

9.09. ±

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1장.

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#1.1.1(a).

$$A = \{1, 2, 3\}$$

(b)

$$-5 < x < 5$$

$$B = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

(c)

$$x = \frac{-(-3) \pm \sqrt{9 - 4 \cdot 1 \cdot (-1)}}{2}$$

$$= \frac{-3 \pm \sqrt{13}}{2}$$

$$C = \left\{ \frac{-3 - \sqrt{13}}{2}, \frac{-3 + \sqrt{13}}{2} \right\}$$

#1.1.2.(a)

$$A = \{x \in \mathbb{N} \mid x \leq 5\}$$

(b)

$$B = \{x \in \mathbb{N} \mid x = 2n, n \in \mathbb{N}\}$$

(c)

$$(x - 1 + \sqrt{2})(x - 1 - \sqrt{2}) = 0$$

$$x^2 + (-1 + \sqrt{2} - 1 - \sqrt{2})x + (1 - 2) = 0$$

$$x^2 - 2x - 1 = 0$$

$$C = \{x \in \mathbb{R} \mid x^2 - 2x - 1 = 0\}$$



#1.1.3(b).  $P(B) = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$

(b).  $C = P(\{a, b\})$   
 $= \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$P(C) = \{\emptyset, \{\emptyset\}, \{\{a, b\}\}, \{\emptyset, \{a, b\}\}\}$

#1.2.3(a).  $A \cap B = B \cap A$

$x \in (A \cap B)$

$\Leftrightarrow x \in A \text{ and } x \in B$

$\Leftrightarrow x \in B \text{ and } x \in A$

$\Leftrightarrow x \in (B \cap A)$

$\therefore A \cap B = B \cap A$

(b).  $A \cap (B \cap C) = (A \cap B) \cap C$

$x \in A \cap (B \cap C)$

$\Leftrightarrow x \in A \text{ and } x \in B \cap C$

$\Leftrightarrow x \in A \text{ and } (x \in B \text{ and } x \in C)$

$\Leftrightarrow (x \in A \text{ and } x \in B) \text{ and } (x \in C)$

$\Leftrightarrow x \in (A \cap B) \text{ and } x \in C$

$\Leftrightarrow x \in (A \cap B) \cap C$

$\therefore A \cap (B \cap C) = (A \cap B) \cap C$



(c).

$$x \in A \cap (B \cup C)$$

$$\Leftrightarrow x \in A \text{ and } (x \in B \text{ or } x \in C)$$

$$\Leftrightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$$

$$\Leftrightarrow x \in A \cap B \text{ or } x \in A \cap C$$

$$\Leftrightarrow x \in (A \cap B) \cup (A \cap C)$$

$$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(d)

$$(A \cap B)^c$$

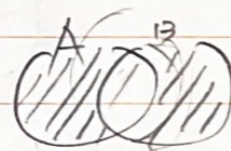
$$x \in (U - (A \cap B))$$

$$\Leftrightarrow x \notin (A \cap B)$$

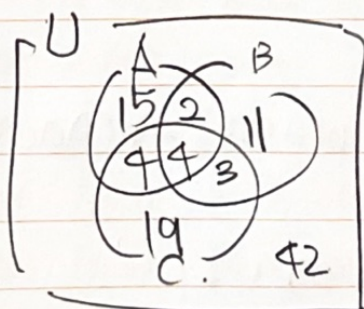
$$\Leftrightarrow x \notin A \text{ or } x \notin B$$

$$\Leftrightarrow \{x \mid x \in A^c \cup B^c\}$$

$$\therefore (A \cap B)^c = A^c \cup B^c$$



#11.3.2 (a)



$$100 = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) + n((A \cap B \cap C)^c)$$

$$= 25 + 20 + 30 - 6 - 7 - 8 + n(A \cap B \cap C) + 42$$

$$50 = 15 - 21 + n(A \cap B \cap C) \quad n(A \cap B \cap C) = 4$$

+4.

