Logistic Regression Model

Cost Function

For linear regression, we had our cost function as:

$$J(\theta) = \frac{1}{m} \cdot \sum_{i=1}^{m} \frac{1}{2} \cdot (h_\theta(x^{(i)}) - y^{(i)})^2$$

We'll use an alternateway of writing the cost function. Instead of using the squared error term as above, we'll use:

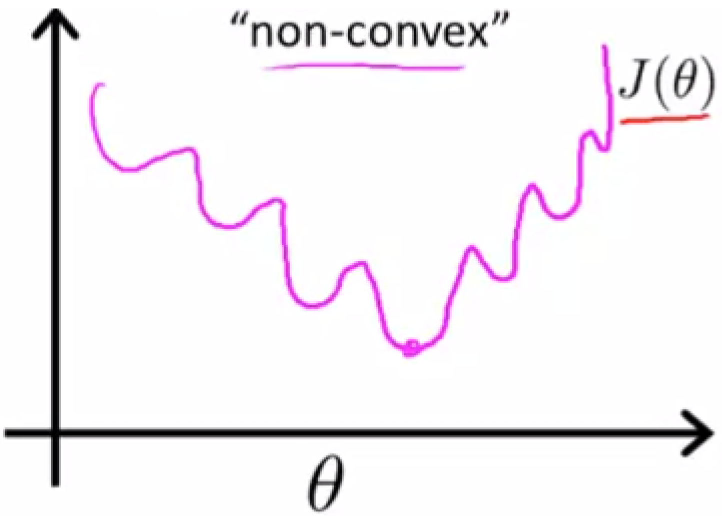
$$J(\theta) = \frac{1}{m} \cdot \sum_{i=1}^{m} cost(h_\theta(x^{(i)}), y^{(i)})$$

$$=> cost(h_\theta(x), y) = \frac{1}{2}(h_\theta(x) - y)^2$$

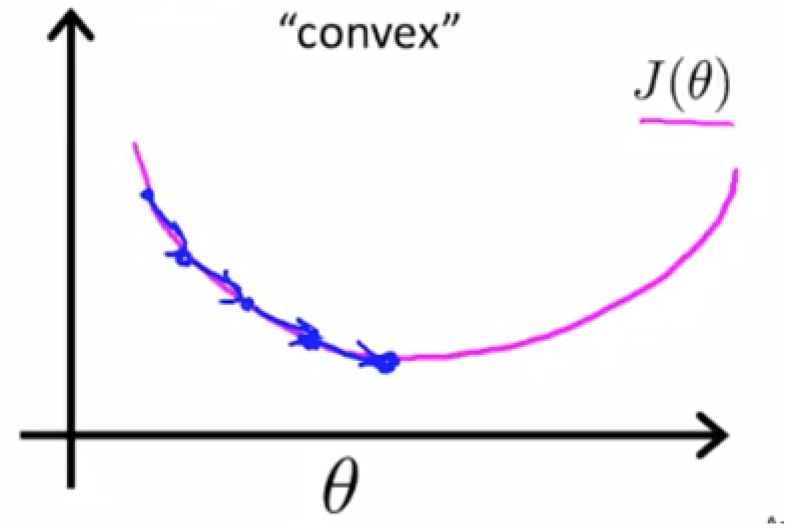
If we plot this function for logistic regression, when

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

We'll get a plot of non-convex function like this:

