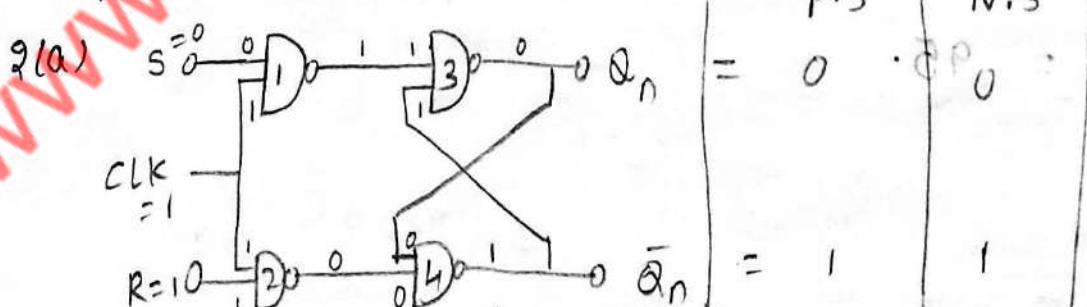
present

(NAND)

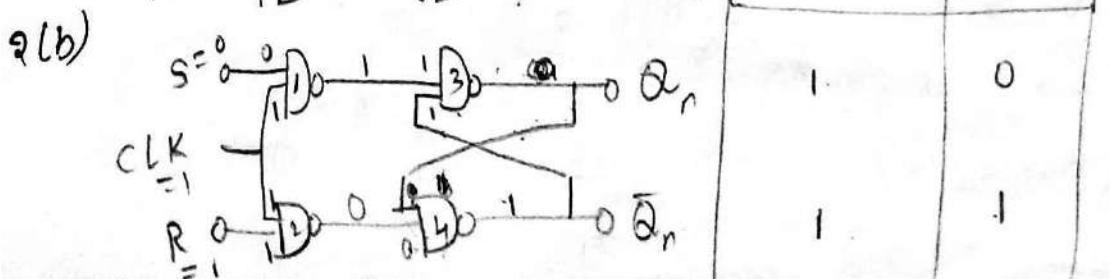
Basic Construction of S-R Flip Flop

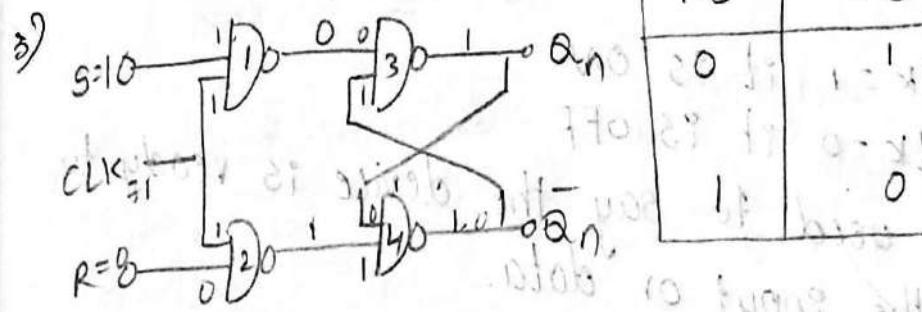


1(a) & 1(b) are no change condition $P.S = N.S \Rightarrow Q_n = Q_{n+1}$

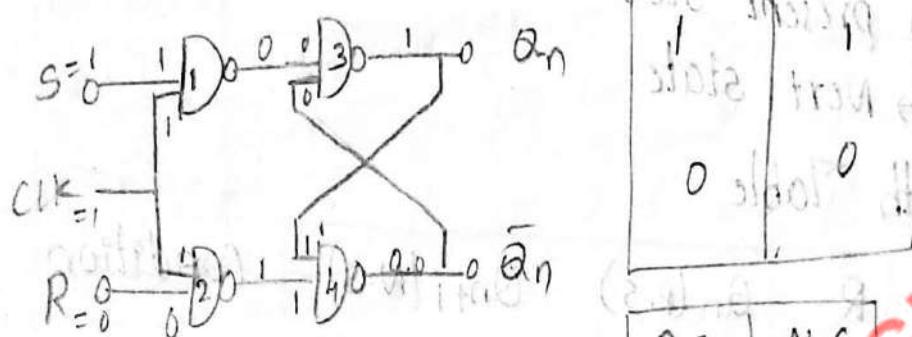


Reset condition (output=0)

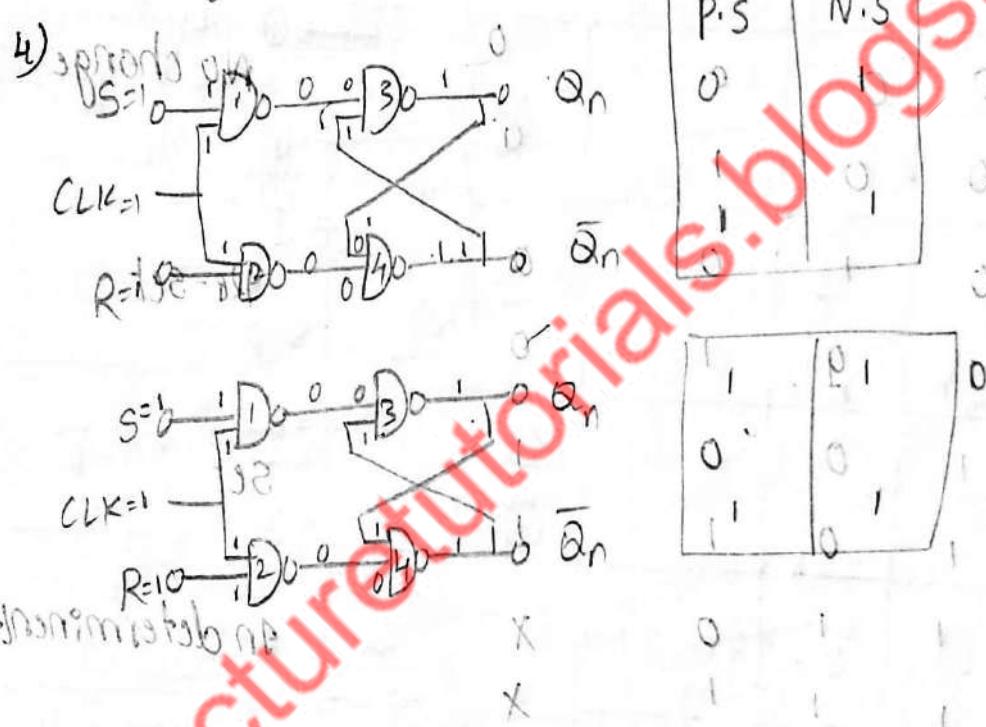




Set state
(output=1)



In determinate condition



In determinate condition

Flip flop:

→ flip flop is a single bit storage device
→ flip flop (flip flops) are four

→ Types of flip flop [Set, Reset]

* S-R flip flop [Set, Reset]

NAND			
A	B	AB	$\bar{A} \cdot \bar{B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

* J-K flip flop

* T flip flop

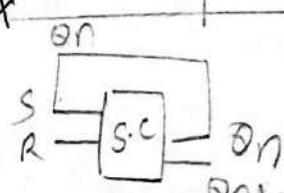
* D flip flop

1. when $CLK = 1$ it is ON
2. when $CLK = 0$ it is OFF
3. CLK is used to say the device is ready to accept the input or data.
4. $PS \rightarrow$ present state
5. $N.S \rightarrow$ Next state

Date Truth Table

SNO	S	R	$Q_n (P.S.)$	$Q_{n+1} (N.S.)$	Condition
0	1	0	0	0	→ ① No change
1	0	0	1	0	→ ②
2	0	1	0	0	→ ③ Re-set
3	0	0	1	0	→ ④ Set
4	1	0	0	1	→ ⑤
5	1	0	1	1	→ ⑥
6	1	1	0	X	In determinent
7	1	1	1	X	

Characteristic Eqⁿ



SR Q_n → Q_{n+1}

Guard (6, 4, 7, 5)

$$(S\bar{R} + \bar{S}R) \cdot (\bar{Q}_n + Q_n)S\bar{R}$$

$$\bar{S}(R + \bar{R})$$

pair (1, 5)

$$(\bar{S}R + S\bar{R})Q_n$$

$$\bar{R}(\bar{S} + S)$$

$$\bar{R}Q_n$$

X	6	X	7
1	4	①	5

$$Q_{n+1} = S + \bar{R} Q_n$$

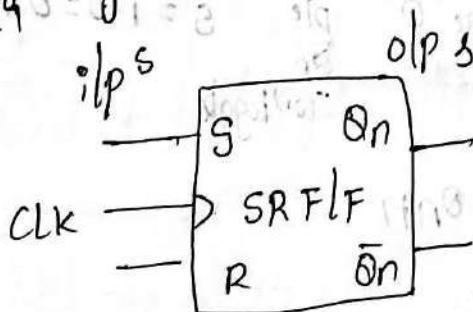
Minimized Truth Table

S R	$Q_{n+1} =$
0 0	Q_n
0 1	0
1 0	1
1 1	X

Excitation Table

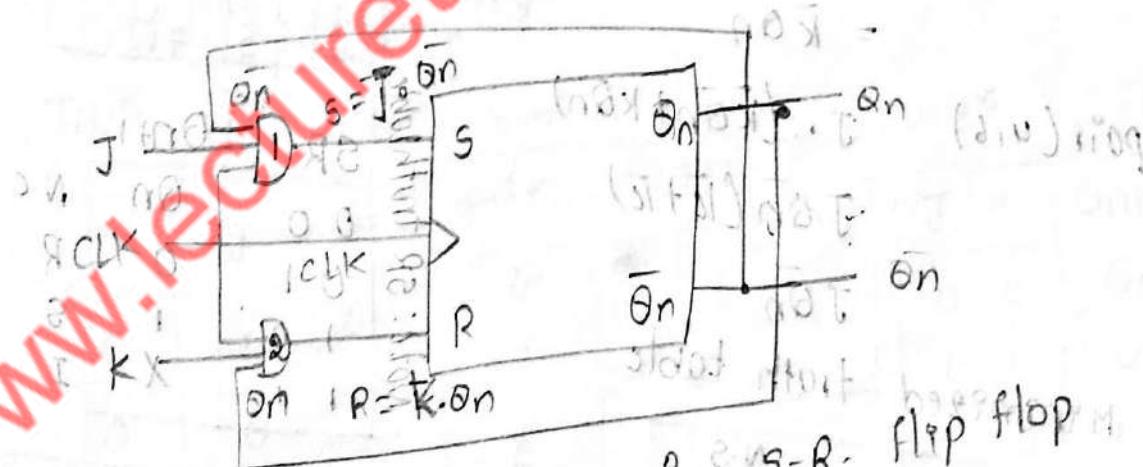
Q_n	Q_{n+1}	S	R
0	0	0 0	
		0 1	
		0 X	
0	1	1 0	
	0	0 1	
1	0	0 0	1
	1	0 1	0
	0	1 0	
	1	X 0	

Date 9/8/19 Symbol



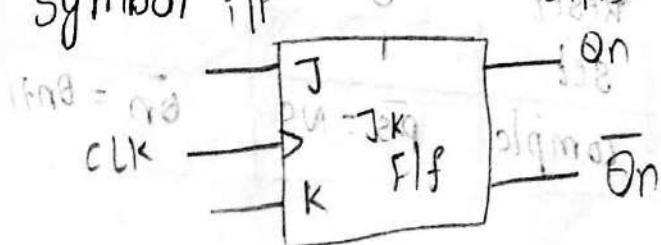
J-K- flip flop

AND Gate



circuit diagram for S-R- flip flop

Symbol ip



Truth Table

Explation table

and truth table
border
conditions present

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J	K	S	R	P.S $Q_n = \bar{Q}_n$	N.S Q_{n+1}	C	$S = J\bar{Q}_n$	$R = K\bar{Q}_n$
0	0	0	0	0	1	0	$S = 0 \cdot 1 = 0$	$R = 0 \cdot 0 = 0$
1	0	0	0	1	0	<u>1</u> ^{new}	$S = 0 \cdot 0 = 0$	$R = 0 \cdot 1 = 0$
0	1	0	0	0	1 ^{N.C}	<u>1</u> ^{new}	$S = 0 \cdot 1 = 0$	$R = 1 \cdot 0 = 0$
0	1	0	1	1	0 ^{R.S}	<u>0</u> ^{new}	$S = 0 \cdot 0 = 0$	$R = 1 \cdot 1 = 1$
1	0	1	0	0	1 ^{set}	<u>1</u> ^{new}	$S = 1 \cdot 1 = 1$	$R = 0 \cdot 0 = 0$
1	0	0	0	1	0 ^{N.C}	<u>0</u> ^{new}	$S = 1 \cdot 0 = 0$	$R = 0 \cdot 1 = 0$
1	1	1	0	0	1 ^{set}	<u>1</u> ^{new}	$S = 1 \cdot 1 = 1$	$R = 1 \cdot 0 = 0$
1	1	0	1	1	0 ^{R.C}	<u>0</u> ^{new}	$S = 1 \cdot 0 = 0$	$R = 1 \cdot 1 = 1$

characteristic equation

(0) toggle

$$J_K Q_n \text{ if } p.s \rightarrow Q_{n+1}$$

pair(1,5) rows. cols

$$(\bar{J} + J) \cdot \bar{K} Q_n$$

$$= \bar{K} Q_n$$

pair(4,6) $J \cdot (\bar{K} \bar{Q}_n + K \bar{Q}_n)$

$$J \bar{Q}_n (\bar{K} + K)$$

$$J \bar{Q}_n$$

Minimized truth table

		K 0	K 1	K 0	K 1	X 0	X 1
		0	1	0	1	0	1
		J 0	J 1	0	1	0	1
J	0	0	1	0	1	0	1
J	1	1	0	1	0	1	0

SR		Q_{n+1}	
0	0	Q _n	N.C
0	1	0	R
1	0	1	S
1	1	X	I

J	K	$Q_{n+1} =$			
0	0	Q _n	N.C	$P.S = N.S$	$Q_n = Q_{n+1}$
0	1	0	Reset	0	
1	0	1	Set	1	
1	1	Q _n	complc	$\bar{P.S} = N.S$	$\bar{Q}_n = Q_{n+1}$

Excitation Table

t.me/jntukonlinebits

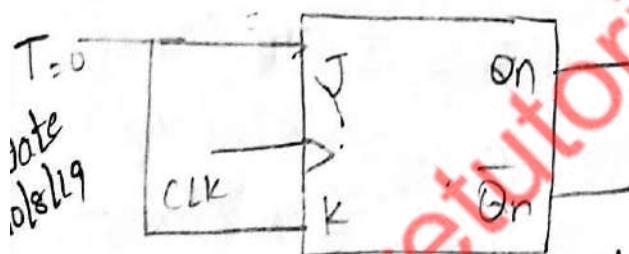
K	0	-	X	0	-	X	-	-	-	0	0	0	
J	0	0		0	1	-	-	0	1	X	0	1	X
Qn	0			1			0		1		1		
Qn+1	0			1			0		1		1		
On	0	.	.	0			-		-		-		

T F/f / Toggle flip flop

T flip flop

olp olp

1 2
T \bar{Q}_n, \bar{Q}_n



two outputs

It has single input and we get two outputs.

