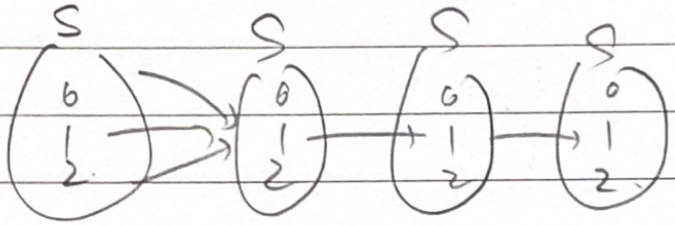
11.18 ± 확률과정 과제

#5.2.1 (1) $P(X_0=1, X_1=1, X_2=1)$

$= p_1 \times p_{11} \times p_{11}$

$= 0.4 \times 0.2 \times 0.2 =$ **0.016**

#5.2.1(2)



$P(X_1=1, X_2=1, X_3=1)$

$= (p_0 p_{01} + p_1 p_{11} + p_2 \cdot p_{21}) \times p_{11} \times p_{11}$

$= (0.6 \times 0.1 + 0.4 \times 0.2 + 0 \times 1) \times 0.2 \times 0.2$

$= 0.14 \times 0.04 =$ **0.0056**

#5.2.2(1)

	1	2
1	0.95	0.05
2	0.8	0.2

(1: 정상, 2: 고장)

#5.2.2(2) $P(X_0=1, X_1=1, X_2=1, X_3=1)$

$= 1 \times p_{11} \times p_{11} \times p_{11}$

$= 0.95^3 =$ **0.857375**

#5.2.3(1)

[WX2] [BX2]

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? 나올 경우의 수 개수.

$S = \{0, 1, 2\}$

$P =$

	0	1	2
0	$\frac{1}{3}$	$\frac{2}{3}$	0
1	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{1}{6}$
2	0	$\frac{2}{3}$	$\frac{1}{3}$

$p_{00} = 1 \times \frac{1}{3} = \frac{1}{3}$.

$p_{10} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

$p_{12} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$p_{21} = 1 \times \frac{2}{3} = \frac{2}{3}$.

#52. 3(2) $P(X_0=2, X_1=2, X_2=2)$

~~$1 \times \frac{1}{3} \times \frac{1}{3}$~~ $= 1 \times p_{22} \times p_{22}$

$= 1 \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

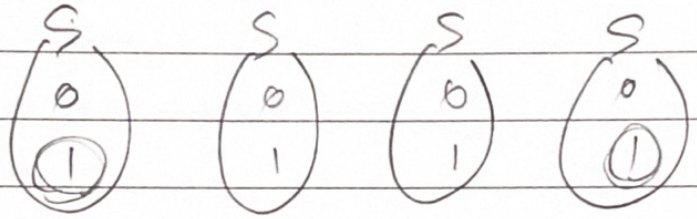
#52. 3(3) $P(X_0=2, X_1=1, X_2=0)$

$= p_2 \times p_{21} \times p_{10}$

$= 1 \times \frac{2}{3} \times \frac{1}{6} = \frac{1}{9}$.

#5 3.1

$P(X_3 = 1 \mid X_0 = 1) = p_{11}^{(3)}$



$$P^2 = \begin{pmatrix} 1 & 0 \\ 0.5 & 0.5 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0.5 & 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 0+0 \\ 0.5+0.25 & 0+0.25 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0.75 & 0.25 \end{pmatrix}$$

$$P^3 = P^2 \cdot P$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{3}{4} + \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{7}{8} & \frac{1}{8} \end{pmatrix}$$

$$p_{11}^{(3)} = P^{(3)}{}_{11} = \frac{1}{8}$$

$$P(X_6 = 1 \mid X_0 = 0) = p_{01}^{(6)}$$

$$= P^{(6)}{}_{01}$$

$$P^6 = P^3 \times P^3$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{7}{8} & \frac{1}{8} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{7}{8} & \frac{1}{8} \end{pmatrix}$$

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$= \begin{pmatrix} 1 & 0 \\ \frac{63}{64} & \frac{1}{64} \end{pmatrix}$

95
6
950

$P^{(6)}_{01} = \frac{63}{64}$

#5.3.2

$p_0 = 0.4$

$p_1 = 0.6$

$P(X_2=0) = p_0 \times p_{00}^{(2)} + p_1 \times p_{10}^{(2)}$

$= 0.4 \times 1 + 0.6 \times 0.75$

$= 0.4 + 0.450 = 0.850$

$P(X_4=1) = p_0 \times p_{01}^{(4)} + p_1 \times p_{11}^{(4)}$

$P^4 = P^2 \times P^2$

$= \begin{pmatrix} 1 & 0 \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ \frac{15}{16} & \frac{1}{16} \end{pmatrix}$

