Optimisation of Solar Photovoltaic Technologies on a Domestic Property

Group Number 9
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Abstract

Optimisation of solar technologies on a domestic property was undertaken to minimise the time until return on investment (ROI). Technologies explored were both building integrated photovoltaics (BIPVs) and traditional photovoltaic solar modules. Subsystem 1 used both Genetic Algorithm and Particle Swarm to optimise the configuration of semi-transparent BIPV panels in the windows of one room in the house. A payback time of 22 years and 250 kWh was achieved with the optimal configuration of panel, while remaining within the defined constraints for light levels, energy produced and cost. Subsystem 2 optimised PV module resistance values on both the roof and wall façade of the building, taking into account the obstacles and shading present. Then the most efficient values were selected with both Genetic Algorithm, Particle Swarm and a multi-objective Genetic Algorithm were tested. After parameters were tuned a total of 3504 kWh/year could be achieved. As this was surplus of the required amount, subsystem 2 was used to supplement the use of BIPVs in subsystem 1. Together both subsystems were combined to calculate an overall solution. The outcomes were 2975 kWh of energy generated a year, a total cost of £8000 and a payback time of 7 years.

Figure 1 domestic property Subsystem 1 (blue) and Subsystem 2 (red)

1. Introduction

Photovoltaic solar technologies provide a potential solution for low carbon energy generation. Conventional solar panels require a lot of space to be cost effective. The optimal use of novel building integrated photovoltaics (BIPVs) could overcome this issue by retrofitting semi-transparent into pre-existing windows in combination with optimised conventional PV panels on the roof and wall façade. The main objective of the system optimisation is to produce enough energy to cover the electricity consumption of the property while keeping installation costs affordable and minimising the number of panels used. Other case studies have explored optimised solar and wind renewable energy solutions (1), we plan to extend this approach to the domestic property.

2. Subsystem 1 – Window Integrated Photovoltaics

Subsystem 1 focused on the use of Building Integrated Photovoltaics (BIPVs) which could be implemented in the pre-existing window frames of the domestic property. BIPV technology is fairly new and has the potential to revolutionise the use of solar technology in buildings without sufficient accessible wall and roof space required for traditional PVs. The BIPVs panels came in a range of power ratings, with varying transparencies these can be found in **APPENDIX**.

The main objective of subsystem 1 was to find the best combination of panels in 5 windows belonging to one room that minimises the number of years it takes to achieve a return on investment (ROI) for the installation. The main interests of the subsystem were which windows were most effective and what power rating satisfied the requirements and had and optimum solution.

2.1. Optimisation formulation

$$\min_{\mathbf{x}} \quad f(x) = \frac{C(x)}{P(x)} \tag{2.1.0}$$

where $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5) \in x \in \{0, 0.042, 0.061, 0.087, 0.104\}$ (2.1.1)

subject to

$$g_1(x):\ 6 - L(x) \le 0 \tag{2.1.2}$$

$$g_2(x):\ C(x) - 4000 \le 0 \tag{2.1.3}$$

$$g_3(x):\ 100 - E(x) \le 0 \tag{2.1.4}$$

$$C(x) = \sum_{i=1}^{5} c_i(x_i) \tag{2.1.5}$$

where

$$c_i(x_i) = \begin{cases} 600\left(\frac{A_p}{A_i}\right) + 500 & if\ x_i > 0 \\ 0 & if\ x_i = 0 \end{cases} \tag{2.1.6}$$

$$E(x) = \sum_{i=1}^{5} I_i \times x_i \times A_i \quad 2.1.7$$

$$P(x) = E(x) \times FIT \quad 2.1.8$$

$$L(x) = \sum_{j=1}^{24} Q_j\,(lx_j) \quad 2.1.9$$

where $$Q_j(lx_j) = \begin{cases} 1 & if\ lx_j > 500 \\ 0 & if\ lx_j \leq 500 \end{cases} \quad 2.1.10$$

where $$lx_j = \sum_{k=1}^{5} \frac{lm_k}{A_w \times d_k^2} \quad 2.1.11$$

where $$lm_k = I \times t(x) \times A_w \quad 2.1.12$$

where $$I = \begin{cases} I_{w,h} & if\ k > 3 \\ I_{r,h} & if\ k \leq 2 \end{cases} \quad 2.1.13$$

and $$t(x) = 38.395x_k^2 - 13.6x_k + 1.0007 \quad 2.1.14$$

2.2. Modelling approach

The x variables corresponded to the power ratings (kWh) of the panels installed in windows 1-5. These values are obtained from the material specification sheet provided by manufacturers(2) .