Truth table

Assumption

using J-K F/f

T	J	K	Qn	Qn+1
0	0	0	0	0 } $\rightarrow 0$
0	0	0	1	1 } N.C
0	1	1	0	1 } $\rightarrow 2$
1	1	1	1	0 } comp
1	1	1	1	1

J	K	Qn	Qn+1
0	0	Qn NC	Qn NC
0	1	0 R	0 R
1	0	1 S	1 S
1	1	Qn comp	Qn comp

Minimized truth table

D/I/P N.S

T Q_{n+1}

0 0

0 1

1 0

1 T

D/I/P	N.S	
T	Q _{n+1}	
0	0	
0	1	
1	0	Q _n N.C
1	T	Q _n COMP

Excitation table

jntukonlinebits

D/I/P	Q _{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

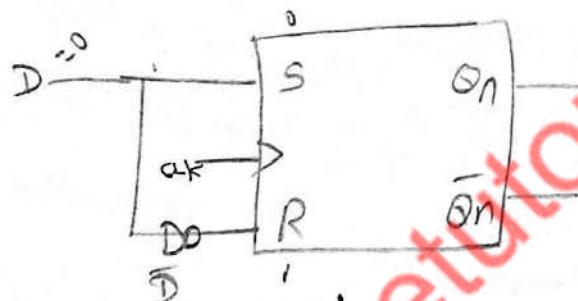
Characteristic equation

$$Q_{n+1} = \bar{T}Q_n + T\bar{Q}_n$$

D flip flop (or) delay
flip flop

Q	0	0	0	Q _{n+1}
T	0	0	1	
T	1	1	1	

This D flip flop can
be constructed by
using S-R flip flop



Truth Table

D/I/P	S=D	R=bar{D}	(P.S) Q _n	(N.S) Q _{n+1}
0	0	1	0	0] Reset
0	0	1	1	0]
1	1	0	0	1] Set
1	1	0	1	1]

S.R. F/F Truth table

Minimized truth table

<http://ntfukonlinebits.com>

S	R	Qn+1	
0	0	Qn	N.C
0	1	0	R
1	0	1	S
1	1	X	inde

D	Qn	Qn+1
0	0	0
1	1	1

Excitation table

characteristic equation:

Qn	Qn+1	D
0	0	0
0	1	1
1	0	0
1	1	1

D	Qn	Qn+1
0	0	0
1	0	1
2	1	0
3	1	1

D	Qn	Qn+1
0	0	1
1	1	0

Date
13/8/19

Conversion of flip flops

1. Convert S-R flip flop to D-flip flop (or)

1. convert S-R flip flop to D-flip flop using S-R flip flop.

Construct D flip flop is going to construct take the
solution 1. which flip flop is going to use for the construction
truth table of that flip flop

2. which flip flop is going to use for the construction
take excitation table of the flip flop

D	Qn			Qn+1			condi	S R X
	P-S	N-S	Qn	Qn+1	condi	condi		
0	0	10	0	0	Reset	X0		0 X0
1	0	1	0	1		1		1 0
2	1	0	1	1	Set	0 1		0 1
3	1	1	1	X		0 X		X 0

Truth table of
D flip flop

Qn	Qn+1	S R D
0	0	0 X0
0	1	1 0
1	0	0 1
1	1	X 0

excitation table
of SR flip flop

S-R flip flop \rightarrow D flip flop

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S K-map

D	Qn	$\bar{Q}n$	Qn
$\bar{Q}n$	0	1	
Qn	1	0	X
$\bar{Q}n$	1	X	0

$$S = D(\bar{Q}n + Qn)$$

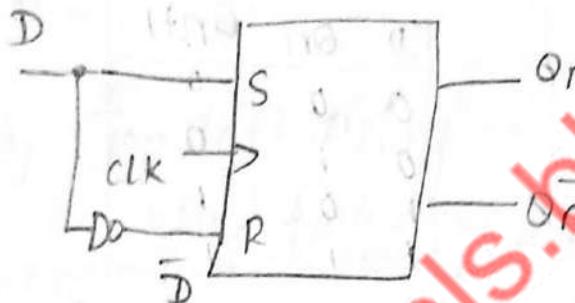
$$S = D$$

R- K-map

D	Qn	$\bar{Q}n$	Qn
\bar{D}	0	1	
D	1	0	D
\bar{D}	1	2	3

$$R = \bar{D}(\bar{Q}n + Qn)$$

$$= \bar{D}$$



D-flip flop

2. Convert S-R flip flop to J-K flip flop
 present (S R) flip flop \rightarrow JK flip flop (construct)
 Truth table
 Excitation table
 Truth table of J-K flip flop
 Excitation table of S-R flip flop

J	K	present Qn	nexts Qn+1	condi tion	S R
0	0	0	0	N.C	0 X
1	0	1	1		X 0
2	0	0	0		0 X
3	1	1	0	Reset	0 1
4	1	0	1	Set	1 0
5	1	1	1		X 0 X
6	1	0	1	Comp	1 0
7	1	1	0		0 1

Qn	Qn+1	S R
0	0	0 X
0	1	1 0
1	0	0 1
1	1	X 0

$$S = \sum_m (U, 6) + d(1, 5)$$

$$R = \sum_m (3, 7) \quad \text{tutor/antukonlinebits}$$

S K-map

		K-Qn			
		K̄Qn	Qn	K̄Qn	Qn
		00	01	11	10
J	0	X			
	1	1	1	3	2
J̄	0	0	1	3	2
	1	1	X	7	6

R K-map

		K̄Qn			
		K̄Qn	Qn	K̄Qn	Qn
		00	01	11	10
J	0	X		1	X
	1	0	1	3	2
J̄	0	1		3	2
	1	1	1	7	6

pair(U, 6)

$$= J \cdot (\bar{K}Qn + K\bar{Q}n)$$

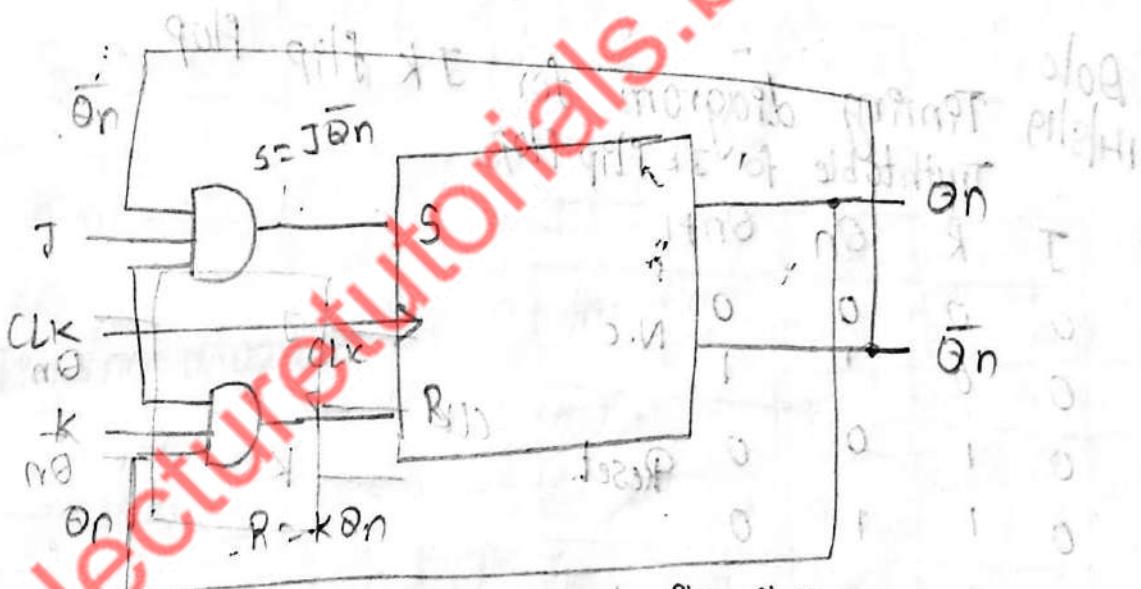
$$= J\bar{Q}n(\bar{K} + K)$$

$$S = J\bar{Q}n$$

pair(3, 7)

$$= (\bar{J} + J)\bar{Q}n$$

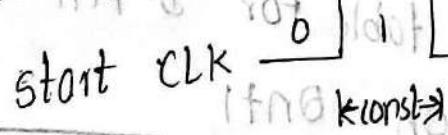
$$R = K\bar{Q}n$$



J-K flip flop

Timing Diagrams
⇒ S-R flip flop (or) edge trigger flop flop

start CLK



konst

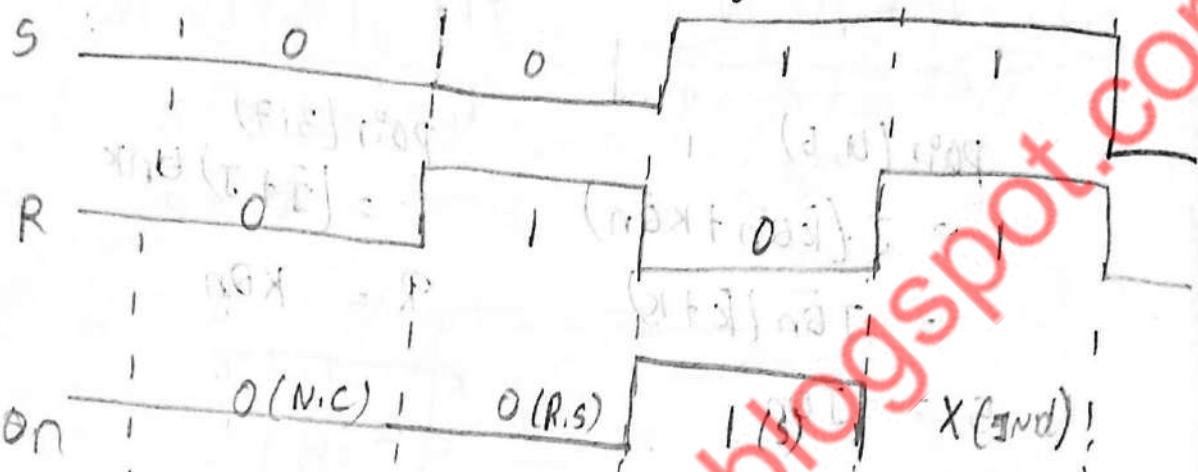
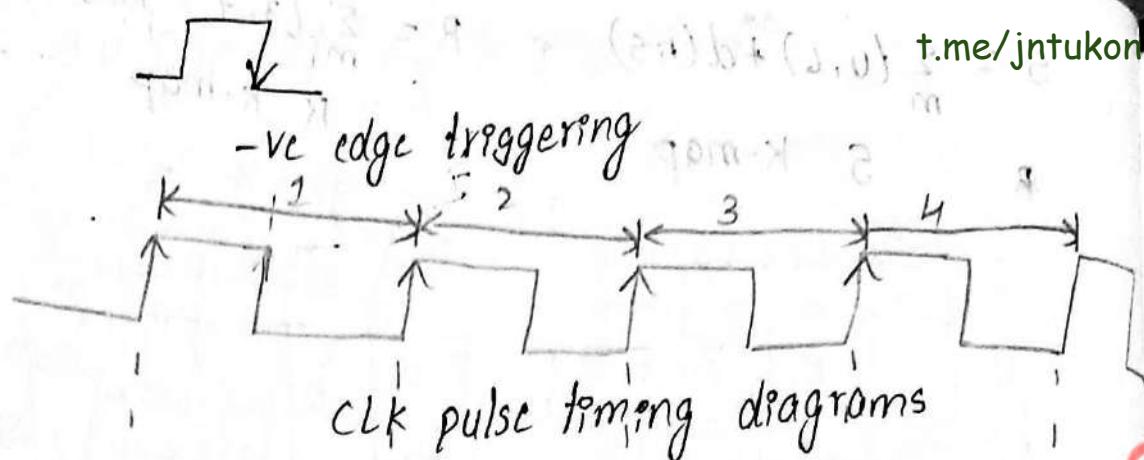
OFF → ON (triggering)

minimized
truth table

S	R	Qn(t)
0	0	Qn(t)
0	1	0
1	0	1
1	1	X

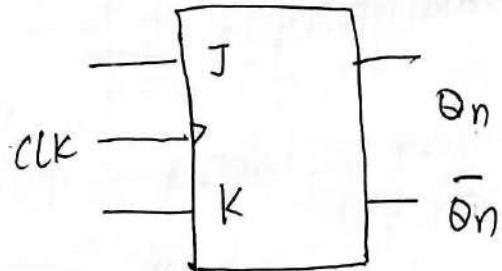


+ve edge triggering



Date : 14/8/19 Timing diagram for JK flip flop
 Truth table for JK flip flop

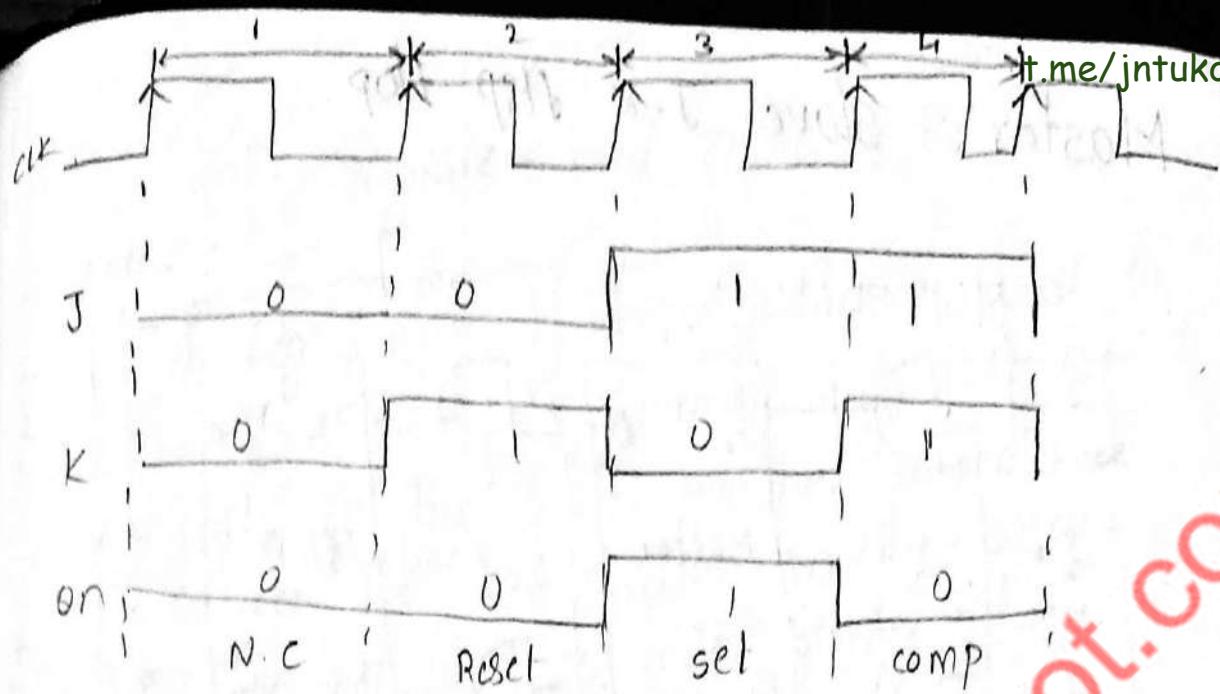
J	K	Qn	Qn+1
0	0	0	0
0	0	1	1 N.C.
0	1	0	0 Reset.
0	1	1	0
1	0	0	1 set
1	0	1	1
1	1	0	1 complete
1	1	1	0



