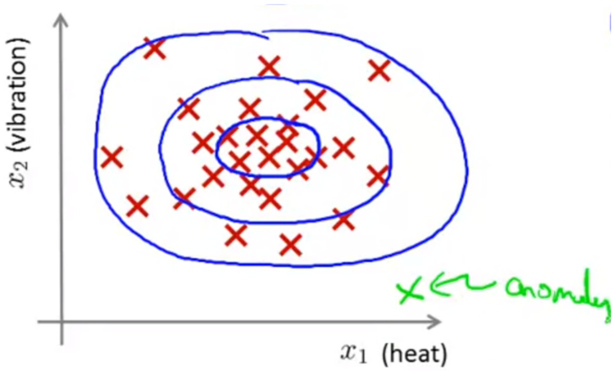
Density Estimation

Dataset: $x^{(1)}, x^{(2)}, \ldots, x^{(m)}$

Objective is to find if x_{test} is anomalous

We can detect anomalies with some classification like:

$$p(x_{test}) \begin{cases} < \epsilon & \text{anomaly} \\ \geq \epsilon & \text{OK} \end{cases}$$



Use Cases

- Fraud Detection
  - $x^{(i)}$ = Features of user i's activities
  - Model $p(x)$ from data
  - Identify unusual users by checking which have $p(x) < \epsilon$
- Manufacturing
- Monitoring Data Centre Computers

  $x(i)$ = Features of machine i

  x_1 = Memory Use

  x_2 = Number of disk accesses per second

  x_3 = CPU Load

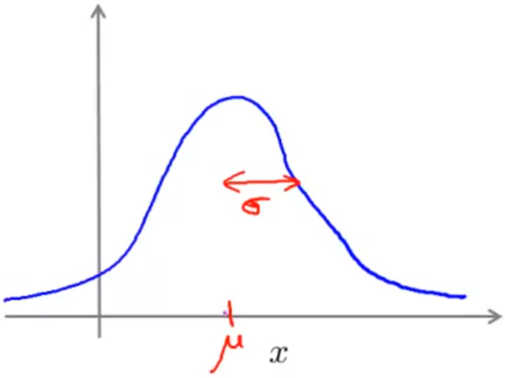
  x_4 = CPU Load / Network Traffic

  ... Other features

Gaussian Distribution

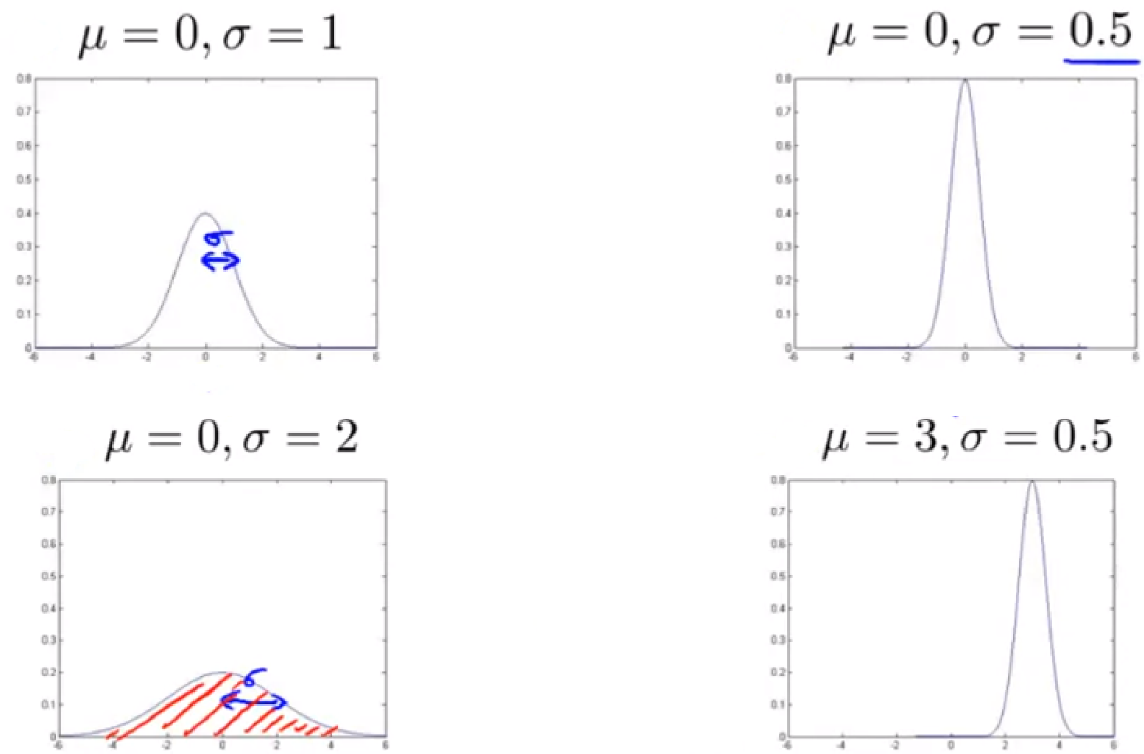
Say $x \in \mathbb{R}$. If x is a distributed Gaussian with mean μ, variance σ^2. We can write this as:

$$x \sim \aleph(\mu, \sigma^2)$$



$$\text{Gaussian Distribution}, p(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(- \frac{(x-\mu)^2}{2\sigma^2} \right)$$

Examples



Parameter Estimation

$$\mu = \frac{1}{m} \sum_{i=1}^{m} x^i$$

$$\sigma^2 = \frac{1}{m} \sum_{i=1}^{m} (x^{(i)} - \mu)^2$$

Algorithm

We can model $p(x)$ as:

$$
\begin{aligned}
p(x) &= p(x_1; \mu_1, \sigma_1^2) \cdot p(x_2; \mu_2, \sigma_2^2) \dots p(x_n; \mu_n, \sigma_n^2) \\
&= \prod_{j=1}^{n} p(x_j; \mu_j, \sigma_j^2)
\end{aligned}
$$

Where,

- $x_1 \sim \aleph(\mu_1, \sigma_1^2)$
- $x_2 \sim \aleph(\mu_2, \sigma_2^2)$

  $\vdots$

- $x_n \sim \aleph(\mu_n, \sigma_n^2)$

So the algorithm would be:

1. Choose features x_i that you think might be indicative of anomalous examples
2. Fit parameters $\mu_1, \dots, \mu_n, \sigma_1^2, \dots, \sigma_n^2$

$$\mu_j = \frac{1}{m} \sum_{i=1}^{m} x_j^{(i)}$$

$$\sigma_j^2 = \frac{1}{m} \sum_{i=1}^{m} (x_j^{(i)} - \mu_j)^2$$

3. Given new example x, compute $p(x)$

$$p(x) = \prod_{j=1}^{n} p(x_j, \mu_j, \sigma^2) = \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_j} exp\Big(-\frac{(x_j - \mu_j)^2}{2\sigma_j^2}\Big)$$

Anomaly if $p(x) < \epsilon$