The objective function $f(x)$ (equation 3.1.0) calculates the number of years that it takes for a return on investment of the configuration and is the ratio of upfront cost $C(x)$ (equation 3.1.5) over the yearly payback $P(x)$ (equation 3.1.8).

$C(x)$ (equation 3.1.5) was calculated by the sum of the number of panels used for that configuration of windows. Each panel cost £400 (2) with an additional installation cost of £500 per window for installation(3). An x value of 0 implied that the window remained unchanged so incurred no costs.

$P(x)$ (equation 3.1.8) was calculated by the product of yearly energy generated by the array configuration, $E(x)$ (equation 3.1.7) and the feed in tariff (FIT) rate (4). $E(x)$ was calculated using the yearly solar irradiance values on both the wall façade and roof area as well as the power ratings of the panels taken from the product specification sheet (2). These were acquired from a PV SOL simulation that took into account the geometry, shading and geographical location of the property.

$g_1(x)$ (equation 3.1.2) states that the number of hours of 'quality' light must be greater than 6. The 'quality' of light was determined by being greater than standard office lighting conditions of *500 lux* (5). The irradiance values selected for this constraint was an average day in January, obtained from PV SOL. The Light levels in Lux falling on the work area for each hour of the day was calculated. This is calculated by assuming the windows as point sources, with the illuminance of each source taken as a product of irradiance, transparency and panel area. Transparency ($t(x)$, equation 3.1.14) was determined using a linear regression model using transparency and power data from the specification sheet (2). The minimum number of hours required were based on the subjective design requirements determined by communicating with our 'client'.

$g_2(x)$ (equation 3.1.3) states that cost of this subsystem must be less than £4000**.** This value was taken as a maximum cost due to the subjective design requirements of the project put forward by our 'client'. This was calculated by the cost function $C(x)$ described earlier.

$g_2(x)$ (equation 3.1.4) states that the yearly energy generated by the subsystem must be greater than 100 kWh. This constraint is in place to ensure that the solution does produce some energy.

2.3. Explore the problem space

In order to understand the problem better a full factorial experiment was conducted. For every combination of x the $C(x)$, $E(x)$ and $L(x)$ was plotted against the objective function $f(x)$ (figure 1).

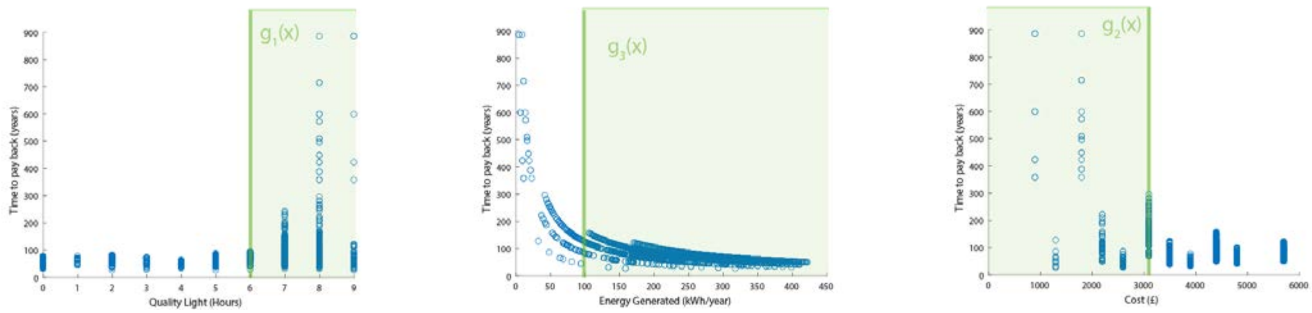


Figure 2 L(x), E(x) and C(x) with constraints from left to right

Upon visual inspection of the range space it was noted that due to the complex and discrete nature of the problem space, no clear gradients were found in light quality and cost. There did seem to be some exponential relationships with energy and the objective function, however the complexity and layers of values implied that it would be difficult to solve with gradient based solutions.

