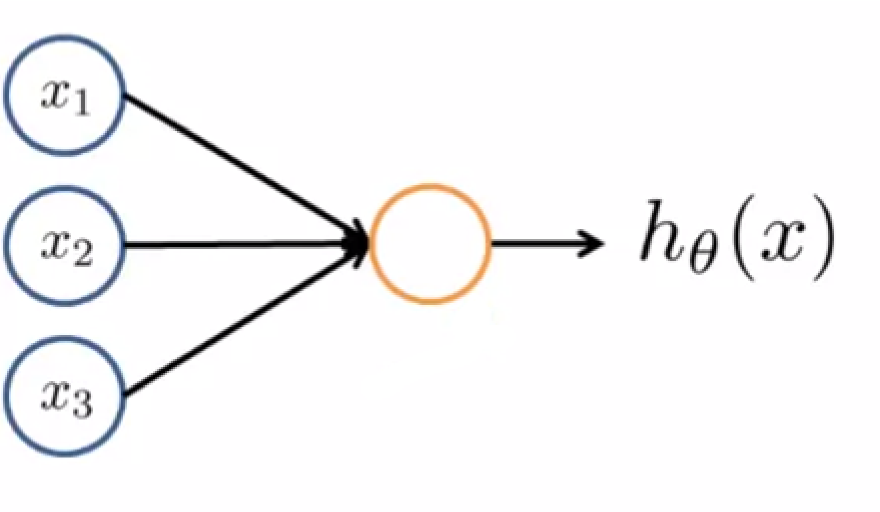
Neural Networks



Here,

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

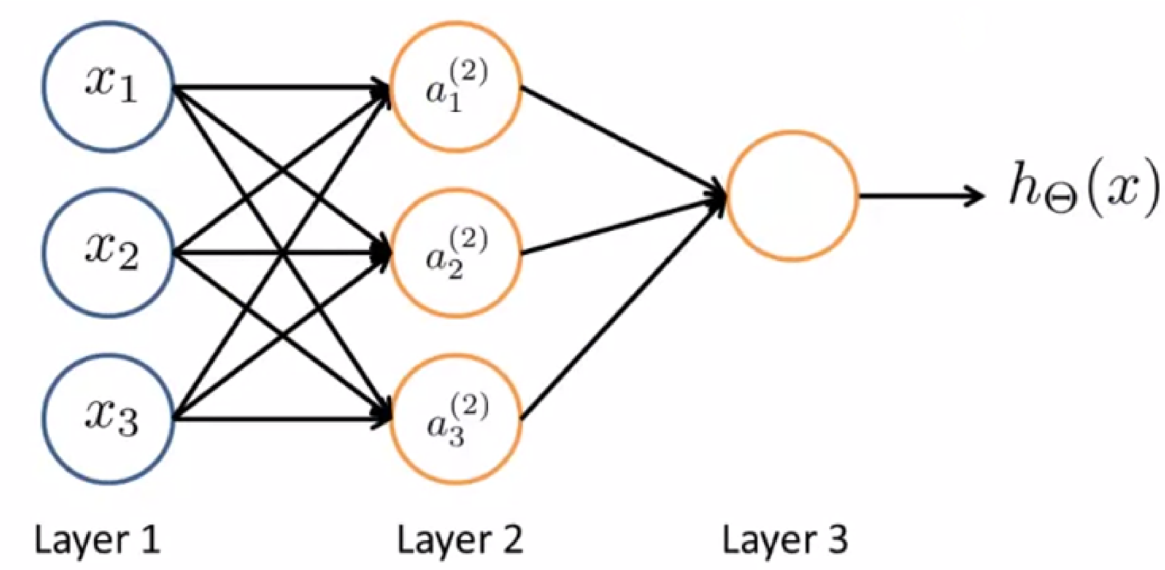
$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \text{(Weights) } \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

For Bias, we can have an extra node x_0 as input. The above represents an artificial neuron with a **Sigmoid (Logistic) Activation Function**.

$$g(z) = \frac{1}{1+e^{-Z}}$$

Hidden Layer

A neural network can have one or more hidden layers, and these layers can have bias such as $a_0^{(2)}$



Here,

- $a_i^{(j)}$ = "Activation" of unit i in layer j
- $\theta^{(j)}$ = Matrix of weights controlling function mapping from layer j to layer $(j+1)$

Computations on the above represented neural network:

$$\begin{aligned}
a_1^{(2)} &= g(\theta_{10}^{(1)}x_0 + \theta_{11}^{(1)}x_1 + \theta_{12}^{(1)}x_2 + \theta_{13}^{(1)}x_3) \\
a_2^{(2)} &= g(\theta_{20}^{(1)}x_0 + \theta_{21}^{(1)}x_1 + \theta_{22}^{(1)}x_2 + \theta_{23}^{(1)}x_3) \\
a_3^{(2)} &= g(\theta_{30}^{(1)}x_0 + \theta_{31}^{(1)}x_1 + \theta_{32}^{(1)}x_2 + \theta_{33}^{(1)}x_3) \\
h_\theta(x) = a_1^{(3)} &= g(\theta_{10}^{(2)}a_0^{(2)} + \theta_{11}^{(2)}a_0^{(2)} + \theta_{12}^{(2)}a_0^{(2)} + \theta_{13}^{(2)}a_0^{(2)})
\end{aligned}$$

If the network has s_j units in layer j, s_{j+1} units in layer $(j+1)$, then $\theta^{(j)}$ will be of dimension $s_{j+1} * (s_j + 1)$

Vectorized Implementation (Forward Propagation)

Let's assign:

$$\begin{aligned}
a_1^{(2)} &= g(\theta_{10}^{(1)}x_0 + \theta_{11}^{(1)}x_1 + \theta_{12}^{(1)}x_2 + \theta_{13}^{(1)}x_3) = g(z_1^{(2)}) \\
a_2^{(2)} &= g(\theta_{20}^{(1)}x_0 + \theta_{21}^{(1)}x_1 + \theta_{22}^{(1)}x_2 + \theta_{23}^{(1)}x_3) = g(z_2^{(2)}) \\
a_3^{(2)} &= g(\theta_{30}^{(1)}x_0 + \theta_{31}^{(1)}x_1 + \theta_{32}^{(1)}x_2 + \theta_{33}^{(1)}x_3) = g(z_3^{(2)}) \\
h_\theta(x) = a_1^{(3)} &= g(\theta_{10}^{(2)}a_0^{(2)} + \theta_{11}^{(2)}a_0^{(2)} + \theta_{12}^{(2)}a_0^{(2)} + \theta_{13}^{(2)}a_0^{(2)}) = g(z^{(3)})
\end{aligned}$$

Therefore, we now have:

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

Now we can vectorize the computation as:

$$z^{(2)} = \theta^{(1)}x = \theta^{(1)} \cdot a^{(1)}$$

$$a^{(2)} = g(z^{(2)})$$

For adding the bias in the hidden layer,

$$a_0^{(2)} = 1$$

$$z^{(3)} = \theta^{(2)} \cdot a^{(2)}$$

$$h_\theta(x) = a^{(3)} = g(z^{(3)})$$