AMD-USD Exchange Rate Modeling

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Abstract

The analysis of FX rates can provide a new perspective of a country's economic standing/health. However, many people in the industry still argue that it is hard to model the FX rates, brining the example of Random Walks outperforming most in terms of directional accuracy. This study will focus on the Armenian dram–US dollar (AMD–USD) exchange rate (hereafter AMD-USD FX rate) rate, since there are no public models for the aforementioned FX rate.

Keywords: Economics, Simple Linear Regression, Multiple Linear Regression, Autoregressive (AR), Time-fitted, Random Walk

Contents

1 Introduction

The AMD-USD FX rate plays a central role in Armenia's economy, influencing import costs, export competitiveness, inflation dynamics, and monetary policy transmission. Since energy, machinery, and food account for a large share of Armenia's imports, fluctuations in the AMD–USD FX rate can have immediate welfare implications for households and firms.

Moreover, the relative predictive power of macroeconomic factors (e.g., money supply, interest rates, inflation) versus purely autoregressive specifications remains underexplored for the Armenian context.

This paper addresses these gaps by examining four competing models for monthly AMD–USD returns over January 2010–March 2025: (i) a multivariate linear regression (MLR) incorporating key macro indicators; (ii) a MLR model with subsetted variables; (iii) a MLR model wiht simple time-trend specification; (iv) a MLR model wiht an autoregressive (AR(1)) component.

Using a 70/30 train–holdout split and evaluating out-of-sample R^2, adjusted R^2, and directional accuracy, we compare forecasting performance and draw implications for the Central Bank of Armenia's policy toolkit.

Our contributions are threefold:

(1) extending the sample through March 2025 to capture recent policy changes and shocks; (2) measuring how much adding macroeconomic data improves forecasts compared to just using past exchange-rate values; (3) providing actionable insights on which model best balances forecasting accuracy and interpretability in the Armenian setting.

1.1 Known Models

Currently commercial banks and top financial audit institutions (like the Big 4) have either dedicated research teams to model the FX rates, or outsource them. Some may even leverage paid data like currency premium, however this study will take into account as to what can be available to the public for free. Another point of concern is that FX rate models struggle to outperforms models that leverage (1) random walks and (2) some sort of mean-reversion [EW24] [Che+17]

1.2 The Structure of the Paper

This project draft paper consists of the following parts:

- Literature Review
- Methodology
- Developed Models Descriptions
- Results
- Conclusion

The **Literature Review** section is devoted to explaining the background information needed to familiarize oneself wiht this analysis, including the models and economic variables.

In the **Methodology** section, we discuss how we are going to analyze the FX rates and evaluate the models.

In the **Developed Models Descriptions** we will go over my developed models and discuss them thoroughly.

We will summarize our testing in the **Results** section.

Finally, in the **Conclusion** section, we will summarize our work, and give our recommendation to those who were interested in this paper.

2 Literature Review

2.1 Random Walk Definition and Directional Accuracy

A *random walk* for the exchange rate X_t is defined by

$$X_t \;=\; X_{t-1} + \varepsilon_t,$$

where $\{\varepsilon_t\}$ is a sequence of independent, identically distributed innovations with $E[\varepsilon_t] = 0$ and a continuous, symmetric distribution.

Our forecasted change is

$$\widehat{\Delta X}_t = \widehat{X}_t - X_{t-1} = \varepsilon_t.$$

We define *directional accuracy* (DA) as the probability that the forecasted direction matches the realized direction:

$$\mathrm{DA} = \Pr\bigl(\mathrm{sign}(\widehat{\Delta X}_t) \;=\; \mathrm{sign}(\Delta X_t)\bigr).$$

Proposition. For the random-walk model, using symmetry we get The directional accuracy is $\mathrm{DA} = 0.5$.

2.2 VIF and Multiple Linear Regression

The multiple regression model extends to k regressors:

$$Y_t \;=\; \beta_0 + \sum_{i=1}^{k} \beta_i X_{i,t} + u_t, \quad E[\varepsilon_t \mid X_{1,t}, \ldots, X_{k,t}] = 0.$$

Multicollinearity arises when regressors are highly correlated, inflating estimator variances. For regressor X_j, define the variance inflation factor

$$\mathrm{VIF}_j \;=\; \frac{1}{1 - R_j^2},$$

where R_j^2 is the R^2 from regressing X_j on all other regressors. A rule of thumb is that $\mathrm{VIF}_j > 10$ indicates serious multicollinearity.

2.3 Overall F-Test for Joint Significance

To assess whether all k predictors jointly contribute to the model, we test

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_k = 0 \quad \text{vs.} \quad H_A : \text{at least one } \beta_i \neq 0.$$

Choose a significance level α (e.g. 0.05), compute the F-test p-value, and then:

$$\text{If } p < \alpha, \quad \text{reject } H_0 \quad \Longrightarrow \quad \text{the predictors are jointly significant at level } \alpha.$$

Otherwise, do not reject H_0, implying no evidence that the predictors as a group improve the model.