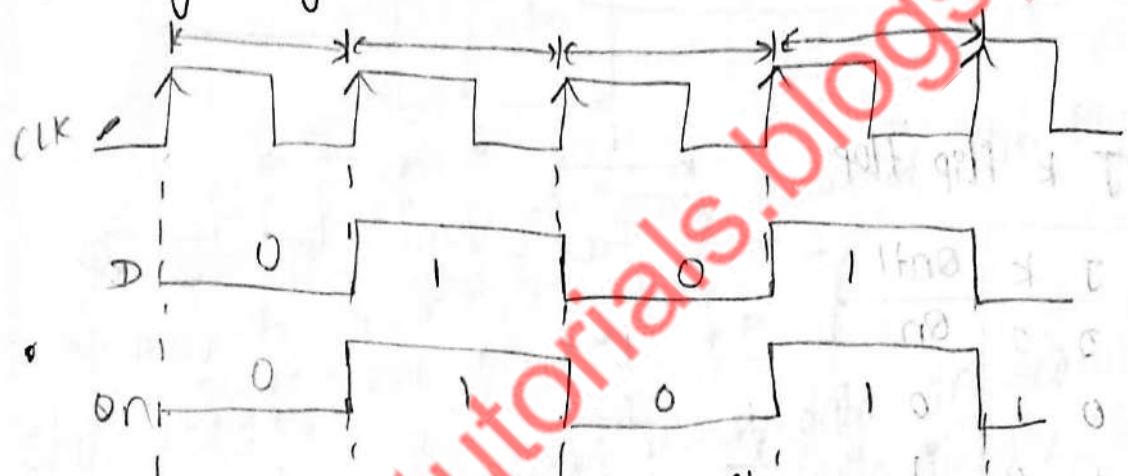
Minimized truth table for D-Flop flop

D	Qn	Qn+1
0	0	0
0	1	0
1	0	1
1	1	1

Reset is Qn+1 = 0
 Set



Timing diagram for D flip flop



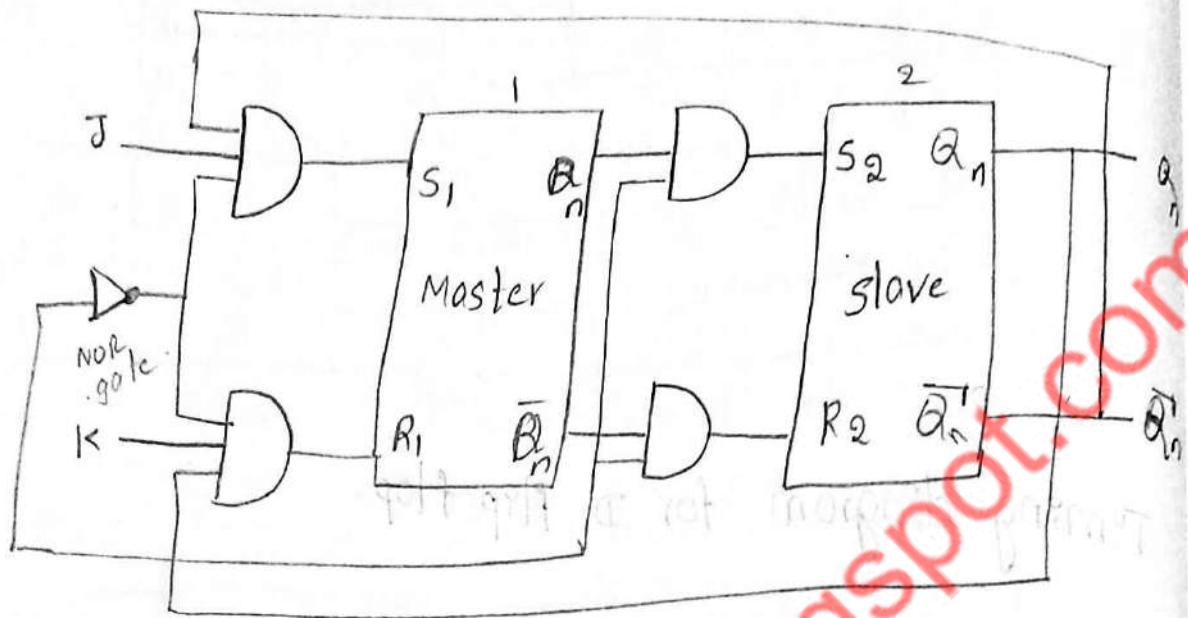
Timing diagram for T flip flop

T	Qn	On	P-S	N-S
0	0	0	0	N.C.
0	1	1	1	0
1	0	0	0	1 comp

minimized truth table

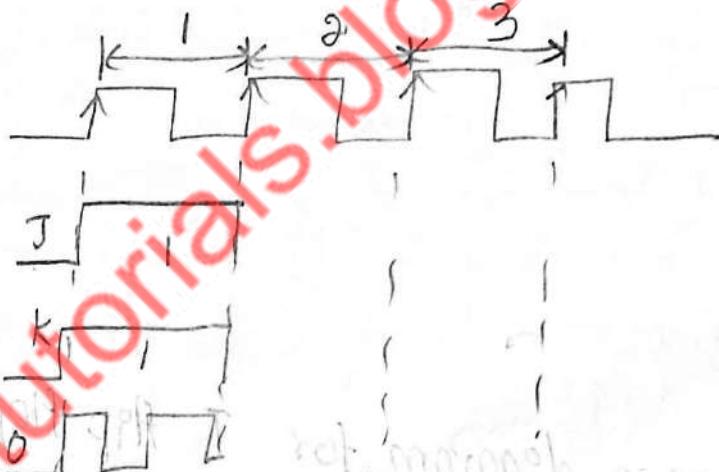
Master's slave J-K flip flop

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J K flip flop

J	K	Qn
0	0	Qn
0	1	0
1	0	1
1	1	\bar{Q}_n



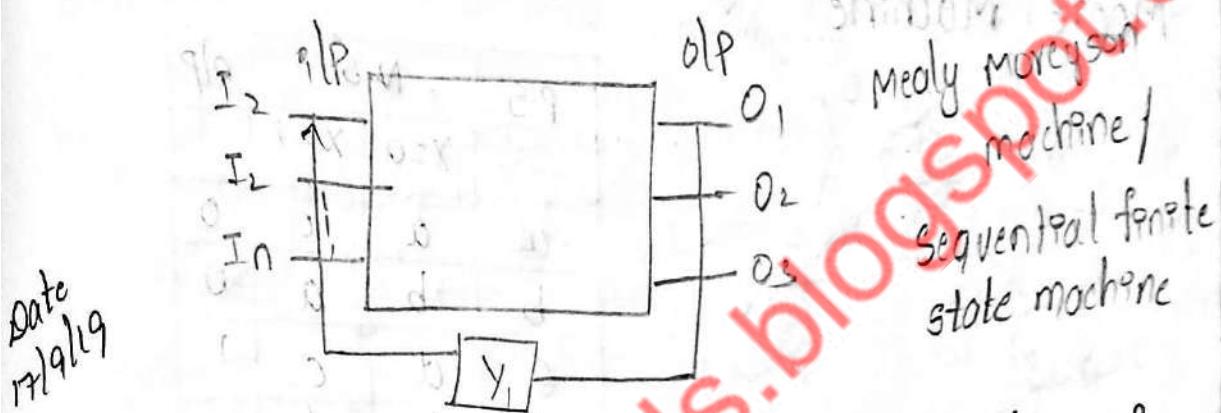
Racearound problem

Here when the values of J K are both "1" we get the toggle or complement as output

while occurring this output an error race around problem occurred to avoid this "Master slave JK flip flop is used".

Finite State Machine

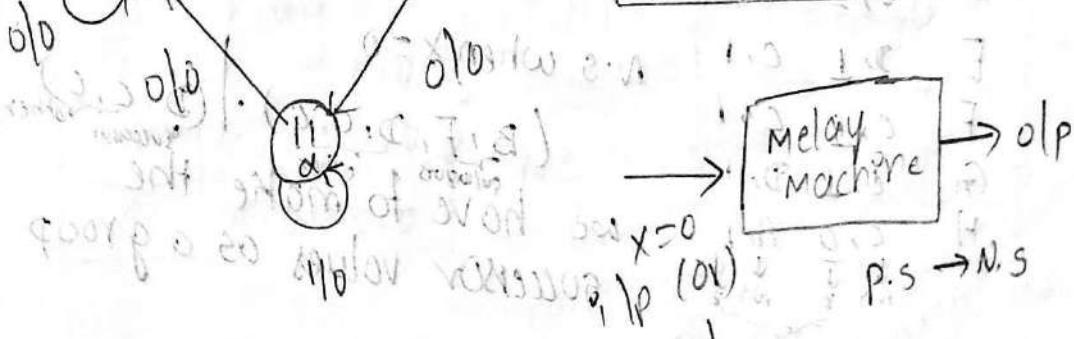
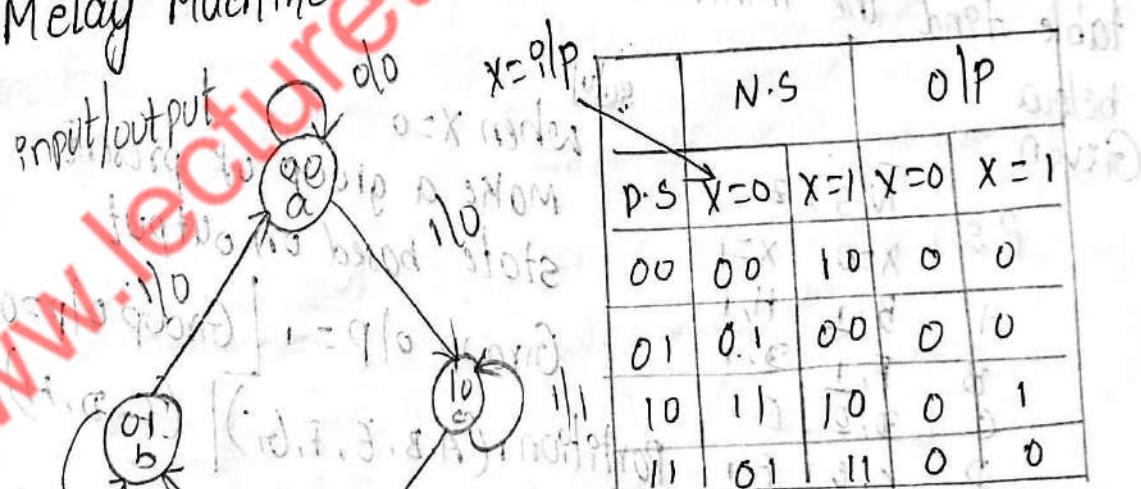
Finite state machine is a model which is used to describe the synchronous sequential machine. It is a machine with fixed no. of states i_1, i_2, \dots, i_n = input variables; $o_1, o_2, o_3, \dots, o_n$ = output variables $y_1, y_2, y_3, \dots, y_n$ are state variables.



State Diagrams

State diagram is the pictorial representation of states, present state & next state

Mealy Machine



Excitation table

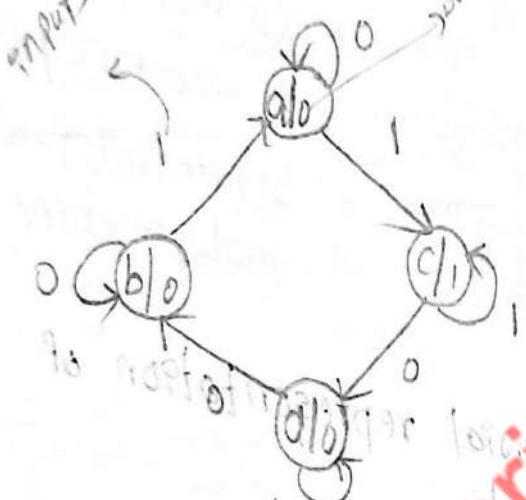
N.S		output		
P.S	X=0	X=1		
00	00	0	10	Q _{0P}
01	01	0	00	0
10	11	0	10	1
11	01	0	11	0

Note

If they didn't mention any model it is melay otherwise moore

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Moore Machine



State table

P.S	N.S		o/p
	X=0	X=1	
a	a	c	0
b	b	a	0
c	d	c	1
d	b	d	0

what are the conditions to minimize the state table find the minimized state table for the given below.

Given

P.S	N.S, Z		o/p
	X=0	X=1	
A	B, 1	H, 1	
B	F, 1	D, 1	
C	D, 0	E, 1	
D	C, 0	F, 1	
E	D, 1	C, 1	
F	C, 1	C, 1	
G	C, 1	D, 1	
H	C, 0	A, 1	

when X=0

make a group of present state based on output

Group o/p = 1 Group o/p = 0

partition: (A, B, E, F, G) (C, D, H)

N.S when X=0

(B, F, D, C, C) . | (D, C, C)

we have to make the successor values as a group

partition 2

$$(A+B)(E+F+G)(C)(DH) \quad [\because B \text{ is } \text{sum} \text{ of } \text{bits}]$$

F is a successor of B

N.S when $X=1$

$$(HD) \underbrace{(CD)}_{\text{successor}} \underbrace{(E)}_{\text{other}} \underbrace{(FA)}_{\text{successors}}$$

HD are successor of AB
E is not successor of C

partition 3

$$(AB)(EFG)(C)(DH)(CD)(H)$$

minimized state table

P.S	N.S, Z	
	X=0	X=1
A	B, 1	H, 1
B	F, 1	D, 1
C	D, 0	E, 1
E	D, 1	C, 1
H	C, 0	A, 1

Date
19/9/2019

Derive a circuit that realizes the finite state machine defined by the state assignment table below using J-K flip flop.

when $X=0$

Group 0/p = 1

Group 0/p = 0

P.S	N.S, Z	
	X=0	X=1
A	B, 0	E, 0
B	E, 0	D, 0
C	D, 1	A, 0
D	C, 1	E, 0
E	B, 0	D, 0

partition 1 (C, D)

N.S when $X=0$

(D, C)

(A, B, E)

(B, E, B)

partition 2

(E) (D) (C)

N.S when $X=1$

(A, E)

(AB) E

(E, D, D)

partition 3

D, C

(AB) E

Minimized truth table

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P.S	N.S, Z	
	X=0	X=1
A	B, 0	C, 0
C	D, 1	A, 0
D	C, 1	E, 0
E	B, 0	D, 0

Implement the finite state machine using J-K flip flops [the given state table is reduced state table for the above reduced table]

Q1P soln	X	P.S		N.S		Q1P _Z	J _P	K _P	J _Q	K _Q
		Q _P	Q _Q	Q _P	Q _Q					
0	0	A	00	B	01	0	0	X	1	X
1	0	B	01	B	01	0	0	X	X	0
2	0	C	10	D	11	1	X	0	1	X
3	0	D	11	C	10	1	X	0	X	1
4	1	A	00	B	01	0	0	X	X	0
5	1	B	01	D	11	0	1	X	X	0
6	1	C	10	A	00	0	X	1	0	X
7	1	D	11	B	01	0	X	1	X	0

Given minimized table

Q _n	Q _{n+1}	J K		P.S	N.S, Z	
		X	X		X=0	X=1
0	0	0	X	A	B, 0	B, 0
0	1	1	X	B	B, 0	D, 0
1	0	X	1	C	D, 1	A, 0
1	1	X	0	D	C, 1	B, 0

Excitation table of JK flip flop

	00	01	11	10
0	X	X	1	
1	1	X	X	1

$$Jq = \bar{x} + \bar{p}$$

	00	01	11	10
0	X	1	X	
1	0	X	1	X

$$Kq = \bar{x} \cdot p$$

	00	01	11	10
0		X	X	
1	0	1	X	X

$$Jp = xq$$

	00	01	11	10
0	X	X	1	2
1	X	X	1	1

$$Kp = x$$

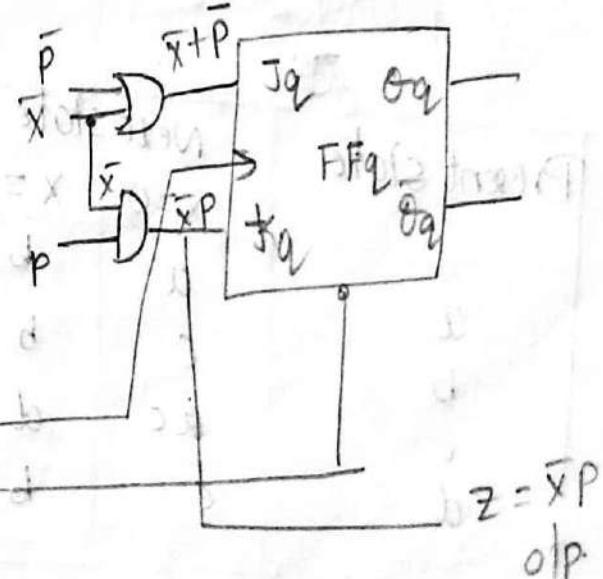
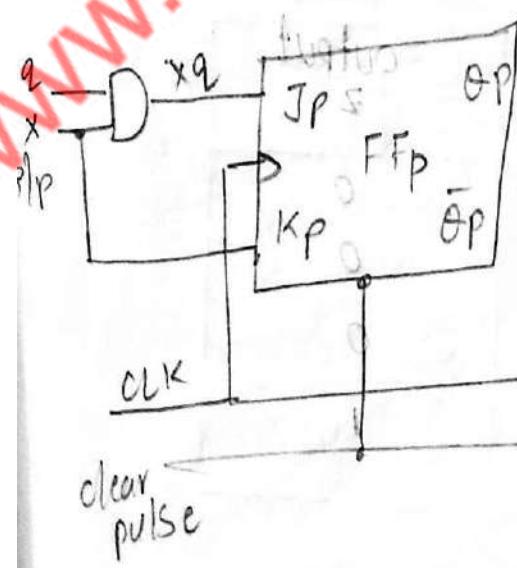
olp: z

