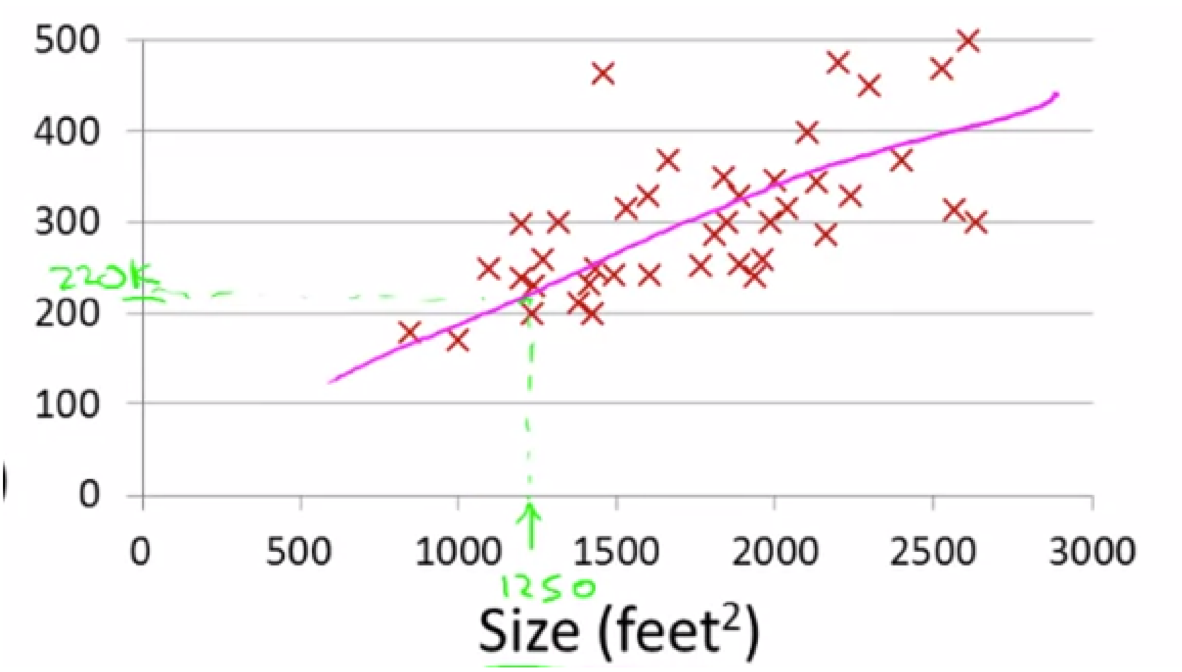
Model and Cost Function

Let's take an example of a sample dataset which contains the housing prices in a particular city.



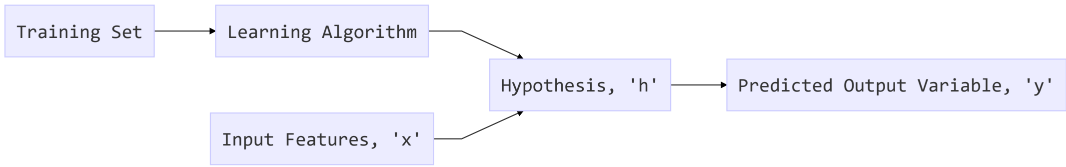
This is an example of:

- **Supervised Learning**: Given the "correct answer" for each example in the dataset
- **Regression Problem**: Objective is to predict real-valued output

Size in ft^2 (x)	Price in $100 (y)
2104 ($x^{(1)}$)	460 ($y^{(1)}$)
1416 ($x^{(2)}$)	232 ($y^{(2)}$)
1534 ($x^{(3)}$)	315 ($y^{(3)}$)
852 ($x^{(4)}$)	178 ($y^{(4)}$)
...	...

Notation:

- ***m***: Number of training examples
- ***x's***: "Input" Variable
- ***y's***: "Output" Variable
- ***(x, y)***: Single Training Example
- ***($x^{(i)}$, $y^{(i)}$)***: i$^{(th)}$ Training Example



What is Hypothesis?