2.4 Autoregressive (AR(1)) Model

We consider the AR(1) model

$$X_t = \phi \, X_{t-1} + \varepsilon_t,$$

where $|\phi| < 1$ and $\{\varepsilon_t\}$ is i.i.d. with $E[\varepsilon_t] = 0$ and a continuous, symmetric distribution.

Note that when $\phi = 1$ this reduces to the random-walk case with DA $= 0.5$; for $\phi \neq 1$, the directional accuracy depends jointly on ϕ and the distribution of X_{t-1} and ε_t.

2.5 Key Economic Factors

The economic factors below have been selected by referencing different studies, but predominantly the [Wil08] study commissioned by the World Bank

M2 Money Supply M2 comprises currency in circulation, demand deposits, savings deposits, small time deposits, and retail money-market mutual funds. It is a broad measure of the money stock used to gauge liquidity and inflationary pressures.

GDP and CPI Gross Domestic Product (GDP) measures the total value of goods and services produced. To obtain real GDP, nominal GDP is deflated by the Consumer Price Index (CPI):

$$\text{Real GDP}_t = \frac{\text{Nominal GDP}_t}{\text{CPI}_t/100}.$$

Trade Balance The trade balance is the difference between exports and imports of goods and services:

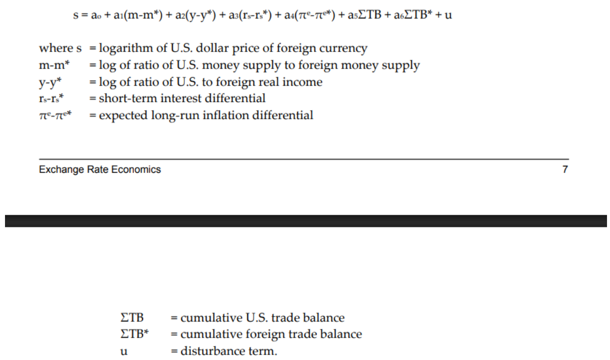
$$\text{Trade Balance}_t = \text{Exports}_t - \text{Imports}_t.$$

A positive (negative) value denotes a surplus (deficit).

Inflation and Refinancing Rate Inflation is the rate of change of the CPI:

$$\pi_t = \frac{\text{CPI}_t - \text{CPI}_{t-1}}{\text{CPI}_{t-1}}.$$

The refinancing (policy) rate is the interest rate at which the central bank lends to commercial banks; it influences short-term market rates and, indirectly, inflation and economic activity.



$$s = a_0 + a_1(m-m^*) + a_2(y-y^*) + a_3(r_s-r_s^*) + a_4(\pi^e-\pi^{e*}) + a_5\Sigma TB + a_6\Sigma TB^* + u$$

where s = logarithm of U.S. dollar price of foreign currency
$m-m^*$ = log of ratio of U.S. money supply to foreign money supply
$y-y^*$ = log of ratio of U.S. to foreign real income
$r_s-r_s^*$ = short-term interest differential
$\pi^e-\pi^{e*}$ = expected long-run inflation differential

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ΣTB = cumulative U.S. trade balance
ΣTB^* = cumulative foreign trade balance
u = disturbance term.

Figure 1: Photo from WorldBank's Document [Wil08]

3 Methodology

3.1 Background

Our method of evaluation for the different models will be revolved around comparative analysis of the R^2, adjusted R^2 and directional accuracy metrics. Our data will range from Jan 2010 - Mar 2025, specifically holding out the last 30 percent for out of sample testing.

3.2 Developed Models and their Description

After conducting a thorough literature review, I have decided to develop the following models:

3.2.1 "All-Variables" Model

This model includes all the economic variables mentioned above. No prior variable selection will be made.

3.2.2 "Subsetted-Variables" Model

We will perform simple linear regression to test for independent variable predictability, then with the subsetted variables we will remove those that have a VIF score over 10. After this we should have a guarantee that we reject the null hypothesis of the overall F-test.

3.2.3 AR(1) Fitted Model

One of my hypothesis is that lag-1 plays a significant predicability role for the AMD-USD FX Rates, thus we will combine the variables from the 3.2.2 section model and add an autoregressive component.

3.2.4 Time-fitted Model

Another hypothesis of mine is that the FX-rate has a moving trend, thus we will combine the variables from 3.2.2 model and add a linear scale for time.