	00	01	11	10
0	0	1	1	2
1	0	5	7	6

$$z = \bar{x}p$$

logic circuit

Here x, p, q are inputs.



$$z = \bar{x}p$$

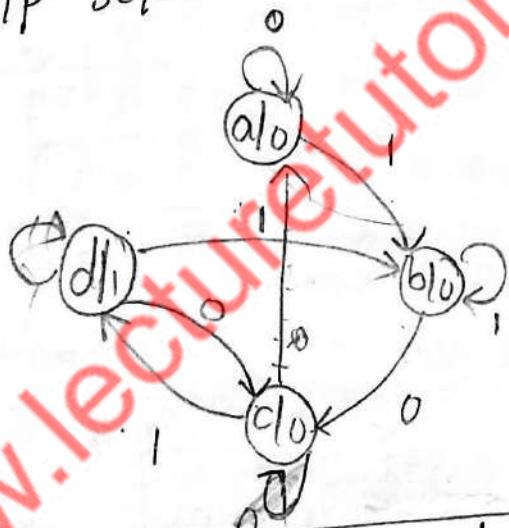
olp

- Assignment*
- If the simple example explain the difference between Mealy and Moore type machine (Unit-II)
 - Draw the diagram of Moore type FSM (finite state machine) for serial adder (Unit-II).
 - Draw the diagram of Mealy type state machine for serial adder and explain its operation (Unit-II).
 - What are the capabilities and limitations of finite state machine explain (Unit-II).

Date 2019 Sequence Detector

Design a Moore type sequence detector to detect a serial input sequence of 101

o/p sequence 101
 $s_1 s_2 s_3$



Excitation table of d Ff

on	on + 1	D
0	0	0
0	1	1
1	0	0
1	1	1

Present state	Next state		Output z
	$x=0$	$x=1$	
a	a	b	0
b	c	b	0
c	ac	d	0
d	c	b	1

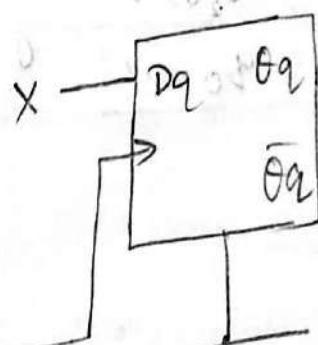
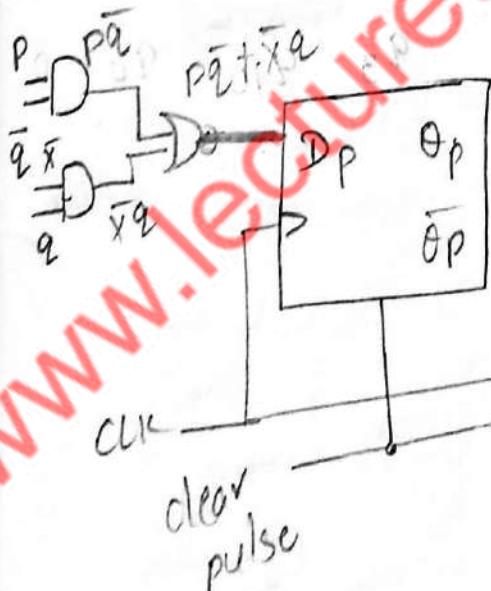
Xq/p	$P.S$	$N.S$	D_P	D_q
	$p \cdot q$	$p+q+1$		
0	0 a	00	0 0	0 1
1	0 b	01	1 0	0
2	0 c	10	1 0	0
3	0 d	11	1 0	0
4	1 a	00	b 0 1	1
5	1 b	01	b 0 1	1
6	1 c	10	d 1 1	1
7	1 d	11	b 0 1	1

X	$p \cdot q$	$p+q+1$	$p \cdot q$
0	00	01	1
1	01	11	1
2	10	10	1

X	$p \cdot q$	$p+q+1$	$p \cdot q$
0	00	01	11
1	01	11	10
2	10	10	10

$$D_P = p \cdot q + \bar{p} \cdot \bar{q}$$

$$D_q = X$$



Date
21/9/19 Melay to Moore Machine

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P.S	N.S , z	
	X=0	X=1
q ₀	q _{3,0}	q _{1,1} , q _{1,0}
q ₁	q _{0,1}	q _{3,1} , q _{0,0}
q ₂	q _{2,1} , q _{1,1}	q _{2,1} , q _{2,0}
q ₃	q _{1,1} , q _{0,1}	q _{0,1}

q _{0,1}	q _{1,1}	q _{11,1}
q _{0,1}	q _{1,0}	q _{10,0}
No change		
q _{3,0}	q _{2,1}	q _{21,1}
q _{3,0}	q _{2,0}	q _{20,1}
No change		

State table

P.S	N.S , z		O/P
	X=0	X=1	
q ₀	q _{3,0}	q _{11,1}	1
q ₁₀	q _{0,1}	q _{3,0}	0
q ₁₁	q _{0,1}	q _{3,0}	1
q ₂₀	q _{21,1}	q _{20,0}	0
q ₂₁	q _{21,1}	q _{20,0}	1
q ₃	q _{10,0}	q _{0,1}	0

Moore Machine

P.S	X=0	X=1	O/P
q ₀	q ₃	q ₁₁	1
q ₁₀	q ₀	q ₃	0
q ₁₁	q ₀	q ₃	1
q ₂₀	q ₂₁	q ₂₀	0
q ₂₁	q ₂₁	q ₂₀	1
q ₃	q ₁₀	q ₀	0

Date
16/8/19

Registers and Counters

Registers

A register is a device which is used to store the data in binary format and shifts the binary bits in the serial manner and also we can collect the information in the same binary format. It is called bit by bit.



store the binary data

temporary

flip flops

Register is formed by using the no. of flip flop.

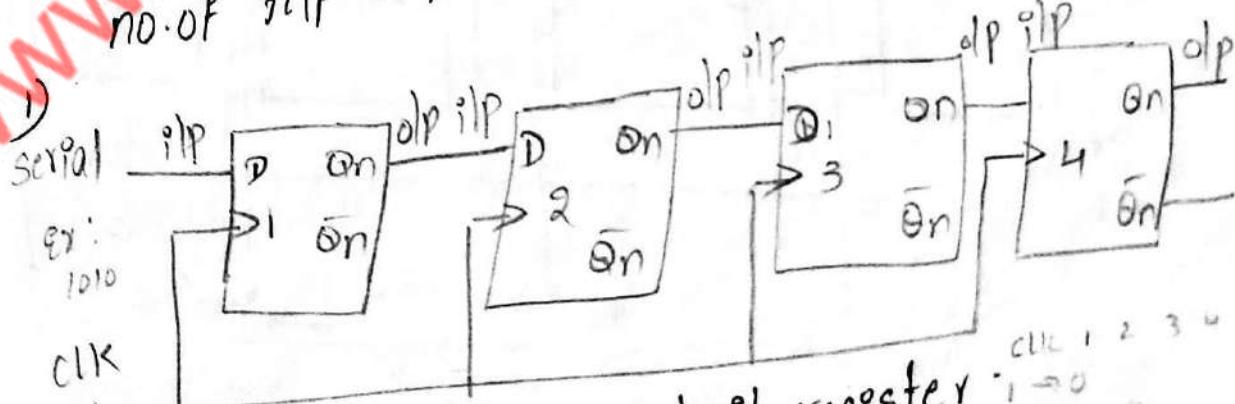
it may be D or T flip flop.

shift register

It shifts the given binary data in serial manner and we can collect the data in the

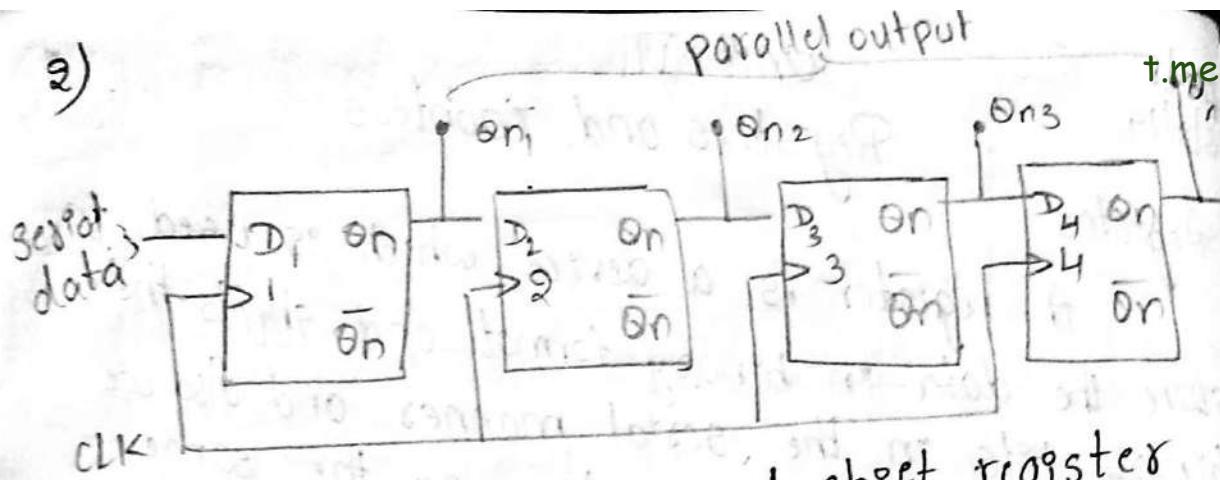
same binary manner

4 bit register no. of flip flops = 4 ; we may use "D" flip flop



Serial in - serial out shift register
 \therefore in means = entry of data

clk : 1 2 3 4
 1 → 0
 2 → 0
 3 → 0
 4 → 0

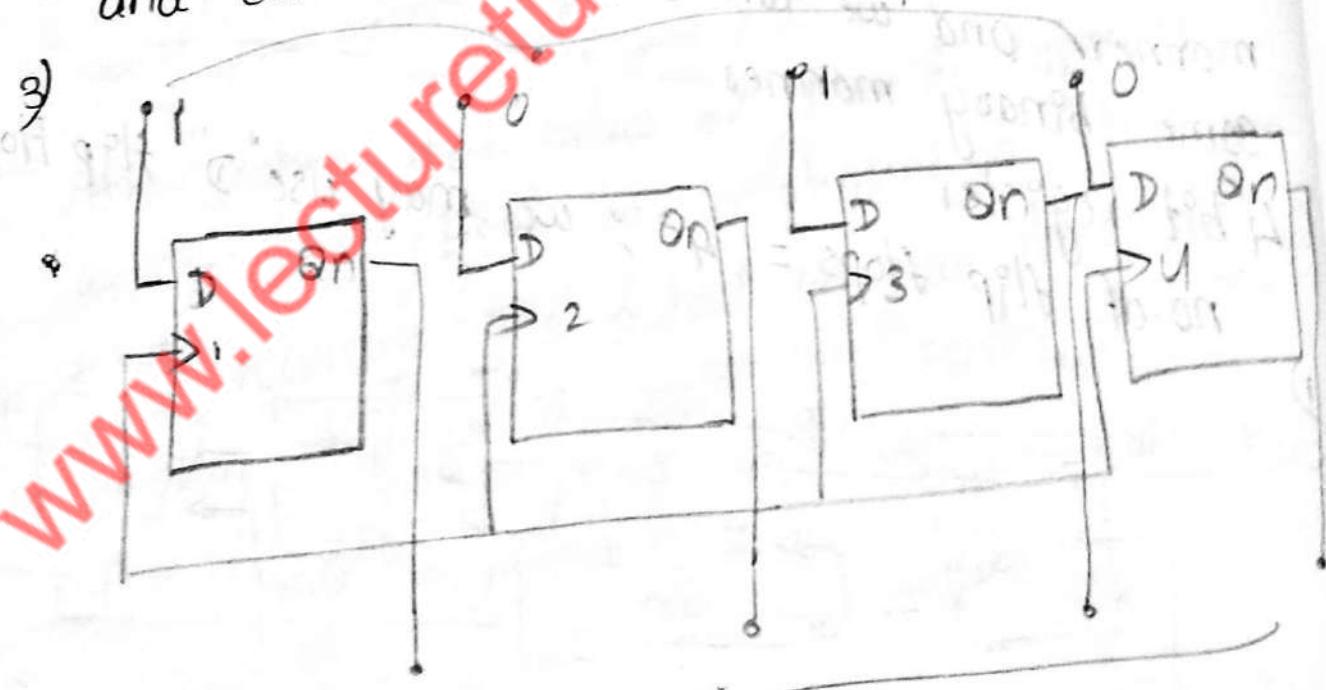


Serial - in - parallel - out shift register

CLK	ops across f/f			
	D ₁	D ₂	D ₃	D ₄
0	-	-	-	-
1	0	-	-	-
2	1	0	-	-
3	0	1	0	-
4	1	0	1	0

