

11.18.2021

DATE

#13.9.9

$$f(a+h, b+h) \doteq f(a, b) + f_x(a, b)h_1 + f_y(a, b)h_2$$

$$f(2.2, 4.9) = f(2+0.2, 5-0.1)$$

$$\doteq f(2, 5) + f_x(2, 5) \cdot 0.2 + f_y(2, 5) \cdot (-0.1)$$

$$= 6 + 1 \cdot 0.2 + 1 \cdot (-0.1) = 6.3$$

#13.5b(2)

$$u_x = u_r \cdot r_x + u_\theta \cdot \theta_x$$

$$u_r = 2r \sin 2\theta$$

$$u_\theta = 2r^2 \cos 2\theta$$

$$r_x = \frac{2x}{2\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}}$$

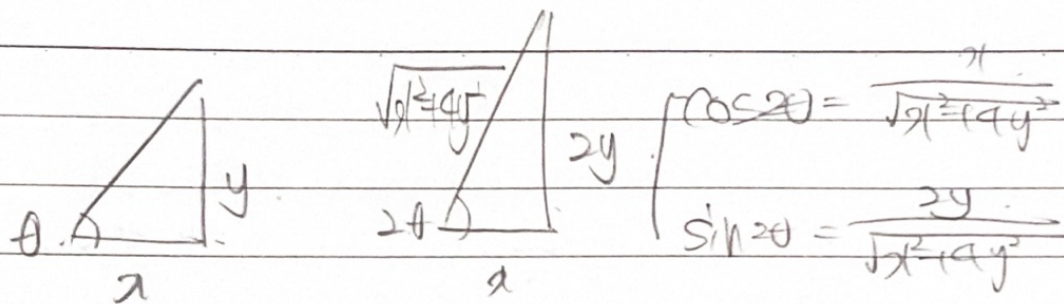
$$r_y = \frac{y}{\sqrt{x^2+y^2}}$$

$$\theta_x = \frac{y \cdot \frac{1}{x}}{1 + \left(\frac{y}{x}\right)^2}$$

$$= \frac{\frac{y}{x}}{1 + \frac{y^2}{x^2}} = \frac{\frac{y}{x}}{\frac{x^2 + y^2}{x^2}} = \frac{y}{x} \cdot \frac{x^2}{x^2 + y^2} = \frac{xy}{x^2 + y^2}$$

$$\theta_y = \frac{\frac{1}{x}}{1 + \left(\frac{y}{x}\right)^2}$$

$$= \frac{\frac{1}{x}}{1 + \frac{y^2}{x^2}} = \frac{1}{x} \cdot \frac{x^2}{x^2 + y^2} = \frac{x}{x^2 + y^2}$$



$$\begin{aligned}
 U_x &= U_r \cdot r_x + U_\theta \cdot \theta_x \\
 &= \cancel{2\sqrt{x^2 + y^2}} \cdot \frac{2y}{\sqrt{x^2 + 4y^2}} \cdot \frac{x}{\sqrt{x^2 + y^2}} \\
 &\quad + 2(x^2 + y^2) \cdot \frac{x}{\sqrt{x^2 + 4y^2}} \cdot \frac{-y}{y^2 + 1} \\
 &= \frac{4xy}{\sqrt{x^2 + 4y^2}} - \frac{2xy(x^2 + y^2)}{\sqrt{x^2 + 4y^2}(y^2 + 1)} \\
 &= \frac{4xy^3 + 4xy - 2x^3y - 2xy^3}{(y^2 + 1)\sqrt{x^2 + 4y^2}} \\
 &= \frac{4xy + 2xy^3 - 2x^3y}{(y^2 + 1)\sqrt{x^2 + 4y^2}} \\
 &= \frac{2xy(2 + y^2 - x^2)}{(y^2 + 1)\sqrt{x^2 + 4y^2}}
 \end{aligned}$$

$$\begin{aligned}
 U_y &= U_r \cdot r_y + U_\theta \cdot \theta_y \\
 &= \cancel{2\sqrt{x^2 + y^2}} \cdot \frac{2y}{\sqrt{x^2 + 4y^2}} \cdot \frac{y}{\sqrt{x^2 + y^2}} \\
 &\quad + 2(x^2 + y^2) \cdot \frac{x}{\sqrt{x^2 + 4y^2}} \cdot \frac{1}{1 + y^2} \\
 &= \frac{4y^3}{\sqrt{x^2 + 4y^2}} + \frac{2x^2(x^2 + y^2)}{(1 + y^2)\sqrt{x^2 + 4y^2}} \\
 &= \frac{4y^3 + 4y^5 + 2x^4 + 2x^2y^2}{\sqrt{x^2 + 4y^2}(1 + y^2)}
 \end{aligned}$$

#3.6.5(1)

$$\vec{T} = (T_x, T_y, T_z) \\ = \left(\frac{x}{(x^2-2y^2+4z^2)^{3/2}}, \frac{-4y}{(x^2-2y^2+4z^2)^{3/2}}, \frac{8z}{(x^2-2y^2+4z^2)^{3/2}} \right)$$

$$\vec{T}(1,2,1)$$

$$= \left(\frac{1}{(1-8+4)^{3/2}}, \frac{-8}{9}, \frac{-8}{9} \right)$$

$$= \left(\frac{1}{9}, \frac{-8}{9}, \frac{-8}{9} \right)$$

$$= \frac{1}{9} (1, -4, -4)$$

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#3.6.9

평면의 방정식을 $f(x,y,z)$ 라고 하자.

$$\vec{f} = \left(\frac{1}{2}x, 2y, \frac{2}{3}z \right)$$

$$\vec{f}(-2, 1, -3) = \left(-1, 2, -\frac{2}{3} \right)$$

$$\text{평면의 법선 벡터} = \left(-1, 2, -\frac{2}{3} \right)$$

$$-(x+2) + 2(y-1) - \frac{2}{3}(z+3) = 0$$

$$-x + 2y - \frac{2}{3}z - 2 - 2 - 2 = 0$$

$$-x + 2y - \frac{2}{3}z - 6 = 0$$

$$\text{평면의 방정식: } -x + 2y - \frac{2}{3}z = 6$$