Constraints $g_1(x)$ and $g_2(x)$ appeared active wrt cost and light quality as they appeared to eliminate possible minima from the range space. Constraint $g_3(x)$ seemed inactive wrt Energy production as it did not eliminate any optimum solutions. However, an optimiser would be needed to test this in more detail. Each constraint was removed independently and tested with the Genetic Algorithm (GA) solver. As predicted, removing $g_1(x)$ changed the optimum solution. When $g_1(x)$ was removed the optimisers tended to full power and zero transparency as light requirements were not constricting the model. When $g_2(x)$ was removed, the optimum solution did not change dramatically. As predicted removing $g_3(x)$ had no effect on the solution. Multiple runs and only significant changes of the optimum were used to test for constraint activity. This was because of the stochastic nature of the solver, producing slightly different answers each time. Therefore, to simplify the formulations, constraint $g_2(x)$ and $g_3(x)$ can be removed from the formulation.

2.4. Optimisation

Optimise problems can be solved using gradient-based or heuristic optimiser algorithms. Gradient based methods must be continuous, convex and differentiable (6). The initial exploration of the problem space showed that this problem has none of these features. A literature study of similar projects have shown that Genetic Algorithm (GA) and Particle Swarm (PS) optimisers have been shown to be effective at solving similar problems(1).

The optimisers were set up by modelling the discrete variables as continuous and applying the upper and lower bounds of the variables described in equation 3.1.0. For GA, the initial conditions were varied and tested however it was found not to have a significant effect on the solution. The results of each optimiser are shown in Table 1. Figure 2 shows the both the solution panel configurations and the explored range space of each solver. It should be noted at this point that both solvers are stochastic (rely a degree of randomness) so it is unlikely to produce exactly repeatable results.

Solver	Time to solve (s)	x_1 (kWh/m²)	x_2 (kWh/m²)	x_3 (kWh/m²)	x_4 (kWh/m²)	x_5 (kWh/m²)	$f(x)$ Years	$C(x)$ Cost (£)	$E(x)$ Energy (kWh)	$L(x)$ Light (hrs)
Genetic Algorithm	7.094	0	0.104	0	0	0.104	**22.581**	2600	298.3	6
Particle Swarm	0.76804	0	0.104	0	0	0	**21.192**	1300	158.92	6

Table 1 Optimisers and Optimum of GA and PS

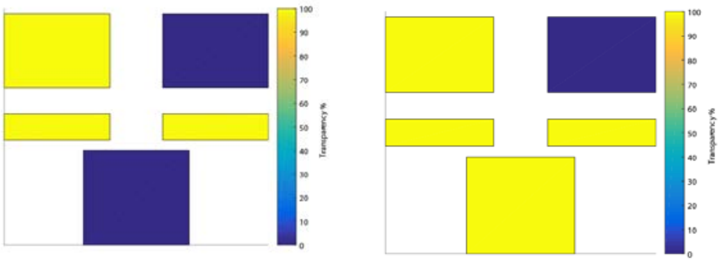
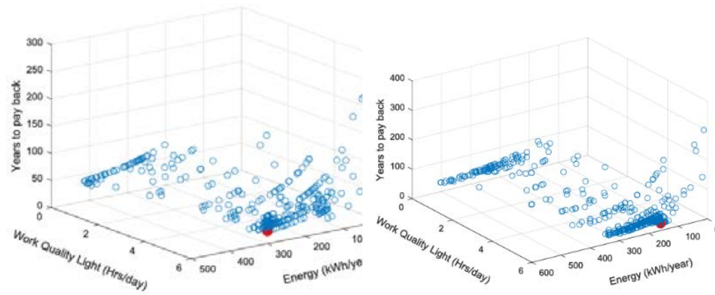


Figure 3 Optimal Panel configuration of GA and PS (left to right)



Optimiser explored Range Space (left to right) Solution in red

Post optimal sensitivity analysis was conducted and the results shown in Table 2. This showed that the variables that had the greatest effect on the output function were x_1 and x_2. Despite the optimisers provided containing many 0 values. For the sake of the sensitivity analysis the x values were set to 0.104 so that the affect of each could be seen.

Variable	x_1	x_2	x_3	x_4	x_5
% Change in $f(x)$ with -10% perturbation	3.0630	3.0630	0.7277	0.7277	2.6756

Table 2 Post Optimal Sensitivity Analysis

2.5. Discussion

Both Genetic Algorithm (GA) and Particle Swarm (PS) Optimisers were conducted in order to determine the combination of windows that satisfied the required constraints and achieved an optimal number of years for ROI. Both solvers provide acceptable optimal solutions of 22.6 and 21.2 Years respectively. Although both methods are stochastic, it was found that PS was far more consistent and repeatable than GA. In addition, the GA took significantly longer to solve compared to Particle Swarm. Often on first trial the GA no feasible answers were found, leading to even more prolonged run time to repeat the solver.

