RECURRENT NEURAL NETWORKS

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1. Introduction

Throughout this handout, we use bold face lower case letters to represent vectors, and bold face capital letters to represent matrices. Most of the time, we will blur the dustinction between matrices and vectors, except when it makes the exposition clearer.

Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a function, and consider a class $\mathcal{C}$ of functions $g : \mathbb{R}^n \to \mathbb{R}^m$, which we assume to be parametrized by $\mathbb{R}^\ell$, in the sense that there is a function $F(\mathbf{x}, \mathbf{w}) : \mathbb{R}^n \times \mathbb{R}^\ell \to \mathbb{R}^n$ such that $\mathcal{C} = \{\mathbf{x} \mapsto F(\mathbf{x}, \mathbf{w}) : \mathbf{w} \in \mathbb{R}^n\}$. The generic problem of machine learning is to find a vector $\mathbf{w} \in \mathbb{R}^\ell$ such that $D(f(\mathbf{x}), F(\mathbf{x}, \mathbf{w}))$ is as small as possible, where D represents a notion of distance between functions, for example,

$$D(f(\mathbf{x}), F(\mathbf{x}, \mathbf{w})) = \min\{\|f(\mathbf{x}) - F(\mathbf{x}, \mathbf{w})\| : \mathbf{x} \in \mathbb{R}^n\}$$

Note that since the minimum is computed over all possible vlues of $\mathbf{x}$, the value of D in te equation above depends on $\mathbf{w}$. Define

$$\ell(\mathbf{w}) = D(f(\mathbf{x}), F(\mathbf{x}, \mathbf{w}))$$

Given the definition of D, we see that $\ell(\mathbf{x}) \geq 0$, and therefore ℓ has a global minimum (which is not necessarily unique). In general, the global minimum of ℓ is not attained, in the sense that if $\epsilon = \min\{\ell(\mathbf{w}) : \mathbf{x} \in \mathbb{R}^n\}$, then there does not necessarily exist a value $\mathbf{w}_0 \in \mathbb{R}^\ell$ such that $\epsilon = \ell(\mathbf{w}_0)$. However, it can be approximated, that is to say there is a sequence $\mathbf{w}_i$ such that $\ell(\mathbf{w}_i) \to \epsilon$ as $i \to \infty$. We may not be able to achieve the minimum with a specific value of $\mathbf{w}$, but we can get as close to it as we want: for every $\varepsilon > 0$, there is some natural number $N_\varepsilon > 0$ such that $\|\epsilon - \ell(\mathbf{w}_i)\| < \varepsilon$ for every $i > N_\varepsilon$. Since we can choose ε to be as small as we want, if we choose it to be smaller than the smallest number which can be represented with a `float32` (say), then from the point of view of any program, $\ell(\mathbf{w}_i) = \epsilon$ when $i > N_\varepsilon$.

The function ℓ we defined above is called a *loss function*. There are many functions which are suitable for ℓ. In fact, ℓ does not need to represent a distance at all. In order to draw a meaningful conclusion from the minimum value of ℓ, all we need to know is that $\ell(\mathbf{w}) = 0$ implies $f(\mathbf{x}) = F(\mathbf{x}, \mathbf{w})$ for every $\mathbf{x}$, or at the very least $\|f(\mathbf{x}) - F(\mathbf{x}, \mathbf{w})\| \leq \varepsilon$ for arbitrary ε. In every neural network we define, we specify a class of functions $F(\mathbf{x}, \mathbf{w})$, and a definition for $\ell(\mathbf{w})$ based on F. In tensorflow, the function we are trying to approximate is given in the form of a relation $f(\mathbf{x}) = \mathbf{y}$. We provide *placeholders* for the values of $\mathbf{x}$ and $\mathbf{y}$. The function $F(\mathbf{x}, \mathbf{w})$ is defined by a computation graph. For example, here is a linear regression,

