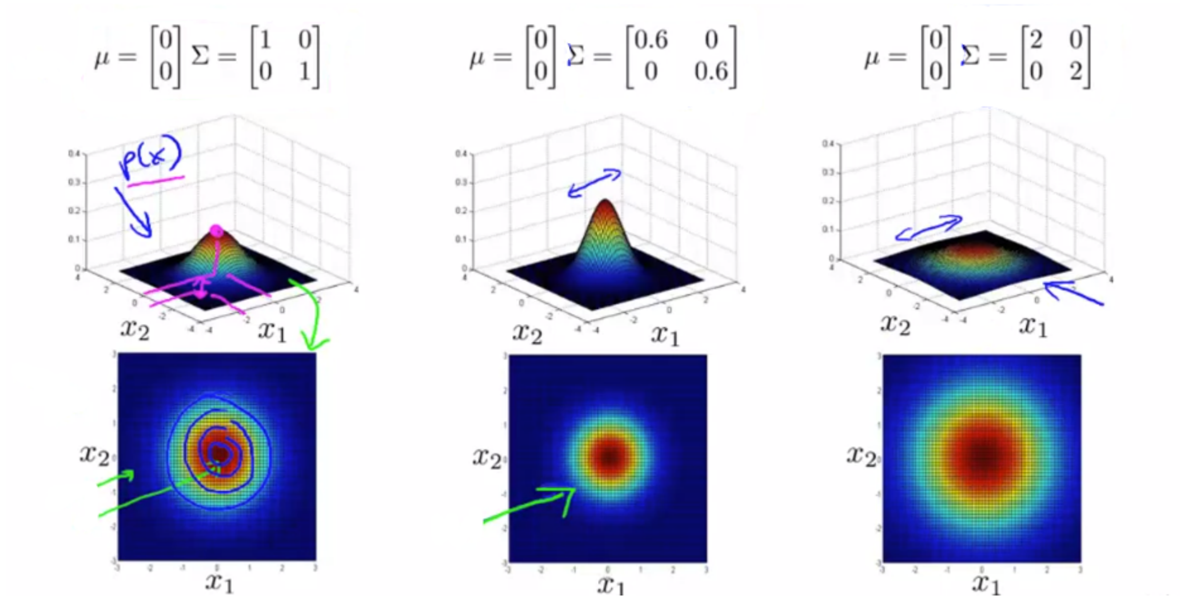
Multivariate Gaussian Distribution

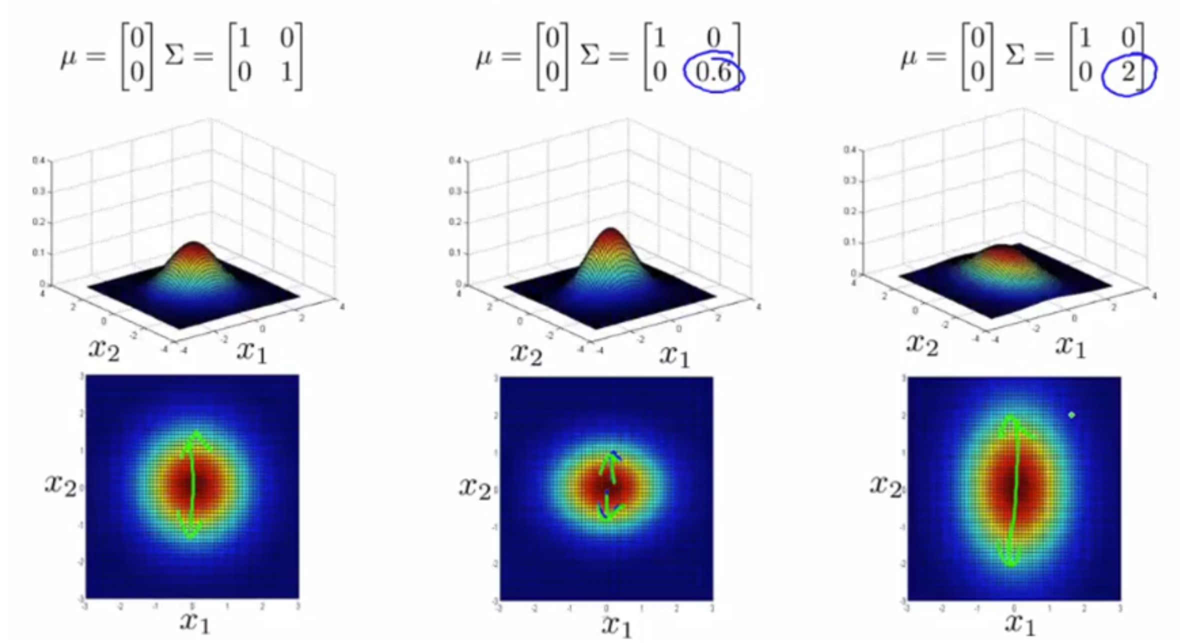
We have $x \in \mathbb{R}^n$. Now instead of modelling $p(x_1), p(x_2), \ldots, \text{etc}$ separately, model $p(x)$ all in one go.