The biggest challenge of the subsystem was its complexity. It consisted of many complex models that were each based on many assumptions and due to the number of parallel calculations being made, each model had to be simplified. This at times over simplified the entire model, diminishing a level of validity of the overall subsystem. This study has informed decision making for the retrofit installation of BIPVs in the windows of a room in a domestic property. It can be scaled to other rooms in the house or in fact to other similar buildings. The model only concentrated on five windows on the property as these were connected to one room, hence making the light calculations simpler. However, in order to scale the model to include all the windows in the house which might have multiple sources of natural light, a more detailed simulation or dataset for light level sensitivity should be used.

3. Subsystem 2 – Wall and Roof Photovoltaic Modules

3.1. Subsystem 2 – Wall and Roof Photovoltaic Modules

Subsystem 2 was about the optimizing of the best coordinate positions on the roof and facade wall of a residential home to place solar panel modules. These solar modules were of a standard size and are connected via the standard and most efficient circuit configuration. This was (for a set of eight single diode solar cells within the module) two sets of four cells connected in series and this pair then connected in parallel – making a 4x2 cell matrix per module. These sets of settings and parameters were evaluated using a computer simulation and 3D model software called PV*SOL(), which takes current market values and the analysis of the building's architecture and geographical location. The development of this subsystem involved a detailed and complex process of understanding the design space to explore the impact of variables and how they may implement the maximum energy generation achievable for a given solar module.

$$\min_{\mathbf{x}} \quad Es(x) = 124.7 - 0.03685Irr(x) \quad - 1.043R(x)^2 - 4.239R(x) \quad Irr(x) \quad - 0.4944Irr(x)^2$$

where $\mathbf{x} = (x_6, x_{7,}x_8, x_{9,}x_{10}, x_{11,}x_{12})$

$$\mathbf{Irr} = (Irr_1, Irr_1 \ldots Irr_{52}) \in x \in \mathbb{R}$$
$$\mathbf{R} = (R_1, R_2 \ldots R_{52}) \in x \in \mathbb{R}$$

subject to
$$g_1(R, Irr): \frac{0.7036}{R \times Irr} - e(R, Irr) \leq 0$$
$$g_2(x): R - 0.650 \leq 0$$
$$g_3(x): 800 - Irr \leq 0$$
$$g_4(x): E(x) - 2800 \leq 0$$

The objective function above is expressed in terms of the average yearly irradiance incident on the module at its position inclusive of the effect of shading, *I,* and the optimal resistance, *R,* value required to meet a maximum power point voltage (obtained through simulation of a MATLAB simulation). The aim for this subsystem is to find the optimal energy that each solar module can generate in its given position whilst considering the shading effect on irradiation. For example, if we have an irradiance value for a given solar module position on the roof or wall, we could use the objective function to find what its maximum energy generation can be if it is tuned to a resistance that allows the module to reach a standard maximum power point voltage. Each solar module can be tuned by a variable resistor. The model should explore the most energy dense and efficient areas; such that as few modules can be used as possible. This will intern ensure for a lower upfront cost, payback time and surface area. The assumptions and parameter of the objective are that the weather conditions are neglected but the PV*SOL *()* software based on the geographical location of the property was to model the climate on the seasonal data from the previous year. This would mean that uncertainty enters the model produced due to the need uncertainty of disturbances to solar irradiance throughout the year or maintenance of the solar modules installed. The modules are installed at the best tilt angle ensuring for standardised and efficient solar capture. All solar modules are of the same surface area = 0.9012m^2. Any heat energy loss from currents are neglected. The consumer demand energy is in kWh and includes the required energy for the self-storage of energy for the battery that supports the inverter for the solar module system. Further to note is the area of the roof and wall facade. *The main variables for the system were determined through the experimental simulation data and used for the meta model fitness functions. The remaining variables used can be found in the Appendix nomenclature of this report.* The resistance is calculated as below through the MATLAB simulation, this is based on the series-parallel resistor configuration within the cell (7).