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in which we try to approximate a function $f : \mathbb{R}^{10} \to \mathbb{R}^{15}$ using a function of the form $F(\mathbf{x}, \mathbf{W}\mathbf{b}) = \mathbf{W}\mathbf{x} + \mathbf{b}$. Here $\mathbf{W}$ is an 15×10 matrix, and $\mathbf{b} \in \mathbb{R}^{15}$

```
x = tf.placeholder(shape=[None, 10])
y = tf.placeholder(shape=[None, 15])
W = tf.Variable(shape=[None, 10, 15])
b = tf.Variable(shape=[None, 15])
F = tf.matmul(x, W) + b
loss = tf.mean_square_error(y, F)
```

We are mainly interested in classification problems, or more generally problems which can be expressed as classification problems. Let A and B be finite sets, and let $f : A \to B$ be any function. In order to use neural networks, which represent functions between topological vector spaces, to represent function between ordinary sets, we must first translate the sets A and B into vector spaces. The most straightforward (and freest) way to interpret a set A as a vector space is to use A as the basis for a vector space. Write $A = \{a_1, ..., a_n\}$, and let $\{\mathbf{e}_1, ..., \mathbf{e}_n\}$ represent the standard basis for $\mathbb{R}^n$. Let $e : A \to \mathbb{R}^n$ be defined by $e(a_i) = \mathbf{e}_i$. Similarly, we can write $B = \{b_1, ..., b_m\}$, and define $e : B \to \mathbb{R}^m$ by $e(b_i) = \mathbf{e}_i$. This specific function is called *one-hot encoding*, and is present in tensorflow as tf.onehot. Since we are mapping a finite set to an infinte set, the function e has no inverse, but it has a "pseudo-inverse" given by the *argmax()* function defined by $argmax(\mathbf{v}) = i$ where i is smallest such that $v_i \geq v_j$ for every $j \neq i$. In other words, *argmax* computes the index of the largest component of a vector.

Given $\mathbf{v} \in \mathbb{R}^m$ is the output of $F(\mathbf{x}, \mathbf{w})$, then if $\mathbf{v}$ has the property that $\sum v_i = 1$, and $v_i \geq 0$ for every i, then we can interpret $\mathbf{v}$ as a probability distribution, where each component v_i represents the probability that $\mathbf{v} = \mathbf{e}_i$. The code, in tensorflow, which approximates a function $A \to B$ can be written as:

```
x_in = tf.placeholder(shape=[None, 1])
x = tf.one_hot(x_in, 10)
y_in = tf.placeholder(shape=[None, 1])
y = tf.one_hot(y_in, 15)
W = tf.Variable(shape=[None, 10, 15])
b = tf.Variable(shape=[None, 15])
F = tf.softmax(tf.matmul(x, W) + b)
loss = tf.cross_entropy(y, F)
```

The softmax function in the penultimate line is defined by $\sigma(\mathbf{v}) = \mathbf{w}$ where

$$w_i = \frac{e^{v_i}}{\sum e^{v_j}}$$

Note that $\sum w_j = 1$, which means that the output of the softmax function can be interpreted as a probability distribution. This makes it a popular choice for classification problems with many classes. If the target space is $\mathbb{R}^2$, then this reduces to a binomial distribution. The cross-entropy function in the last line is defined by $C(\mathbf{v}, \mathbf{w}) = -\sum(v_i \log(w_i))$. Intuitively, C measures the number of vector components that are necessary to differentiate between the distribution $\mathbf{v}$ and the distribution $\mathbf{w}$. Therefore, the optimal value for C to have is 1: measuring any bit should allow us to differentiate between $\mathbf{v}$ and $\mathbf{w}$. Note that $C(\mathbf{v}, \mathbf{w}) = 1$ if and only if $\log C(\mathbf{v}, \mathbf{w}) = 0$, so that the logarithm of the cross entropy function is a candidate for a distance function. The vector $\mathbf{w}$ here should be the output of the model, whereas $\mathbf{v}$ should be the true value. By definition, $w_i = \frac{e^{v_i}}{\sum e^{v_j}}$, so that $\log(w_i) = \log e^{w_i} - \log(\sum e^{v_j}) = w_i - \log(\sum e^{v_j})$. By definition again, $v_i = 1$ and $v_j = 0$ if $i \neq j$. Therefore, $C(\mathbf{v}, \mathbf{w}) = w_i + \log(\sum e^{w_j})$