In this register the data entered in serial
and collected in parallel out

parallel in

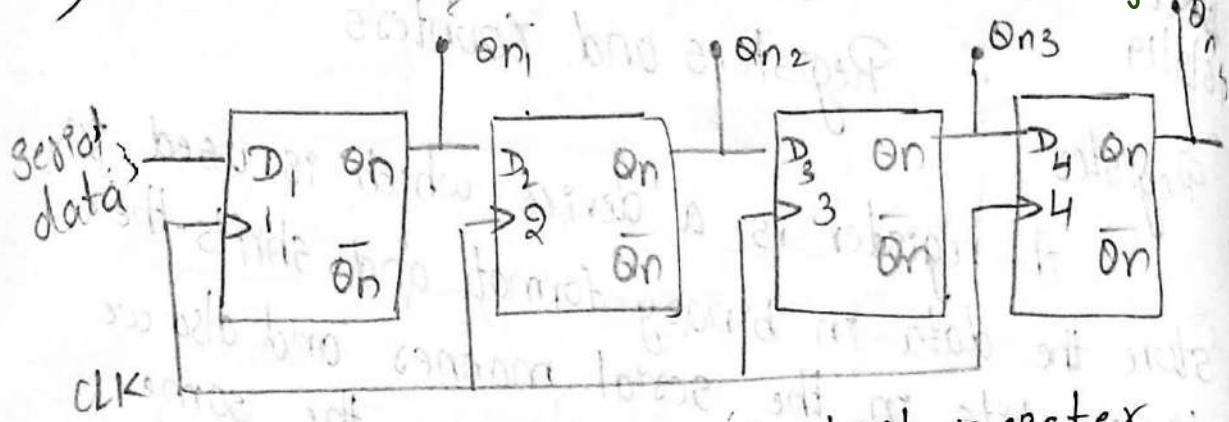


Parallel - in - parallel - out shift register

2)

parallel output

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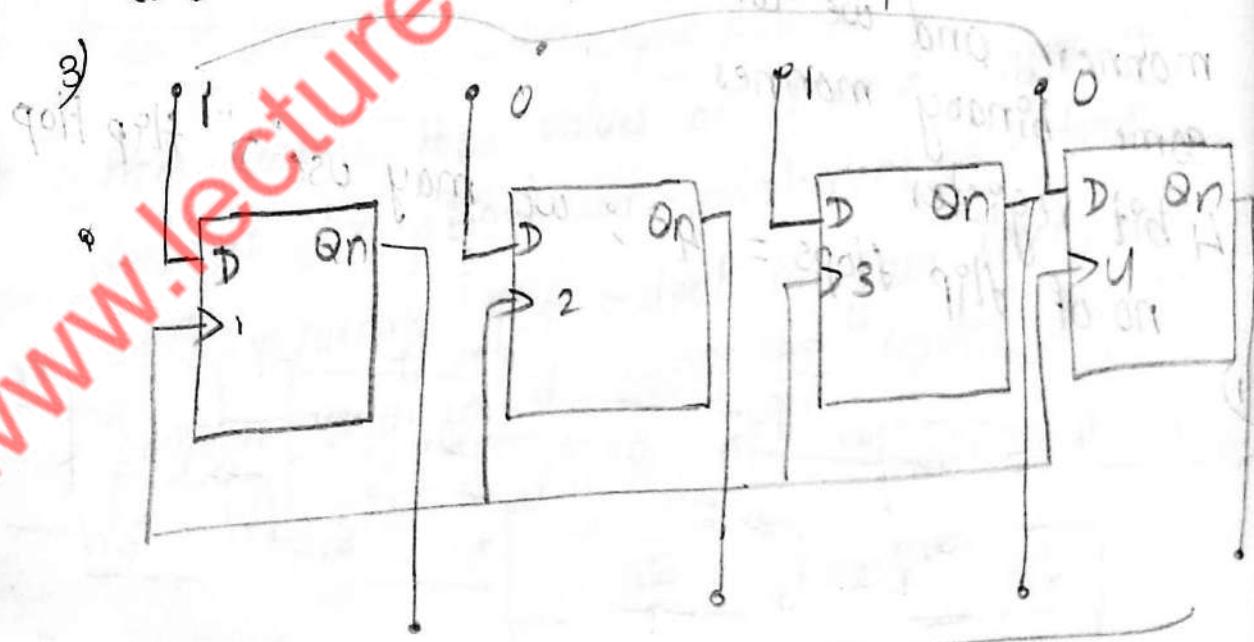


Serial - in - parallel - out shift register

		ops across f/f					
4	3	2	1	D_1	D_2	D_3	D_4
4	3	2	1	0	-	-	-
1	0	1	0	0	-	-	-
2	1	0	-	-	-	-	-
3	0	1	0	-	-	-	-
4	1	0	1	0	-	-	-

equivalent state
 $1010 \rightarrow$
 In this register, the data entered in serial
 and collected in parallel out

parallel in



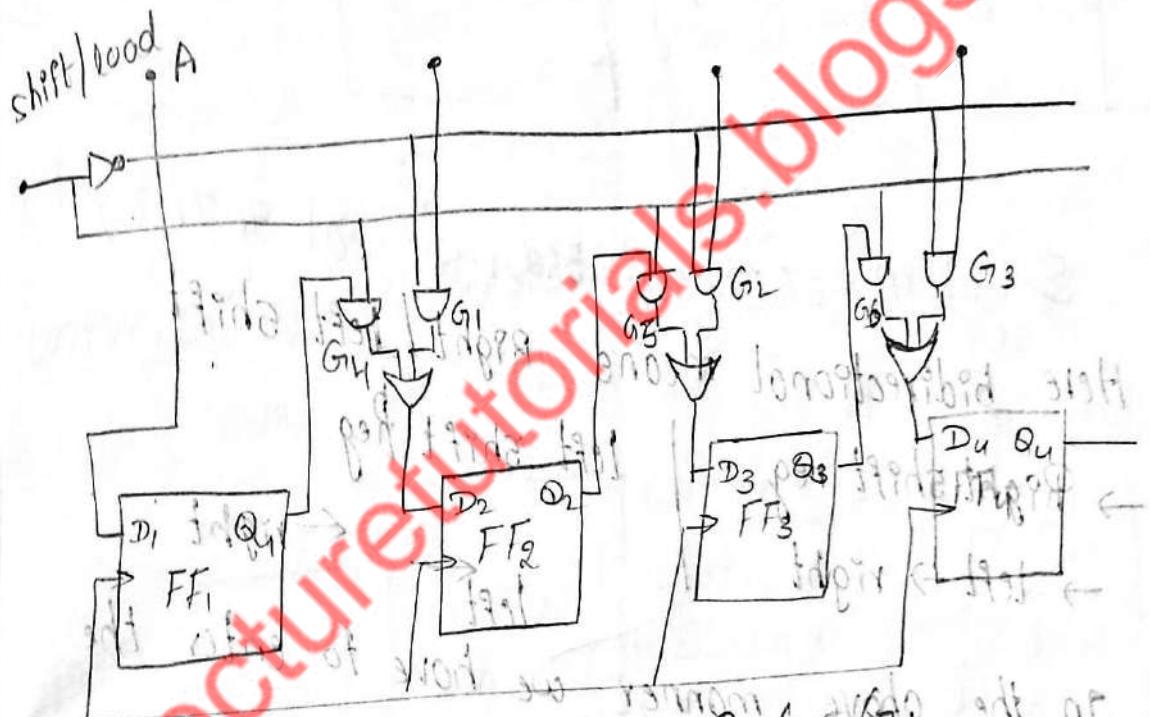
parallel out

Parallel - in - parallel - out shift register

CLK	D ₁	D ₂	D ₃	D ₄
0	-	-	-	-
1	1	0	1	0
2	Q ₁ 1	Q ₂ 0	Q ₃ 1	Q ₄ 0

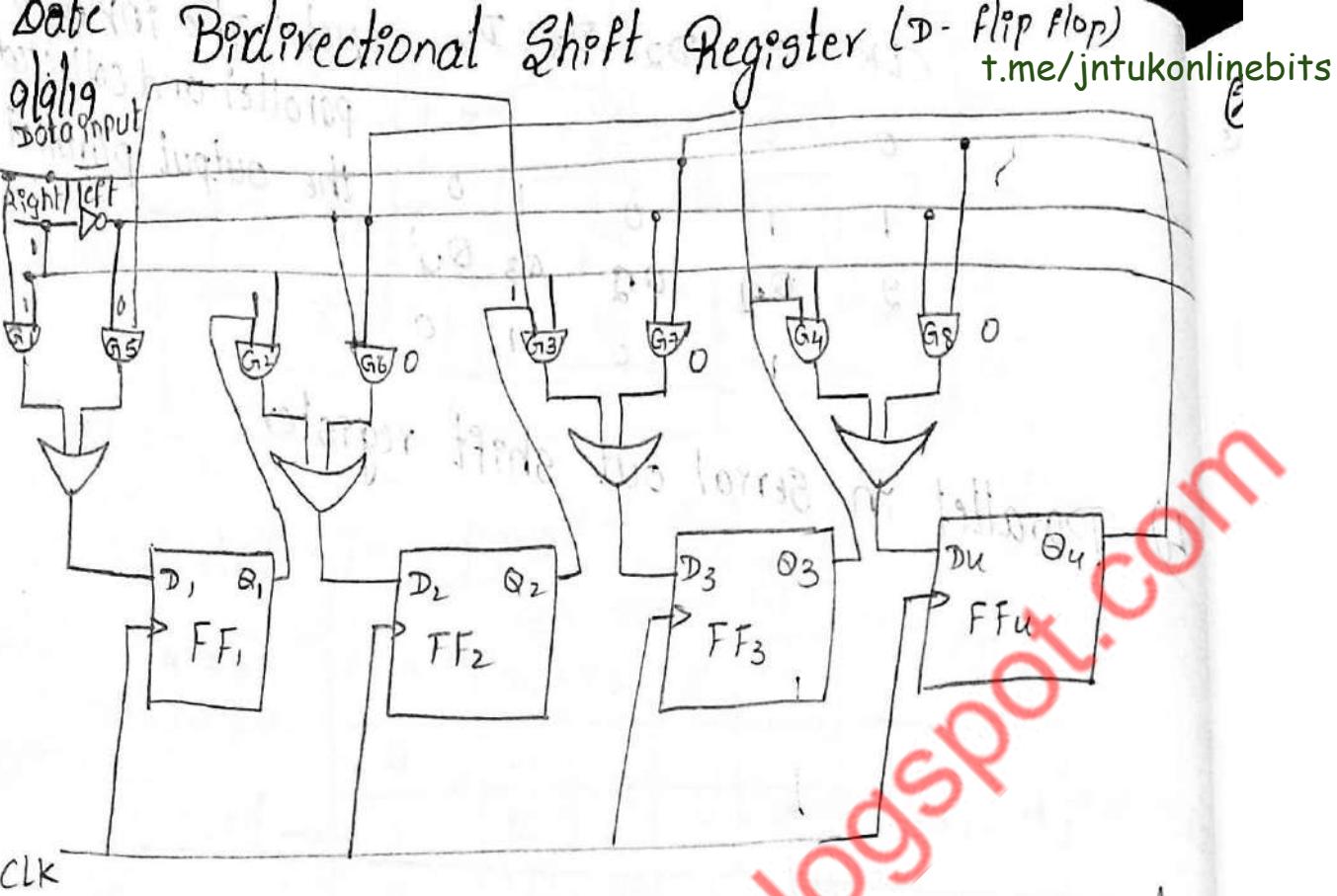
Note: Then output is parallel and collected
the output parallel

4) Parallel-in. Serial-out shift register



shift load = 0 0 0 0
0 0 0 0 disable enable
state before shift
shift load = 0 0 0 0
state after shift

shift load = 0 0 0 0
state after shift
shift load = 0 0 0 0
state after shift



Serial-in - Serial out Bidirectional shift register

Here bidirectional means Right / left shift

→ Right shift reg | left shift Reg

→ left → right

left

right

In the above manner we have to enter the data according to arrow marks

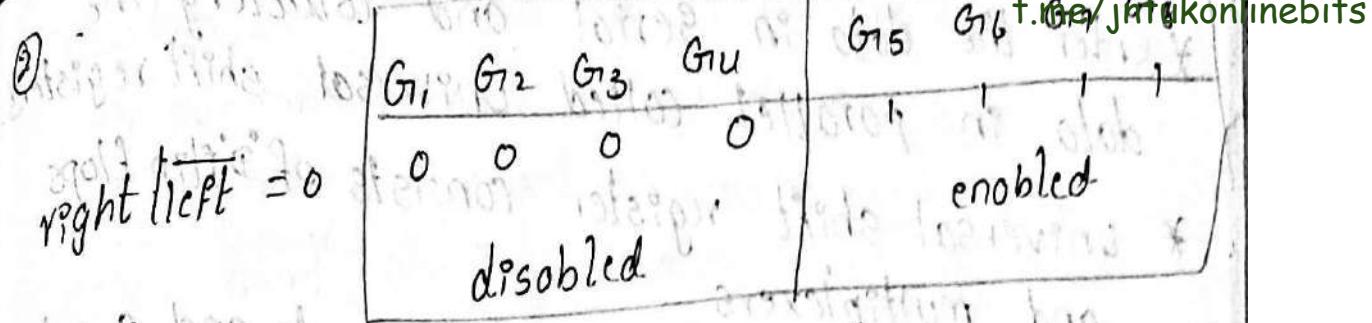
	G ₁	G ₂	G ₃	G ₄	G ₅	G ₆	G ₇	G ₈
Right / left = ,	1	1	1	1	0	0	0	0
enabled state							disabled state	

example

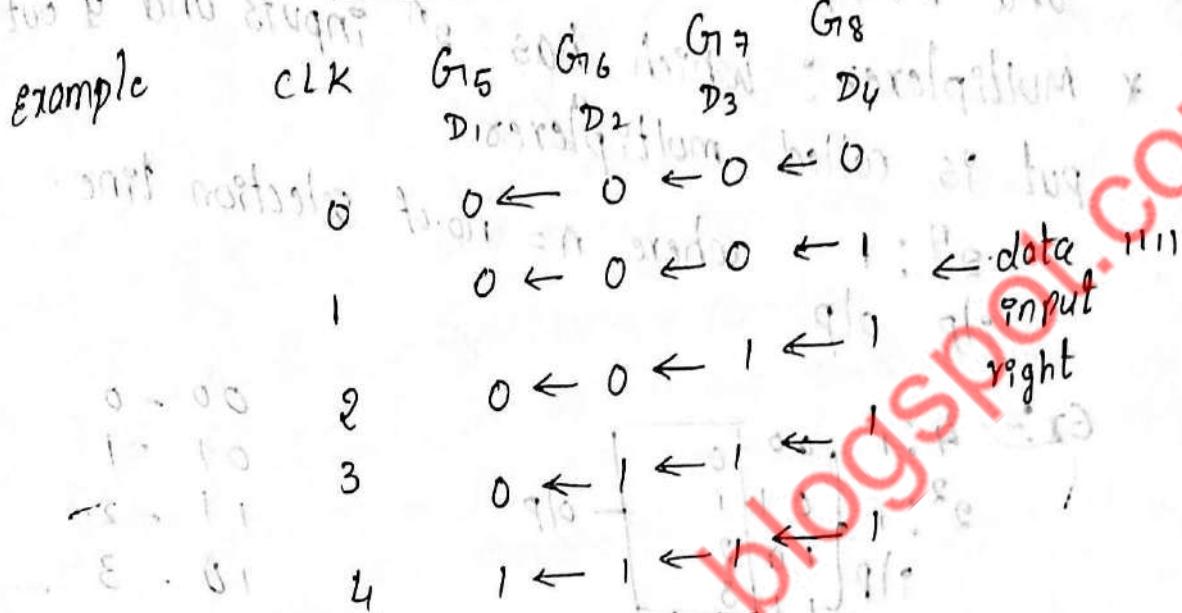
initial state
 $\begin{matrix} CLK \\ 0 \\ \downarrow \end{matrix} \rightarrow \begin{matrix} D_1 \\ 0 \\ \downarrow \end{matrix} \rightarrow \begin{matrix} D_2 \\ 0 \\ \downarrow \end{matrix} \rightarrow \begin{matrix} D_3 \\ 0 \\ \downarrow \end{matrix} \rightarrow \begin{matrix} D_4 \\ 0 \\ \downarrow \end{matrix} \rightarrow \text{initial state}$

