11.29.金 | 개념수2 마지

#13.7.1(3) $f_x = 2x + 2\sin y$.

$f_y = 2x\cos y$.

$2x\cos y = 0$. $x = 0$ or $y = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi \cdots$

$f_x(x, \frac{1}{2}\pi) = 2x + 2 = 0$. $x = -1$

$f_x(x, \frac{3}{2}\pi) = 2x - 2 = 0$. $x = 1$.

$f_x(0, y) = 2\sin y = 0, y = 0, \pi, 2\pi \cdots$

해: $(0, 0)$. $(0, \pi)$. $(0, 2\pi)$. $\cdots$

$(-1, \frac{1}{2}\pi)$. $(-1, \frac{5}{2}\pi)$. $(-1, \frac{9}{2}\pi) \cdots$

$(1, \frac{3}{2}\pi)$. $(1, \frac{7}{2}\pi)$. $(1, \frac{11}{2}\pi) \cdots$

$f_{xx} = 2$.

$f_{xy} = 2\cos y$

$f_{yy} = -2x\sin y$.

$H = \begin{vmatrix} 2 & 2\cos y \\ 2\cos y & -2x\sin y \end{vmatrix}$

$= -4x\sin y - 4\cos^2 y$

$H(0, 0) = 0 - 4 < 0$. 안장점

$H(0, \pi) = 0 - 4 < 0$. 안장점

$H(-1, \frac{1}{2}\pi) = 4 \cdot 1 - 4 \cdot 0$
$= 4 > 0.$ $(-1)^n, \frac{1}{2}n\pi.$

$f_{xx}(-1, \frac{1}{2}\pi) = 2 > 0. \rightarrow$ 극소

$H(-1, \frac{5}{2}\pi) = 4 \cdot 1 - 4 \cdot 0$
$= 4 > 0.$

$f_{xx}(-1, \frac{5}{2}\pi) = 2 > 0 \rightarrow$ 극소

$H(1, \frac{3}{2}\pi) = -4 \cdot (-1) - 4 \cdot 0$
$= 4 > 0.$

∴ 안장점은 $(0, n\pi)$

극대점은 없다

극소점은 $(-1, \frac{1}{2}\pi + 2n\pi)$
$(1, \frac{3}{2}\pi + 2n\pi)$ (단, n은 정수)

#13.7.5.(2) 1. 영역 내부
2. 영역 경계.

$f_x = 1 - y.$

$f_y = 1 - x.$

후보지: $(1,1)$

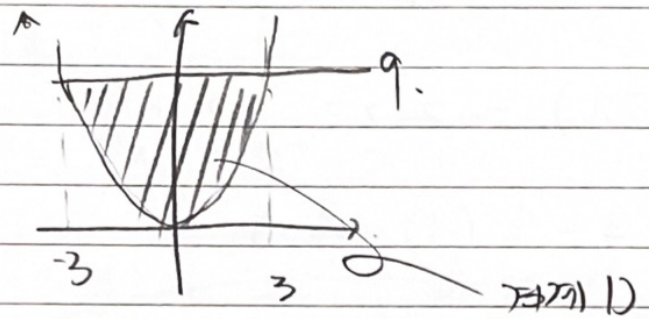
(내부) $f_{xx} = 0$ $f_{yy} = 0.$ $f_{xy} = -1.$

$$H = \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}$$

$= 0 - (+1)(+1)$

$= -1 < 0$.

∴ 내부에는 극값이 존재하지 않는다.



ⅰ). $y = 9$.

$f(x, 9) = x + 9 - 9x$

$= -8x + 9$.

$-8 \cdot 3 + 9 = -24 + 9 = -15$. $(M = 33, m = -15)$

$-8 \cdot (-3) + 9 = 24 + 9 = 33$.

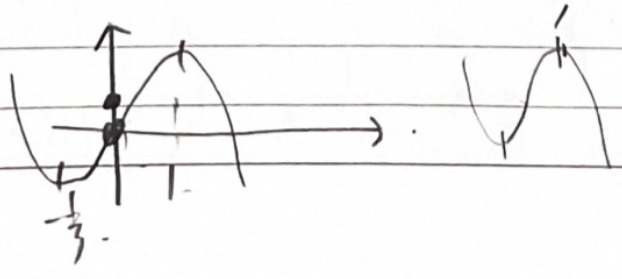
ⅱ). $y = x^2$

$f(x, x^2) = x + x^2 - x^3$.

$= -x^3 + x^2 + x$ $= g(x)$ 라 하자

$g'(x) = -3x^2 + 2x + 1$

$= (3x + 1)(-x + 1) = 0$. $x = -\frac{1}{3}, 1$.



$g(-\frac{1}{3}) = +(+\frac{1}{2\eta}) + \frac{1}{9} - \frac{1}{3}$

$= \frac{1+3-9}{2\eta} = -\frac{5}{2\eta}$.

$g(1) = -1+1+1 = 1$. $\left(\begin{matrix} M=1, \\ m=-\frac{5}{2\eta} \end{matrix}\right)$

∴ 최댓값 = 33.
최솟값 = -15.

#14.3.3(3) $\int_0^1 \int_1^2 \frac{xe^x}{y}\, dy\, dx = \int_0^1 xe^x [\ln|y|]_1^2\, dx$

$= \int_0^1 xe^x \ln 2\, dx$

$= \ln 2 \left([xe^x]_0^1 - \int_0^1 e^x dx \right)$

$= \ln 2 \left(e - [e^x]_0^1 \right)$

$= \ln 2 (e - e) = 0$.

x | 2 3
x | e^x
1 | e^x

#14.1.4(2) $\int_0^1 \int_{-3}^{3} \frac{x}{1+x^2} \cdot y^2\, dy\, dx$.

$= \int_0^1 \frac{x}{1+x^2} \left[\frac{1}{3}y^3\right]_{-3}^{3} dx$.

$= \int_0^1 \frac{2x}{1+x^2} \cdot \frac{1}{2}(\ln 4 + \ln 9)\, dx$.

$= \frac{18}{9} \cdot \frac{1}{2} \left[\ln|1+x^2|\right]_0^1$

$= 9(\ln 2 - \ln 1) = 9\ln 2$