$$Rmod = \frac{Rcell}{2} \qquad\qquad R = \frac{x_7}{x_6}$$

The consumer demand energy = 3000 kWh. The design and environmental obstructions on the roof and wall, such as the windows are a key design constraint and these relative positional coordinates for solar module placement must be deducted from the available data point achievable. This was done through a variable called the obstruction factor. This variable is between 0 and 100 and reflects the severity of the obstruction. The second key constraint that was added through the derivation of theory and was the energy generating efficiency of each solar module. This was calculated via the following:

$$e \quad = x_6 x_7 / I(x_8, x_9, x_{10}) R(x_6, x_7)$$

3.2. Modelling

The area of the roof and wall was divided into area elements that represent a solar module. The area of each solar module was used to populate a grid of centre points that represent the centre point position of each corresponding solar module. Each of these positions had a related value of shading frequency. The variable shading frequency represents the average percentage of shading that a solar module will get throughout the year. The average yearly values for variables for this were obtained from the PV*SOL model. This data was then recorded. As shown in the graphical representation, figure 1. The model of the roof and wall is looked at on a 2D basis. The shading frequency negatively effects the irradiance upon each module area

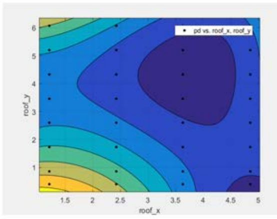


Figure 1

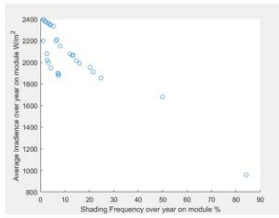


Figure 2

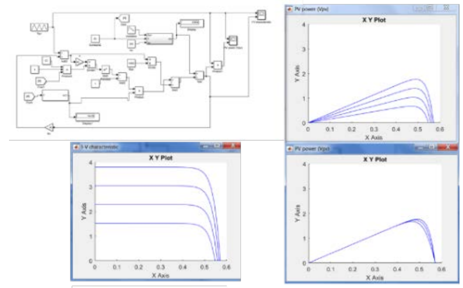
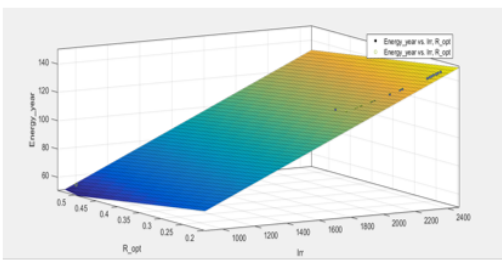
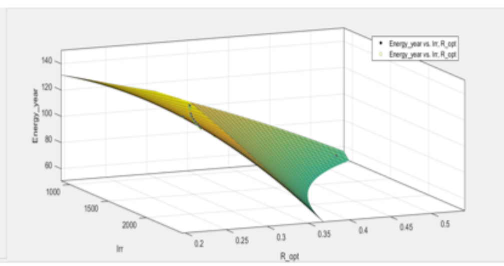


Figure 3,4,5,6

Given a set of solar modules if one module is reaching a higher voltage than another, if the lower voltage is below a minimum then the associated solar module may not generate enough electricity and turn itself off as well as the modules around even if they are reaching higher voltages. If each module voltage generation is maximized via the tuning of a variable resistor then it is possible to ensure that even if the power generated by a solar module is low relative to a neighbor, the energy will still be generated and maximized to its full potential. Design of experiments was conducted in two layers for this optimization subsystem network model. A one diode model acting as a black box was built from the available toolbox blocks from the MATLAB/Simulink library. This was modelled against a modified version of a simple current solar cell model (7). The final objective function is a result of curve fitting linear regression to a polynomial degree.





Linear model Polynomial model 1:
Goodness of fit:
SSE: 157.7
R-square: 0.29854
Adjusted R-square: 0.59854

Linear model Polynomial model 2:
Goodness of fit:
SSE: 0.004128
R-square: 0.98564
Adjusted R-square: 0.98564

3.3. Explore Problem Space

To produce a realistic model for a solar module that allows the maximisation of energy in terms of irradiance (considering the shading effect) and resistance, it was necessary to determine the most influential parameters. Sensitivity analysis was conducted by looking at correlations and relationships between variables.