data input 1111 left

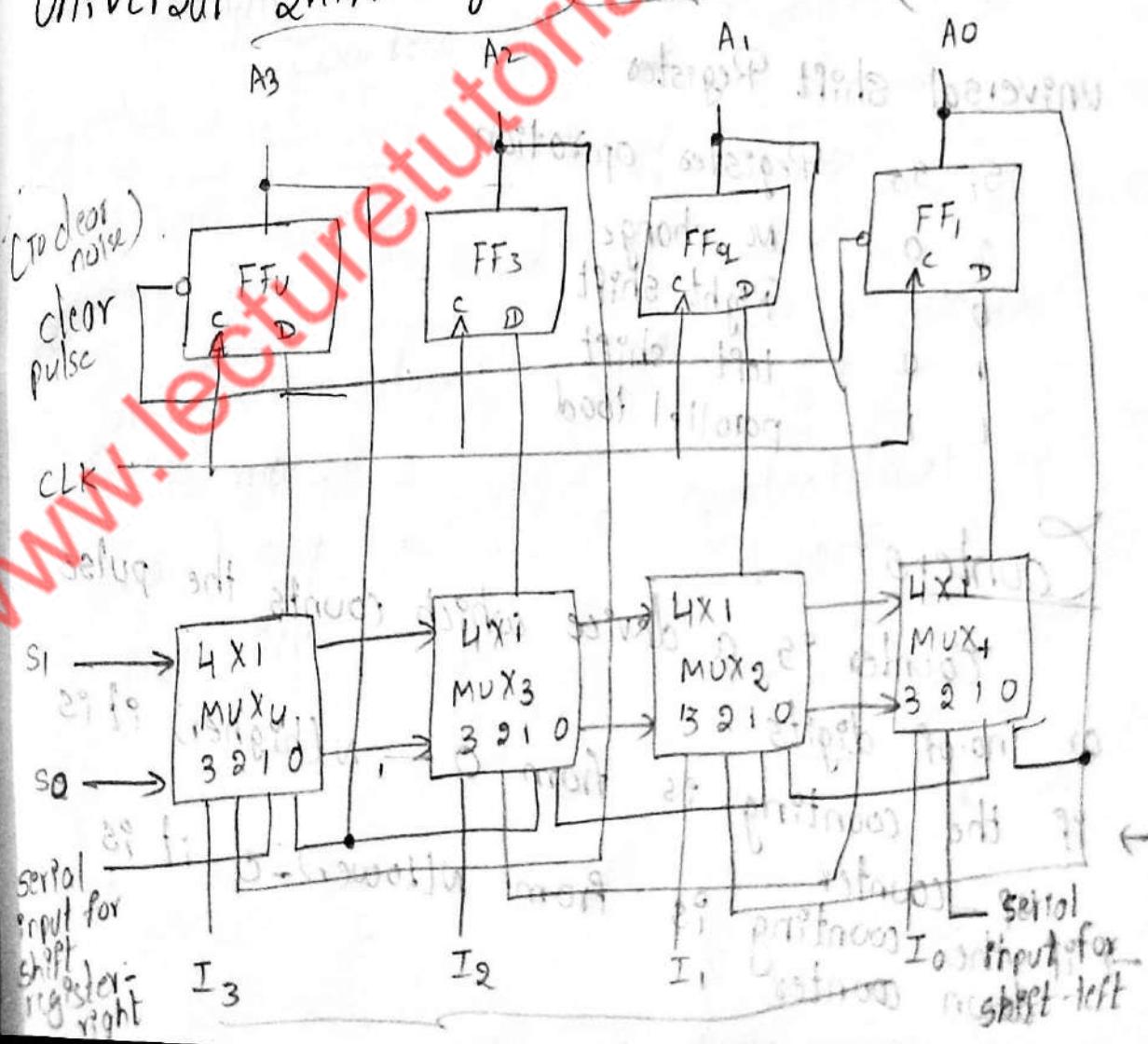
$2 \rightarrow 1 \rightarrow 1 \rightarrow 0 \rightarrow 0$
 $3 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 0$
 $4 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1$



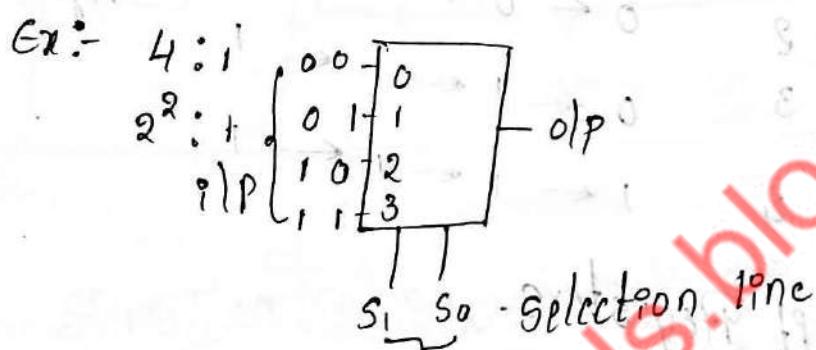
example



universal Shift Registers parallel outputs



- * Enter the data in serial and collecting the data in parallel called universal shift registers.
- * universal shift register consists of D flip flops and multiplexers
- * Multiplexer: which has 2^n inputs and 1 output is called multiplexer



00 - 0
01 - 1
11 - 2
10 - 3

Universal Shift Register

S, S₀ Register operation

0 0	No change
0 1	Right shift
1 0	Left shift
1 1	parallel load

Counters

Counter is a device which counts the pulse

or no. of digits

- if the counting is from 0 - N (higher) it is up counter
- if the counting is from N (lower) - 0 it is down counter

Counters are of two types

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→ Synchronous counters

→ Asynchronous counters

CLK decimal count

1 0

2 1

3 2

4 3

5 4

6 5

7 6

8 7

9 8

10 9

Asynchronous counters

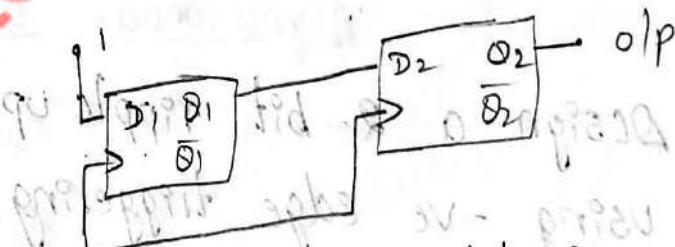
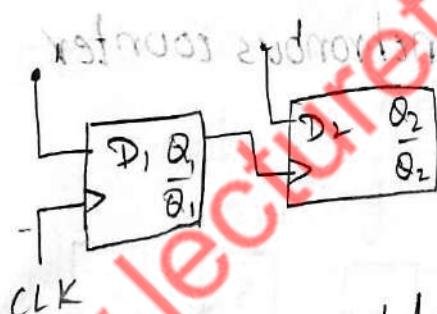
In asynchronous counters, the output of first flip flop is given to the CLK of second flip flop and propagation delay is occurred

Synchronous Counters

In synchronous counters, the CLK pulse is same and no propagation delay

Asynchronous
Counters

Synchronous
Counters



propagation delay
is more

count easy

cost less

propagation delay is
less

count difficult
cost more

$$S = 2^{\log_2 10} - 1$$

$$S = \frac{1}{2} (10 - 1)$$

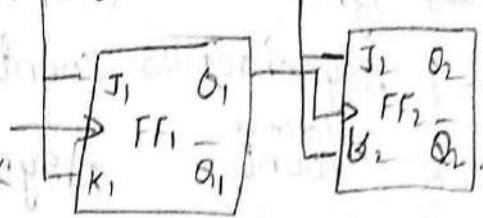
$$(10 - 1) \times 2$$

principle of operation of asynchronous counters

1. Design a two bit ripple up-counter using
J-K flip flop by negative edge triggering

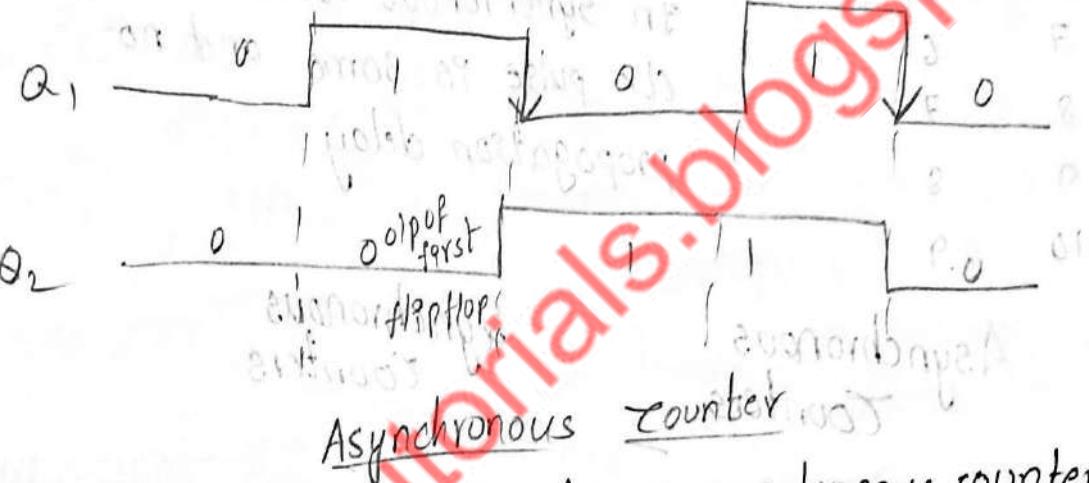
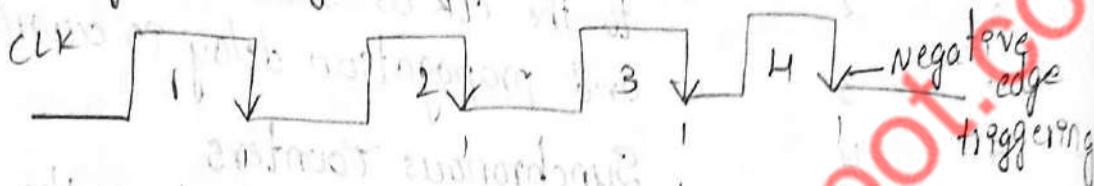
no. of bits $n = 2$

no. of flip flops $= 2$

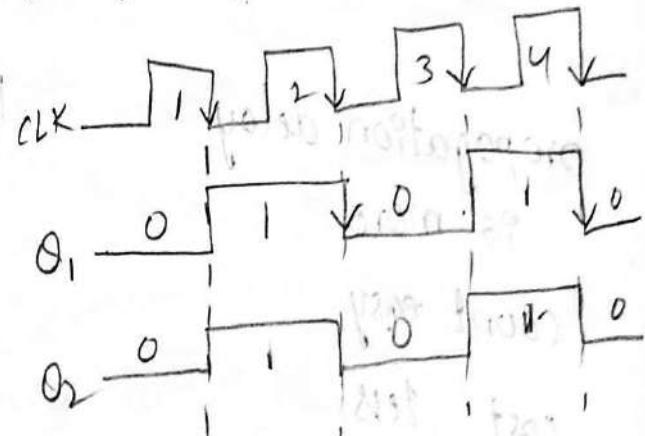
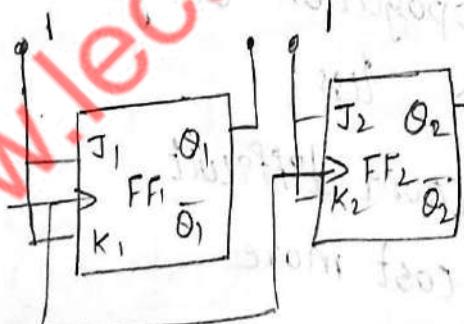


(JK F/F)

By using 'neg' edge triggering



2. Design a 2-bit ripple up-synchronous counter using -ve edge triggering



$n = \text{no. of bits} = 2$

no. of F/F = 2

(JK F/F)

synchronous counter by -ve edge triggering

Design of Synchronous Counters
It is represented as bit counter or divide by time/intukononlinebits

bit counter or modulo N counter or mod N counter.

Design a decade counter or mod 10 counter using T flip flop.

Counters.

n bit counter

Modulo - N counter

Mod - N counter

Divide by N counter

$N = \text{No. of Counter}$.

$N = 0 \text{ to } N-1$

Ex: $N = 10$

Decade Counter

Count the numbers $= N = 10$

Counting sequence $(0-(N-1))$
 $(0-9)$

No. of f/f required to construct the counter

No. of f/f required
 $n = \text{No. of f/f}$

$n=4$



Mod = 10 counter

$N = 10 (0-9)$

Counting sequence

No. of f/f's are
 n .

1	1	1	1
0	1	1	1
X	X	X	X
X	X	1	1
1	1	1	1

inst 10 digits

$$2^n \geq N$$

$$n=0 \quad 2^0 \neq 10$$

$$n=1 \quad 2^1 \neq 10$$

$$n=2 \quad 2^2 \neq 10$$

$$n=3 \quad 2^3 \neq 10$$

$$n=4 \quad 2^4 \geq 10$$

CLK	Dc no	present state				Next state			
		Q _D	Q _C	Q _B	Q _A	Q _{D+1}	Q _{C+1}	Q _{B+1}	Q _{A+1}
0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	1	0	0	1	1
2	2	0	0	1	1	2	0	0	0
3	3	0	1	0	1	3	0	0	1
4	4	0	1	0	0	4	0	1	0
5	5	0	1	0	0	5	0	0	1
6	6	0	1	1	0	6	0	0	0
7	7	0	1	1	1	7	0	0	0
8	8	1	0	0	0	8	0	0	0
9	9	1	0	0	1	9	0	0	1
10	9	1	0	0	0	10	0	0	0

CLK	Dc no	T _D				T _C				T _B				T _A				
		T _D	T _C	T _B	T _A	T _D	T _C	T _B	T _A	T _D	T _C	T _B	T _A	T _D	T _C	T _B	T _A	
0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0
1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1
2	2	0	1	0	1	2	0	0	1	1	0	0	1	1	0	1	1	0
3	3	0	1	0	0	3	0	1	0	0	1	0	0	1	0	1	0	1
4	4	0	1	1	0	4	0	1	0	0	1	0	0	1	0	1	0	0
5	5	0	1	1	1	5	0	1	1	0	0	1	1	0	0	1	1	1
6	6	0	1	1	1	6	0	1	1	0	0	1	1	0	0	1	1	1
7	7	0	1	1	1	7	0	1	1	0	0	1	1	0	0	1	1	1
8	8	1	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0
9	9	1	0	0	1	9	0	0	1	0	0	1	0	0	0	1	0	0
10	9	1	0	0	0	10	0	0	0	0	0	0	0	0	0	0	0	0

Excitation table of T_D/_C/_B/_A

P-S N-S N-T
0 0 0 0
0 1 0 1
1 0 1 0
1 1 1 1

T_A

Q _D	Q _C	Q _B	Q _A	T _A
0	0	1	1	0
1	1	1	1	1
2	1	0	1	1
3	0	1	1	1

T_B

Q _D	Q _C	Q _B	Q _A	T _B
0	0	1	1	0
1	1	1	1	1
2	1	0	1	1
3	0	1	1	1

double octet

guard's primary

guard

$$\text{Pos r} = Q_D \bar{Q}_C \bar{Q}_B \bar{Q}_A$$

$$= Q_D \bar{Q}_A$$

$$T_C = Q_B Q_A$$

$$T_D = Q_D Q_A + Q_C Q_B Q_A$$

$$T_B = Q_D Q_B$$

$$T_A = Q_D Q_C$$

Guard's of Q_D & Q_C & Q_B & Q_A

guard

$$(Q_B \bar{Q}_A + \bar{Q}_B Q_A) (Q_B \bar{Q}_A + \bar{Q}_B Q_A)$$

$$= (\bar{Q}_D + \bar{Q}_C + \bar{Q}_B + \bar{Q}_A) (Q_B \bar{Q}_A + \bar{Q}_B Q_A)$$

