Assignment 2

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1)

a)

Top down

A natural number is in Q1a if and only if:

1. $n = 2$ or
2. $n-3 \in Q1a$

Bottom up

Define the set Q1a to be the smallest set contained in N and statisifying the following conditions

1) $2 \in Q1a$ and
2) if $n \in Q1a$ then $n-3 \in Q1a$

2)3
5)3
8

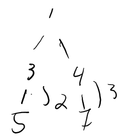
Rule of Inference

$$\frac{}{2 \in Q1A} \qquad \frac{n \in Q1a}{n+3 \in Q1a}$$

b) $\{2n+3m+1 \mid n,m \in N\}$

1
3 4

To down



Top down:

$n \in Q1b$ if

$n = 1$ or

$n-2 \in Q1b$ or

$n-3 \in Q1b$

Bottom up

Define the set Q1b to be the smallest set contained in N and statisifying the following conditions

$1 \in Q1b$ and

if $n \in Q1b$ then $n+2 \in S$ and

if $n \in Q1b$ then $n+3 \in S$

Rule of inference

$$\frac{}{1 \in Q1b} \qquad \frac{n \in Q1b}{n+2 \in Q1b} \qquad \frac{n \in Q1b}{n+3 \in S}$$

3) $\{n, 2n+1\} \mid n \in N\}$

Top down

A pair (n,m) are $\in Q1c$ if and only if:

$n=0$ and $m=1$ or

$(n-1, m-2) \in S$

Bottom up

Define the set Q1c to be the smallest set contained in N and statisifying the following conditions

$(0,1) \in Q1c$ and

if $(m,n) \in Q1c$ then

$(m+1, n+2) \in Q1c$

Rule of inference

$$\frac{}{(0,1) \in Q1c} \qquad \frac{(m,n) \in Q1c}{(m+1, n+2) \in Q1c}$$

d) $\{n, n^2 \mid n \in N\}$

A pair $(n,m) \in Q1d$ if and only if

$n=0$ and $m=0$ or

$m = n^2$

so

$n^2+2n+1 = m+2n+1$

$n = 0$ and $m = 0$ or $\quad n^2+2n+1 = m+2n+1$

$(n-1, m-2n+1) \in S$

Bottom-up

Define the set Q1b to be the smallest set contained in N and statisifying the following conditions

$(0,0) \in Q1d$ and

if $(m,n) \in S$ then

$(m+1)(m+2n+1) \in S$

Rule of inference

$$\frac{}{(0,0) \in Q1d} \qquad \frac{(m,n) \in Q1d}{(m+1, m+2n+1) \in Q1d}$$

[6 marks] Prove1 that if e ∈ LcExp (see Definition 1.1.8 — Essentials of Programming Languages page 9), then there are the same number of left and right parentheses in e. (Hint: first show that such a representation exists and then show there are infinitely many)

LcExp ::= Identifier
 ::= (lambda (Identifier) LcExp)
 ::= (LcExp LcExp)

Theorem: Let e be a LcExp as defined by definition 1.1.8

Proof: This proof is by induction on the structure of e where e is an element of LcExp.

Induction Hypothesis(IH): For any an element of LcExp they will always have the same number of left and right parentheses.

Inductive proof: we must prove that e has a balanced number of left and right parenthese for the base case and we then prove that this holds true for all other elements of LcExp.

Base Case: If e is an Identifier then there are no parentheses which means they are balanced.

Based on Definition 1.1.8 there are a number of cases we need to look at to prove that our brackets hold for all elements of LcExp

1) The element can be in the form (lambda (Identifier) LcExp) then by definition the outer expression has the same number of left and right parentheses. We proved in the base case that an identifier has no parentheses, so we have matching there as well. Finally the subexpression LcExp will have balanced if it is an Idenfiter, as previously shown, or if it is in the form (lambda (Identifier) LcExp) . We will also show that it will be balanced in the form (LcExp LcExp)
2) The element can be in the form (LcExp LcExp). This outer form has a balanced number of left and right parentheses. If the inner LcExp's must be in the base case, the form (lambda (Identifier) LcExp) or the form (LcExp LcExp). Since we have shown that each of these forms have a balanced number of parentheses then regardless of the combination of LcExp and LcExp that the element has the same number of left and right parentheses.

Therefore we have shown that for every element of LcExp the number of left and right parentheses will be balanced.