2. Dealing with sequences

Let A be a finite set. We define the set A^* of all finite sequences of elements of A to be the smallest set X with the property that the empty sequence ϵ is in X, and whenever $w \in X$ and $a \in A$, the sequence $aw \in X$. If A and B are two finite sets, and $f : A \to B$ is any function, then we can extend f extends uniquely to a function $f^* : A^* \to B^*$, which is defined by $f(a_1 a_2, ..., a_n) = f(a_1)f(a_2)\cdots f(a_n)$. This f^* is the only function extending f which satisfies $f(vw) = f(v)f(w)$.

Let S be another finite set with a distinguished element $\perp \in S$, and let $f : A \times S \to B \times S$ be a function. We can use f to define a function $f^* : A^* \to B^*$ in several ways. Consider an auxiliary function $f^* : A^* \to B^* \times S$ by defining First we define $f^*(\epsilon) = (\epsilon, \perp)$. If $a \in A$ then $f^*(a) = f(a, \perp)$. If $v = aw$, then by induction we can compute $f^*(w) = (u, s)$. Write $f(a, s) = (b, s')$, and define $f^*(aw) = (bu, s')$. We can produce a function $f^* : A^* \to B^*$ by taking $f^*(w) = \pi_2 g(w)$.

We can use f to define a function $f^* : A^* \to B$. Consider an auxiliary function $f^* : A^* \to B \times S$ by defining First we define $f^*(\epsilon) = (b_0, \perp)$. If $a \in A$ then $f^*(a) = f(a, \perp)$. If $v = aw$, then by induction we can compute $f^*(w) = (u, s)$. Write $f(a, s) = (b, s')$, and define $f^*(aw) = (b, s')$. We can produce a function $f^* : A^* \to B$ by taking $f^*(w) = \pi_2 g(w)$.

Let S be another finite set with a distinguished element $\perp \in S$, and let $f : A \times S \to B \times S$ be a function. We can use f to define a function $f^* : A \to B^N$ for every N in several ways. Consider an auxiliary function $f^* : A \to B^N \times S$ by defining. If $a \in A$ then $f^*(a)_1 = f(a, \perp)$. Suppose $f^*(a)_n = (w, s)$, and $f(a, s) = (b, s')$, then $f^*(a)_{n+1} = (bw, s')$. This is an example of a one-to-many function. If A has a distinguished element a_0, then we can define $f^*(a)_1 = f(a, \perp)$, and $f^*(a)_n = (w, s)$, and $f(a_0, s) = (b, s')$, then $f^*(a)_{n+1} = (bw, s')$. This is an example of a one-to-many function. Note that this is a special case of the many to many function, where $f^*(a)$ is just defined as $f^*(aaa \cdots a)$, or $f^*(aa_0 \cdots a_0)$.

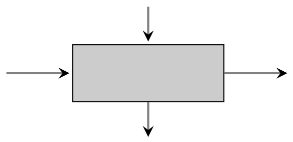
3. The base of recurrent networks

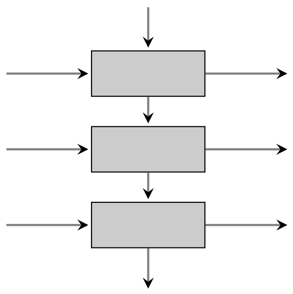
Consider a function $f : U \times S \to V \times S$, where U, V and S are finite dimensional vector spaces. Note that every vector space has a a distinguished element 0, the zero-vector. Let $u_1, ..., u_n$ be a finite sequence of vectors in U. We construct a sequence of vectors in V as follows. Write $f(u_1, 0) = (v_1, s_1)$, and for every i, if $f(u_i, s_{i-1}) = (v_i, s_i)$. Note that this is a state machine, close to a deterministic finite automaton, except that here the state space S is infinite.