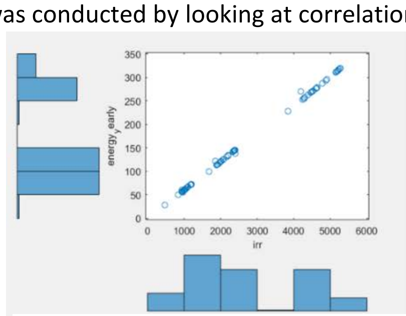


Figure 8: Energy vs Irradiance relative to shading

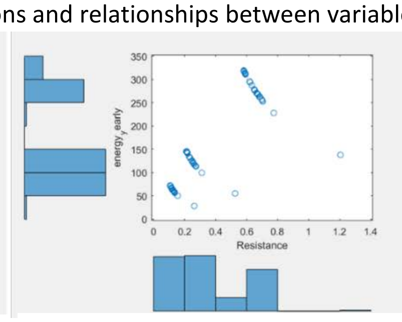


Figure 9: Energy vs Resistance relative to shading

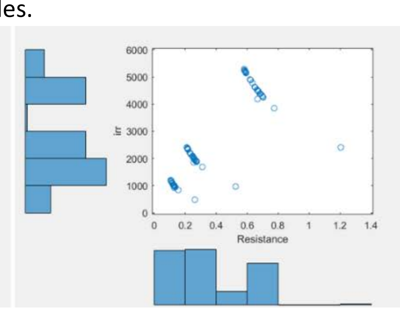


Figure 10: Irradiance vs Resistance relative to shadina

For the final model after training and testing the resistance variable was scaled for units purposes. The obtained results of shading frequency, irradiance, module positioning and general module circuit configuration data was recorded which act as the input variables. The data obtained from experiments conducted using a MATLAB SIMULINK model of a single diode solar cell to be connected in the relative series-parallel 4x2 solar module array act as initial element points later and test data. The aim of the simulation is to maximise every solar cell and module configuration and generate the module resistance, then the irradiance and resistance for modules and from the model created could be used to get the maximum energy generated by each module on the building. When testing the constraint activity applied, the obstruction constraint was not active if applied after the PSO and it limited the number of possible modules in the range space. When applying an efficiency above 10 or below 4 the constraint was not active. An average efficiency for a solar module in between 6 and 12 percent so the constraint was applied at 6- actively eliminating 4 modules. The direction of the objective function changes in local areas but in general the relationships between energy and irradiance is positively linear monotonic and the between energy and resistance it is non monotonic.

3.4. Optimisation Analysis and Discussion

Both gradient decent and non-gradient (Particle Swarm and Genetic Algorithm) methods of solving for the optimisation of the model were tested. However, the gradient methods did not give reproducible or realistic results because local minima where discovered at which seemed random points. This was because when the constraints where applied, a minimum was found between locals of a discrete variable. Finding the minima instead needed to be understood by a non-gradient population-based method. Both Particle swarm optimization and genetic algorithm was applied to the model. The genetic algorithm method centred around a discrete population method. However, there is a continuous nature between the irradiance and energy output variable. The tests of this method showed no change in energy between certain boundaries within the model. The only way to find a realistic method using a genetic algorithm was to open up the lower and upper bounds of the irradiance variable to values as high as 4000. For the module positions and previous work completed on the relationship between irradiance and shading frequency for the building in question, these values were unrealistic. The final, quickest and more suitable method for optimising the objective function and model behaviour was Particle Swarm optimisation (PSO). PSO explores a optimisation problem through a population based method, were in this case we can model a particle in the swarm as one of the possible modules on the roof or wall. The PSO code produced for this optimisation problem reaches the minima via a continuous nature in that each particle in given initial vector positions and velocities which are updated to reach the better point in the population by the comparison of each particle to its neighbour relative to the objective function.

The design obstruction constraint is applied before the PSO is conducted to search the test data for the initial positions, directions and velocities of the particles. The efficiency constraint is applied within the objective function in that is passed into the PSO run code. This constraint is achieved through a for loop penalty being applied, such that efficiencies of modules below 6% are removed from the population solution space. The PSO finds the module number that can achieve the most maximum power point generated energy. The PSO then searches through the particles and orders then from highest to lowest in terms of their energy generated. This descending energy output is the cumulatively summed up until the reach energy required by the subsystem to reach is met.

