No.

#10.3.2(2)

$\frac{1}{12} - \frac{3}{12}$

$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n$$

$$= 1 + \frac{-1}{1}x + \frac{1}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{-1}{5!}x^5 + \frac{1}{6!}x^6 \cdots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

$$= 1 + \frac{-1}{2!}x^2 + \frac{1}{4!}x^4 + \frac{-1}{6!}x^6$$

$\frac{7}{6} + \frac{3}{6}$

$$e^{-x}\cos x = 1 + x(-1) + x^2\left(\frac{1}{2!} - \frac{1}{2!}\right) + x^3\left(\frac{-1}{3!} + \frac{1}{2!}\right)$$

$$+x^4\left(\cancel{\frac{1}{4!}}_{\frac{1}{24}} + \cancel{\frac{1}{2!}\cdot\frac{-1}{2!}}_{-\frac{1}{4}} + \cancel{\frac{1}{4!}}_{\frac{1}{24}}\right) + \cancel{x^6\left(\frac{1}{6!} + \frac{1}{4!}\cdot\frac{-1}{2!} + \frac{1}{2!}\cdot\frac{1}{4!} - \frac{1}{6!}\right)} + \cdots x^5\left(-\frac{1}{5!} + \frac{-1}{3!}\cdot\frac{-1}{2!}\right)$$

$\frac{1}{12} + \frac{1}{120} \quad \frac{108}{120} \quad \frac{9}{10}$

$$= 1 - x + 0\cdot x^2 + \frac{1}{3}x^3 - \frac{1}{6}x^4 + \frac{9}{10}x^5$$

$$\therefore e^{-x}\cos x = 1 - x + \frac{1}{3}x^3 - \frac{1}{6}x^4 + \frac{9}{10}x^5 + \cdots$$

수렴구간. $(-\infty, \infty)$.

No.

#10.3.2.(9)

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}$$

I) $n=0$. $a_0 = 1$

$$\frac{1}{1} = 1.$$

II) $n=1$. $a_1=1$

$4+4\cdot 1\cdot(+1) = 8 > 0$.

$$\frac{2x\cdot(1-x)-(1+x^2)(-1)}{(1-x)^2} = \cancel{\frac{2x-x^2}{(1-x)^2}} =$$

$$= \frac{2x-2x^2+1+x^2}{(1-x)^2}$$

$$= \frac{-x^2+2x+1}{(1-x)^2}$$

$$\cancel{= \frac{x^2-2x+1}{(x-1)}}$$

$a_1 = -0+0+1 = 1$

III) $n=2$.

$$\frac{(-2x+2)(1-x)^2 + (-x^2+2x+1)\,2(1-x)(-1)}{(1-x)^{4\,3}}$$

$a_2 = 2\cdot 1 - (1)\cdot 2 = 0$.

No.

$$\frac{1+x^2}{1-x} = \frac{\frac{1}{1-x}}{\frac{1}{1+x^2}}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3+x^4+\cdots$$

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} = 1-x^2+x^4-x^6+x^8+\cdots$$

$$(1-x^2+x^4-x^6+x^8\cdots)\overline{)\,(1+x+x^2+x^3+x^4+\cdots)}$$

$$\frac{1+x^2}{1-x} = \frac{(1-x)^2+2x}{1-x} = \frac{1}{1-x} + \frac{x^2}{1-x}$$

$$= 1-x + \frac{2x}{1-x}$$

$\frac{1}{1-x}$ $\sum_{n=0}^{\infty} x^n$ $\qquad$ $1+x+2\sum_{n=2}^{\infty} x^n$

$$\frac{x^2}{1-x} = \sum_{n=0}^{\infty} x^{n+2}$$

$$\therefore \frac{1+x^2}{1-x} = 1+x+2(x^2+x^3+x^4+\cdots+x^n).$$

수렴 구간. $-1<x<1$.

#10.3.3(2)

$$e^{x^3} = \sum_{n=0}^{\infty} \frac{1}{n!} x^{3n}$$

$$= \cancel{x} \; 1 + x^3 + \frac{1}{2!}x^6 + \frac{1}{3!}x^9 + \cdots$$

$$e^{x^3} - 1 = x^3 + \frac{1}{2!}x^6 + \frac{1}{3!}x^9 + \cdots$$

$$\lim_{x\to 0} \frac{e^{x^3}-1}{x^3} = \lim_{x\to 0} \frac{x^3 + \frac{1}{2!}x^6 + \frac{1}{3!}x^9 + \cdots}{x^3} = 1.$$

$\therefore 1.$

#10.4.2(2)

$$(1-x)^{\frac{1}{2}} = \sum_{n=0}^{\infty} {}_{\frac{1}{2}}C_n x^n.$$

$$= {}_{\frac{1}{2}}C_0 + {}_{\frac{1}{2}}C_1 x + {}_{\frac{1}{2}}C_2 x^2$$

$$= 1 + \frac{\frac{1}{2}}{1} x + \frac{\frac{1}{2}\cdot\left(-\frac{1}{2}\right)}{2!} x^2.$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2.$$

$x = 0.02$ $\quad 1 + \frac{1}{2}0.02 - \frac{1}{8}\cdot 0.0004$

$= 1 + 0.01 - \cancel{0}.00005 = 1.00995$

$\therefore 1.00995.$

No.

#10.9.9.(1)

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\frac{\sin x}{x} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n}$$

$$\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n} \, dx.$$

$$= \left[\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \frac{1}{2n+1} \cdot x^{2n+1} \right]_0^1.$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \frac{1}{(2n+1)}.$$

$$= 1 - \frac{1}{3 \cdot 3!} + \frac{1}{5 \cdot 5!} - \frac{1}{6 \cdot 6!} + \frac{1}{7 \cdot 7!}$$

$$\doteq 1 - 0.0555 + 0.0016$$

$$= 0.9461.$$

$\therefore 0.946.$

#10.4.5(1).

$$\sum_{n=0}^{\infty} \frac{1}{2^n} - \sum_{n=0}^{\infty} \frac{1}{3^n}.$$

$$= \frac{1}{1-\frac{1}{2}} - \frac{1}{1-\frac{1}{3}}.$$

$$= \frac{1}{\frac{1}{2}} - \frac{1}{\frac{2}{3}}$$

$$= 2 - 1.5 = \frac{1}{2}.$$

$\therefore \frac{1}{2}.$