9.23 土.
정석 ± 숙제

#10.1.1(7)

$$\lim_{n\to\infty}\left|\frac{(x+3)^{n+1}}{3^{n+1}}\cdot\frac{3^n}{(x+3)^n}\right|$$

$$=\lim_{n\to\infty}\left|\frac{x+3}{3}\right|$$

$$\left|\frac{x+3}{3}\right|<1.$$

$$-3<x+3<3$$

$$-6<x<0. \qquad \therefore -6<x<0.$$

#10.1.2(6)

$$f(x)=\frac{1+x}{1-x}=\sum_{n=0}^{\infty}a_n(x-a)^n. \qquad \frac{1-x+2x}{1-x}=1+2\frac{x}{1-x}$$

$$a_n=\frac{f^{(n)}(a)}{n!} \qquad \frac{1(1-x)+(1+x)(+1)}{(1-x)^2}=\frac{2}{(1-x)^2}$$

$$=2\left(\frac{1}{1-x}\right)^2.$$

$$\left(\frac{1}{1-x}\right)^2=\left(\sum(-1)^n x^n\right)^2$$

$$=\sum_{n=0}^{\infty}(-1)^n x^n\cdot\sum_{n=0}^{\infty}(-1)^n x^n$$

DATE. NO.

$f'(x) = \frac{(1-x) - (1+x)(-1)}{(1-x)^2}$

$= \frac{1+2\cancel{x}\cdots}{(1-x)^2}$

$= 2\frac{1}{(1-x)^2} = 2\left(\frac{1}{1-x}\right)^2$

$= \left(\frac{1}{1-x}\right)^2 = 2\left(\sum_{n=0}^{\infty}(-1)^n x^n\right)^2$

적분하면

$f(x) =$

$\frac{1+x}{1-x} = \frac{1}{1-x} \cdot \frac{1}{\frac{1}{1+x}}$

$= \sum x^n \cdot \sum \frac{1}{\sum(-1)^n x^n}$

$= \frac{\sum x^n}{\sum(-1)^n x^n}$

$\frac{1-x+2x}{1-x} = 1 + \frac{2x}{1-x}$

$\frac{1}{1-x} + \frac{x}{1-x} = \sum x^n + x\sum x^n$

$= \sum x^n + \sum x^{n+1}$

$= \sum x^n(1+x)$

$$f(x) = \frac{1+x}{1-x}$$

$$= \frac{1}{1-x} + \frac{x}{1-x}$$

$$= \sum_{n=0}^{\infty} x^n + x\sum_{n=0}^{\infty} x^n$$

$$= \sum_{n=0}^{\infty} (x^n + x^{n+1})$$

$$= \sum_{n=0}^{\infty} x^n(1+x)$$

$$\lim_{n\to\infty} \left|\frac{x^{n+1}}{x^n}\right| = |x| < 1. \quad \therefore -1 < x < 1$$

$$\therefore f(x) = \frac{1+x}{1-x} = \sum_{n=0}^{\infty} x^n(1+x). \quad R = 1.$$

#10.1.4(1)

$$\int \frac{x}{1-x^2}\,dx = -\frac{1}{2}\int \frac{-2x}{1-x^2}\,dx$$

$$= -\frac{1}{2}\ln|1-x^2| + C \quad (\text{C는 적분 상수})$$

$$= -\frac{1}{2}\cancel{\ln}\left(-\sum_{n=0}^{\infty} \frac{(x^2)^{n+1}}{n+1}\right) + C$$

$$= \frac{1}{2}\sum_{n=0}^{\infty} \frac{x^{2n+2}}{n+1} + C$$

$$\lim_{n\to\infty} \frac{1}{2} \left| \frac{x^{2n+4}}{n+2} \cdot \frac{n+1}{x^{2n+2}} \right|$$

$$= \lim_{n\to\infty} \frac{1}{2} \left| \frac{x^2}{n+2} \cdot n+1 \right|$$

$$= \frac{1}{2} |x^2| < 1$$

$$|x^2| < 2$$

$$-2 < x^2 < 2$$

$$\therefore 0 < x < 2$$

$$\therefore \frac{1}{2} \sum_{n=0}^{\infty} \frac{x^{2n+2}}{n+1} + C$$ (C는 적분상수).

수렴 반경 : $0 < x < 2$.

#10.2.1(3)

$$f(x) = \sum \frac{f^{(n)}(0)}{n!} x^n$$

$f(x) = \ln(2+x)$ 이라 하자.

$$\frac{f'(x)}{1!} = \frac{1}{x+2}$$

$$\frac{f''(x)}{2!} = \frac{-1}{(x+2)^2} \quad \frac{1}{\cancel{4}\cdot 2}$$

$$\frac{f'''(x)}{3!} = \frac{2(x+2)}{(x+2)^4} = \frac{2}{(x+2)^3} \cdot \frac{1}{\cancel{6}}$$

DATE. NO.

$$f''''(x) = \frac{-6(x+2)^2}{(x+2)^{\cancel{4}}} \cdot \frac{1}{4!}$$

$$= \frac{-6}{(x+2)^4} \quad \frac{1}{4 \cdot 6}$$

$$\therefore \ln(2+x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n(\cancel{x}+2)^n} x^n$$ 수렴구간: ~~[illegible]~~

$-2 < x < 2$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)(\cancel{x}+2)^{n+1}} \cdot \frac{n(x+2)^n}{x^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n}{n+1} \cdot x \cdot \frac{1}{\cancel{x}+2} \right|$$

$$= \left| \frac{x}{\cancel{x}+2} \right| < 1.$$

$$\left|\frac{x}{2}\right| < 1.$$

$$-1 < \frac{x}{2} < 1.$$

$$-2 < x < 2$$

$\cancel{\left|\frac{x}{x+2}\right|} < 1.$

$\cancel{-1 < \frac{x}{x+2} < 1}$

$-x+2 < x < -x-2.$

~~I) $-x+2 < x$~~

~~$-2x < -2$.~~

~~$\therefore x > 1$~~

~~II) $x < -x-2$.~~

~~$2x < -2$.~~

~~$\therefore x < -1$.~~

morning glory

DATE NO.

#10.2.3(4)

풀) $f(x) = \frac{1}{x}$ 이라 하자.

$$Tf(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x)}{n!}(x-1)^n$$

$$f'(x) = -\frac{1}{x^2} \qquad f'(1) = -1$$

$$f''(x) = \frac{-2x}{x^4} = \frac{-2}{x^3} \qquad f''(1) = 2$$

$$f'''(x) = \frac{2}{x^6}\cdot 3x^2 = \frac{6}{x^4} \qquad f'''(1) = 6$$

$$f''''(x) = \frac{-6}{x^8}\cdot 4x^3 = \frac{-24}{x^5} \qquad f''''(1) = -24$$

$$\therefore Tf(x) = \sum_{n=0}^{\infty} (-1)^n (x-1)^n$$