$P(X_4=1) = 0.4 \times 0 + 0.6 \times \frac{1}{16} = 0.037115$

#5.3.3

$P(X_2=0 \mid X_0=1) = p_{10}^{(2)} = p^2_{10}$

$P(X_4=1 \mid X_0=2) = p_{21}^{(4)} = p^4_{21}$

$P^2 = P \times P$

$$= \begin{pmatrix} 1. & 0. & 0 \\ 0.18 & 0.26. & 0.54. \\ 0.3. & 0.16. & 0.52 \end{pmatrix}$$

$P^4 = P^2 \times P^2$

$$= \begin{pmatrix} 1. & 0. & 0. \\ 0.324 & 0.1156 & 0.432 \\ 0.1684 & 0.324 & 0.3616 \end{pmatrix}$$

$P^2_{10} =$ 0.18

$P^4_{21} =$ 0.324

#34 $P(X_2=2) = p_{02}^{(2)} + p_{12}^{(2)} + p_{22}^{(2)}$

$= 0.3 \times 0 + 0.3 \times 0.54 + 0.4 \times 0.52$

$= 0 + 0.162 + 0.208 =$ 0.37

$P(X_4=1) = p_{01}^{(4)} + p_{11}^{(4)} + p_{21}^{(4)}$

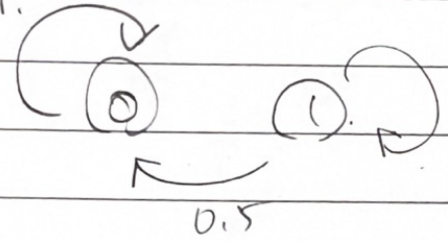
$= 0.3 \times 0 + 0.3 \times 0.1156 + 0.4 \times 0.324$

$= 0 + 0.05268 + 0.1296 =$ 0.18228

11.24. 금. 유한마하 과제 DATE. NO.

#5.4.1(1)

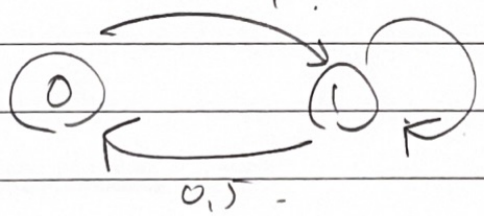
1.



0 ← 1.

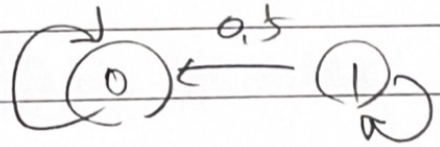
상호작용이지 않으므로 (가물각이다.)

#5.4.1(2)



0 ↔ 1. 상호작용이므로 (가역적이다.)

#5.4.2(1)



1 ← 0. 가물각.

d(1) = 1 → 비가역적 (예고각 X)

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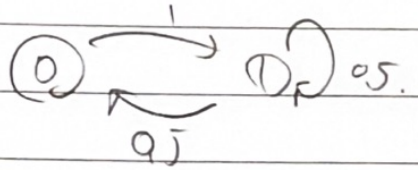
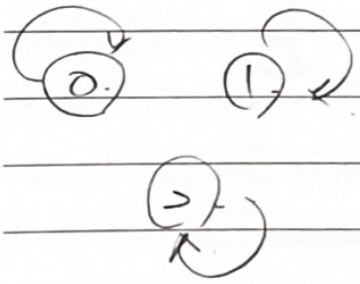
#5.4.2 (2)



0 ⇔ 1. 가능.

d(0) = 1
d(1) = 1 → 비가역 (기억)

#5.4.3(1)



가역적이다.

d(0) = 1.

d(1) = 1.

d(2) = 1 (2개 = 1).

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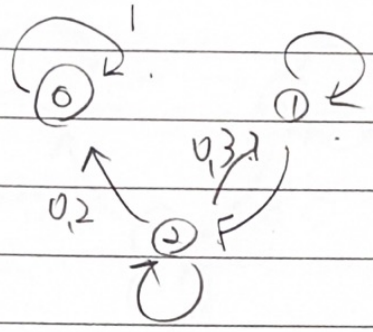
#5.4.3(1)



0. X 1.

1. ↔ 2.