$$T_A = \bar{Q}_D \bar{Q}_A$$

$$T_B = \bar{Q}_D Q_B$$

$$T_C = Q_B \bar{Q}_A$$

$$T_D = Q_D Q_A + Q_C Q_B Q_A$$

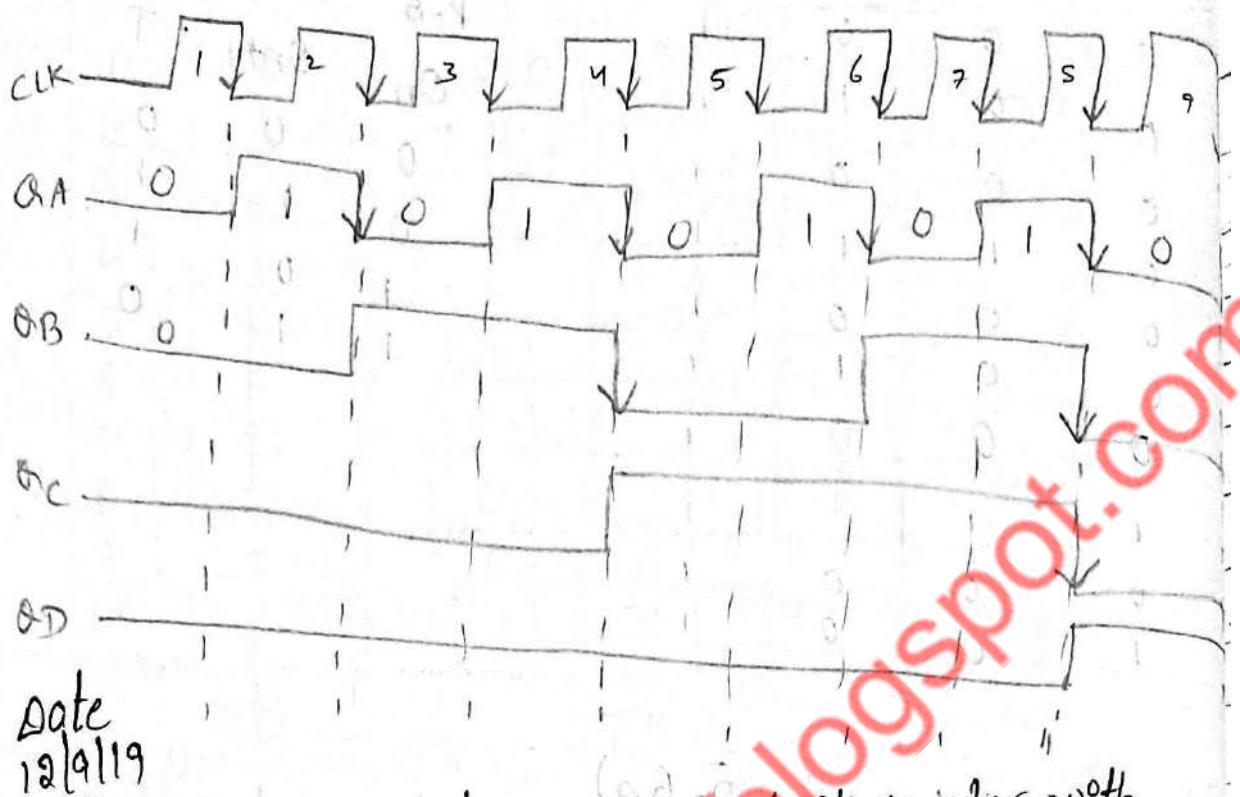


$$T_C = Q_B Q_A$$

$$T_D = Q_D Q_A$$

Timing Diagram

t.me/jntukonlinebits



Date
12/11/19

Design a 10 bit ring and shift counter with
D flip flops

10 bit shift and ring counter
It is called shift counter because it shifts the
value from present to next
It is called ring counter when the cycle is
repeated after counting the maximum value

10 bit counter ; $N = 10$ (0 - 9)

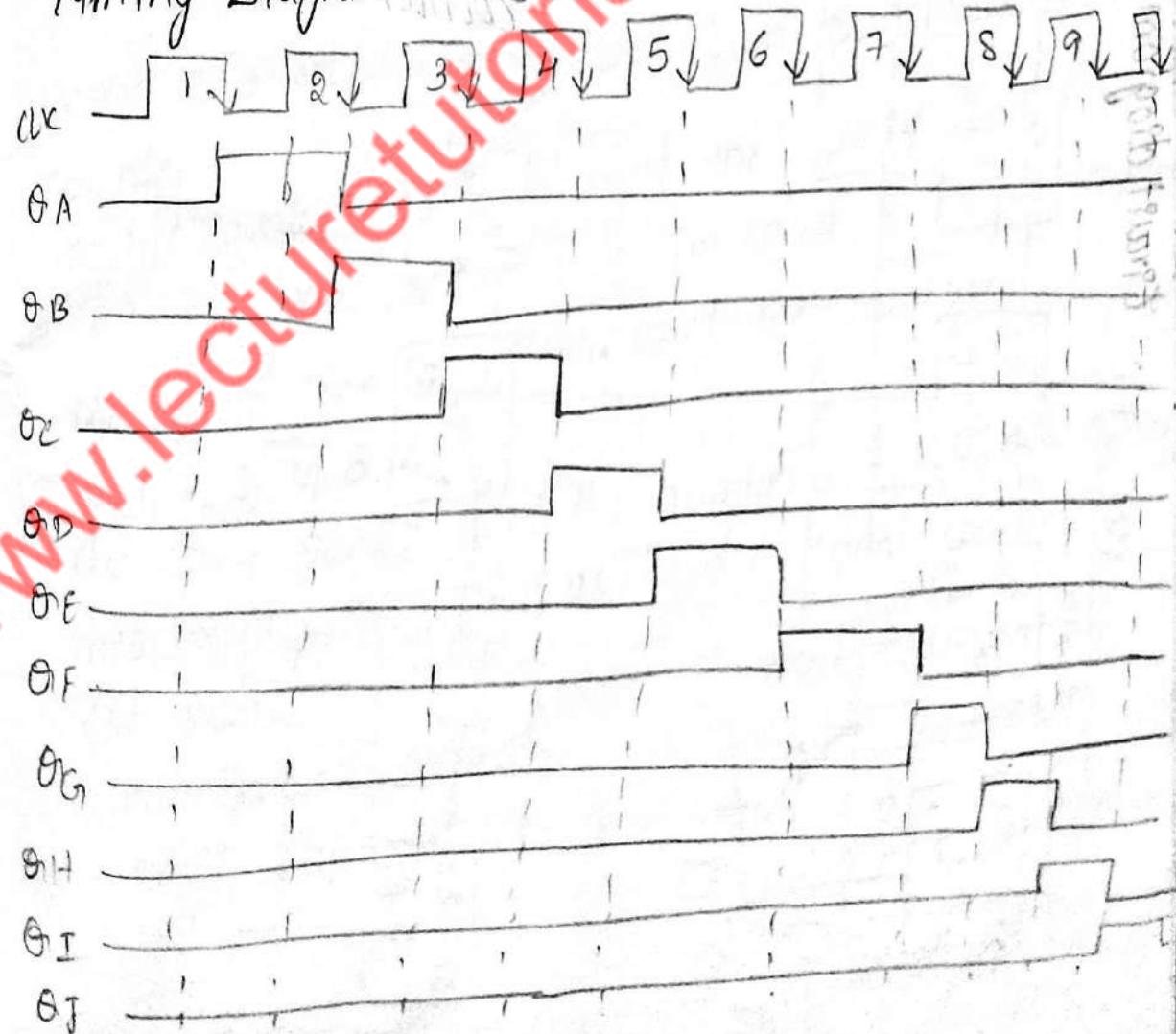
Shift counter Ring counter



0 - 0
1
2
3
4
5
6
7
8
9

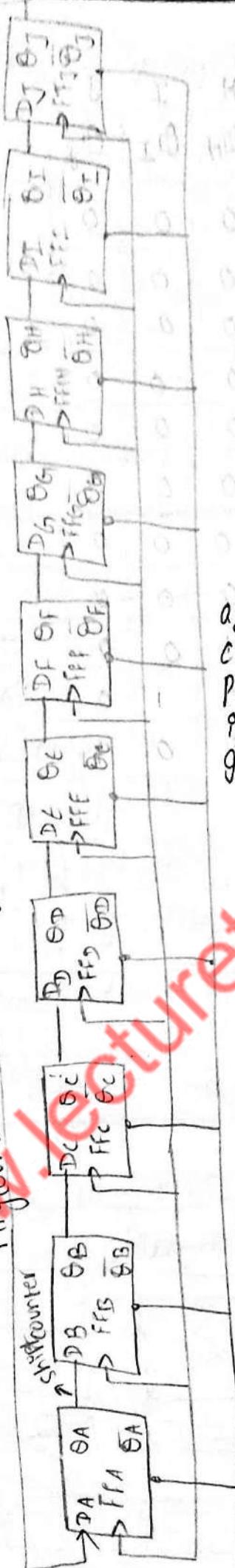
clk Decimal num ber	A	B	C	D	E	F	G	H	I	J
	Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_I	Q_J
0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0
2	1	0	1	0	0	0	0	0	0	0
3	2	0	0	1	0	0	0	0	0	0
4	3	0	0	0	1	0	0	0	0	0
5	4	0	0	0	0	1	0	0	0	0
6	5	0	0	0	0	0	1	0	0	0
7	6	0	0	0	0	0	0	1	0	0
8	7	0	0	0	0	0	0	0	1	0
9	8	0	0	0	0	0	0	0	1	0
10	9	0	0	0	0	0	0	0	0	1

Timing Diagram:



Twisted Yang Counter counter.me/jntukonlinebits

~~Lesson 1~~ Current program for shift and ring counter



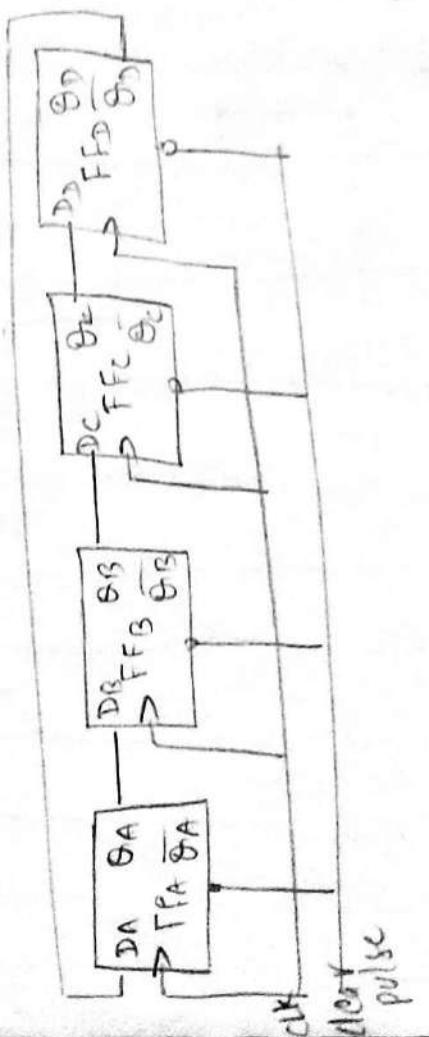
clear pulse/preset

CLK	Q _A	Q _B	Q _C	Q _D	\bar{Q}_D
0	0	0	0	0	1
1	1	0	0	0	1
2	1	1	0	0	1
3	1	1	1	0	1
4	1	1	1	1	0
5	0	1	1	1	0
again	0	0	1	1	0
CLK pulse	0	0	0	1	0
95 given	0	0	1	0	1

S. blogspot

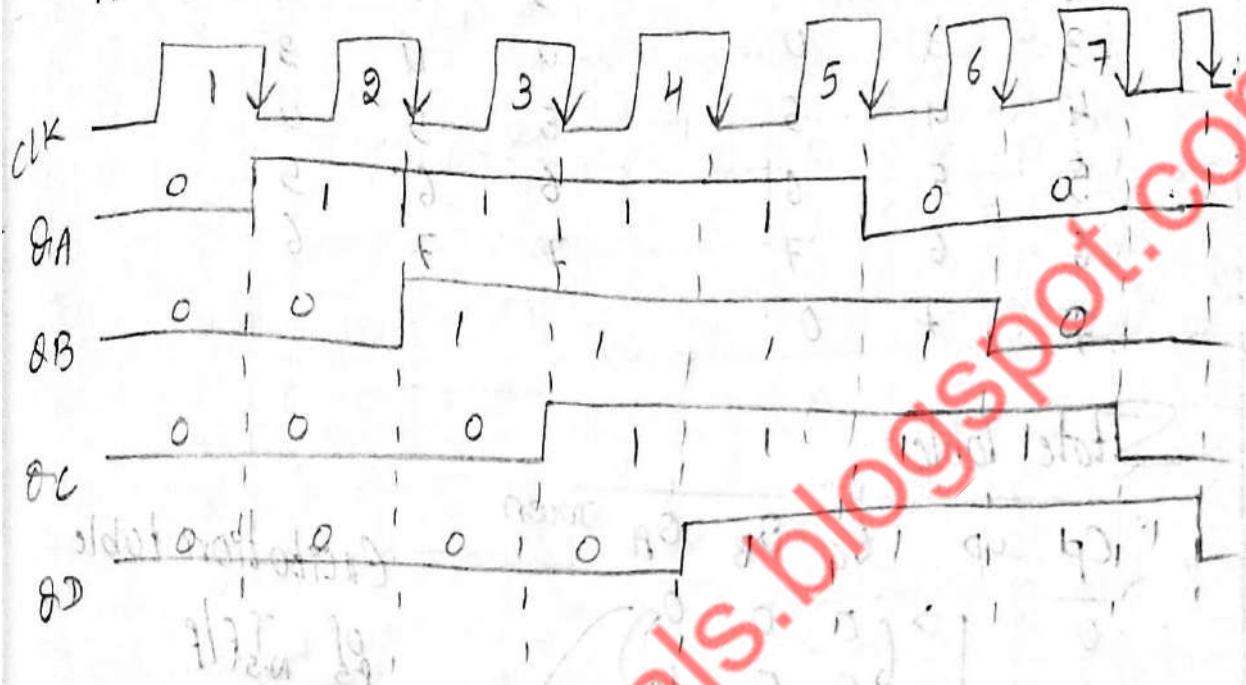
for first diagram for Johnson

~~8~~ Circuit diagram for Johnson Counter



Design a 4-bit Johnson Counter or a 4-bit shift and twisted ring counter with D flip flops
 4 bit Johnson counter / 4 bit and twisted ring counter

$$N=4 \text{ (0-3)} \quad [4+4=8(0-7)]$$



Design a 3-bit [down-5] up-down synchronous counter

3-bit counter enable to count the numbers as $2^3 = N$

3-bit counter enable to count 8 i.e., (0-7)

$2^3 = 8$ you can be able to count the numbers

The up-counter can able to count the numbers

0, 1, 2, 3, 4, 5, 6, 7, 0..