It maps from input features to output variables. It can be represented as

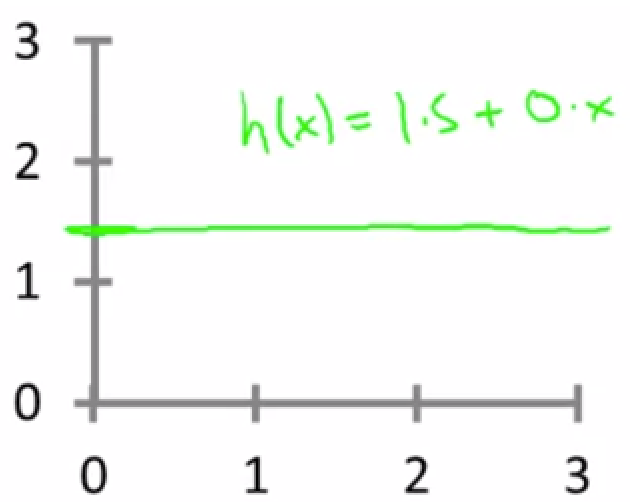
$$h_\theta(x) = \theta_0 + \theta_1 x$$

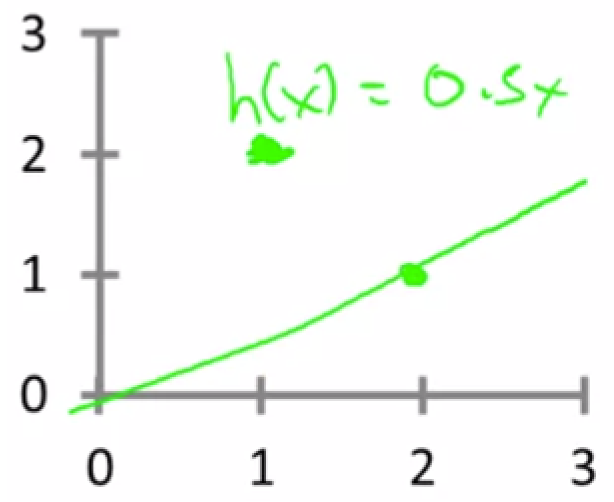
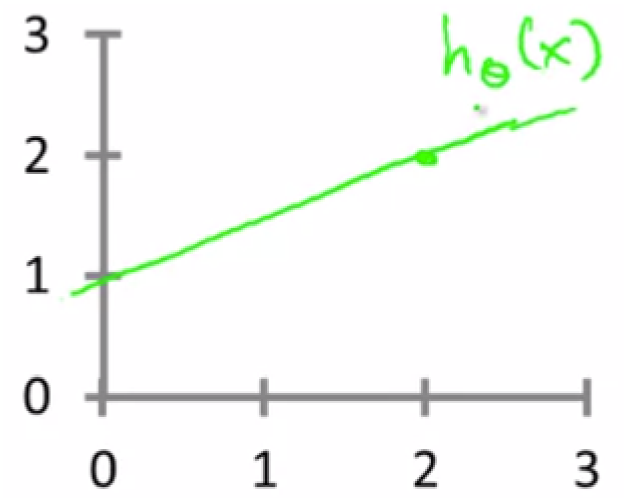
The shorthand notation for the above equation is

$$h(x) = \theta_0 + \theta_1 x$$

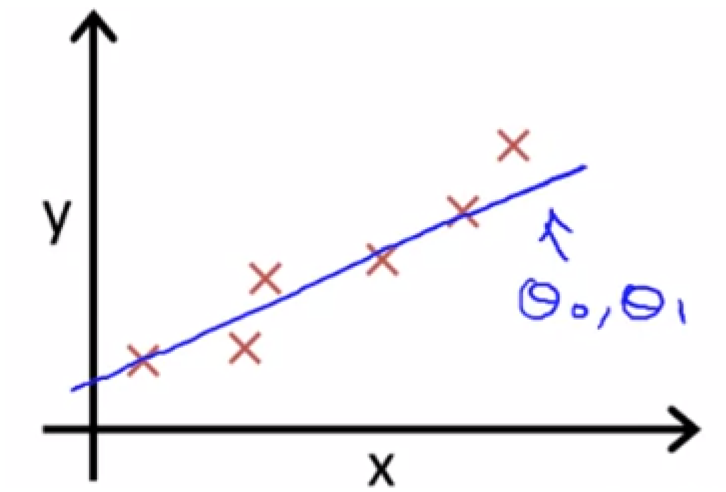
Here, θ_0 and θ_1 are the parameters

The black line represents the Hypothesis. This model is called **Univariate Linear Regression** or *Linear Regression with One Variable*. Below are some of the examples for certain values of the parameters θ_0 and θ_1.