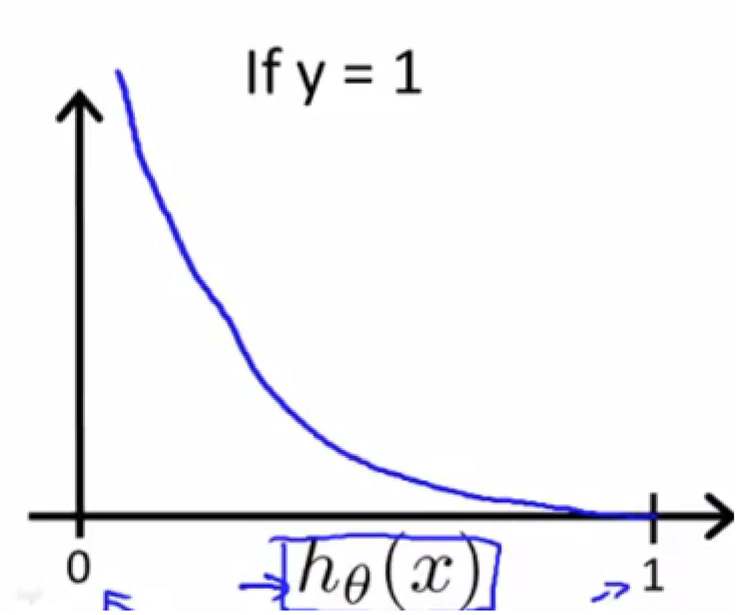


This function will have many local optima, and we cannot run gradient descent on this sort of function. Instead, we want a convex function which will guarantee that gradient descent will converge on the global minimum.



We'll therefore use,

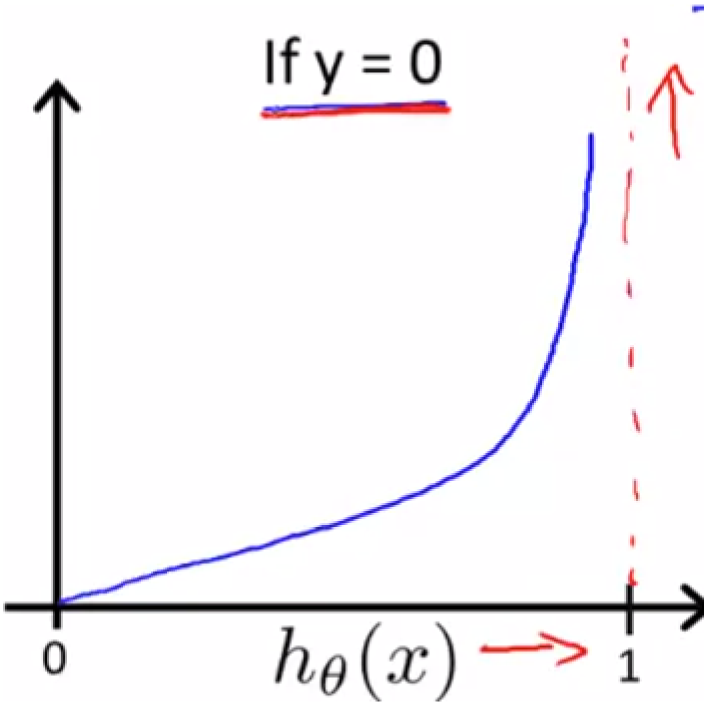
$$cost(h_\theta(x), y) = \begin{cases} -log(h_\theta(x)) & \text{if } y = 1 \\ -log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



If $y = 1$ and $h_\theta(x) = 1$, then $cost = 0$

But as $h_\theta(x) \to 0$, $cost \to \infty$

It captures intuition that if $h_\theta(x) = 0$, [predict $P(y = 1|x; \theta) = 0$], but $y = 1$, we'll penalize the learning algorithm by a very large cost. It's like the probability of a patient to have a malignant tumor is 0, even though the value of $y = 1$.



Therefore,

$$cost(h_\theta(x), y) = \begin{cases} 0 & \text{if } h_\theta(x) = y \\ \infty & \text{if } y = 0 \text{ and } h_\theta(x) \to 1 \\ \infty & \text{if } y = 0 \text{ and } h_\theta(x) \to 0 \end{cases}$$

> In logistic regression, the cost function for our hypothesis outputting (predicting) $h_\theta(x)$ on a training example that has label $y \epsilon \{0, 1\}$ is:
>
> $$cost(h_\theta(x), y) = \begin{cases} -log(h_\theta(x)) & \text{if } y = 1 \\ -log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$
>
> Which of the following are true? Check all that apply.
>
> - [x] If $h_\theta(x) = y$, then $cost(h_\theta(x), y) = 0$ (for $y = 0$ and $y = 1$)
> - [x] If $y = 0$, then $cost(h_\theta(x), y) \to \infty$ as $h_\theta(x) \to 1$
> - [ ] If $y = 0$, then $cost(h_\theta(x), y) \to \infty$ as $h_\theta(x) \to 0$
> - [x] Regardless of whether $y = 0$ or $y = 1$, if $h_\theta(x) = 0.5$, then $cost(h_\theta(x), y) > 0$