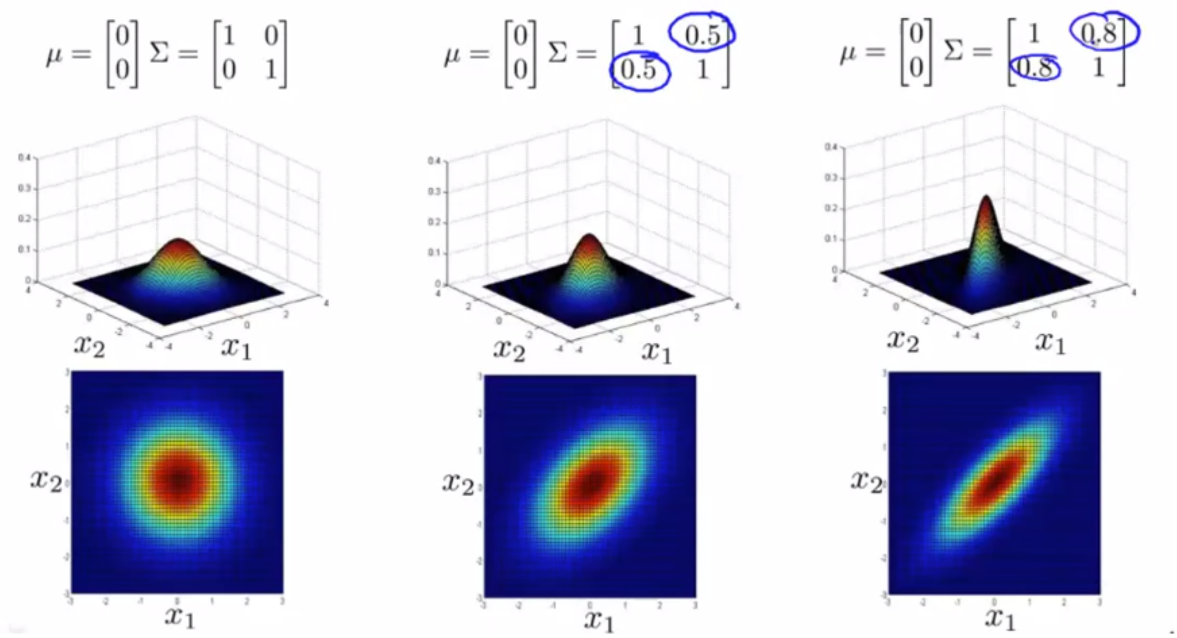
Parameters: $\mu \in \mathbb{R}^n, \sum \in \mathbb{R}^{n \times n}$ (covariance matrix)

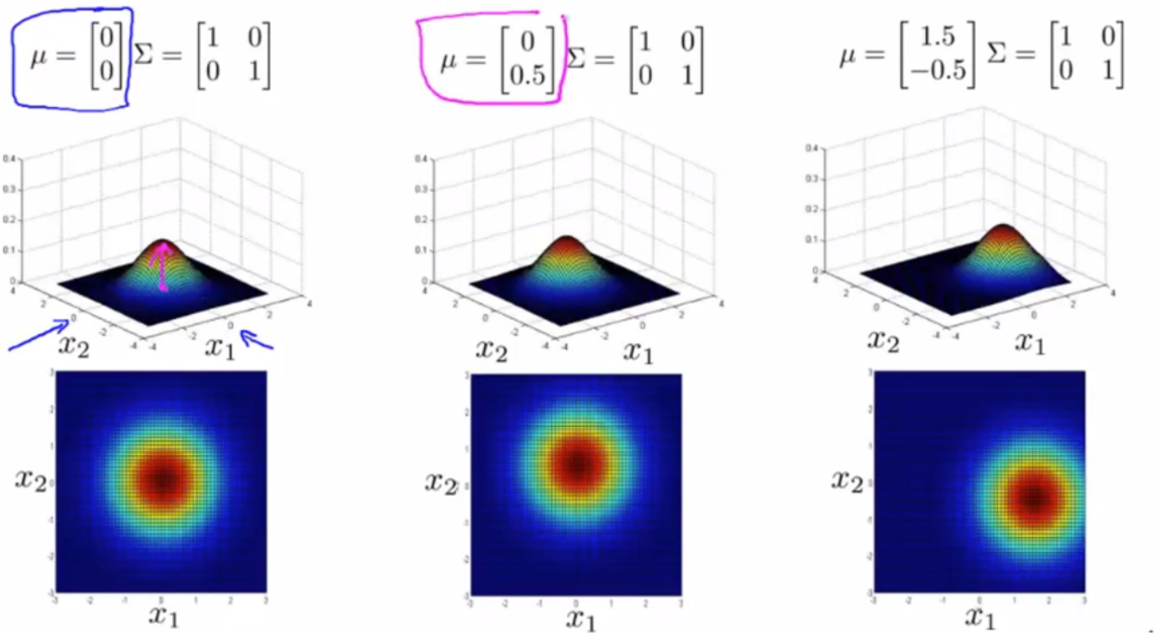
Formula for Multivariate Gaussian Distribution:

$$p(x; \mu, \Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} exp\Big(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x - \mu) \Big)$$







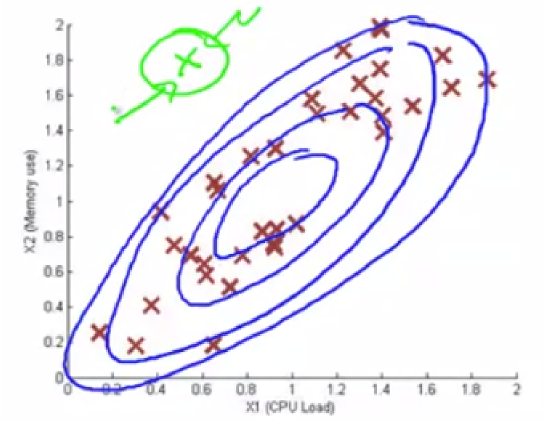


Parameter Fitting

Given training set $\{x^{(1)}, x^{(2)}, \ldots, x^{(n)}\}$

$$\mu = \frac{1}{m}\sum_{i=1}^{m} x^{(i)} \qquad \Sigma = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)} - \mu)(x^{(i)} - \mu)^T$$

Anomaly Detection with Multivariate Gaussian



1. Fit model $p(x)$ by setting μ and Σ

$$\mu = \frac{1}{m}\sum_{i=1}^{m} x^{(i)} \qquad \Sigma = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)} - \mu)(x^{(i)} - \mu)^T$$

2. Given a new example x, compute

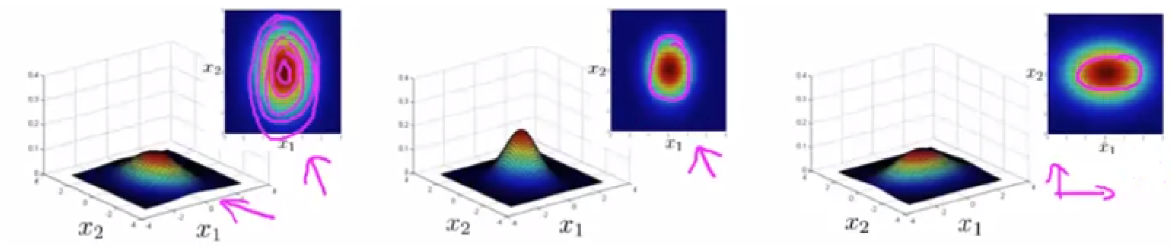
$$p(x) = \frac{1}{(2\pi)^{\frac{n}{2}}\,|\Sigma|^{\frac{1}{2}}} exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right)$$

Flag an anomaly if $p(x) < \epsilon$

Relation to Original Model

We have the original model as:

$$p(x) = p(x_1; \mu_1, \sigma_1^2) \times p(x_2; \mu_2, \sigma_2^2) \times \cdots \times p(x_n; \mu_n, \sigma_n^2)$$



The original model corresponds to multivariate Gaussian, where the contours of the Gaussian are always axis alligned. Which means we will have our co-variance matrix like:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3^2 & \vdots & 0 \\ 0 & 0 & 0 & \dots & \sigma_n^2 \end{bmatrix}$$

Original Model	Multivariate Gaussian
Manually create features to capture anomalies where x_1, x_2 take unusual combination of values	Automatically captures correlations between features
Computationally cheaper (scales better to large values of n)	Computationally more expensive due to computation of Σ^{-1}
Works okay even if m (Training set size) is small	Must have $m > n$, or else Σ is non-invertible