Parameters	Graph
$\theta_0 = 1.5$ $\theta_1 = 0$ $h(x) = 1.5 + (0 * x)$	

Parameters	Graph
$\theta_0 = 0$ $\theta_1 = 0.5$ $h(x) = 0 + (0.5 * x)$	
$\theta_0 = 1$ $\theta_1 = 0.5$ $h(x) = 1 + (0.5 * x)$	

Choosing the Values of θ_0 and θ_1



- Choose θ_0 and θ_1 so that $h_\theta(x)$ is close to y for our training examples
- Formulation:

$$minimize_{\theta_0 \theta_1} \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

  Where,

  - $h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$

- $m =$ Number of training examples
- Cost Function (Squared Error Function):

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

- Therefore, in Linear Regression, the goal is:

$$minimize_{\theta_0 \theta_1} J(\theta_0, \theta_1)$$

Note: There are other cost functions that work pretty well, but the Squared Error Cost function is the one most commonly used for *Regression* problems

Cost Function Intuition 1

Let's have a simplified version of the cost function, where the hypothesis is (assuming $\theta_0 = 0$):

$$h_\theta(x) = \theta_1 x$$

We have the Cost Function,

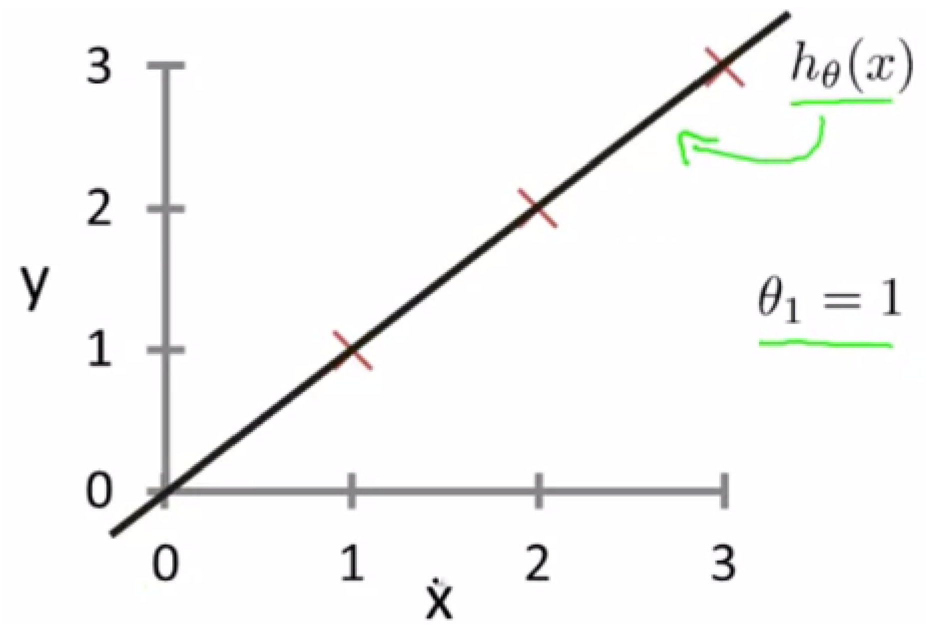
$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

And our goal is:

$$minimize_{\theta_1} J(\theta_1)$$

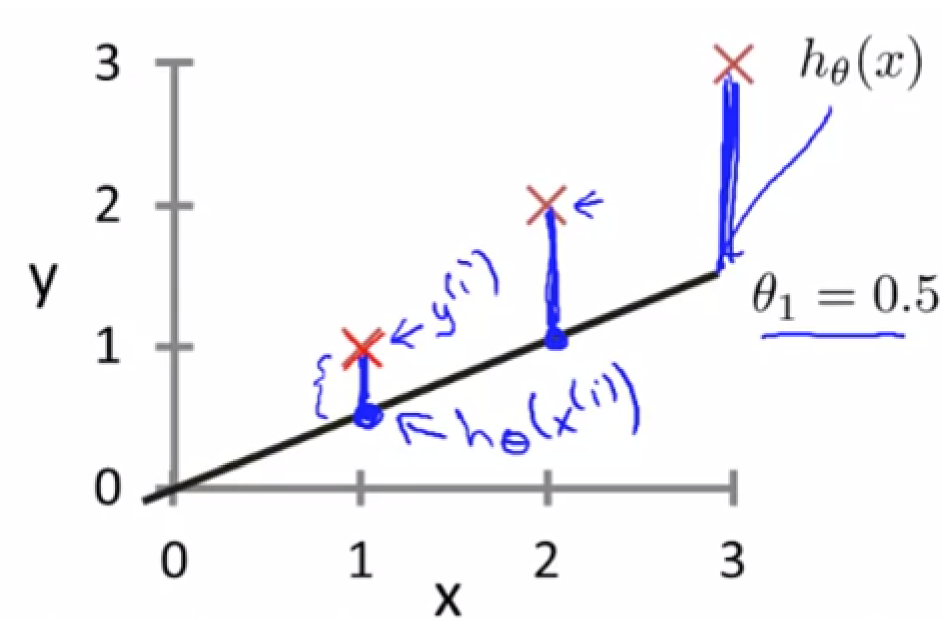
When $\theta_1 = 1$:

$$\begin{aligned} J(\theta_1) &= \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2 \\ &= \frac{1}{2m} \sum_{i=1}^{m} (\theta_1 x^{(i)} - y^{(i)})^2 \\ &= \frac{1}{2*3} [(1*1-1)^2 + (1*2-2)^2 + (1*3-3)^2] \\ &= \frac{1}{6}(0^2 + 0^2 + 0^2) \\ &= 0 \end{aligned}$$



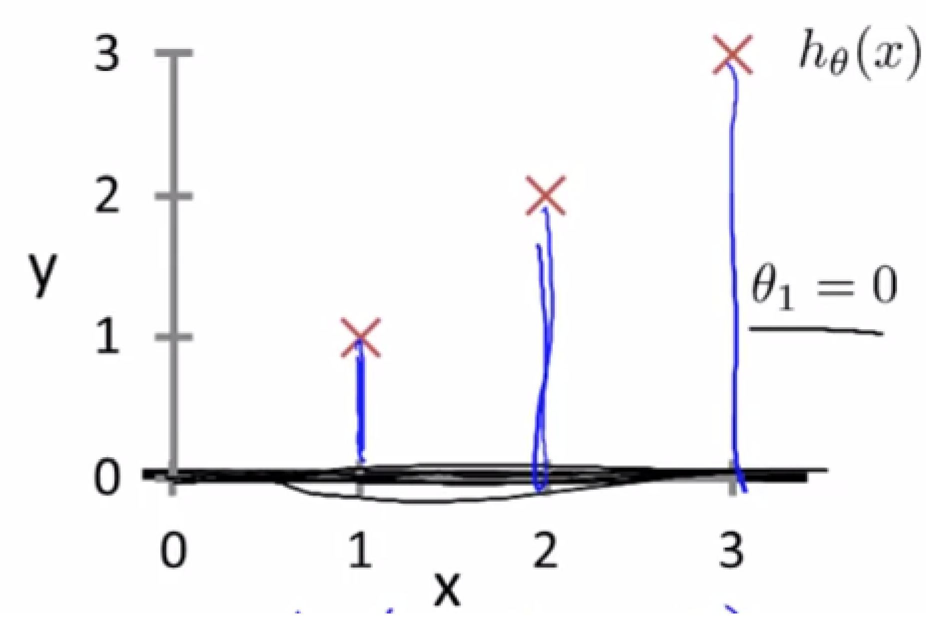
When $\theta_1 = 0.5$:

$$\begin{aligned}
J(\theta_1) &= \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 \\
&= \frac{1}{2m}\sum_{i=1}^{m}(\theta_1 x^{(i)} - y^{(i)})^2 \\
&= \frac{1}{2*3}[(0.5*1-1)^2 + (0.5*2-2)^2 + (0.5*3-3)^2] \\
&= \frac{1}{2*3}[(0.5-1)^2 + (1-2)^2 + (1.5-3)^2] \\
&= \frac{1}{2*3}[0.5^2 + 1^2 + 1.5^2] \\
&= \frac{1}{6}*3.5 \\
&= 0.58
\end{aligned}$$