The benefit of using PSO over GA is firstly, that only the best particle gives out information to the other particles. Moreover, PSO allows for effectively two sets of populations (the pbest and the current position) which allows for more exploration and diversity within the constraints. For this particular design optimisation problem, the GA test failed mainly due to the elimination early of underachieving elements. However, the output of this algorithm requires the optimisation problem to find the maximum energy that can be generated by a module with a relative irradiance, whilst considering all other modules and placing them in a descending order from highest achievable energy generation (with associated tuned resistance) to the lowest. This means that there is a need to retain information of many possible elements of the range space such that the optimums may be found for the 52 possible area space positions on the roof and wall. The PSO convergence behaviour was plotted to determine the how many iterations the algorithm required. The fitness function converges from -162 kWh to -187kWh to the nearest kWh. To view a possible, the trade-off within this subsystem of the total payback time of the system for different module configurations/layouts vs the total surface area used up for these configurations a simple genetic algorithm multi-objective experiment of the test data was executed and the following pareto front points achieved, where objective 1 is total surface area and objective 2 is payback time.

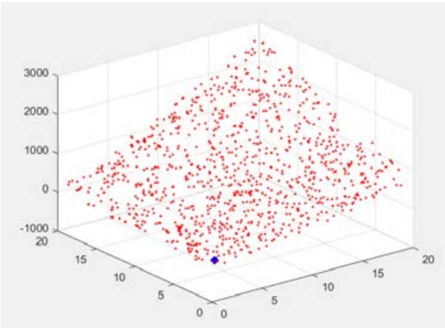


Figure 10: no constraint PSO

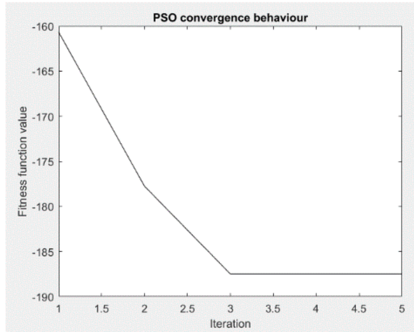


Figure 11: PSO convergence

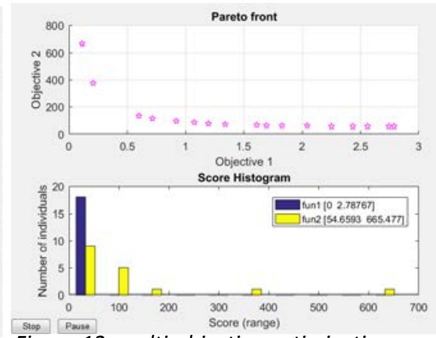


Figure 12: multi-objective optimization, surface area and payback time

4. System-level optimisation

The system level optimisation took both subsystems into account. First subsystem 1 ran to obtain an optimal solution for the windows. The energy output of this solution was used as an input value to subsystem 2. Subsystem 2 then calculated an optimum solution for the wall and roof, given the energy it needed in order to reach the total energy required to cover the usage of the house. Once both solutions are obtained they combined to find the total number of years to reach a ROI.

4.1. System level formulation

$$\min_{\mathbf{x}} \quad f(x, Irr, R) = \frac{Cost(x,Irr,R)}{Payback(x,Irr,R)} \qquad 4.1.0$$

where $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5) \in \mathcal{x} \in \{0, 0.042, 0.061, 0.087, 0.104\}$ 4.1.1

and $\mathbf{Irr} = (Irr_1, Irr_1 \ldots Irr_{52}) \in \mathcal{x} \in \mathbb{R}$ 4.1.1

and $\mathbf{R} = (R_1, R_2 \ldots R_{52}) \in \mathcal{x} \in \mathbb{R}$ 4.1.1

subject to

$h_1(x)\text{: } Energy(x) - 3000 = 0$ 4.1.2

$g_1(x)\text{: } Cost(x) - 10000 \leq 0$ 4.1.3

$g_2(x)\text{: } 6 - L(x) \leq 0$ 2.1.2

$g_3(x)\text{: } C(x) - 4000 \leq 0$ 2.1.3

$g_4(x)\text{: } 100 - E(x) \leq 0$ 2.1.4

$g_5(R, Irr)\text{: } \frac{0.7036}{R \times Irr} - e(R, Irr) \leq 0$ 3.1.?

$g_6(x)\text{: } R - 0.650 \leq 0$ 3.1.?

$g_7(x)\text{: } 800 - Irr \leq 0$ 3.1.?

$g_8(x)\text{: } E(x) - 2800 \leq 0$ 3.1.?