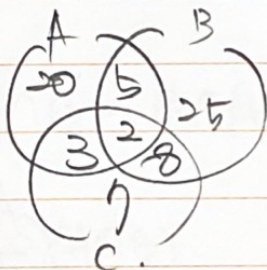


$$15 + 11 + 19 = 45 \text{ (total)}; \therefore 45 \text{ total.}$$

(b)  $2 + 4 + 3 = 9; \therefore 9 \text{ total.}$

(c)  $n(A - C) = 17; \therefore 17 \text{ total.}$

#1.3.3(a)



$$n(A \cup B) = 30 + 40 - 10 = 60; \therefore 60.$$

(b).

$$n((A \cup C)^c)$$

$$= n(U - n(A \cup C))$$

$$= 100 - (30 + 40 - 5) = 65; \therefore 65.$$

(c)

$$n(A \cup B \cup C)$$

$$= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$= (30 + 40 + 20) - (10 + 5 + 10) + 2.$$

$$= 90 - 25 + 2 = 67.$$

$$\therefore 67$$



(d)

$$\begin{aligned} n(A \cup (C \cap B)) &= n(A \cup (C - B)) \\ &= 30 + 9 = 39. \quad (\because 3) \end{aligned}$$

#1.9.3

$$n(A \times A) = n(A) \times n(A) = 9. \quad \because n(A) = 3$$

$$A \times A = \{(a, b) \mid a \in A, b \in A\}$$

$$(0, 1), (1, 2) \text{ 이 } A \times A \text{ 이고 } A = \{0, 1, 2\}$$

$$\therefore (0, 0), (0, 2), (1, 0), (1, 1), (2, 0), (2, 1), (2, 2)$$

#1.49

$$A \times (B \cap C) = (A \times B) \cap (A \times C) \text{ 증명}$$

$$(x, y) \in A \times (B \cap C)$$

$$\Leftrightarrow x \in A \text{ and } y \in (B \cap C)$$

$$\Leftrightarrow x \in A \text{ and } (y \in B \text{ and } y \in C)$$

$$\Leftrightarrow (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \in C)$$

$$\Leftrightarrow (x, y) \in A \times B \text{ and } (x, y) \in A \times C$$

$$\Leftrightarrow (A \times B) \cap (A \times C)$$

$$\therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$$



#1.4.5

$$(A \times C) \cap (B \times D) = (A \cap B) \times (C \cap D) \quad \text{3B5}$$

$$(x, y) \in (A \times C) \cap (B \times D).$$

$$\rightarrow (x \in A \text{ and } y \in C) \text{ and } (x \in B \text{ and } y \in D)$$

$$\rightarrow (x \in A \text{ and } x \in B) \text{ and } (y \in C \text{ and } y \in D)$$

$$\rightarrow (x \in A \cap B) \text{ and } (y \in C \cap D)$$

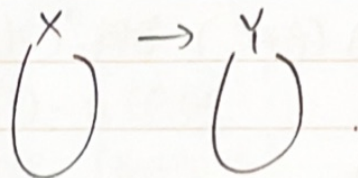
$$\rightarrow (x, y) \in (A \cap B) \times (C \cap D)$$

$$\therefore (A \times C) \cap (B \times D) = (A \cap B) \times (C \cap D)$$

#1.5.2

$$n(X) = m.$$

$$n(Y) = n.$$



$$\therefore n^m \text{ 7H}$$

#1.5.9(a)

$$\begin{aligned} f \circ g &= f(g(x)) \\ &= 3(5x)^2 + 2 \\ &= 15x^2 + 2 \end{aligned}$$

(b)

$$\begin{aligned} g \circ f &= g(f(x)) \\ &= 5(3x^2 + 2) \\ &= 15x^2 + 10 \end{aligned}$$

(c) 
$$\begin{aligned} f \circ (g \circ f) &= f(g(f(x))) \\ &= f(15x^2 + 10) \\ &= 3(15x^2 + 10)^2 + 2 \\ &= 3(225x^4 + 300x^2 + 100) + 2 \\ &= 675x^4 + 900x^2 + 302. \end{aligned}$$

(d) 
$$\begin{aligned} (f \circ g) \circ f &= f(g(x)) \cdot f(x) \\ &= f(g(f(x))) \\ &= 675x^4 + 900x^2 + 302. \end{aligned}$$