The down counter count the numbers 7, 6, 5, 4, 3, 2, 1, 0

The down counter known as bi-directional counter

This counter is known as
(up-down)

Counting = N . sequence

No. of flip flops = $n = 3$

Mode signal	
up	down = 1
→ 1	0
ON	OFF
up counter	
down counter	
up down = 0	
→ 0	1
OFF	ON

Date
13/9/19

UP Counter

	P.S	N.S		
0	0	1	1	
1	1	2	2	
2	2	3	3	
3	3	4	4	
4	4	5	5	
5	5	6	6	
6	6	7	7	
7	7	0		

Down Counter

	P.S	N.S	
0	0	7	
1	1	0	
2	2	1	
3	3	2	
4	4	3	
5	5	4	
6	6	5	
7	7	6	

State Table

Cp	UP	θ_C	θ_B	θ_A	DOWN
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	1	0	0	0	
5	1	0	1	0	
6	1	1	0	0	
7	1	1	1	1	

Excitation table

of N.S

θ_n	θ_{anti}	T
0	0	0
0	1	1
1	0	1
1	1	0

For K-map

$$T_A = \sum_m (0, 1, 2, 4, 5, 15)$$

$$T_B = \sum_m (0, 2, 4, 6, 9, 11, 13, 15)$$

$$T_C = \sum_m (0, 4, 11, 15)$$

Exco

Excitation table

present state

next state

t.me/jntukonlinebits

\bar{Q}_D	Q_C	Q_B	Q_A	no. of row	$Q_C + 1$	$Q_B + 1$	$Q_A + 1$	T_C	T_B	T_A
0	0	0	0	0	0	1	1	1	1	1
1	0	1	0	1	1	0	0	0	0	1
2	0	2	0	1	2	0	0	1	1	0
3	0	3	0	1	3	0	1	0	2	0
4	0	4	1	0	4	0	1	1	3	0
5	0	5	1	0	5	1	0	0	4	0
6	0	6	1	1	6	1	0	1	5	0
7	0	7	1	1	7	1	1	0	6	0
8	1	0	0	0	8	0	0	1	1	0
9	1	1	0	0	9	0	1	0	2	0
10	1	2	0	1	10	0	1	1	3	0
11	1	3	0	1	11	1	0	0	4	1
12	1	4	1	0	12	1	0	1	5	0
13	1	5	1	0	13	1	1	0	6	0
14	1	6	1	1	14	1	1	1	7	0
15	1	7	1	1	15	0	0	0	1	0

\bar{Q}_D	Q_A	T_A	\bar{Q}_B	Q_B	T_B	\bar{Q}_C	Q_C	T_C
00	00	01	11	10	$\bar{Q}_D \bar{Q}_B Q_A$	00	01	11
01	01	10	15	17	$\bar{Q}_D Q_B \bar{Q}_A$	01	10	10
11	11	10	13	15	$\bar{Q}_D \bar{Q}_B Q_A$	11	12	12
10	10	11	14	16	$\bar{Q}_D Q_B \bar{Q}_A$	10	11	11

$$q_1 \rightarrow \bar{Q}_D Q_A$$

$$q_2 \rightarrow \bar{Q}_D \bar{Q}_A$$

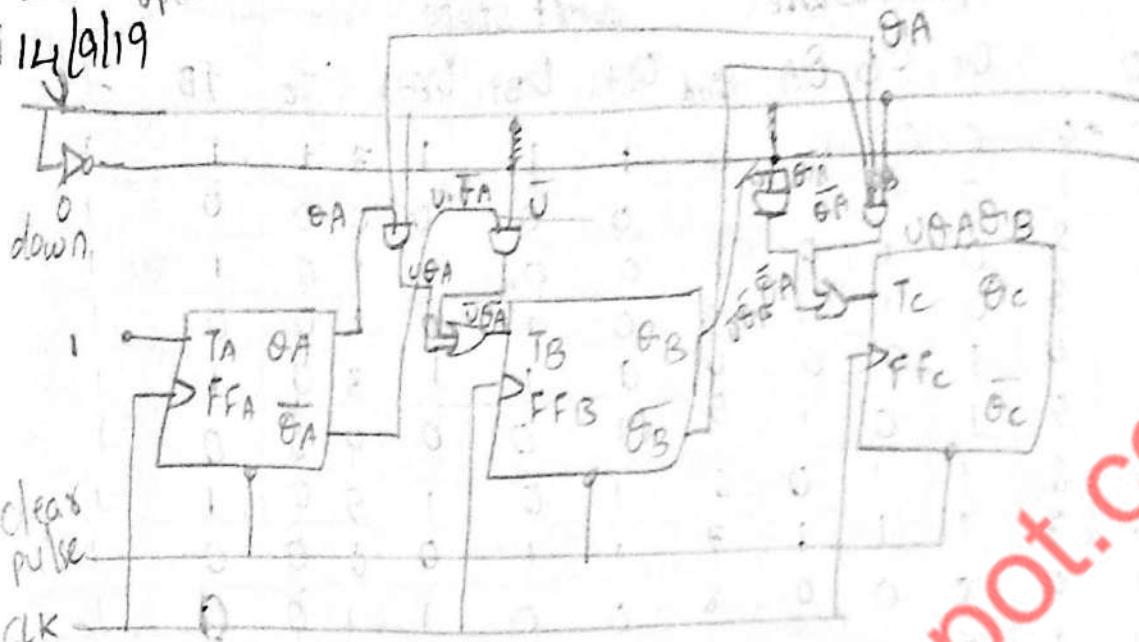
$$p_1 \rightarrow \bar{Q}_D \bar{Q}_B \bar{Q}_A$$

$$p_2 \rightarrow \bar{Q}_D Q_B Q_A$$

$$T_B = \bar{Q}_D Q_A + \bar{Q}_D \bar{Q}_A$$

$$T_C = \bar{Q}_D \bar{Q}_B \bar{Q}_A + \bar{Q}_D Q_B Q_A$$

Date 14/9/19 up/down = 1



logic diagram using T- flop flop

Design divide by "6" counter using

Mod 6 Counter

divide by 6 counter (01) MOD 6 - Counter

no. of counts = 6 ; counting range $0 - (N-1)$
 $(0-5)$

$$n = \text{no. of flip flops} \quad 2^n \geq N$$

$$n=1 \quad 2^1 \not\geq 6 \quad \text{No. of flip flops} = 3$$

$$n=2 \quad 2^2 \not\geq 6$$

$$n=3 \quad 2^3 \geq 6$$

$$\text{no. of bits} = \text{no. of flip flops} = 3$$

Note

$$\text{MOD } -6(0-5) ; n=3 \left(\begin{smallmatrix} 0 & 1 \\ 6,7,8 & 0 \end{smallmatrix} \right)$$

don't care

$$-5(0-4) ; n=3 \left(\begin{smallmatrix} 0 & 1 \\ 5,6,7 & 0 \end{smallmatrix} \right)$$

don't care

P.S	N.S
0	1
1	2
2	3
3	4
4	5
5	0

D.no	CLK u P _i s 1				N's				Tc	TB	TA
	Qc	QB	QA		Q _{c+1}	Q _{B+1}	Q _{A+1}				
0	0	0	0	1	0	0	1	0	0	0	1
1	1	0	0	1	2	0	1	0	0	1	1
2	2	0	1	0	3	0	1	1	0	0	1
3	3	0	1	1	4	1	0	0	1	1	1
4	4	1	0	0	5	1	0	1	0	0	1
5	5	1	0	1	0	0	0	0	1	0	1
6	6	x	x	x							
7	7	x	x	x							

Excitation table of T flipflop

$$Q_n = Q_{n+1} \quad T \quad T_A = \sum_m (0, 1, 2, 3, 4, 5)$$

$$T_B = \sum_m (1, 3)$$

$$T_C = \sum_m (3, 5)$$

		JA			
		QB	QA	QB	QA
QC		00	01	11	10
0	0	1	1	3	2
1	1	1	x	x	6
	u	5	7	x	6

		QB QA			
		QC	QB	QA	
QC		00	01	11	10
0	0	0	1	1	2
1	1	0	1	x	x
	u	5	7	x	6

		QB QA			
		QC	QB	QA	
QC		00	01	11	10
0	0	0	1	3	2
1	1	0	1	x	x
	u	5	7	x	6

$$T_A = 1$$

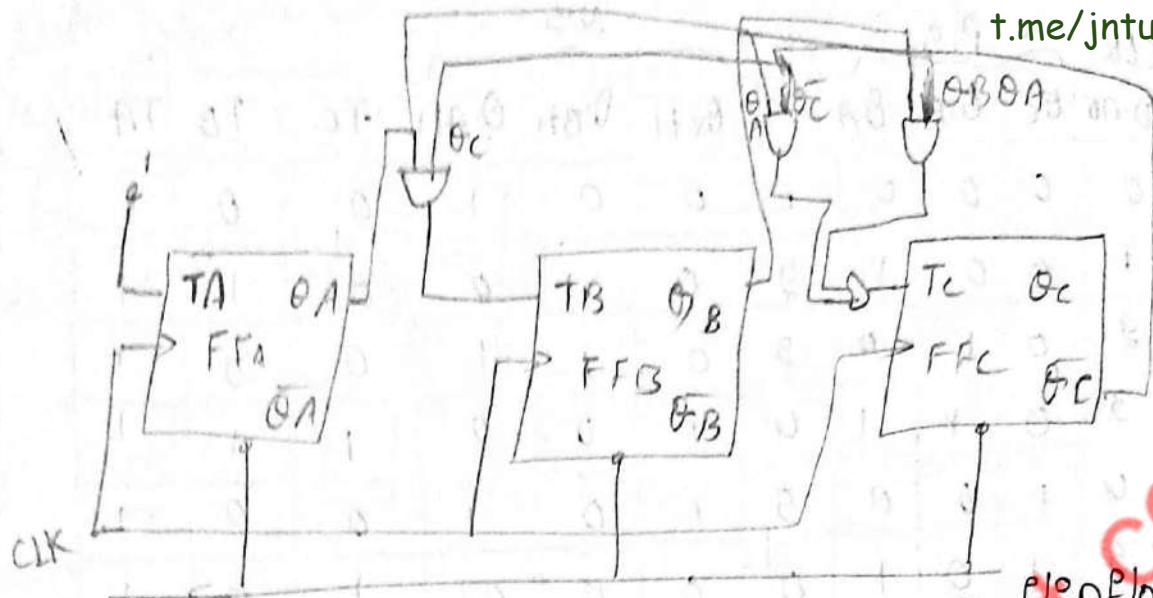
$$T_B = \bar{Q}_C \cdot Q_A$$

$$T_C = P_2 + P_1 \rightarrow (\bar{Q}_C Q_A + Q_B Q_A)$$

$$P_1 (\bar{Q}_C Q_A) + P_2 C$$

$$(\bar{Q}_C + Q_C)(Q_B Q_A)$$

$$\bar{Q}_C Q_A + Q_B Q_A$$



Date Design Mod-12 Counter using T- flipflops

16/9/19 The total no of counts $N = 12$

counting sequence - 0 to $N-1$

$$0 \text{ to } (12-1)$$

$$0 \text{ to } 11 \quad (12, 13, 14, 15)$$

don't cares

No. of flip flops - $n - 2^n \geq N$

$$1 \quad 2^1 \geq 12$$

$$2 \quad 2^2 \geq 12$$

$$3 \quad 2^3 \geq 12$$

$$4 \quad 2^4 \geq 12$$

T flip flop

P.S N.S

Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

No. of flip flops = No. of binary bits
 $n = 4$; $0 - (2^n - 1)$

P.S N.S

$$0 \quad 1$$

$$1 \quad 2$$

$$2 \quad 3$$

$$3 \quad 4$$

$$4 \quad 5$$

$$5 \quad 6$$

$$6 \quad 7$$

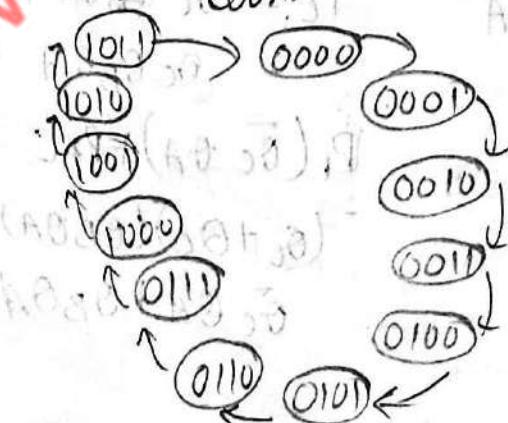
$$7 \quad 8$$

$$8 \quad 9$$

$$9 \quad 10$$

$$10 \quad 11$$

$$11 \quad 0$$



D.NO	PS				N.S				TD	Time/jntukonlinebits	
	θ_D	θ_B	θ_C	θ_A	θ_{D+1}	θ_{C+1}	θ_{B+1}	θ_{A+1}		T _E	P _{ELOP}
0	0	0	0	0	1	0	0	0	1	0	0
1	0	0	0	2	0	0	1	0	0	0	1
2	0	0	1	0	3	0	0	1	1	0	0
3	0	0	2	0	4	0	1	0	0	1	1
4	0	1	0	0	5	0	1	0	1	0	0
5	0	1	0	1	6	0	1	1	0	0	1
6	0	1	1	0	7	0	1	1	1	0	0
7	0	1	1	1	8	1	0	0	0	1	1
8	1	0	0	0	9	1	0	0	1	0	0
9	1	0	0	1	10	1	0	1	0	0	1
10	1	0	1	0	11	1	0	1	1	0	0
11	1	0	1	1	0	0	0	0	1	0	1

$$TA = \sum_m (0, 1, 2, \dots, 11)$$

$$TB = \sum_m (1, 3, 5, 7, 9, 11)$$

$$TC = \sum_m (3, 7) ; TD = \sum_m (7, 11)$$

		TA			
θ_D	θ_B	00	01	11	10
00	10	11	13	12	
01	14	15	17	16	
11	X ₁₂	X ₁₃	X ₁₅	X ₁₄	
10	8	9	11	10	

Octent (1)

$$TA = 1$$

		TC			
θ_D	θ_B	00	01	11	10
00	0	1	(1)	2	
01	u	5	(1)	6	
11	X ₁₂	X ₁₃	X ₁₅	X ₁₄	
10	8	9	11	10	

$$\text{pair}(3, 7); TC = \overline{\theta_D} \theta_B \theta_A$$

		TB			
θ_D	θ_B	00	01	11	10
00	00	0	1	3	2
01	01	u	5	(1)	6
11	X ₁₂	X ₁₃	(X ₁₅)	X ₁₄	
10	8	9	11	10	

$$\text{Octent } TB = \theta_A$$

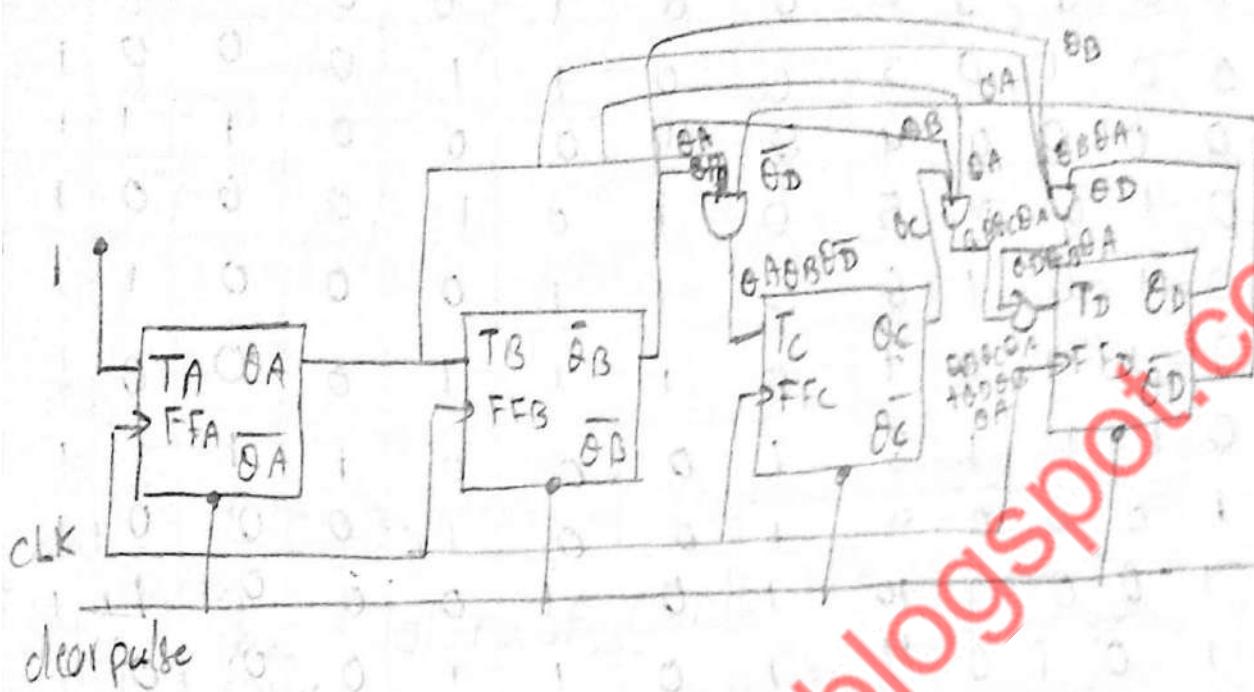
TD

		TD			
θ_D	θ_B	00	01	11	10
00	0	1	3	2	
01	u	5	(1)	6	
11	X ₁₂	X ₁₃	(X ₁₅)	X ₁₄	
10	8	9	11	10	

$$\text{pair}(7, 5); \text{pair}(15, 11)$$

$$TD = \theta_C \theta_B \theta_A + \theta_D \theta_B \theta_A$$

$$T_A = 1^\circ; T_B = \theta_A^\circ; T_C = \bar{\theta}_D \theta_B \theta_A^\circ; T_D = \theta_D \theta_B \theta_A^\circ$$



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