The set $\{0,1\}^*$ of all finite sequences of 0's and 1's is countable, and S, being a real vector space, is uncountable. Therefore, we can encode every element $w \in \{0,1\}^*$ as a vector in S. Informally, S is enough to encode any finite state space, and every possible value for the content of the tape of a Turing machine. We get:

Theorem 3.1. *Recurrent neural networks are Turing-complete.*

which explains why recurrent neural networks seem to be able to produce results that other networks can't. Training a recurrent network is the same as producing a Turing machine.





4. Back Propagation

In general, in order to minimize a function $f(w)$ of one variable, we make use of the fact that the extreme values of f all have $f'(x) = 0$. If f is a function of two or more variables, then if x is such that $f(x)$ is a local max, or minimum, then all partial derivatives $\frac{\partial f}{\partial w_i}(x) = 0$. If $f : \mathbb{R}^n \to \mathbb{R}$ is a function, then the *gradient* of f is the vector $\nabla f(x) = \left(\frac{\partial f}{\partial w_1}(x), ..., \frac{\partial f}{\partial w_n}(x)\right)$. One interesting property of $\nabla f(x)$ is that, as a vector, it "points" in the direction in which f decreases the most. That is to say, there is a good chance that $f(x - \alpha \nabla f(x)) \leq f(x)$. The chance is better the smaller α is. We see that if we iterate that process, we get a sequence $x_1, ..., x_n, ...$ such that $f(x_{i+1}) \leq f(x_i)$ for every i. Since $m \leq f(x_i) \leq f(x_0)$ for every i, we get a bounded decreasing sequence of real number, which converges.

Caveat: this ideal situation doesn't allways happen.

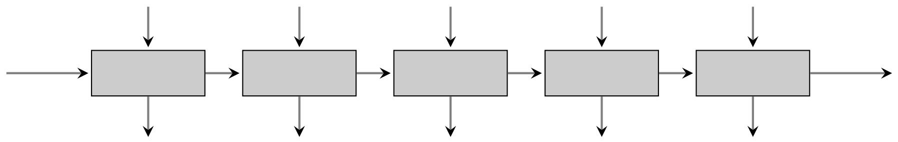
This iterative process is *(Stocastic) gradient descent*.

Suppose f is a complsition of two functions, and that we can write $f(x, w) = g(h(x, w_1), w_2)$. Then we can write $\nabla_w f(x, w) = \nabla g(h(x, w_1), w_2) \cdot \nabla h(x, w_1)$. Each level of composition corresponds to a layer of neural network. The chain rule transforms function composition into a product. If a network becomes deep, then the derivative of the loss function becomes a long product. When gradients become small in norm (like they do when we approach a min), then the gradient becomes very close to 0, and the update rule for gradient descent stops changing the weights.

This is the vanishing gradient problem for very deep networks, and makes convergence slow.

5. Backward Propagation Through Time

How do we train recurrent neural networks? we can use back propagation, just like a regular neural network. Heuristically, we unroll the recurrent network "infinitely" many times, until it looks like an ordinary very deep neural network. In practice, since computers only have a finite amount of resources, we only unroll the network a large but finite number of times, and treat it like an ordinary neural network.



6. Types of Recurrent Cells

There are three main types of recurrent cells which are used to build recurrent neural networks. Let us describe them, and give some of their basic properties, along with a naïve implementation in tensorflow.