1. → 0. 가분적

$d(0) = 1$.

$d(1) = 1$

$d(2) = 1$. 주기 = 1.

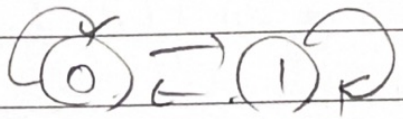
#5.4.4(1)

가분적이고 비주기적이므로 에르딕이 아니다

#5.4.4(2)

가분적이고 비주기적이므로 에르딕이 아니다.

#5.5.1(1)



0 ↔ 1.

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0 ⟷ 1. 상호 도달. → 기약성

$d(0) = 1$.
$d(1) = 1$. } 비주기성

∴ 에르고딕. → 정상 확률을 찾자.

$\pi_0 + \pi_1 = 1$.

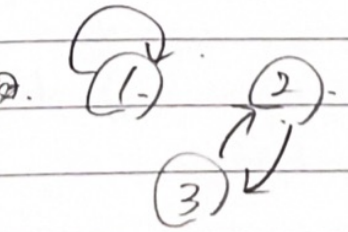
$(\pi_0 \ \pi_1)\begin{pmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{pmatrix} = (\pi_0 \ \pi_1)$

$0.7\pi_0 + 0.5\pi_1 = \pi_0$ $0.3\pi_0 = 0.5\pi_1$

$0.3\pi_0 + 0.5\pi_1 = \pi_1$ $0.5\pi_1$

$\frac{3}{5}\pi_1 + \pi_1 = 1$ $\pi_1 = \frac{5}{8}$, $\pi_0 = \frac{3}{8}$

#5.5.1(2)



기약성이 아니므로 에르고딕이 아니고
따라서 극한 확률은 존재하지 않는다.

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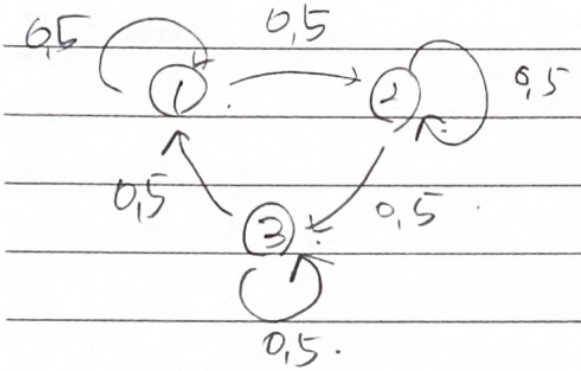
5.1.(3)



1 ↔ 2
2 ↔ 3
3 ↔ 1

상태도달 → 기약적

$d(1) = 1$.
$d(2) = 1$.
$d(3) = 1$.

→ 비주기적

마르코프 체인에서 극한 분포가 존재한다.

$$(\pi_1 \ \pi_2 \ \pi_3) \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \end{pmatrix} = (\pi_1 \ \pi_2 \ \pi_3)$$

$\pi_1 + \pi_2 + \pi_3 = 1$

$$\begin{cases} 0.5\pi_1 + 0.5\pi_2 = \pi_1 \\ \pi_2 + \pi_3 = \pi_2 \\ \pi_1 + \pi_3 = \pi_3 \end{cases}$$

$\pi_1 = \pi_2$
$\pi_2 = \pi_3$
$\pi_3 = \pi_1$

$\therefore \pi_1 = \pi_2 = \pi_3 = \frac{1}{3}$

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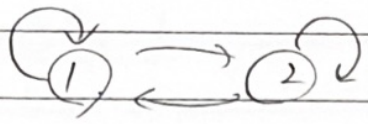
#5.5.2

$S = \{1, 2\}$

정상 상태

	1	2
1	0.95	0.05
2	0.8	0.2



상호 도달 → 가능

$d(1) = 1$
$d(2) = 1$ 주기 = 1 → 비주기

결론: 이 체인은 한 정상 분포가 존재

$(\pi_1, \pi_2) \begin{pmatrix} 0.95 & 0.05 \\ 0.8 & 0.2 \end{pmatrix} = (\pi_1, \pi_2)$

$\begin{cases} 95\pi_1 + 5\pi_2 = 100\pi_1 \\ 80\pi_1 + 20\pi_2 = 100\pi_2 \end{cases}$ $\pi_1 + \pi_2 = 1$

$\pi_1 = \pi_2$ $\pi_1 = 0.5$, $\pi_2 = 0.5$

(0, 1) (0.5)