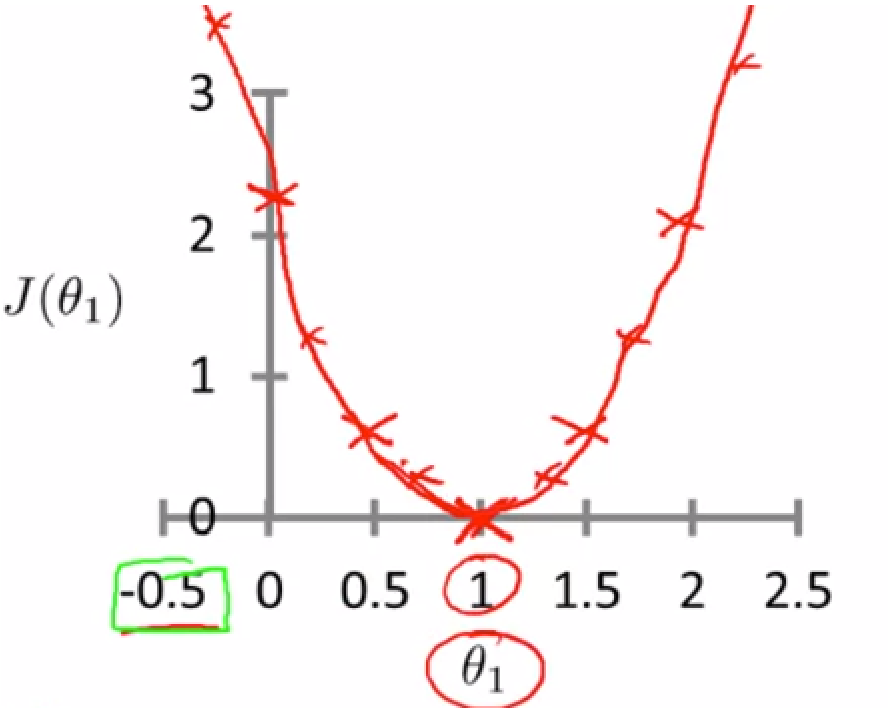


When $\theta_1 = 0$:

$$\begin{aligned}
J(\theta_1) &= \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 \\
&= \frac{1}{2m}\sum_{i=1}^{m}(\theta_1 x^{(i)} - y^{(i)})^2 \\
&= \frac{1}{2*3}[(0*1-1)^2 + (0*2-2)^2 + (0*3-3)^2] \\
&= \frac{1}{2*3}[(1^2 + 2^2 + 3^2] \\
&= \frac{1}{6}*14 \\
&= 2.3
\end{aligned}$$



Plot for $J(\theta_1)$ as a function of parameter θ_1:



The value that minimizes $J(\theta_1)$ here is $J(\theta_1) = 1$ for this particular dataset

Cost Function Intuition 2

Problem Formulation:

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters: θ_0, θ_1

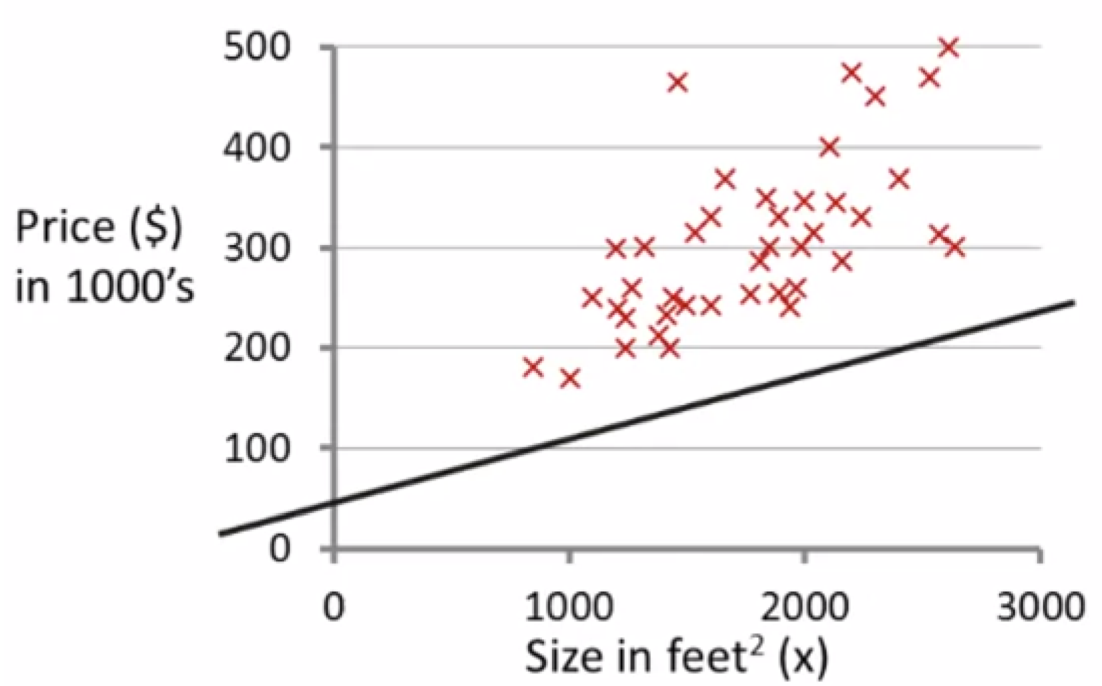
Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