6.1. **Basic Recurrent Cell.**

We begin with the most basic of recurrent cell. Abstractly, a function $f : U \times S \to V \times S$ can be defined using two functions $u : \mathbb{R}^n \times \mathbb{R}^\ell \to \mathbb{R}^n$ and $v : \mathbb{R}^n \times \mathbb{R}^\ell \to \mathbb{R}^\ell$. We se ethe former function as providing the output, and the latter function as a state updating function. The most common definition for u and v for basic recurrent cells is as follows: $u(x, s) = f(A_u x + B_u s)$, where A nad B are matrices, and f is a non-linear function, and $v(x, s) = \tanh(A_v x + B_v s)$. Te parameters in this definition are A_u, B_u, A_v and B_v. In tehsorflow, the code looks like:

```
def basic_rnn_cell(input_tensor, state_tensor):
  A_u = tf.Variable(shape=[N, L])
  B_u = tf.Variable(shape=[L, L])
  A_v = tf.Variable(shape=[N, L])
  B_v = tf.Variable(shape=[L, L])
  output_tensor = tf.relu(tf.matmul(input_tensor, A_u) +
                          tf.matmul(state_tensor, B_u))
  new_state_tensor = tf.tanh(tf.matmul(input_tensor, A_v) + \
                                tf.matmul(state_tensor, B_v))
  return output_tensor, new_state_tensor
```

6.2. **Long Short-term Memory Cell.**

There are a few problems with the architecture described above. First, it suffers greatly from the vanishing gradient problem, since there is no way to prevent gradients from becoming very small. Secondly, simple recurrent networks have a hard time remembering facts about the input sequence. To remedy this situation, long short-term memory cells were introduced. The overall structure of an LSTM cell is almost the same as the basic RNN cell. This time we take the previous output into account, which gives the structure as $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^\ell \to \mathbb{R}^m \times \mathbb{R}^\ell$ which can be divided as two function $u : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^\ell \to \mathbb{R}^m$ and $v : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^\ell \to \mathbb{R}^\ell$. To define the functione u and v, we use auxiliary functions F, I, and O defined as follows

$$\begin{aligned}
F(x, y) &= \sigma(A_F x + B_F y + b_F) \\
I(x, y) &= \sigma(A_I x + B_I y + b_I) \\
O(x, y) &= \sigma(A_O x + B_O y + b_O) \\
S(x, y) &= \sigma(A_O x + B_O y + b_O)
\end{aligned}$$

The state update is given by $v(x, y, s) = F(x, y) \circ s + I(x, y) \circ \sigma_v(A_v x + B_v y + b_v)$, where $\circ$ denotes pointwise multiplication of vectors, and finally, the output can be defined as $u(x, y, s) = O(x, y) \circ \sigma_u(v(x, y, s))$.

```
def lstm_gate(input_tensor, previous_output, port_op):
  A = tf.Variable(shape=[N, L])
  B = tf.Variable(shape=[L, L])
  b = tf.Variable(shape=[L, L])
  x = tf.matmul(input_tensor, A)+ tf.matmul(previous_output, B) + b
  return post_op(x)

def lstm_cell(input_tensor, output, state):
  F = lstm_gate(input_tensor, output, tf.sigmoid)
  I = lstm_gate(input_tensor, output, tf.sigmoid)
  O = lstm_gate(input_tensor, output, tf.sigmoid)
  S = lstm_gate(input_tensorm output, tf.tanh)
  new_state = tf.mul(output, F) + tf.mul(I, S)
  output = tf.mul(O, tf.tanh(new_state))
  return output, new_state
```

6.3. **Gated Recurrent Unit Cell.** A common variant of the long short term memory cell is the *gated recurrent unit cell*, more commonly known as GRU cells. The philosophy behind their design is similar to the long short term memory. Once again, each step of the computation takes into account a state vector and the output of the previous iteration. GRU's are functions $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^\ell \to \mathbb{R}^m \times \mathbb{R}^\ell$, which can be divided as two function $u : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^\ell \to \mathbb{R}^m$ and $v : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^\ell \to \mathbb{R}^\ell$. To define the functione u and v, we use auxiliary functions U and R defined as follows

$$\begin{aligned} U(x, y) &= \sigma(A_U x + B_U y + b_U) \\ R(x, y) &= \sigma(A_R x + B_R y + b_R) \end{aligned}$$