4 Results

```
Residuals:
     Min       1Q   Median       3Q      Max 
-15.2745  -5.6120  -0.4111   5.1143  23.5872 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.095e+03  6.431e+01 -17.022  < 2e-16 ***
m_diff       1.639e+02  6.994e+00  23.440  < 2e-16 ***
y_diff      -2.773e+00  3.335e+00  -0.831 0.407418    
rs_diff      6.996e+00  1.560e+00   4.486 1.67e-05 ***
pi_diff      1.090e+01  2.936e+00   3.713 0.000311 ***
TB_cum_arm  -3.300e-03  1.276e-03  -2.587 0.010866 *  
TB_cum_us   -2.416e-05  6.556e-06  -3.685 0.000343 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 2: All Variables Model Info

```
Residuals:
     Min       1Q   Median       3Q      Max 
-15.9443  -5.8727  -0.0441   5.1967  24.2597 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -9.940e+02  5.908e+01 -16.824  < 2e-16 ***
m_diff       1.508e+02  6.394e+00  23.590  < 2e-16 ***
rs_diff      1.005e+01  1.386e+00   7.254 3.96e-11 ***
pi_diff      7.558e+00  2.948e+00   2.564   0.0116 *  
TB_cum_arm  -7.864e-03  1.890e-04 -41.615  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 3: Subsetted Variables Model Info

```
Residuals:
     Min       1Q   Median       3Q      Max 
-15.4556  -3.6110   0.2531   2.4586  28.6629 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -4.138e+02  7.895e+01  -5.241 6.85e-07 ***
USD_lag1     5.850e-01  6.468e-02   9.045 3.03e-15 ***
m_diff       6.293e+01  1.092e+01   5.762 6.45e-08 ***
rs_diff      3.625e+00  1.295e+00   2.798  0.00598 ** 
pi_diff      2.586e+00  2.361e+00   1.095  0.27560    
TB_cum_arm  -3.285e-03  5.276e-04  -6.225 7.19e-09 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 4: AR(1) Fitted Model Info

```
Residuals:
    Min      1Q  Median      3Q     Max 
-15.294  -5.562  -0.266   4.411  23.530 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.528e+03  1.569e+02  -9.741  < 2e-16 ***
time_idx     3.928e-07  1.077e-07   3.647 0.000391 ***
m_diff       1.549e+02  6.198e+00  24.994  < 2e-16 ***
rs_diff      6.325e+00  1.670e+00   3.787 0.000238 ***
pi_diff      1.009e+01  2.895e+00   3.485 0.000685 ***
TB_cum_arm  -3.074e-03  1.326e-03  -2.319 0.022080 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 5: Time Fitted Model Info

Model	R^2	Adj. R^2	Dir. Accuracy
All predictors	0.4568	0.3889	0.6296
Subsetted variables	0.5970	0.5647	0.5741
AR(1) fitted	0.9510	0.9460	0.5370
Time fitted	0.0195	−0.0806	0.6852

Table 1: Model performance metrics

Significance Threshold (5% level, one-sided):

$$\hat{p}_1 \;>\; 0.5 \;+\; z_{0.95}\sqrt{\frac{0.5\,(1-0.5)}{183\times 0.3}} \;=\; 0.5 \;+\; 1.645\sqrt{\frac{0.25}{54}} \;\approx\; 0.612.$$

We can see that all variables model fails in the overall f-test, and has mediocre R^2 metrics. For the subsetted variables, my initial assumption that it would pass the overall f-test was correct. But more specifically, the AR(1) model has very high OS R^2 values, and the time fitted model has near 70 percent directional accuracy, confirming my two hypothesis about each of the models. However, we should also note that under alpha=0.05, the AR(1) also fails in the Overall F-test

5 Conclusion

5.1 Summary

In this paper, we compared four models for forecasting monthly AMD–USD returns over January 2010–March 2025. The AR(1) specification achieved the highest out-of-sample R^2 (0.9510), while the time-fitted model delivered the best directional accuracy (0.6852). The subsetted-variables model struck a balance between fit (adj. $R^2 = 0.5647$) and statistical significance, outperforming the “all-variables” approach. These results suggest that simple autoregressive dynamics and time trends capture much of the short-term movement in the AMD–USD rate, but that careful variable selection remains valuable for interpretability.

5.2 Future Work and Reading

For those who are interested in future work around this theme one may include more comprehensive data, like currency premium, or analyzing the correlations between country-specific ETFs and their FX rates. Furthermore, adding data until Jan 2000 would be desirable.

Data Availability

The datasets used in this study are publicly accessible:

- Monthly U.S. dollar–Armenian dram exchange rates: Federal Reserve Economic Data (FRED)
- Macroeconomic indicators for Armenia: Central Bank of Armenia
- Supplementary economic time series: Trading Economics

References

[Che+17] Yin-Wong Cheung et al. *Exchange rate prediction redux: New models, new data, new currencies*. ECB Working Paper 2018. European Central Bank, 2017.

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