And our goal is:

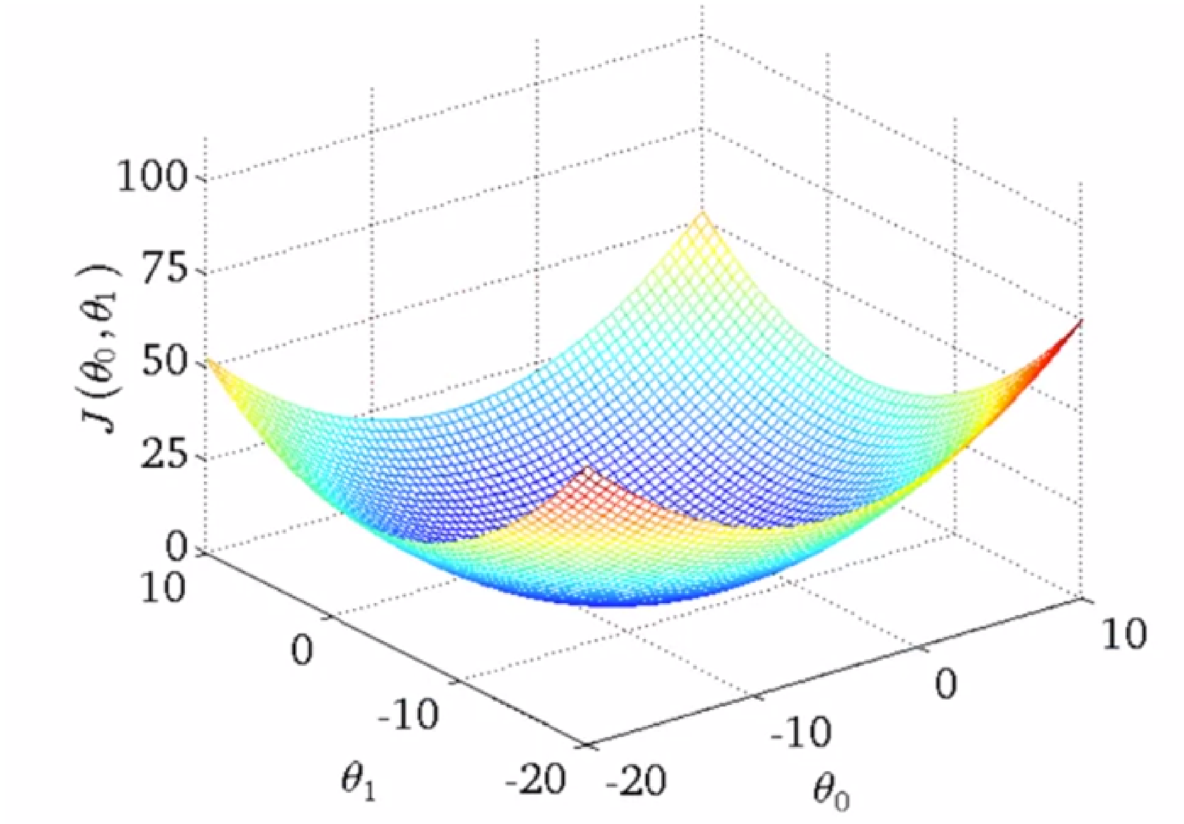
$$minimize_{\theta_0, \theta_1} J(\theta_0, \theta_1)$$

Let's make a random hypothesis on a training set of housing prices:

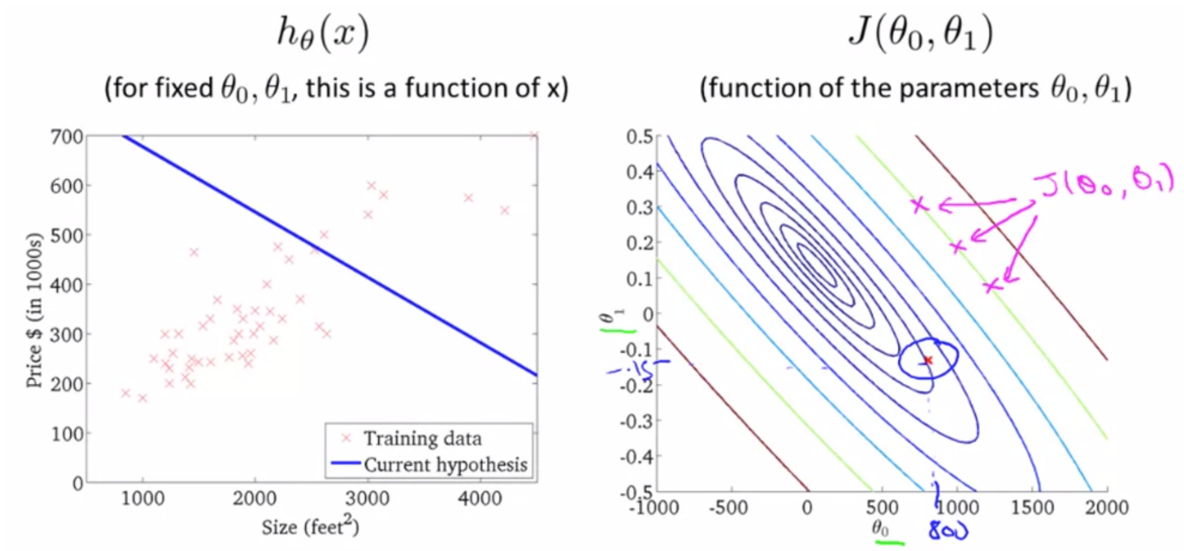


Here,
$\theta_0 = 50$
$\theta_1 = 0.06$
$h_\theta(x) = 50 + 0.06x$

Plot for $J(\theta_1)$ as a function of parameter θ_1 and θ_1:

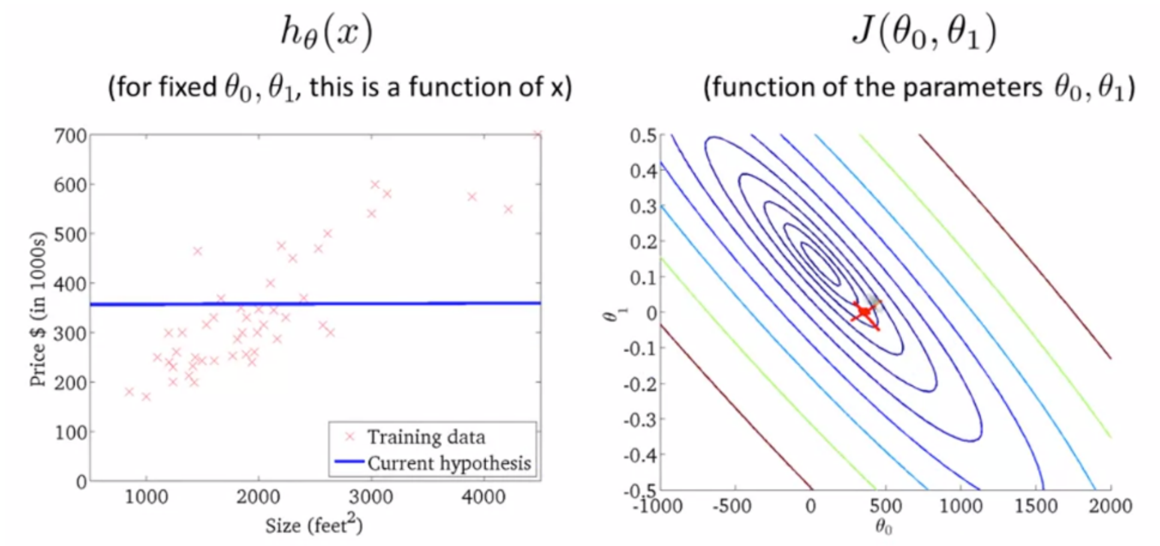


Example 1:



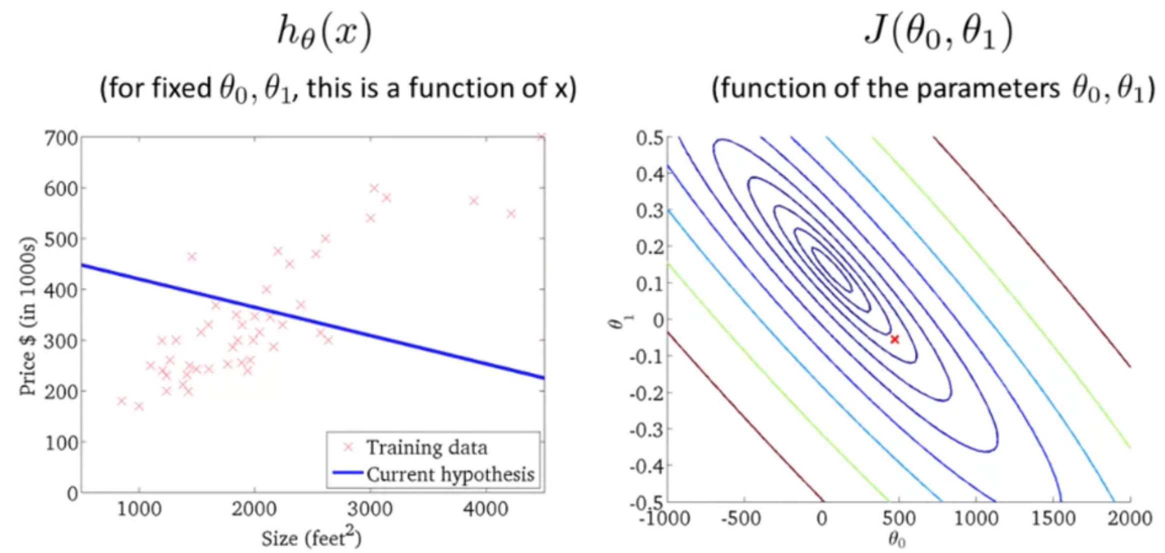
Selected point (cost) will have $\theta_0 = 800$ and $\theta_1 = -0.15$, which isn't a good fit for the data

Example 2:



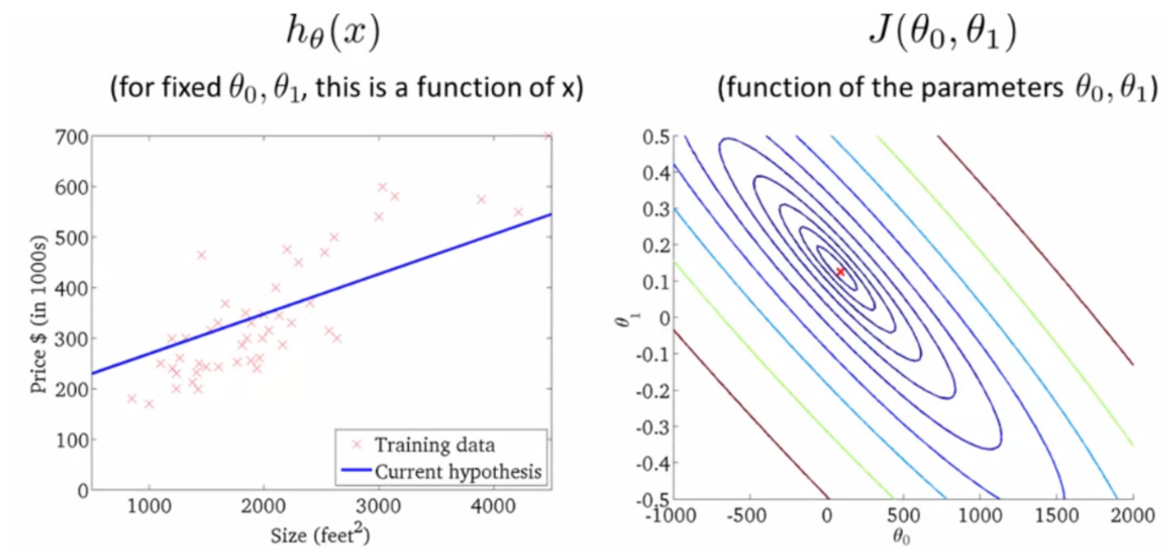
Selected point (cost) will have $\theta_0 = 360$ and $\theta_1 = 0$, which isn't a good fit for the data

Example 3:



Selected point (cost) will have $\theta_0 = 500$ and $\theta_1 = 0$, which isn't a good fit for the data

Example 4:



Selected point (cost) will have $\theta_0 = 250$ and $\theta_1 = 0.15$, which looks like a good fit for the data, even though it's not quite at the minimum