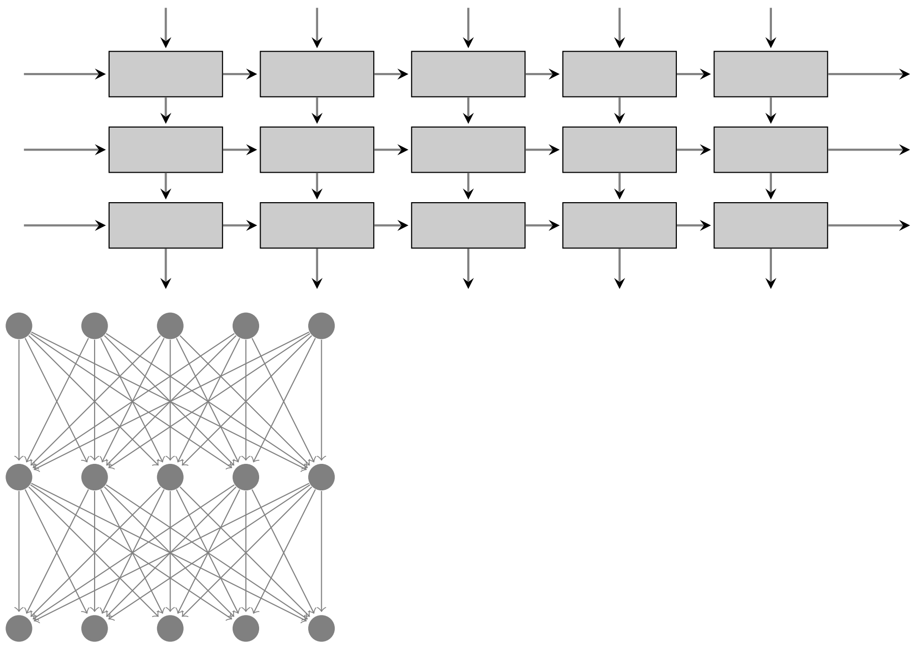
The state update is given by $v(x, y, s) = U(x, y) \circ s + (1 - s) \circ \sigma_h(A_v x + B_v(R(x, y) \circ y) + b_v)$.

```
def gru_gate(input_tensor, previous_output, port_op):
  A = tf.Variable(shape=[N, L])
  B = tf.Variable(shape=[L, L])
  b = tf.Variable(shape=[L, L])
  x = tf.matmul(input_tensor, A)+ tf.matmul(previous_output, B) + b
  return post_op(x)

def gru_cell(input_tensor, output, state):
  U = gru_gate(input_tensor, output, tf.sigmoid)
  R = gru_gate(input_tensor, output, tf.sigmoid)
  O = gru_gate(input, tf.mul(R, output))
  return tf.mul(R, output) + tf.mul((1-R)O)
```

7. Examples

7.1. **Many-to-One.** The Buzzometer sentiment analysis tool uses a bi-directional GRU model, in which the output a 4-layer bidirectional recurrent network is fed into a two-layer fully connected network which separates the input into four classes, corresponding to neutral, positive, negative and irrelevant messages. The only non-linearities in the network are inside the GRU cells. Graphically, the network can be represented as:



The base input ofor this network is a (unicode) character, which is one-hot encoded before being fed into the recurrent layers. For training, the network is unrolled to a length of 256 characters, which is about twice the length of an average message in our database, and all strings are padded or truncated to a length of 256 characters.

```
SEQ_LENGTH = 256
E_DIM = 128
STATE_DIM = 512
NUM_CLASSES = 4
def inference():
    model_input = tf.placeholder('uint8', shape=[None, SEQ_LENGTH])
    _ = tf.one_hot(Globals.model_input, depth=E_DIM, axis=-1)
    _ = tf.reshape(_, [-1, SEQ_LENGTH, E_DIM])
    fw = multi_layer_rnn(N_LAYERS, STATE_DIM)
    bw = multi_layer_rnn(N_LAYERS, STATE_DIM)
    output, _ = tf.nn.bidirectional_dynamic_rnn(fw, bw, _, dtype=tf.float32)
    fw_output = tf.reshape(output[0][:, -1:], [-1, STATE_DIM])
    bw_output = tf.reshape(output[1][:, :1], [-1, STATE_DIM])
    f = project(fw_output, E_DIM)
    b = project(bw_output, E_DIM)
    e = tf.add(f, b)
    Globals.model_output = project(e, NUM_CLASSES)
    Globals.prediction = tf.cast(tf.argmax(Globals.model_output, 1), tf.uint8)
    return Globals.model_input, Globals.model_output
```

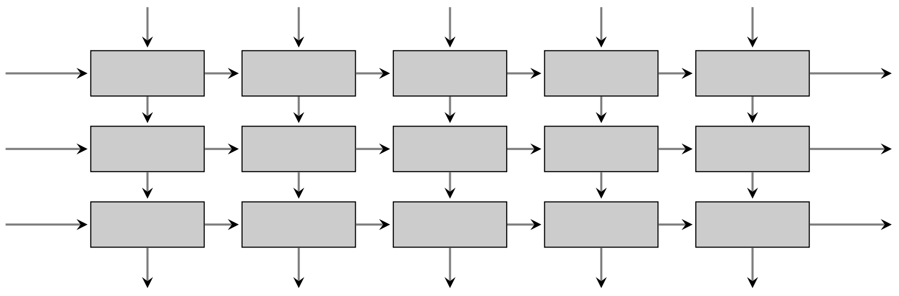
	Negative	**Neutral**	**Positive**	**Irrelevant**
Negative	291	113	50	127
Neutral	108	113	50	85
Positive	0	0	0	0
Irrelevant	0	0	0	0

	Negative	**Neutral**	**Positive**	**Irrelevant**
Negative	292	52	36	55
Neutral	61	176	21	62
Positive	5	15	65	15
Irrelevant	33	28	14	70

	Negative	**Neutral**	**Positive**	**Irrelevant**
Negative	319	38	11	43
Neutral	20	188	2	20
Positive	10	18	91	10
Irrelevant	27	27	7	169

	Negative	**Neutral**	**Positive**	**Irrelevant**
Negative	334	4	4	9
Neutral	3	283	2	7
Positive	1	0	125	2
Irrelevant	1	0	4	221

	Negative	**Neutral**	**Positive**	**Irrelevant**
Negative	321	16	3	20
Neutral	8	286	4	6
Positive	4	2	125	16
Irrelevant	11	9	1	168