$Cost(x, Irr, R) = C(x) + Cs(Irr, R)$ 4.1.4

$where\ C(x)\ and\ Cs(Irr, R)\ are\ defined\ in\ relevant\ subsystems$

$Payback(x, Irr, R) = P(x) + Ps(Irr, R)$ 4.1.5

$where\ P(x)\ and\ PS(Irr, R)\ are\ defined\ in\ relevant\ subsystems$

The total amount of energy production for the whole system $g_1(x)$ was selected by looking at the average energy consumption for a property that size and energy performance band (8). The overall cost of the whole system, ($g_2(x)$) was set as less than £10,000 due to this being a subjective design constraint of our 'client'.

When the whole system was run, a total of 2950 kWh was produced by the system, costing £8842 and taking 7.75 years to achieve ROI. All constraints were satisfied by solution

5. Conclusion

The optimisation of the best configuration of solar windows to solar panel modules has involved complex meta modelling and data exploring. The overall, outcome of our system provide the optimal configuration of solar windows, the combinations relative opacity and for this being priority the best configuration of module numbers and their relative positions. In the future the modelling of these technologies may be able to be applied to the neighbouring buildings to help make more residential properties more off the national grid.

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Appendix A. Nomenclature

Define all symbols that you use, particularly for the mathematical model development. Make sure you use a consistent nomenclature and set of symbols in subsystems. (It may also be convenient to divide the symbols list to subsystems.)

Variables	Definition	Units	Value
x	Power value of panel (1-5)	kW/m^2	-
A_1	Area of window 1	m^2	1.44
A_2	Area of window 2	m^2	1.44
A_3	Area of window 3	m^2	0.5
A_4	Area of window 4	m^2	0.5
A_5	Area of window 5	m^2	1.8
A_w	Area of working space	m^2	9
I_1	Yearly Irradiance value on window 1	kWh/m^2	1061.16
I_2	Yearly Irradiance value on window 2	kWh/m^2	1061.16
I_3	Yearly Irradiance value on window 3	kWh/m^2	744.54
I_4	Yearly Irradiance value on window 4	kWh/m^2	744.54
I_5	Yearly Irradiance value on window 5	kWh/m^2	744.54
FIT	Feed in tariff	$£/kWh$	0.386
$I_{w,h}$	Irradiance values for wall for each hour of the day	kWh/m^2	-
$I_{r,h}$	Irradiance values for roof for each hour of the day	kWh/m^2	-
Q_i	Quality of light	-	{0,1}
lx	Lux value	lx	-
d_1	Distance from window 1 to working space	m	0.4
d_2	Distance from window 2 to working space	m	1.5
d_3	Distance from window 3 to working space	m	1.5
d_4	Distance from window 4 to working space	m	2
d_5	Distance from window 5 to working space	m	2
Imax	Maximum current for a given power rating	A	-
Vmax	Maximum voltage for a given power rating	V	-
Xpos	X position of panel	m	-
Ypos	P position of panel	m	-
SA	Surface area of panel array	m^2	-
Sfreq	Shading frequency of panel	$\%$	-
OF	Obstruction factor	$\%$	-
Irr	Variables of Irradiance values for each module (1-52)	kWh/m^2	-
R	Variable of resistance value for each module (1-52)	Ω	-
Functions	Definition	*Units*	Value
$f(x)$	Window objective function, the number of years until ROI	*Years*	-
$C(x)$	Function that calculates the cost of the array	*£*	-
$E(x)$	Function that calculates the energy output of the window array	*kWh*	-
$P(x)$	Function that calculates the financial payback of the array	*£/Year*	-

$L(x)$	Function that calculates the number of hours of light that qualifies as sufficient to work during the day	*Hours*	-
t(x)	Function that calculates transparency (Coefficients from linear regression)	*%*	-
Et()	Function that calculates yearly energy produced by wall and roof modules		
Cs(SA)	Cost function for wall and roof panels	*£*	-
e(Rmax,Irr)	Efficiency function of the panels	*%*	-
Ps(Cs)	Payback function for yearly FIT payback	*£*	-
T(Irr,R)	Time function for payback on ROI for roof and wall panels	*Years*	-

Note:

1. Upper limit: 8 pages for groups of 2 members (11 pages for the group of 3), excluding References and Appendix of Nomenclature.

2. Except for ‘Appendix A. Nomenclature’, no other Appendices are accepted.