7.2. **Many-to-many.**

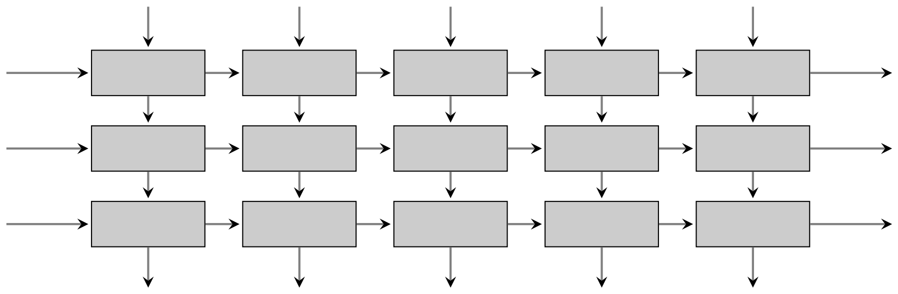
```
SEQ_LENGTH = 256
E_DIM = 128
STATE_DIM = 512
N_LAYERS = 3

def inference():
    model_input = tf.placeholder('uint8', shape=[None, SEQ_LENGTH])
    _ = tf.one_hot(Globals.model_input, depth=E_DIM, axis=-1)
```

```
        encode = multi_layer_rnn(N_LAYERS, STATE_DIM)
        state_tuple = tuple(tf.unstack(Globals.initial_state, axis=0))
        output, state = tf.nn.dynamic_rnn(encode, _,
                                          dtype=tf.float32,
                                          initial_state=state_tuple)
        output = tf.reshape(output, [-1, STATE_DIM])
        output = project(output, E_DIM)
        out = tf.cast(tf.argmax(output, 1), tf.uint8)
        out = tf.reshape(out, [-1, SEQ_LENGTH])
        Globals.generated_sequence = out
        Globals.generated_characters = tf.nn.softmax(output)
        Globals.model_output = output
        Globals.state = state

def generate_text(length, session=None):
    generated_text = ''
    character = [[ord(' ')]]
    istate = np.zeros([N_LAYERS, 1, STATE_DIM])
    while len(generated_text) < length:
        feed_dict = {Globals.model_input: character,
                     Globals.initial_state: istate}
        next_char, state = session.run([Globals.generated_characters,
                                        Globals.state],
                                       feed_dict=feed_dict)
        next_char = np.asarray(next_char).astype('float64')
        next_char = next_char / next_char.sum()
        op = np.random.multinomial
        next_char_id = op(1, next_char.squeeze(), 1).argmax()
        next_char_id = next_char_id if chr(next_char_id) in \
                          string.printable else ord(" ")
        generated_text += chr(next_char_id)
        character = [[next_char_id]]
        istate = state
    return generated_text
```

7.3. **Many-to-one-to-many.** As an example of a recurrent neural network performing a task that ordinary neural network should not be able to perform, we present a sorting function which uses GRU cells. The architecture of the network is the reason we call it a many-to-one-to-many network. One level takes a list of numbers, and produces a vector which we can see as representing all the numbers in the list, and a second layer uses this output vector as input, and produces the original list, sorted in increasing order. The training was done using sequences of natural numbers $n \in \{1, ..., 32\}$, and each of the training sequences of length 32. After about 6 days of training, the accuracy of the network stabilized at around 95%, and it is worth noting that eventhough it took a long time to breach the 90% accuracy mark, very early on in the training did the network output increasing sequences.



```
SEQ_LENGTH = 256
E_DIM = 128
STATE_DIM = 512
N_LAYERS = 4

def inference():
    model_input = tf.placeholder('uint8', shape=[None, SEQ_LENGTH])
    _ = tf.one_hot(Globals.model_input, depth=E_DIM, axis=-1)
    _ = tf.reshape(_, [-1, SEQ_LENGTH, E_DIM])
    encode = multi_layer_rnn(N_LAYERS, STATE_DIM)
    encoded_input, state = tf.nn.dynamic_rnn(encode,
                                             _,
                                             dtype=tf.float32)
    Globals.encoder_output = state
    with tf.variable_scope('decoder'):
        training_decoder_input = tf.zeros_like(Globals.model_input)
        _ = tf.one_hot(training_decoder_input, depth=E_DIM, axis=-1)
        _ = tf.reshape(_, [-1, SEQ_LENGTH, E_DIM])
        decode = multi_layer_rnn(N_LAYERS, STATE_DIM)
        decoded_output, state = tf.nn.dynamic_rnn(decode, _,
                                                  dtype=tf.float32,
                                                  initial_state=state)
        decoded_output = tf.reshape(decoded_output, [-1, STATE_DIM])
        output = project(decoded_output, E_DIM)
        out = tf.cast(tf.argmax(output, 1), tf.uint8)
        out = tf.reshape(out, [-1, SEQ_LENGTH])
        Globals.training_decoder_input = training_decoder_input
        Globals.model_output = output
        Globals.prediction = out
        Globals.decoder = decode
        Globals.decoder_input = _
```